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ICSE Examination Paper 2026

Class X

MATHEMATICS

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately.
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.

(i) $(x+3), 1, (3x-7)$ and -5 are in proportion. The value of x is:

(a) -1

(b) 1

(c) -5

(d) 5

Sol :

Since the four terms are in proportion, the product of the extremes is equal to the product of the means.

$$(x+3)\times(-5) = 1\times(3x-7)$$
$$-5x-15 = 3x-7$$
$$-5x-3x = -7+15$$
$$-8x = 8$$
$$x = \frac{8}{-8} = -1$$

Thus (a) is the correct option.

(ii) The marked price of a refrigerator is ₹ 12,000 and GST paid by the customer is ₹ 2,160. The rate of GST is:

(a) 5%

(b) 12%

(c) 18%

(d) 28%

Sol :

$$\text{Rate of GST} = \left(\frac{\text{GST Paid}}{\text{Marked Price}}\right)\times 100$$
$$= \left(\frac{2160}{12000}\right)\times 100$$
$$= \frac{216}{12} = 18\%$$

Thus (c) is the correct option.

(iii) Rakhi's mobile number has the following integers: 1,6,9,8,9,1,7,8,9. The mode of the above given data is :

(a) 1

(b) 6

(c) 8

(d) 9

Sol :

The mode is the value that appears most frequently in a set of observations. Counting the frequency of each digit:

1 appears 2 times
6 appears 1 time
7 appears 1 time
8 appears 2 times
9 appears 3 times

As the digit 9 has the highest frequency, it is the mode of the data.
Thus (d) is the correct option.

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(iv) A and B opened a recurring deposit account in a bank which is paying simple interest at 9% per annum. A deposited ₹1,500 for one year and B deposited ₹1,200 for 15 months. The amount invested by:

(a) A is ₹ 27 more than B

(b) A is ₹ 300 more than B

(c) A is ₹ 300 less than B

(d) Both A and B are same (₹18,000)

Sol :

The total amount invested in a recurring deposit is the product of the monthly installment (P) and the total number of months (n).
For A : 1500 × 12 months = ₹18,000.
For B : 1200 × 15 months = ₹18,000.
Both A and B invested the same total principal.
Thus (d) is the correct option.

(v) Find the equation of a line whose y-intercept is 6 and is parallel to x-axis.

(a) $y = 6$

(b) $x = 6$

(c) $x + y = 6$

(d) $y - x = 6$

Sol :

Any line parallel to the x-axis is a horizontal line with a slope (m) equal to 0. Using the slope-intercept form $y = mx + c$:
Given $m = 0$

$$y = (0)x + 6$$

$$y = 6$$

Thus (a) is the correct option.

(vi) Asha buys ₹20 shares of a company which pays 9% dividend at such a price that she gets a return of 12% on her investment. At what price did she buy each share?

(a) ₹20

(b) ₹15

(c) ₹25

(d) ₹18

Sol :

The annual income per share is the same whether calculated on the nominal value or the market value.
Dividend % on NV = Profit % on MV.
Let the Market Value (purchase price) be x.

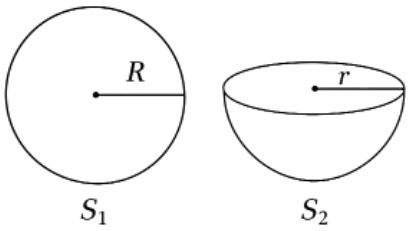
$$\frac{9}{100} \times 20 = \frac{12}{100} \times x$$

$$1.8 = 0.12x$$

$$x = \frac{1.8}{0.12} = ₹15$$

Thus (b) is the correct option.

(vii) The total surface area of a solid sphere (S_1) and a solid hemisphere (S_2), as shown in the diagram, are equal. The ratio of radii R and r is:



(a) 1 : 1

(b) 2 : 1

(c) $\sqrt{3} : 2$

(d) $2 : \sqrt{3}$

Sol :

Total surface area of a solid sphere (S_1) $= 4\pi R^2$
Total surface area of a solid hemisphere (S_2) $= 3\pi r^2$
Given,

$$4\pi R^2 = 3\pi r^2$$

$$\frac{R^2}{r^2} = \frac{3}{4}$$

Taking the square root on both sides:

$$\frac{R}{r} = \frac{\sqrt{3}}{2} \Rightarrow R : r = \sqrt{3} : 2$$

Thus (c) is the correct option.

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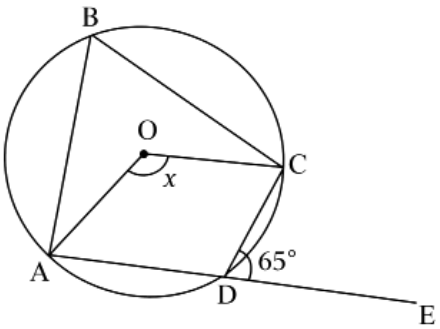
(viii) In the given diagram, O is the centre of the circle and $ABCD$ is a cyclic quadrilateral. If $\angle CDE = 65^\circ$, then the value of x is:



(a) 32.5°

(b) 65°

(c) 115°

(d) 130°

Sol :

In a cyclic quadrilateral, the exterior angle is equal to the interior opposite angle. Therefore, $\angle ABC = \angle CDE = 65^o$. The angle subtended by an arc at the center is double the angle subtended by the same arc at the circumference.

$$x = 2\times \angle ABC$$

$$x = 2\times 65^o = 130^o$$

Thus (d) is the correct option.

(ix) The nature of roots of quadratic equation $3x^2 - 6x - 3 = 0$ are:

(a) Real and equal

(b) Real, distinct and rational

(c) Real, distinct and irrational

(d) No real roots

Sol :

The nature of roots is determined by the discriminant $D = b^2 - 4ac$.

Comparing $3x^2 - 6x - 3 = 0$ with $ax^2 + bx + c = 0$, we get $a = 3, b = -6, c = -3$.

$$D = (-6)^2 - 4(3)(-3)$$

$$D = 36 + 36 = 72$$

Since $D > 0$, the roots are real and distinct. Furthermore, since 72 is not a perfect square, the roots are irrational.
Thus (c) is the correct option.

(x) **Assertion (A) :** If a die is rolled, the probability of getting a number greater than 6 is $\frac{1}{6}$.

Reason (R) : There are six possible outcomes when rolling a die, $\{1,2,3,4,5,6\}$.

(a) (A) is true and (R) is false.

(b) (A) is false and (R) is true.

(c) Both (A) and (R) are true and (R) is the correct explanation of (A).

(d) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Sol :

A standard die has the faces $\{1,2,3,4,5,6\}$. There is no number greater than 6 on a standard die, so the number of favorable outcomes is 0.

$$\text{Probability (number} > 6) = \frac{0}{6} = 0$$

Therefore, Assertion (A) is false. However, Reason (R) correctly lists the six possible outcomes, so it is true.
Thus (b) is the correct option.

(xi) If the areas of two similar triangles are in the ratio 9 : 64, then the ratio of their corresponding altitudes is:

(a) 3 : 8

(b) 2 : 1

(c) 9 : 64

(d) 8 : 3

Sol :

For two similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding altitudes.

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{h_1}{h_2}\right)^2$$

Given the ratio of areas is 9 : 64,

$$\frac{9}{64} = \left(\frac{h_1}{h_2}\right)^2$$

Taking the square root of both sides

$$\frac{h_1}{h_2} = \sqrt{\frac{9}{64}} = \frac{3}{8}$$

The ratio of the altitudes is 3 : 8.
Thus (a) is the correct option.

(xii) What must be added to $x^3 + 7x^2 + 3x + 2$ so that the result is completely divisible by $(x+2)$?

(a) -40

(b) -16

(c) 16

(d) 40

Sol :

Let $f(x) = x^3 + 7x^2 + 3x + 2$

According to the Remainder Theorem, when $f(x)$ is divided by $(x+2)$, the remainder is $f(-2)$.

$$f(-2) = (-2)^3 + 7(-2)^2 + 3(-2) + 2$$

$$= -8 + 7(4) - 6 + 2$$

$$= -8 + 28 - 6 + 2 = 16$$

Since the remainder is 16, we must add a value k such that the new remainder becomes 0.

$$16 + k = 0 \Rightarrow k = -16$$

Thus (b) is the correct option.

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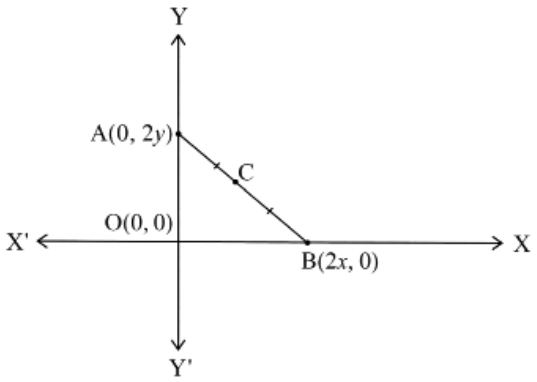
(xiii) In the given diagram, ΔAOB is a right-angled triangle and C is the mid-point of AB. The coordinates of the point which is equidistant from the three vertices of ΔAOB is :



(a) (x, y)

(b) (y, x)

(c) $\left(\frac{x}{2}, \frac{y}{2}\right)$

(d) $\left(\frac{2x}{3}, \frac{2y}{3}\right)$

Sol :

In a right-angled triangle, the point equidistant from all three vertices is the circumcenter, which always lies at the midpoint of the hypotenuse. The vertices are $O(0,0)$, $A(0,2y)$ and $B(2x,0)$. The hypotenuse is AB. The coordinates of the midpoint C are calculated using the midpoint formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$C = \left(\frac{0 + 2x}{2}, \frac{2y + 0}{2}\right)$$

$$C = (x, y)$$

Thus (a) is the correct option.

(xiv) Given matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 2 & -4 \end{bmatrix}$.

Product AB is a matrix of order:

(a) 2×2

(b) 2×1

(c) 1×2

(d) Product AB is not possible

Sol :

The order of matrix A is 2×2 (2 rows and 2 columns). The order of matrix B is 1×2 (1 row and 2 columns). For matrix multiplication AB to be defined, the number of columns in the first matrix (A) must be equal to the number of rows in the second matrix (B). Number of columns in $A = 2$. Number of rows in $B = 2$. Since $2 \neq 1$, the multiplication is not possible.

Thus (d) is the correct option.

(xv) **Assertion (A):** The 9th term of a Geometric Progression (G.P.) $6, -12, 24 - 48...$ is a positive term.

Reason (R): The value of $(-2)^8$ is always positive.

(a) (A) is true and (R) is false.

(b) (A) is false and (R) is true.

(c) Both (A) and (R) are true and (R) is the correct explanation of (A).

(d) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Sol :

In the given G.P., the first term $a = 6$ and the common ratio $r = \frac{-12}{6} = -2$.

The formula for the n^{th} term of a G.P. is $a_n = a \cdot r^{n-1}$. For the 9th term $(n = 9)$:

$$a_9 = 6 \cdot (-2)^{9-1}$$

$$a_9 = 6 \cdot (-2)^8$$

Since any negative number raised to an even power results in a positive value, $(-2)^8$ is positive. Multiplying this by the positive first term (6) results in a positive 9th term. Therefore, Assertion (A) is true, and Reason (R) correctly explains why it is positive.

Thus (c) is the correct option.

QUESTION 2.

(i) The fourth and seventh terms of an Arithmetic Progression (A.P.) are 60 and 114 respectively.

Find the:

(a) First term and common difference.

(b) Sum of its first 10 terms.

Sol :

(a) Let the first term of the A.P. be a and the common difference be d. The formula for the n^{th} term is $T_n = a + (n-1)d$.

From the given information:

$$T_4 = a + 3d = 60 \quad \ldots(1)$$

$$T_7 = a + 6d = 114 \quad \ldots(2)$$

Subtracting equation (1) from equation (2) :

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$$(a+6d)-(a+3d)=114-60$$
$$3d=54$$
$$d=18$$

Substituting the value of d into equation (1) :

$$a+3(18)=60$$
$$a+54=60$$
$$a=6$$

The first term is 6 and the common difference is 18.

(b) To find the sum of the first 10 terms ($n=10$), use the sum formula :

$$S_n=\frac{n}{2}[2a+(n-1)\,d]$$
$$S_{10}=\frac{10}{2}[2(6)+(10-1)\,18]$$
$$=5[12+162]=5[174]$$
$$S_{10}=870$$

(ii) Given, $A=\begin{bmatrix}3 & 1\\ 5 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-1 & a\\ 3 & -5\end{bmatrix}$ and product $AB=\begin{bmatrix}b & 7\\ 4 & 5\end{bmatrix}$. Find the values of a and b.

Sol :

First, calculate the product of matrices A and B.

$$AB=\begin{bmatrix}3 & 1\\ 5 & 3\end{bmatrix}\begin{bmatrix}-1 & a\\ 3 & -5\end{bmatrix}$$
$$=\begin{bmatrix}(3)(-1)+(1)(3) & (3)(a)+(1)(-5)\\ (5)(-1)+(3)(3) & (5)(a)+(3)(-5)\end{bmatrix}$$
$$=\begin{bmatrix}-3+3 & 3a-5\\ -5+9 & 5a-15\end{bmatrix}$$
$$AB=\begin{bmatrix}0 & 3a-5\\ 4 & 5a-15\end{bmatrix}$$

Now, equate the elements of this resulting matrix to the given product $\begin{bmatrix}b & 7\\ 4 & 5\end{bmatrix}$:

Equating the first row, first column element ((a_{11}): $b=0$

Equating the first row, second column element (a_{12}):

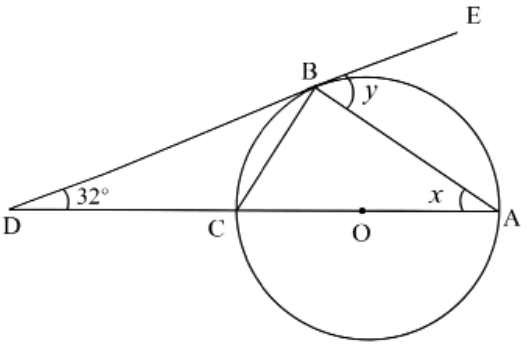
$$3a-5=7$$
$$3a=12$$
$$a=4$$

Verifying with the second row, second column element (a_{22}):

$$5(4)-15=20-15=5$$

This matches the given matrix. Therefore, the values are $a=4$ and $b=0$.

(iii) In the given diagram, O is the centre of the circle and the tangent DE touches the circle at B. If $\angle ADB=32^\circ$. Find the values of x and y.



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Sol :

Finding x :

In $\Delta ODB, OB$ is the radius and DE is the tangent at B. Therefore, $OB\perp DE$, which means $\angle OBD=90^\circ$. In ΔODB the sum of angles is 180° :

$$\angle DOB+\angle ODB+\angle OBD=180^\circ$$
$$\angle DOB+32^\circ+90^\circ=180^\circ$$
$$\angle DOB=180^\circ-122^\circ$$
$$=58^\circ$$

Since COA is a straight line, $\angle AOB$ and ΔDOB form a linear pair:

$$\angle AOB=180^\circ-58^\circ$$
$$=122^\circ$$

In $\Delta OBA, OA=OB$ (radii of the same circle). Thus, ΔOBA is an isosceles triangle where $\angle OAB=\angle OBA=x$.

$$x+x+122^\circ=180^\circ$$
$$2x=58^\circ$$
$$x=29^\circ$$

Finding y :

From the tangent-radius property, we know $\angle OBE=90^\circ$.

From the diagram, $\angle OBE=\angle OBA+\angle ABE$. Substituting the known values:

$$90^\circ=x+y$$
$$90^\circ=29^\circ+y$$
$$y=90^\circ-29^\circ$$
$$y=61^\circ$$

QUESTION 3.

(i) The polynomial $kx^3+3x^2-11x-6$ when divided by $(x+1)$, leaves a remainder of 6.

(a) Find the value of k.

(b) Using the value of k factorise completely the polynomial $kx^3+3x^2-11x-6$.

Sol :

(a) Let $P(x)=kx^3+3x^2-11x-6$. According to the Remainder Theorem, when $P(x)$ is divided by $(x+1)$, the remainder is $P(-1)$.

Given,

$$P(-1)=6$$

$$k(-1)^3+3(-1)^2-11(-1)-6=6$$

$$-k+3+11-6=6$$

$$-k+8=6$$

$$-k=6-8$$

$$-k=-2$$

$$\Rightarrow \quad k=2$$

(b) Now the polynomial is $P(x)=2x^3+3x^2-11x-6$. To factorise completely, we find a factor by trial using the Factor Theorem. For $x=2$:

$$P(2)=2(2)^3+(2)^2-11(2)-6$$

$$=16+12-22-6=0$$

Since $P(2)=0,(x-2)$, is a factor.

Dividing $2x^3+3x^2-11x-6$ by $(x-2)$:

$$2x^3+3x^2-11x-6=(x-2)(2x^2+7x+3)$$

Now, factorizing the quadratic part $2x^2+7x+3$ by splitting the middle term:

$$2x^2+6x+x+3$$

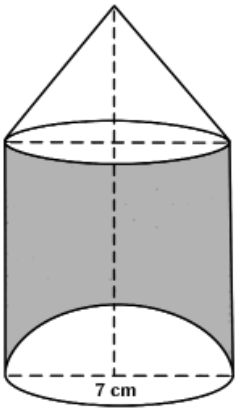
$$2x(x+3)+1(x+3)$$

$$(2x+1)(x+3)$$

The completely factorized polynomial is:

$$(x-2)(2x+1)(x+3)$$

(ii) An eye drop bottle is prepared consisting of a hemisphere, a cylinder and a conical cap, as shown in the given diagram. Height of the cylindrical and conical parts are each, equal to the diameter (7 cm). Find the :



(a) Minimum height of the cylindrical box required to pack this bottle.

(b) Volume of the liquid medicine (shaded part) in the bottle. Give your answer to the nearest whole number. (Use $\pi=\frac{22}{7}$)

Sol :

Given : Diameter $(d)=7$,

So, Radius $(r)=3.5\,\text{cm}=\frac{7}{2}$ cm.

Height of cylinder $(h_{cy})=7$ cm.

Height of cone $(h_{co})=7$ cm.

(a) The bottle is composed of a hemisphere at the base, a cylinder in the middle, and a cone at the top. The total height of the bottle is :

Total Height = Radius of hemisphere + Height of cylinder + Height of cone

Total Height = 3.5 + 7 + 7 = 17.5 cm

The minimum height of the cylindrical box required is 17.5 cm.

(b) The liquid medicine (shaded part) occupies the cylindrical portion but is displaced by the hemisphere at the bottom.

Volume of liquid = Volume of cylinder – Volume of hemisphere

$$V=\pi r^2h_{cy}-\frac{2}{3}\pi r^3$$

$$=\pi r^2\left(h_{cy}-\frac{2}{3}r\right)$$

$$=\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times\left(7-\frac{2}{3}\times\frac{7}{2}\right)$$

$$=\frac{22}{7}\times\frac{49}{4}\times\left(7-\frac{7}{3}\right)$$

$$=\frac{11\times7}{2}\times\left(\frac{21-7}{3}\right)$$

$$=\frac{77}{2}\times\frac{14}{3}$$

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$$= \frac{77 \times 7}{3} = \frac{539}{3}$$

$$V \approx 179.66...$$

To the nearest whole number, the volume is 180 cm^3.

(iii) Use ruler and compass for the following construction:

(a) Construct an equilateral triangle ABC of side 5 cm.

(b) Construct the circumcircle of ΔABC.

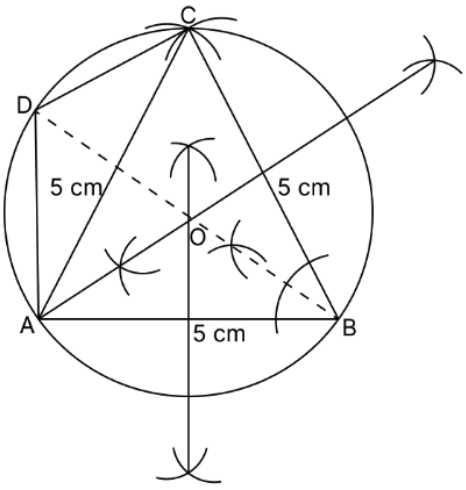
(c) Construct the locus of points which are equidistant from AB and BC. Mark the point where the circumcircle and locus meet, as D.

(d) Give the geometrical name of quadrilateral $ABCD$.

Sol :

Steps of construction :

1. Draw a line segment AB = 5 cm.
2. At point A with radius = 5 cm draw an arc.
3. At point B with radius = 5 cm draw another arc, cutting previous arc at point C.
4. Join AC and BC.
5. Construct perpendicular bisectors of AB and BC, let the bisectors meet at point O.
6. With O as centre and OA as radius draw a circumcircle.
7. Construct angle bisector of triangle ABC, mark the point as D where angle bisector intersects circumcircle.
8. Join AD and CD.



Since, points A, B, C and D lie on the circumcircle, thus ABCD is a cyclic quadrilateral. Hence, ABCD is a cyclic quadrilateral.

SECTION-B

(Attempt any four questions from this section.)

QUESTION 4.

(i) Prove that :

$(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta) = \sin\theta\cos\theta$

Sol :

$$\text{L.H.S.} = (\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)$$

Substitute $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)$$

Take the L.C.M. in both brackets :

$$= \frac{1-\cos^2\theta}{\cos\theta}\ \frac{1-\sin^2\theta}{\sin\theta}$$

Since, $1 - \cos^2\theta = \sin^2\theta$

$$1 - \sin^2\theta = \cos^2\theta$$

So, the expression becomes

$$\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta}{\sin\theta}\right)$$

$$= \frac{\sin^2\theta \cdot \cos^2\theta}{\sin\theta \cdot \cos\theta}$$

$$= \sin\theta\cos\theta = \text{R.H.S}$$

Hence Proved.

(ii) The cost price of a TV set is ₹ 20,000. The shopkeeper marked it for ₹ 24,000. He sells it to a customer at a discount of 10% on the marked price. If the sale is intra-state and the rate of GST is 12%, find the :

(a) Discounted price of the TV set.

(b) Amount paid by the customer to clear the bill.

Sol :

Given : Marked Price (M.P.) = ₹ 24,000;
Discount % = 10%;
GST % = 12%.

(a) Discounted Price = M.P. – Discount

$$= 24000 - (10\% \text{ of } 24000)$$

$$= 24000 - 2400$$

$$= ₹\ 21,600$$

(b) Since it is an intra-state sale, GST is applied to the discounted price.

$$\text{Total GST} = 12\% \text{ of } 21600$$

$$= \frac{12}{100} \times 21600$$

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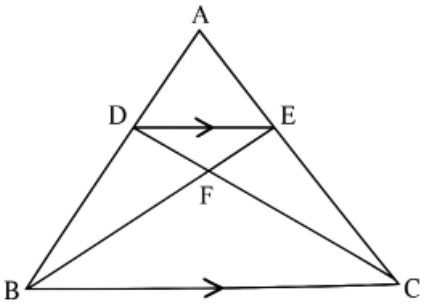
$= ₹\,2,592$

Amount paid by the customer = Discounted Price + GST

Total Bill $= 21600 + 2592$

$= ₹\,24,192$

(iii) In the given diagram, $DE \parallel BC$ and $AD : DB = 2 : 3$.



(a) Prove that : $\Delta ADE \sim \Delta ABC$ and hence find $DE : BC$.

(b) Prove : $\Delta DFE \sim \Delta CFB$.

(c) Given, area of $\Delta DFE = 16$ square units, find the area of ΔCFB.

Sol :

(a) In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ (Corresponding angles are equal)

$\angle AED = \angle ACB$ (Corresponding angles are equal)

Therefore, $\Delta ADE \sim \Delta ABC$ (by AA similarity)

Given,

$$AD : DB = 2 : 3$$

Let $AD = 2x$

and $DB = 3x$

From figure,

$$AB = AD + DB$$

$$= 2x + 3x = 5x$$

We know that,

Corresponding sides of similar triangle are in proportion.

$$\frac{DE}{BC} = \frac{AD}{AB} = \frac{2x}{5x} = \frac{2}{5}$$

$$DE : BC = 2 : 5$$

Hence, $\Delta ADE \sim \Delta ABC$ and $DE : BC = 2 : 5$ proved.

(b) Given, $DE \parallel BC$

In triangle ΔDFE and ΔCFB,

$\angle DFE = \angle CFB$ (Vertically opposite angles are equal)

$\angle DEF = \angle CBF$ (Alternate interior angles are equal)

$\Delta DFE \sim \Delta CFB$ (By AA similarity)

Hence, proved that $\Delta DFE \sim \Delta CFB$.

(c) Let area of ΔCFB be x square units.

We know that,

The areas of two similar triangles are proportional to the squares of their corresponding sides.

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CFB} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CFB} = \left(\frac{2}{5}\right)^2$$

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CFB} = \frac{4}{25}$$

$$\frac{16}{x} = \frac{4}{25}$$

$$x = \frac{16 \times 25}{4}$$

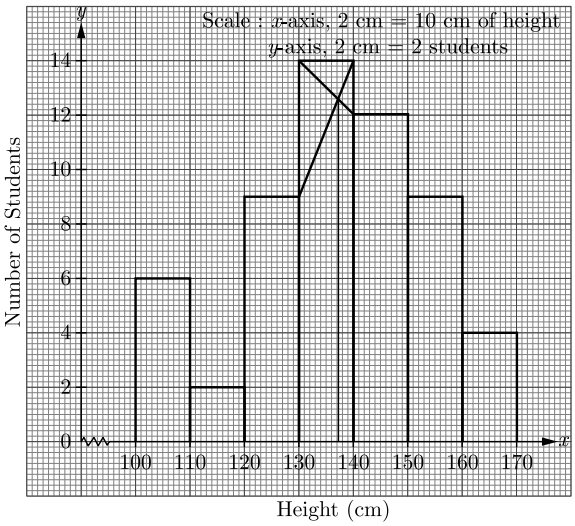
$$= \frac{400}{4}$$

$$x = 100$$

Hence, area of $\Delta CFB = 100$ sq. units.

QUESTION 5.

(i) The histogram drawn on the graph represents the number of students of different heights (in cm).



Using the graph, answer the following :

(a) The number of students whose height is 150 cm and above.

(b) The modal height.

(c) The total number of students.

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Sol :

(a) To find the number of students whose height is 150 cm and above, we sum the frequencies of the class intervals from 150 onwards:

(i) Number of students in 150-160 cm range = 9

(ii) Number of students in 160-170 cm range = 4

Total students = 9 + 4 = 13

(b) The modal class is the interval with the highest frequency, which is 130-140 cm (frequency = 14). To find the mode from the histogram:

(i) A line is drawn from the top-right of the preceding bar (120-130) to the top-right of the modal bar.

(ii) A line is drawn from the top-left of the succeeding bar (140-150) to the top-left of the modal bar.

(iii) The intersection of these two lines is projected down to the x-axis. From the graph, the projection meets the x-axis at 7 small divisions after 130. Since 1 small division = 1 cm, the modal height is 137 cm.

(c) The total number of students is the sum of the frequencies of all the bars in the histogram:

$$\begin{aligned}\text{Total} &= 6+2+9+14+12+9+4\\ &= 56 \text{ students}\end{aligned}$$

(ii) $A(-10,-2)$ and $B(2,10)$ are two end points of a line segment. If AB intersects the x-axis at P, find the:

(a) Ratio in which P divides AB.

(b) Coordinates of point P.

Sol :

(a) Let point at which AB intersects x-axis be $P(x, 0)$ and P divides AB in the ratio $m : n$.

Using section formula,

$$y = \frac{my_2 + ny_1}{m+n}$$

Substituting values we get

$$0 = \frac{m \times 10 + n \times -2}{m+n}$$

$$0 = \frac{10m - 2n}{m+n}$$

$$10m - 2n = 0$$

$$10m = 2n$$

$$\frac{m}{n} = \frac{2}{10}$$

$$= \frac{1}{5}$$

$$m : n = 1 : 5$$

(b) Using section formula,

$$x = \frac{mx_2 + nx_1}{m+n}$$

Substituting values we get:

$$x = \frac{1 \times 2 + 5 \times -10}{1+5}$$

$$= \frac{2-50}{6}$$

$$= \frac{-48}{6}$$

$$x = -8$$

$$P = (x, 0) = (-8, 0)$$

Hence, coordinates of point P are (–8, 0).

(iii) Solve the quadratic equation $(x-2)^2 - 5x - 3 = 0$ and give your answer correct to 3 significant figures.

(Use Mathematical Tables for this question if necessary.)

Sol :

Given,

$$(x-2)^2 - 5x - 3 = 0$$

$$x^2 - 4x + 4 - 5x - 3 = 0$$

$$x^2 - 9x + 1 = 0$$

Comparing $x^2 - 9x + 1 = 0$ with $ax^2 + bx + c = 0$, we get:

$a = 1$, $b = -9$ and $c = 1$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{81-4}}{2}$$

$$= \frac{9 \pm \sqrt{77}}{2}$$

$$x = \frac{9 \pm 8.775}{2}$$

$$x = \frac{9+8.775}{2} \text{ or } \frac{9-8.775}{2}$$

$$= \frac{17.775}{2} \text{ or } \frac{0.225}{2}$$

$$= 8.8875 \text{ or } 0.1125$$

$$x = 8.89 \text{ or } 0.113$$

Hence, $x = 8.89$ or 0.113.

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QUESTION 6.

(i) Kabir bought 120 shares of a company with nominal value ₹100, available at a premium of ₹25. Find :

(a) The money invested by Kabir in buying these shares.

(b) The rate of dividend, if he received ₹1,080 as dividend from these shares after one year.

(c) His rate of return.

Sol :

Given :

Number of shares $(n) = 120$;

Nominal Value $(NV) = ₹100$;

Premium $= ₹25$.

(a) Market Value (MV) per share = NV+Premium

$$MV = 100 + 25 = ₹125$$

$$\text{Total Investment} = n \times MV$$

$$\text{Investment} = 120 \times 125 = ₹15{,}000$$

(b) Let the rate of dividend be d%.

$$\text{Annual Dividend} = n \times 100d \times NV$$

$$1080 = 120 \times 100d \times 100$$

$$1080 = 120 \times d$$

$$d = \frac{1080}{120} = 9$$

The rate of dividend is 9%.

(c)Rate of Return $= \left(\frac{\text{Annual Dividend}}{\text{Total Investment}}\right) \times 100$

$$\text{Rate of Return} = \left(\frac{1080}{15000}\right) \times 100$$

$$= 7.2\ \%$$

(ii) Find the mean of the following frequency distribution using step-deviation method. (Take assumed mean = 28)

Class Interval	Frequency
0 – 8	10
8 – 16	20
16 – 24	14
24 – 32	16
32 – 40	18
40 – 48	22

Sol :

Let Assumed Mean (A) = 28 and Class Width (h) = 8.

Class Interval	Frequency (f)	Mid-value (x)
0 – 8	10	4
8 – 16	20	12
16 – 24	14	20
24 – 32	16	28
32 – 40	18	36
40 – 48	22	44
Total	$\sum f = 100$	

$$\sum fu = -22$$

Using the Step-Deviation Formula:

$$\text{Mean} = A + h\left(\frac{\Sigma fu}{\Sigma f}\right)$$

$$= 28 + 8\left(\frac{-22}{100}\right)$$

$$= 28 - 1.76$$

$$= 26.24$$

(iii) The difference of two natural numbers is 5 and sum of their reciprocals is $\frac{3}{10}$. Find the two numbers.

Sol :

Let the two natural numbers be: x and $x + 5$

Given,

The sum of the reciprocals of two numbers is $\frac{3}{10}$.

$$\frac{1}{x} + \frac{1}{x+5} = \frac{3}{10}$$

$$\frac{x+5+x}{x(x+5)} = \frac{3}{10}$$

$$\frac{2x+5}{x(x+5)} = \frac{3}{10}$$

$$10(2x+5) = 3[x(x+5)]$$

$$20x + 50 = 3x^2 + 15x$$

$$3x^2 + 15x - 20x - 50 = 0$$

$$3x^2 - 5x - 50 = 0$$

$$3x^2 + 10x - 15x - 50 = 0$$

$$x(3x+10) - 5(3x+10) = 0$$

$$(x-5)(3x+10) = 0$$

$$(x-5) = 0 \text{ or } (3x+10) = 0$$

$$x = 5 \text{ or } 3x = -10$$

$$x = 5 \text{ or } x = -\frac{10}{3}$$

Since, the numbers are natural numbers, thus x cannot be negative.

Thus, $x = 5$ and $x + 5 = 5 + 5 = 10$.

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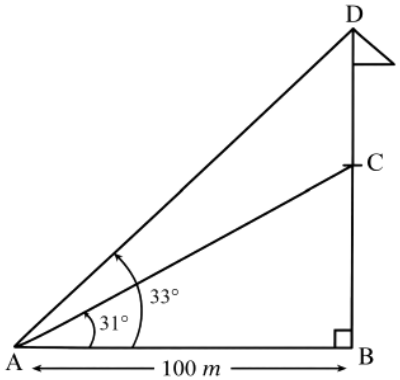
Hence, numbers are 5 and 10.

$$x(x-5)x+(x-5)=103$$

QUESTION 7.

(i) A flagpole is erected at the top of a building. The angle of elevation of the top and foot of the flagpole from a point 100 m away, on the same level as that of the foot of the building, are 33° and 31° respectively. Find the height of the flagpole. Give your answer correct to the nearest metre.

(Use Mathematical Tables for this question.)



Sol :

Let BC be the height of the building and CD be the height of the flagpole. Let A be the point 100 m away from the foot of the building B.

Given :

$$AB = 100 \text{ m}, \angle CAB = 31^\circ$$

and $\angle DAB = 33^\circ$

In right-angled ΔABC :

$$\tan(31^\circ) = AB \times BC$$

$$BC = 100 \times \tan(31^\circ)$$

From mathematical tables,

$$\tan(31^\circ) \approx 0.6009$$

$$BC = 100 \times 0.6009 = 60.09 \text{ m}$$

In right-angled ΔABD :

$$\tan(33^\circ) = AB \times BD$$

$$BD = 100 \times \tan(33^\circ)$$

From mathematical tables,

$$\tan(33^\circ) \approx 0.6494$$

$$BD = 100 \times 0.6494 = 64.94 \text{ m}$$

Height of the flagpole $(CD) = BD - BC$:

$$CD = 64.94 - 60.09$$

$$= 4.85 \text{ m}$$

Rounding to the nearest meter, the height of the flagpole is 5 m.

(ii) Using a graph paper, draw an ogive for the following distribution which shows a record of weight in kilograms of 100 students.

Weight (in kg)	Number of students
35 – 40	4
40 – 45	6
45 – 50	10
50 – 55	24
55 – 60	26
60 – 65	17
65 – 70	8
70 – 75	5

Use your ogive to estimate the following:

(a) the median weight of the students.

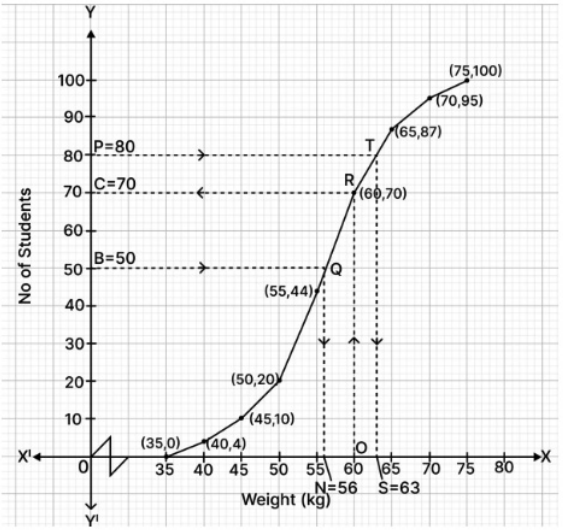
(b) percentage of students whose weight is 60 kg or more.

(c) the weight above which 20% of students lie.

Sol :

The cumulative frequency table for the given continuous distribution is:

Weight (in kg)	Number of students (f)	Cumulative Frequency (CF)
35 – 40	4	4
40 – 45	6	10
45 – 50	10	20
50 – 55	24	44
55 – 60	26	70
60 – 65	17	87
65 – 70	8	95
70 – 75	5	100



(a) Steps of Construction:

1. Since, the scale on x-axis starts at 35, a break (kink) is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 35.
2. Take 1 cm along x-axis = 5 kg (weight)
3. Take 1 cm along y-axis = 10 (students)
4. Plot the points (40, 4), (45, 10), (50, 20), (55, 44), (60, 70), (65, 87), (70, 95), (75, 100) representing upper class limits and the respective cumulative frequencies. Also plot the point representing lower limit of the first class i.e. 35-40.
5. Join these points by a freehand drawing.

The required ogive is shown in figure above.

Here, n (no. of students) = 100

To find the median:

Let B be the point on y-axis representing frequency $= \frac{n}{2} = \frac{100}{2} = 50$

Through B draw a horizontal line to meet the ogive at Q. Through Q, draw a vertical line to meet the x-axis at N. The abscissa of the point N represents.

From graph, $N = 56$

Hence, the median weight = 56 kg.

(b) Percentage of students whose weight is 60 kg or more.

Let O be the point on x-axis representing weight = 60.

Through O draw a vertical line to meet the ogive at R. Through R, draw a horizontal line to meet the y-axis at C. The ordinate of the point C represents.

From graph, $C = 70$

Students weighing more than 60 kg $= 100 - 70 = 30$.

Percentage of students weighing more that 60 kg $= \frac{30}{100} \times 100 = 30\%$

Hence, students weighing more than 60 kg = 30%.

(c) The weight above which 20% of the students lie.

Total number of students = 100

20% of students $= \frac{20}{100} \times 100 = 20$

So, 80 students are below that weight.

Let P be the point on y-axis representing number of students = 80.

Through P draw a horizontal line to meet the ogive at T. Through T, draw a vertical line to meet the x-axis at S. The ordinate of the point S represents weight above which 20% of the students lie.

From graph, $S = 63$

Hence, weight above which 20% students lie = 63 kg.

QUESTION 8.

(i) Rohit and Vinay both opened a recurring deposit account in a bank for 2 years at 8% simple interest. Vinay deposited ₹300 per month. On maturity, Rohit's interest was ₹ 800 more than Vinay's interest. Find the :

(a) interest earned by Vinay.

(b) sum deposited by Rohit every month.

Sol :

(a) Given :

Time $(n) = 2$ years $= 24$ months,

Rate $(r) = 8\%$ p.a.

$P =$ ₹ 300 for Vinay

By formula,

$$I = \frac{P \times n(n+1)}{2} \times \frac{r}{12 \times 100}$$

Substituting values, we get:

$$I = \frac{300 \times 24(24+1)}{2} \times \frac{8}{1200}$$

$$= \frac{300 \times 24(25)}{2} \times \frac{8}{1200}$$

$$= 300 \times 300 \times \frac{8}{1200}$$

$$= 300 \times 2 = ₹\ 600$$

Hence, interest earned by Vinay = ₹ 600

(b) Rohit's interest is ₹ 800 more than Vinay's.

Interest earned by Rohit $= 600 + 800 =$ ₹1400

Let Rohit deposit ₹P per month.

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By formula,

$$I = \frac{P \times n(n+1)}{2} \times \frac{r}{12 \times 100}$$

Substituting values, we get:

$$I = \frac{P \times 24(24+1)}{2} \times \frac{8}{1200}$$

$$1400 = P \times 300 \times \frac{8}{1200}$$

$$1400 = 2P$$

$$P = \frac{1400}{2} = ₹700$$

Hence, Rohit deposits ₹ 700 per month.

(ii) The fourth term of a Geometric Progression (G.P.) is 16 and its seventh term is 128. Find its :

(a) common ratio

(b) first term

Sol :

(a) Let the first term be a and the common ratio be r. The nth term formula is $a_n = ar^{n-1}$.

Given :

$$a_4 = ar^3 = 16 \quad ...(1)$$

$$a_7 = ar^6 = 128 \quad ...(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{ar^6}{ar^3} = \frac{128}{16}$$

$$r^3 = 8$$

$$r = \sqrt[3]{8} = 2$$

Hence, common ratio is 2.

(b) The first term,

Substituting $r = 2$ in equation (1), we get :

$$a(2)^3 = 16$$

$$a(8) = 16$$

$$a = \frac{16}{8} = 2$$

Hence, first term is 2.

(iii) Use graph sheet for this question. Take 2 cm = 1 unit along both x and y axis. Graphically represent parallelogram $OABC$, where $O(0, 0)$, $A(2, 3)$, $B(5, 3)$ and $C(3, 0)$. Reflect $OABC$:

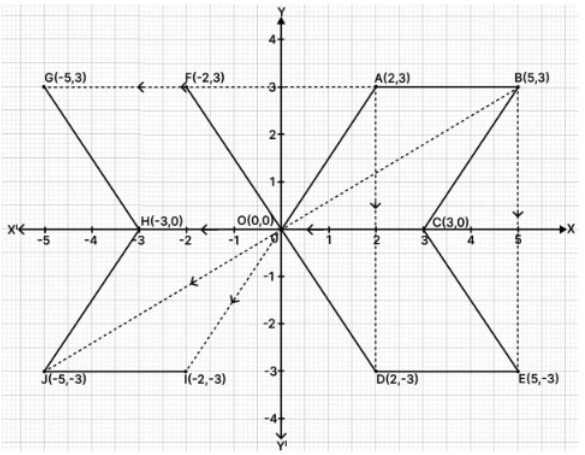
(a) On the x-axis and name its image as $ODEC$.

(b) Through the origin and name its image as $OIJH$.

(c) On the y-axis and name its image as $OFGH$.

Sol :

(a) The point $O(0, 0)$, $A(2, 3)$, $B(5, 3)$ and $C(3, 0)$ are plotted on the graph below:



From graph, on reflecting O, A, B, C on x-axis we get,

$O(0, 0) \Rightarrow O(0, 0)$

$A(2, 3) \Rightarrow D(2, -3)$

$B(5, 3) \Rightarrow E(5, -3)$

$C(3, 0) \Rightarrow C(3, 0)$

(b) From graph, on reflecting O, A, B, C through origin we get,

$O(0, 0) \Rightarrow O(0, 0)$

$A(2, 3) \Rightarrow I(-2, -3)$

$B(5, 3) \Rightarrow J(-5, -3)$

$C(3, 0) \Rightarrow H(-3, 0)$

(c) From graph, on reflecting O, A, B, C on y-axis we get,

$O(0, 0) \Rightarrow O(0, 0)$

$A(2, 3) \Rightarrow F(-2, 3)$

$B(5, 3) \Rightarrow G(-5, 3)$

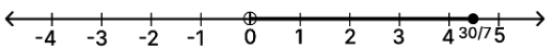
$C(3, 0) \Rightarrow H(-3, 0)$

QUESTION 9.

(i) Solve the following in equation, write the solution set and represent it on the real number line.

$$-1 < \frac{2x-3}{3} - \frac{x}{5} \leq 1, x \in R$$

Sol :



Given,

$$-1 < \frac{2x-3}{3} - \frac{x}{5} \leq 1, x \in R$$

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Solving L.H.S. of the inequation,

$$-1 < \frac{2x-3}{3} - \frac{x}{5}$$

$$-1 < \frac{10x-15-3x}{15}$$

$$-1 < \frac{7x-15}{15}$$

$$-15 < 7x-15$$

$$-15+15 < 7x$$

$$0 < 7x$$

$$x > 0$$

Solving R.H.S. of the inequation,

$$\frac{2x-3}{3} - \frac{x}{5} \le 1$$

$$\frac{10x-15-3x}{15} \le 1$$

$$\frac{7x-15}{15} \le 1$$

$$7x-15 \le 15$$

$$7x \le 15+15$$

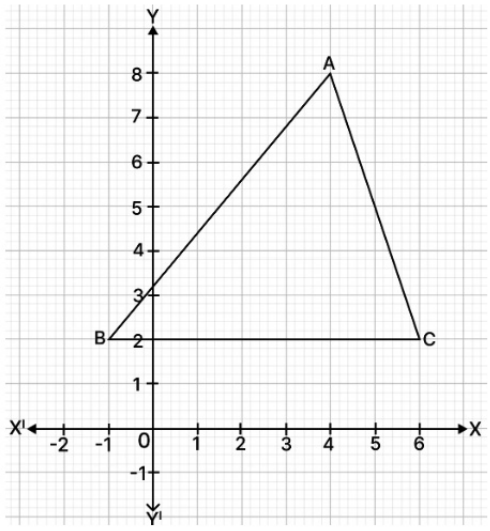
$$7x \le 30$$

$$x \le \frac{30}{7}$$

Hence, $0 < x \le \frac{30}{7}$

Solution set $= \left(0, \frac{30}{7}\right]$

(ii) Use the following graph and answer the given questions :



(a) Write the co-ordinates of points A, B and C.

(b) Find the equation of a line passing through the mid-point of AC and parallel to AB.

Sol :

(a) By observing the vertices of the triangle on the Cartesian grid:

(i) The coordinates of A are (4, 8).

(ii) The coordinates of B are (-1, 2).

(iii) The coordinates of C are (6, 2).

(b) First, calculate the mid-point of AC : Using $A(4, 8)$ and $C(6, 2)$:

$$\text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{4+6}{2}, \frac{8+2}{2}\right)$$

$$= (5, 5)$$

Next, find the slope (m) of the line AB : Using $A(4,8)$ and $B(-1,2)$:

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{8-2}{4-(-1)} = \frac{6}{5}$$

Since the required line is parallel to AB, its slope will also be $m = \frac{6}{5}$.

Now, find the equation of the line passing through the mid-point (5, 5) with slope $\frac{6}{5}$:

$$y-5 = \frac{6}{5}(x-5)$$

$$5y-25 = 6x-30 \qquad \text{multiply by 5}$$

$$6x-5y-5 = 0$$

(iii) A solid wooden toy is prepared by joining a cone, a cylinder and a sphere, as shown in the given diagram. The radius of each of the three solids is 7 cm and heights of each of the cone and the cylinder is 24 cm.

Find:

(a) The total surface area of the given solid.

(b) The cost of painting the total surface at the rate of ₹ 0.50 per cm^2. (Use $\pi = \frac{22}{7}$)

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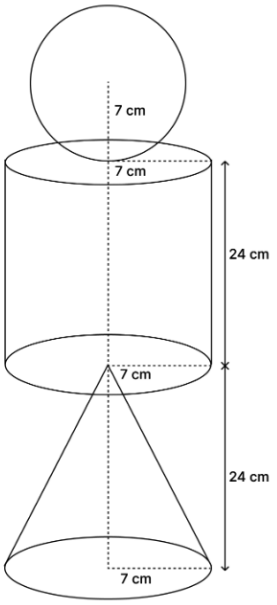

Sol :

Given:
Radius (r) = 7 cm,
Height of cylinder (hcy) = 24 cm,
Height of cone (hco) = 24 cm.

(a) To find the Total Surface Area (TSA) of the composite solid, we sum the exposed surface areas of each part. Since they are joined, the circular bases at the junctions are not part of the external surface.

1. Slant height (l) of the cone:

$$l = r^2 + \text{hco}^2$$
$$= (7)^2 + (24)^2$$
$$= 49 + 576$$
$$= 625$$
$$= 25 \text{ cm}$$

2. Exposed Surface Areas:

(i) **Sphere :** A circular area of πr^2 is hidden where it joins the cylinder.
Area of sphere $= 4\pi r^2 - \pi r^2 = 3\pi r^2$

(ii) **Cylinder :** Both circular bases are hidden (one by the sphere, one by the cone).
Area of cylinder = Curved Surface Area (CSA) $= 2\pi r \times \text{hcy}$

(iii) **Cone :** The base is hidden by the cylinder.
Area of cone = CSA $= \pi rl$

3. Calculation:

$$\text{TSA} = 3\pi r^2 + 2\pi r \times \text{hcy} + \pi rl$$
$$= \pi r(3r + 2\text{hcy} + l)$$
$$= \frac{22}{7} \times 7 \times [3(7) + 2(24) + 25]$$
$$= 22 \times [21 + 48 + 25]$$
$$\text{TSA} = 22 \times 94 = 2068 \text{ cm}^2$$

(b) The cost of painting at ₹ 0.50 per cm^2 :
$$\text{Cost} = 2068 \times 0.50 = ₹ 1,034$$

QUESTION 10.

(i) If $x = \frac{5ab}{a-b}$, $a \neq b$,

(a) Find : $\frac{x}{a}$

(b) Using properties of proportion, find: $\frac{x+a}{x-a}$

Sol :

Given: $x = \frac{5ab}{a-b}$

(a) To find $\frac{x}{a}$, divide both sides by a :

$$\frac{x}{a} = \frac{1}{a} \cdot \frac{5ab}{a-b}$$
$$= \frac{5b}{a-b}$$

(b) To find $\frac{x+a}{x-a}$, we apply the Componendo and Dividendo property to the result from part (a):

$$\frac{x+a}{x-a} = \frac{\frac{a^2+4ab}{a-b}}{\frac{6ab-a^2}{a-b}}$$
$$= \frac{a^2+4ab}{6ab-a^2}$$
$$= \frac{a(a+4b)}{a(6b-a)}$$
$$\frac{x+a}{x-a} = \frac{a+4b}{6b-a}$$

(ii) A survey was conducted on 300 families having 2 children each. The results obtained are given below.

Number of girl child	Number of families
2	95
1	165
0	40
Total	300

If one family is selected at random, find the probability that it will have :

(a) one girl child

(b) one or more girl child

(c) no boy child

Sol :

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(a) **Probability of one girl child :** Number of families with one girl child = 165.

P(one girl child)

$$= \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{165}{300}$$

Simplifying by dividing by 15:

$$P = \frac{11}{20} = 0.55$$

(b) **Probability of one or more girl child :** This includes families with 1 girl or 2 girls.

Number of favorable families = 165+95 = 260

P(one or more girl child) $= \frac{260}{300}$

Simplifying:

$$P = \frac{26}{30} = \frac{13}{15} \approx 0.867$$

(c) **Probability of no boy child :** "No boy child" in a family of 2 children means both children are girls (2 girl children).

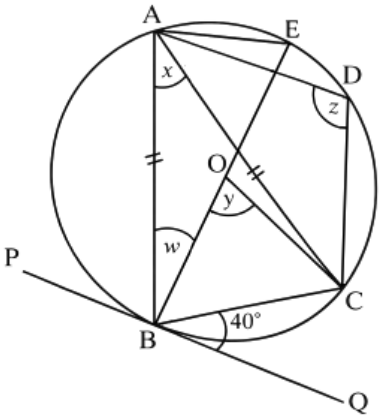
Number of families with 2 girl children = 95

$$P(\text{no boy child}) = \frac{95}{300}$$

Simplifying by dividing by 5:

$$P = \frac{19}{60} \approx 0.317$$

(iii) In the given figure 'O' is the centre of the circle. PQ is a tangent to the circle at B and $AB = AC$. If $\angle CBQ = 40°$, find the unknown angles x, y, z and w.



Sol :

Given,

$$\angle CBQ = 40°$$

In a circle, the angle between a tangent and a chord through the point of contact is equal to the angle in the opposite (alternate) segment of the circle.

$$\angle BAC = \angle CBQ = 40°$$

$$x = 40°$$

Since, $AB = AC$

$\angle ABC = \angle BCA$ (angles opposite to equal sides of a triangle are equal)

In triangle ABC,

$$\angle ABC + \angle BAC + \angle BCA = 180°$$

$$2\angle ABC + 40° = 180°$$

$$2\angle ABC = 180° - 40°$$

$$2\angle ABC = 140°$$

$$\angle ABC = \frac{140°}{2}$$

$$\angle ABC = 70°$$

We know that,

The angle which an arc subtends at the center is double that which it subtends at any point on the remaining part of the circumference.

$$\angle BOC = 2\angle BAC$$

$$y = 2x$$

$$y = 80°$$

In triangle OBC,

$OB = OC$ (Radii of same circle)

$\angle OBC = \angle OCB$ (Angles opposite to equal sides in a triangle are equal)

By angle sum property of triangle,

$$\angle OBC + \angle OCB + \angle BOC = 180°$$

$$2\angle OBC + 80° = 180°$$

$$2\angle OBC = 100°$$

$$\angle OBC = \frac{100}{2} = 50°$$

From figure,

$$w = \angle ABC - \angle OBC$$

$$= 70° - 50° = 20°$$

We know that,

Sum of opposite angles of a cyclic quadrilateral is 180°.

In cyclic quadrilateral $ABCD$,

$$\angle ABC\angle + ADC = 180^o$$

$$70° + z = 180°$$

$$z = 180° - 70° = 110°$$

Hence, $x = 40°$, $y = 80°$, $z = 110°$, $w = 20°$.

END

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ICSE Examination 2025

ICSE Class X

MATHEMATICS

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately.
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.

Choose the correct answers to the questions from the given options.

(Do not copy the questions, write the correct answer only.)

(i) The given quadratic equation $3x^2 + \sqrt{7x} + 2 = 0$ has

(a) Two equal real roots

(b) Two distinct real roots

(c) More than two real roots

(d) No real roots

Sol :

We have $3x^2 + \sqrt{7x} + 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3$, $b = \sqrt{7}$ and $c = 2$

Now $D = b^2 - 4ac$

$= 7 - 4 \times 3 \times 2$

$= 7 - 24$

$= -19$

Since D is negative equation has no real roots.

Thus (d) is correct option

(ii) Mr. Anuj deposits ₹500 per month for 18 months in a recurring deposit account at a certain rate. If he earns ₹570 as interest at the time of maturity, then his matured amount is

(a) ₹$(500 \times 18 + 570)$

(b) ₹$(500 \times 19 + 570)$

(c) ₹$(500 \times 18 \times 19 + 570)$

(d) ₹$(500 \times 9 \times 19 + 570)$

Sol :

Mr. Anuj deposits ₹500 per month for 18 months,

Total amount deposited = ₹ 500×18

Interest = ₹570

Matured amount will be ₹$(500 \times 18 + 570)$.

Thus (a) is correct option

(iii) Which of the following cannot be the probability of any event?

(a) $\frac{5}{4}$

(b) 0.25

(c) $\frac{1}{33}$

(d) 67%

Sol :

Probability is always $p \leq 1$

Thus (a) is correct option

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(iv) The equation of the line passing through origin and parallel to the line $3x+4y+7=0$ is

(a) $3x+4y+5=0$

(b) $4x-3y-5=0$

(c) $4x-3y=0$

(d) $3x+4y=0$

Sol :

Any line parallel to $3x+4y+7=0$ is given by

$$3x+4y+C=0$$

Since it passes through origin, C must be zero.
Thus we have $3x+4y=0$.
Thus (d) is correct option

(v) if $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ then A^2 is equal to

(a) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$

(b) $\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(d) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

Sol :

We have $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

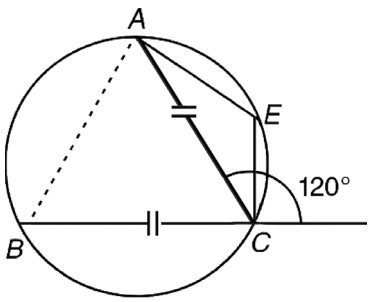
$$A^2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0+1 & 1+0\\ 0+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Thus (c) is correct option

(vi) In the given diagram, chords AC and BC are equal. If $\angle ACD=120^\circ$, then $\angle AEC$ is



(a) 30° (b) 60°

(c) 90° (d) 120°

Sol :

The angle between a tangent and a chord at a point on the circle equals the inscribed angle subtending that same chord from the opposite arc.
In the diagram, CD is tangent at C and AC is the chord, so

$$\angle ACD=\angle AEC$$

Since $\angle ACD$ is given as 120° it follows immediately that

$$\angle AEC=120^\circ$$

Thus (d) is correct option

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(vii) The factor common to the two polynomials x^2-4 and x^3-x^2-4x+4 is

(a) $(x+1)$ (b) $(x-1)$

(c) $(x+2)$ (d) $(x-2)$

Sol :

$$x^2-4=(x-2)(x+2)$$

$$x^3-x^2-4x+4=x^2(x-1)-4(x-1)$$

$$=(x-1)(x^2-4)$$

$$=(x-1)(x-2)(x+2)$$

Thus factor common are $(x+2)$ and $(x-2)$.
Thus (c, d) is correct option

(viii) A man invested in a company paying 12% dividend on its share. If the percentage return on his investment is 10%, then the shares are

(a) At par

(b) Below par

(c) Above par

(d) Can not be determined

Sol :

Let the face value of one share be F.
Dividend per share is 12 % of $F=0.12F$.
Let the cost price (what the man pays) per share be P.
As per question,

$$\frac{0.12F}{P}\times 100\,\%=10\,\%$$

$$\frac{12F}{P}=10$$

$$P = 1.2F$$

Since the cost price $P(= 1.2F)$is more than the face value F , the shares are above par.

Thus (c) is correct option

(ix) **Statement 1** : The point which is equidistant from three non-collinear points D, E and F is the circumcentre of the ΔDEF.

Statement 2 : The incentre of a triangle is the point where the bisector of the angles intersects.

(a) Both the statements are true

(b) Both the statements are false

(c) Statement 1 is true and Statement 2 is false

(d) Statement 1 is false and Statement 2 is true

Sol :

Statement 1 is true: The circumcentre is the point that is equidistant from all three vertices of a triangle.
Statement 2 is also true: The incentre is the point where the angle bisectors of a triangle meet.
Thus (a) is correct option

(x) **Assertion (A)** : If $\sin^2 A + \sin A = 1$ then $\cos^4 A + \cos^2 A = 1$

Reason (R) : $1 - \sin^2 A = \cos^2 A$

(a) (A) is true, (R) is false

(b) (A) is false, (R) is true

(c) Both (A) and (R) are true, and (R) is the correct reason for (A)

(d) Both (A) and (R) are true, and (R) is the incorrect reason for (A)

Sol :

We have $\sin^2 A + \sin A = 1$ (i)

or $\sin A = 1 - \sin^2 A$ (i)

Substituting $1 - \sin^2 A = \cos^2 A$ in above equation we get

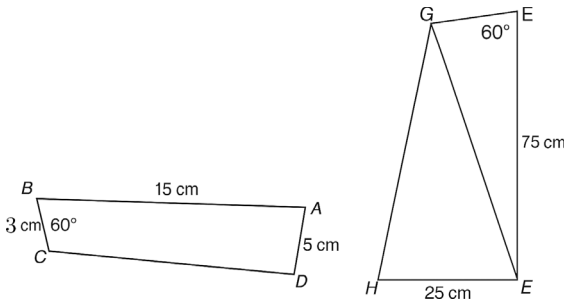
$$\sin A = \cos^2 A \quad \text{(ii)}$$

Substituting above in (i) we get

$$\cos^4 A + \cos^2 A = 1$$

Thus (c) is correct option

(xi) In the given diagram $\Delta ABC \sim \Delta EFG$. If $\angle ABC = \angle EFG = 60°$, then the length of the side FG is



(a) 15cm (b) 20cm

(c) 25cm (d) 30cm

Sol :

Since $\Delta ABC \sim \Delta EFG$

$$\frac{FG}{BC} = \frac{EF}{BA}$$

$$\frac{FG}{3} = \frac{75}{15}$$

$$FG = 15 \text{ cm}$$

Thus (a) is correct option

(xii) If the volume of two spheres is in the ratio 27 : 64, then the ratio of their radii is

(a) 3 : 4 (b) 4 : 3

(c) 9 : 16 (d) 16 : 9

Sol :

$$\tfrac{4}{3}\pi r_1^3 : \tfrac{4}{3}\pi r_2^3 :: 27 : 64$$

$$r_1^3 : r_2^3 :: 27 : 64$$

$$r_1 : r_2 :: 3 : 4$$

Thus (a) is correct option

(xiii) The marked price of an article is ₹ 1375. If the CGST is charged at a rate of 4%, then the price of the article including GST is

(a) ₹55 (b) ₹110

(c) ₹1430 (d) ₹1485

Sol :

If CGST is charged at a rate of 4%, then SGST will be also at a rate of 4% and total GST will be at a rate of 8%.

$$\text{Total Price} = 1375 + 8\% \text{ of } 1375$$

$$= 1375 + 8\%$$

$$= 1375 + 110$$

$$= ₹1485$$

Thus (d) is correct option

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(xiv) The solution set for $0 < -\frac{x}{3} < 2, x \in Z$ is

(a) $\{-5-4-3-2-1\}$

(b) $\{-6-5-4-3-2-1\}$

(c) $\{-5-4-3-2-1\ 0\}$

(d) $\{-6-5-4-3-2-1\ 0\}$

Sol :

$$0 < -\frac{x}{3} < 2,\ x \in Z$$

$$0 < -x < 6$$

$$-6 < x < 0$$

Thus $x \in \{-5-4-3-2-1\}$

Thus (a) is correct option

(xv) **Assertion (A)** : The mean of first 9 natural numbers is 4.5.

Reason (R) :Mean $= \frac{\text{Sum of all the observations}}{\text{Total numer of observations}}$

(a) (A) is true, (R) is false

(b) (A) is false, (R) is true

(c) Both (A) and (R) are true, and (R) is the correct reason for (A)

(d) Both (A) and (R) are true, and (R) is the incorrect reason for (A)

Sol :

$$\text{Mean} = \frac{1+2+3+4+5+6+7+8+9}{9} = 5$$

Assertion is not true.

$$\text{Mean} = \frac{\text{Sum of alll observations}}{\text{Total number of observations}}$$

Reason is true.

Thus (b) is correct option

QUESTION 2.

(i) Solve the following quadratic equation $2x^2 - 5x - 4 = 0$ Give your answer to correct to three significant figures.

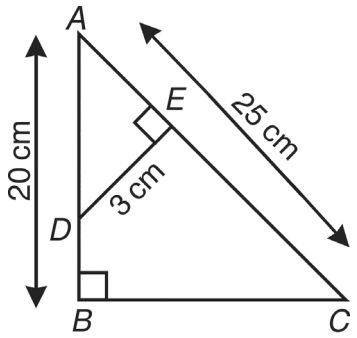
(Use mathematical tables for this question)

(ii) Mrs. Rao deposited ₹250 per month in a recurring deposit account for a period of 3 years. She received ₹10,110 at the time of maturity. Find:

(a) the rate of interest

(b) how much more interest Mrs. Rao will receive if she had deposited ₹50 more per month at the same rate of interest and for the same time.

(iii) In ΔABC, $\angle ABC = 90°$, $AB = 20$ cm, $AC = 25$ cm, DE is perpendicular to AC such that $\angle DEA = 90°$ and $DE = 3$ cm as shown in the given figure.



(a) Prove that $\Delta ABC \sim \Delta AED$.

(b) Find the lengths of BC, AD and AE

(c) If BCED represents a plot of land on a map whose actual area on ground is 576 m2, then find the scale factor of the map.

Sol :

(i) We have $2x^2 - 5x - 4 = 0$

Here $a = 2,\ b = -5,\ c = -4,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$== \frac{5 \pm \sqrt{(-5^2) - 4(2)(-4)}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{57}}{4}$$

$x = 3.137$ or $x = -0.637$

$x = 3.14 - 0.637$

(ii) (a) Mrs. Rao deposits ₹250 per month for 3 years,

Total amount deposited = ₹250×36

= ₹9000

Let the rate of interest be $r\%$ per annum,then

$$\text{SI} = \frac{P \times n(n+1) \times r}{2 \times 12 \times 100}$$

$$= \frac{250 \times 36 \times 37 \times r}{2 \times 12 \times 100}$$

$$= \frac{9000 \times 37 \times r}{2400}$$

$$= \frac{90 \times 37 \times r}{24}$$

$$= \frac{555}{4} r$$

It is given that amount on maturity is ₹10,110,

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thus

$$9000+\frac{555r}{4}=10{,}110$$

$$\frac{555r}{4}=10{,}110-9000$$

$$\frac{555r}{4}=1{,}110$$

$$r=8$$

(b)

$$\text{SI}=\frac{P\times n(n+1)\times r}{2\times 12\times 100}$$

$$=\frac{300\times 36\times 37\times 8}{2400}$$

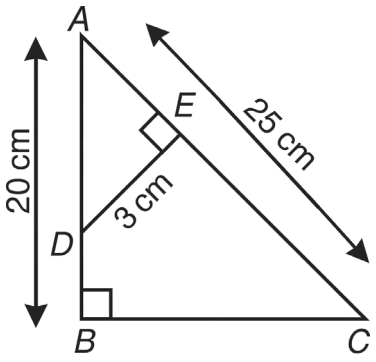
$$=36\times 37$$

$$=1332$$

₹1332

Difference $=1332-1110=$₹222

(iii)



(a) In ΔABC and ΔAED

Due to common angle we have

$$\angle A=\angle A$$

$$\angle DEA=\angle ABC=90^\circ$$

Thus by AA similarity we have

$$\Delta ABC\sim\Delta AED$$

(b) $$BC^2=AC^2-AB^2$$

$$=25^2-20^2=15^2$$

$$BC=15\text{cm}$$

Now due to $\Delta ABC\sim\Delta AED$ we have

$$\frac{DE}{AD}=\frac{BC}{AC}$$

$$\frac{3}{AD}=\frac{15}{25}$$

$$AD=5\text{cm}$$

Again due to $\Delta ABC\sim\Delta AED$ we have

$$\frac{DE}{AE}=\frac{BC}{AB}$$

$$\frac{3}{AE}=\frac{15}{20}$$

$$AE=4\text{cm}$$

(c) $$\text{ar}(BCED)=\text{ar}(\Delta ABC)-\text{ar}(\Delta AED)$$

$$=\frac{1}{2}\times 20\times 15-\frac{1}{2}\times 3\times 4$$

$$=150-6$$

$$=144\text{cm}^2$$

Actual area $=576\text{m}^2$

$$=576\times 10^4\text{cm}^2$$

Let scale factor be k,

$$k^2=\frac{\text{Area of the map}}{\text{Area of actual figure}}$$

$$=\frac{144}{576\times 10^4}$$

$$=\frac{1}{40000}$$

$$k=1:200$$

QUESTION 3.

(i) Use ruler and compass for the following construction. Construct a ΔABC ,where $AB=$ 6cm, $AC=4.5$cm and $\angle BAC=120^\circ$. Construct the ΔABC Measure and write down the length of the radius of the circle

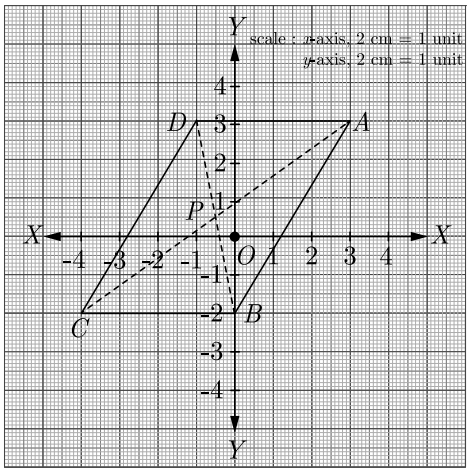
(ii) If $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$, $B=\begin{bmatrix}2&4\\1&2\end{bmatrix}$, and $C=\begin{bmatrix}-5&1\\7&-4\end{bmatrix}$

Find :

(a) $A+C$ (b) $B(A+C)$

(c) $5B$ (d) $B(A+C)-5B$

(iii) In the given graph $ABCD$ is a parallelogram.



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Using the graph, answer the following:

(a) Write down the coordinates of A, B, C, and D

(b) Calculate the coordinates of 'P', the point of intersections of the diagonals AC and BD

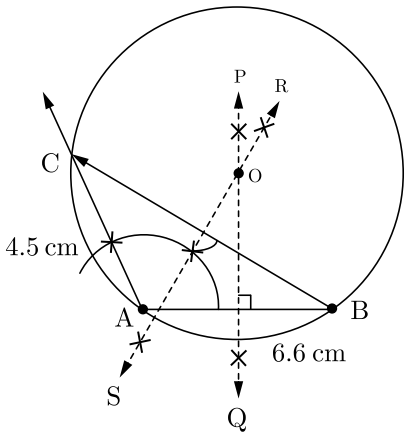
(c) Find the slope of sides CB and DA and verify that they represent parallel lines.

(d) Find the equation of the diagonal AC

Sol :

(i) Steps of construction:

1. Draw a line segment AB of 6 cm.
2. At point A, construct an $\angle BAX$ of 120° using ruler and compass.
3. Taking A as centre, draw an arc of length 4.5 cm which cuts $\overrightarrow{AX}$ at C.
4. Join BC.
5. Draw perpendicular bisectors of sides BC and AB. Which are represented by PQ and RS respectively.
6. PQ and RS are interest at O which circumcentre
7. Taking OA as radius draw a circle which passes through A, B, and C
8. Measures OA, OB or OC which is radius.



(ii)

(a) $$A+C=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-5 & 1\\ 7 & -4\end{bmatrix}$$

$$=\begin{bmatrix}1-5 & 2+1\\ 3+7 & 4-4\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 3\\ 10 & 0\end{bmatrix}$$

(b) $$B(A+C)=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-5 & 1\\ 7 & -4\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}\begin{bmatrix}-4 & 3\\ 10 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(2\times-4)+(1\times10) & (2\times3)+(1\times0)\\ (4\times-4)+(2\times10) & (4\times3)+(2\times0)\end{bmatrix}$$

$$=\begin{bmatrix}2 & 6\\ 4 & 12\end{bmatrix}$$

(c) $$5B=5\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}5\times2 & 5\times1\\ 5\times4 & 5\times2\end{bmatrix}$$

$$=\begin{bmatrix}10 & 5\\ 20 & 10\end{bmatrix}$$

(d) $$=\begin{bmatrix}2 & 4\\ 1 & 2\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-5 & 1\\ 7 & -4\end{bmatrix}\right)-5\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}\begin{bmatrix}-4 & 3\\ 10 & 0\end{bmatrix}-\begin{bmatrix}5\times2 & 5\times1\\ 5\times4 & 5\times2\end{bmatrix}$$

$$=\begin{bmatrix}-8+10 & 6+0\\ -16+20 & 12+0\end{bmatrix}-\begin{bmatrix}10 & 5\\ 20 & 10\end{bmatrix}$$

$$=\begin{bmatrix}2 & 6\\ 4 & 12\end{bmatrix}-\begin{bmatrix}10 & 5\\ 20 & 10\end{bmatrix}$$

$$\begin{bmatrix}-8 & 1\\ -16 & 2\end{bmatrix}$$

(iii) (a) The coordinates are as follows $A(3,3)$, $B(0,-2)$, $C(-4-2)$ and $D(-1,3)$

(b) In any parallelogram, diagonal bisects each other. Using mid point formula we have.

$$P\left(\frac{0+(-1)}{2},\frac{-2+3}{2}\right)$$

$$P\left(\frac{-1}{2},\frac{-1}{2}\right)$$

(c) Slope of Sides

$$\text{Slope of side } CB=\frac{-2-(-2)}{-4-0}=0$$

$$\text{Slope of side } DA=\frac{3-3}{-1-3}=0$$

Slope of side CB = Slope of Side DA

Both side CB and DA are paralle to each other

(d) Equation of diagonal AC,

By two point form of line we have

$$y-3=\frac{-2-3}{-4-3}(x-3)$$

$$y-3=\frac{-5}{-7}(x-3)$$

$$7y-21=5x-15$$

$$5x-7y+6=0$$

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SECTION-B

QUESTION 4.

(i) Solve the following in equation, write the solution set and represent it on the real number line

$2x-\frac{5}{3} < \frac{3x}{5}+10 \le \frac{4x}{5}+11 \; ; \; x \in R$

(ii) The first term of an Arithmetic Progression (A.P.) is 5, the last term is 50 and their sum is 440. Find:
(a) the number of terms
(b) common difference

(iii) Prove that:
$\frac{(\cot A+\tan A-1)(\sin A+\cos A)}{\sin^3 A+\cos^3 A}=\sec A.\operatorname{cosec} A$

Sol :

(i) we have $2x-\frac{5}{3} < \frac{3x}{5}+10 \le \frac{4x}{5}+11$

Now, $2x-\frac{5}{3} < \frac{3x}{5}+10$

$30x-25 < 9x+150$

$21x < 175$

$x < \frac{175}{21}$ ...(i)

Again, $\frac{3x}{5}+10 \le \frac{4x}{5}+11$

Multiple both side by 5 we have

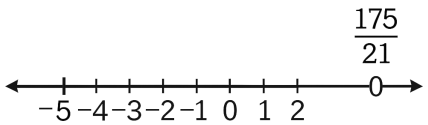
$3x+50 \le 4x+55$

$-5 \le x$ ...(ii)

From (i) and (ii), we get

$-5 \; x < \frac{175}{21}$

Solution on number line



(ii) Consider, first term of AP be $a=5$ and last term of A P be $s=440$.

(a)

Now $440=\frac{n}{2}(a+l)$

$440=\frac{n}{2}(5+50)$

$880=55n$

$n=16$

(b) $l=50$

$a+(n-1)\,d=50$

$5+(16-1)\,d=50$

$15d=45$

$d=3$

(iii) L.H.S $=\frac{(\cot A+\tan A-1)(\sin A+\cos A)}{\sin^3 A+\cos^3 A}$

Using $\cot A=\frac{\cos A}{\sin A}$, $\tan A=\frac{\sin A}{\cos A}$

and $a^3+b^3=(a+b)(a^2+b^2-ab)$

L.H.S

$=\frac{(\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}-1)(\sin A+\cos A)}{(\sin A+\cos A)(\sin^2 A+\cos^2 A-\sin A\cos A)}$

$=\frac{\left(\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}-1\right)}{(1-\sin A\cos A)}$

$=\frac{(\frac{1}{\sin A\cos A}-1)}{(1-\sin A\cos A)}$

$=\frac{(\frac{1-\sin A\cos A}{\sin A\cos A})}{(1-\sin A\cos A)}$

$=\frac{1}{\sin A\cos A}$

$=\operatorname{cosec} A\sec A$

= RHS hence proved.

QUESTION 5.

(i) Using properties of proportion, find the value of 'x'

$\frac{6x^2+3x-5}{3x-5}=\frac{9x^2+2x+5}{2x+5}; \; x \neq 0$

(ii) It is given that $(x-2)$ is a factor of polynomial $2x^3-7x^2+kx-2$

(a) Find the value of 'k'.

(b) Hence, factorize the resulting polynomial completely.

(iii) A solid wooden capsule is shown in Figure 1. The capsule is formed of a cylindrical block and two hemispheres.

Find the sum of total surface area of the three parts as shown in Figure 2. Given, the radius of the capsule is 3.5cm and the length of the cylindrical block is 14cm. $\left(\text{Use } \pi=\frac{22}{7}\right)$

Find the sum of total

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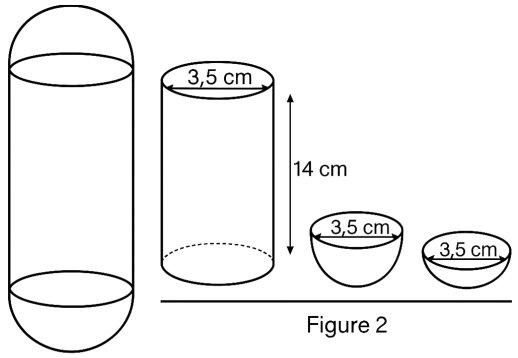

Figure 2

Figure 1

Sol :

(i) $$\frac{6x^2+3x-5}{3x-5}=\frac{9x^2+2x+5}{2x+5}$$

Applying Componendo and Dividendo we have

$$\frac{6x^2+3x-5+3x-5}{6x^2+3x-5-3x+5}=\frac{9x^2+2x+5+2x+5}{9x^2+2x+5-2x-5}$$

$$\frac{6x^2+6x-10}{6x^2}=\frac{9x^2+4x+10}{9x^2}$$

$$9(6x^2+6x-10)=6(9x^2+4x+10) \quad [x\neq 0]$$

$$6(3x^2+3x-5)=2(9x^2+4x+10)$$

$$18x^2+18x-30=18x^2+8x+20$$

$$10x=50$$

$$x=5$$

(ii) (a) We have $f(x)=2x^3-7x^2+kx-2$

Since $(x-2)$ is a factor of polynomial $f(x)$ we have

$$f(2)=0$$

$$2(2)^3-7(2)^2+k(2)-2=0$$

$$16-28+2k-2=0$$

$$2k-14=0$$

$$2k=14$$

$$k=7$$

(b) Now we have $f(x)=2x^3-7x^2+7x-2$

Since $(x-2)$ is a factor of the given polynomial Now,we will apply long division.

$$\begin{array}{r|l} & 2x^2-3x+1 \\ \hline x-2 & 2x^3-7x^2+7x-2 \\ & \underline{2x^3-4x^2} \\ & -3x^2+7x-2 \\ & \underline{-3x^2+6x} \\ & x-2 \\ & \underline{x-2} \\ & 0 \end{array}$$

Thus $f(x)=(x-2)(2x^2-3x+1)$

Now $2x^2-3x+1=2x^2-2x-x+1$

$$=2x(x-1)-1(x-1)$$

$$=(2x-1)(x-1)$$

Factors of polynomial $2x^3-7x^2+7x-2$ are $(x-2),(2x-1),(x-1)$.

(iii) Total surface area of cylindrical block

$$=2\pi rh+2\pi r^2$$

Total surface area of hemispherical part

$$=2\pi r^2+\pi r^2$$

Sum of total surface area of all three parts

$$=2\pi rh+2\pi r^2+2\pi r^2+\pi r^2+2\pi r^2+\pi r^2$$

$$=2\pi rh+8\pi r^2$$

$$=2\pi r(h+4r)$$

We have $r=3.5$ and $h=14$

$$2\pi r(h+4r)=2\times\tfrac{22}{7}\times 3.5(14+4\times 3.5)$$

$$=22(14+14)$$

$$=616\text{cm}^2$$

QUESTION 6.

(i) Use a graph paper for this question taking 2cm = 1 unit along both axes.

(a) Plot $A(1,3)$, $B(1,2)$ and $C(3,0)$

(b) Reflect A and B on the $x-$axis and their images as E and D respectively. write down their coordinates.

(c) Reflect A and B through the origin and name their images as F and G respectively.

(d) Reflect A, B and C on the $y-$ axis and name their images as J, I and H respectively.

(e) Join all the points A, B, C, D, E, F, G, H, I and J in order and name the closed figure so formed.

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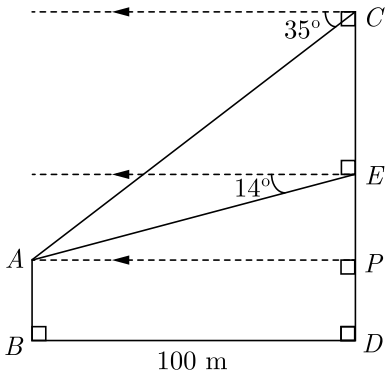
(ii) In the given diagram, AB is a vertical tower 100 m away from the foot of a 30 storied building CD. The angles of depression from the point C and E. (E being the mid-point of CD), are 35° and 14° respectively. (Use mathematical table for the required values rounded off correct to two places of decimals only) Find the height of the:

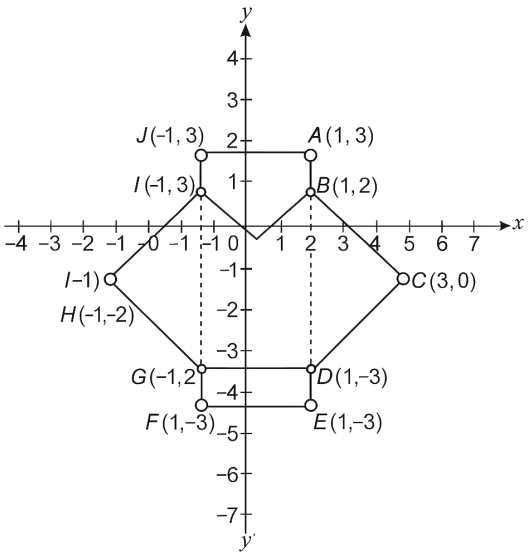
(a) Tower AB

(b) Building CD



Sol :

(i) (a)



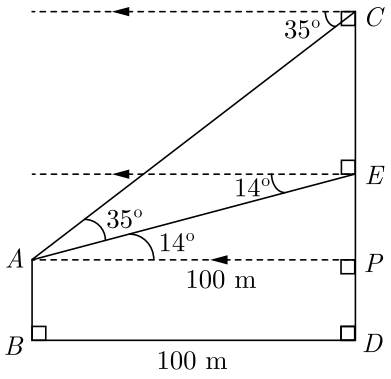
(b) $E(1,-3)$ and $D(1,-2)$

(c) $F(-1,-3)$ and $G(-1,2)$

(d) $J(-1,3)$, $I(-1,2)$ and $H(-3,0)$

(e) 10-Side polygon or Decagon

(ii) (a) We have redrawn the figure as shown below



Let H be the hieght of CD and h be the hieght of $AB = BD$

Since E is mid-point of CD

$$CE = ED = \frac{H}{2}$$

$$PE = \frac{H}{2} - h$$

$$= \frac{H-2h}{2}$$

Now, in ΔEPA

$$\tan 14° = \frac{PE}{AP}$$

$$0.25 = \frac{H-2h}{2 \times 100}$$

$$H - 2h = 50 \quad \text{...(i)}$$

Again, in ΔCAP we have

$$\tan 35° = \frac{PC}{AP} = \frac{H-h}{100}$$

$$0.7 = \frac{H-h}{100}$$

$$H - h = 70 \quad \text{...(ii)}$$

Subtracting equation (i) from equation (ii) we get

$$h = 70 - 50$$

$$h = 20 \quad \text{...(iii)}$$

$$AB = 20 \text{ meters}$$

(b) Putting the value of h in equation (ii), we get

$$H = 70 + 20$$

$$H = 90$$

$$CD = 90 \text{ meters}$$

QUESTION 7.

(i) Use graph paper for this question.

(Take 2 cm = 10 Marks along one axis and 2 cm = 10 students along another axis).

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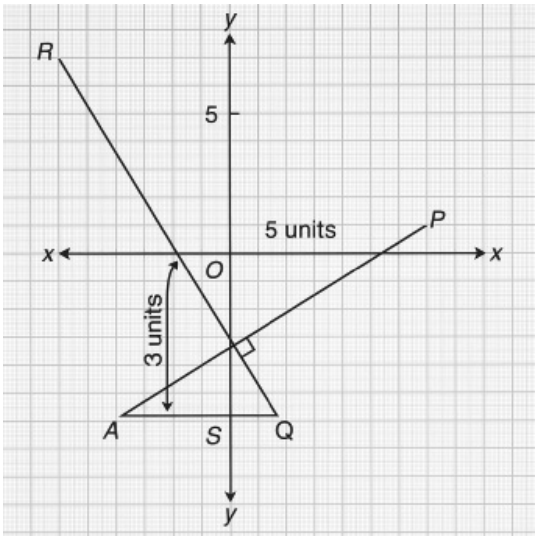
Draw a Histogram for the following distribution which gives the marks obtained by 164 students in a particular class and hence find the mode.

(ii) In the given graph. P and Q are points such that PQ cuts off intercepts of 5 units and 3 units along the $x-axis$ and $y-axis$ respectively. Line RS is perpendicular to PG and passes through the origin. Find the

(a) Coordinates of P and Q

(b) Equation of line RS



(iii) Refer to the given bill.

A customer paid ₹2000 (rounded off to the nearest ₹10) to clear the bill.

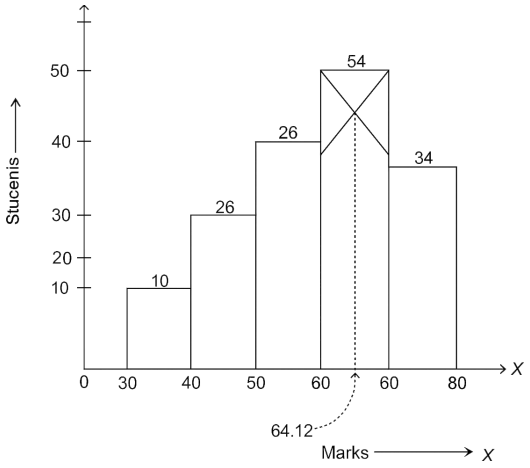
Note: 5% discount is applicable on an article if 10 or more such articles are purchased.

BILL			
Article	**M.P. (₹)**	**Quantity**	**G.S.T.**
A	190	06	12%
B	50	12	18%

Check whether the total amount paid by the customer is correct or not. Justify your answer with necessary working.

Sol :

(i) Scale :
On x-axis : 2cm = Marks
On y-axis : 2cm = 10 Students



Mode = 64.12

(ii)

(a) According to given question, from the given graph $P(5,\ 0)$ and $Q(0,\ 3)$

$$\text{Slope of } PQ = \frac{-3-0}{0-5} = \frac{3}{5}$$

Sinc RS is perpendicular to PQ we have

$$\text{Slope of } RS = \frac{-1}{\left(\frac{3}{5}\right)} \quad \frac{-5}{3}$$

Also, RS line is passing through origin $(0,\ 0)$, by point slope form equation of line RS,

$$y-0 = \frac{-5}{3}(x-0)$$

$$3y = -5x$$

or $$5x+3y = 0$$

(iii) For article A :

$$\text{M.P} = 190 \times 6$$

$$= ₹1140$$

$$\text{GST} = 12\%$$

Amount paid by customer for article

$$A = 1140 + \frac{1140 \times 12}{100}$$

$$= 1140 + 136.8$$

$$= ₹1276.8$$

For article B :

$$\text{M.P} = 50 \times 12$$

$$= ₹600$$

Discount is applicable on article B applicable because number of articles more than 10 are purchased.

$$\text{S.P} = \text{M.P} - \text{Discount}$$

$$= 600 - \frac{600 \times 5}{100}$$

$$= ₹570$$

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GST = 18%

Amount paid by customer for article B

$= 570 + \frac{570 \times 18}{100}$

$= 570 + 102.6$

$= ₹672.6$

Amount paid by customer

$= ₹1276.8 + ₹672.6$

$= ₹1949.4$

Total amount paid by customer is not correct.

QUESTION 8.

(i) A man bought ₹200 shares of a company at 25% premium. If he received a return of 5% on his investment.

Find the :

(a) Market value

(b) Dividend percent declared

(c) Number of shares purchased, if annual dividend is ₹1000.

(ii) For the given frequency distribution, find the :

(a) mean, to the nearest whole number

(b) median

x	10	11	12	13	14	15	16
f	3	2	2	6	3	5	3

(iii) Mr. and Mrs. Das were traveling by car from Delhi to Kasauli for a holiday. Distance between Delhi and Kasauli is approximately 350 km (via NH 152D). Due to heavy rain they had to slow down. The average speed of the car was reduced by 20 km/hr and time of the journey increased by 2 hours. Find :

(a) the original speed of the car.

(b) with the reduce speed, the number of hours they took to reach their destination.

Sol :

(i) Nominal value of shares = ₹200

premium = 25%

(a)

$\text{Market value} = 200 + \frac{25}{100} \times 200$

$= ₹250$

(b)

$\text{Return on investment} = \frac{\text{Dividend} \times \text{N.V}}{100}$

$5\% \text{ of } 250 = \frac{\text{Dividend} \times 200}{100}$

$\frac{5}{100} \times 250 = \frac{5 \times 250 \times 100}{100 \times 200}$

Dividend declared = 6.25%

(c) If dividend is ₹1000 then

₹1000 = 6.25% of ₹200 × Number of shares

$1000 = \frac{625}{10000} \times 200 \times \text{Number of shares}$

$\text{Number of shares} = \frac{1000 \times 10000}{625 \times 200}$

$= 80$

(ii)

x	f	cf	fx
10	3	3	30
11	2	5	22
12	2	7	24
13	6	13	78
14	3	16	42
15	5	21	75
16	3	24	48
	$\sum f = 24$		$\sum fx = 319$

$\text{Mean} = \frac{\sum fx}{f}$

$= \frac{319}{24}$

$= 13.29$

$= 13$ (to the nearest)

(b) $\text{Median} = \frac{\left(\frac{N}{2}\right)^{th}\text{term} + \left(\frac{N}{2}+1\right)^{th}\text{term}}{2}$

$= \frac{12^{th}\text{term} + 13^{th}\text{term}}{2}$

$= \frac{13+13}{2} = 13$

(iii) (a) Let the original speed of car be x km/hr

$\text{Usual time} = \frac{350}{x}$ hours

According to question,

$(x-20)\left(\frac{350}{x}+2\right) = 350$

$350 + 2x - \frac{7000}{x} - 40 = 350$

$2x^2 - 7000 - 40x = 0$

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$$x^2 - 20x - 3500 = 0$$
$$x^2 - 70x + 50x - 3500 = 0$$
$$(x - 70)(x + 50) = 0$$

As $x > 0$, $x = 70$km/hr

Original speed of car is 70 km/hr

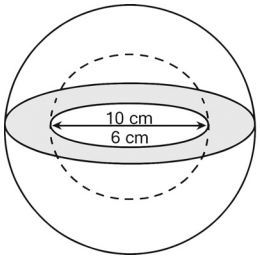
(b)Required number of hours $= \frac{350}{x} + 2$

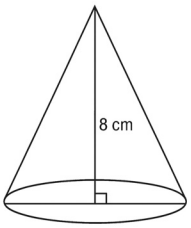
$= \frac{350}{70} + 2$

$=7$ hrs

QUESTION 9.

(i) A hollow sphere of external diameter 10 cm and internal diameter 6 cm is melted and made into a solid right circular cone of height 8 cm. Find the radius of the cone so formed.[Use$\pi = \frac{22}{7}$]





(ii) Ms. Sushmita went to a fair and participated in a game. The game consisted of a box having number cards with numbers from 01 to 30. The three prizes were as per the given table:

Prize	**Number on the card drawn at random is a**
Wall Clock	perfect square
Water Bottle	even number which is also a multiple of 3
Purse	Prime number

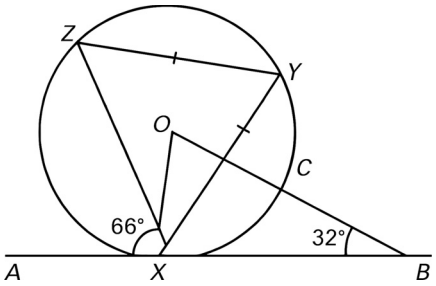
Find the probability of Winning a :

(a) Wall Clock

(b) Water Bottle

(c) Purse

(iii) X, Y, Z and C are the points on the circumference of a circle with centre 'O'. AB is a tangent to the circle at 'X' and $ZY = XY$. Given $\angle OBX = 32°$ and $\angle AXZ = 66°$. Find :



(a) $\angle BOX$

(b) $\angle CYX$

(c) $\angle ZYX$

(d) $\angle OXY$

Sol :

(i) Volume of hollow sphere

$$V_1 = \frac{4}{3}\pi(R_1)^3 - \frac{4}{3}\pi(R_2)^3$$
$$= \frac{4}{3}\pi(R_1^3 - R_2^3)$$

Volume of cone formed from hollow sphere,

$$V_2 = \frac{1}{3}\pi r^2 h$$

According to given conditions in question

$$V_2 = V_1$$
$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi(R_1^3 - R_2^3)$$
$$r^2 h = 4(R_1^3 - R_2^3)$$

Substituting the values we have

$$r^2 8 = 4(5^3 - 3^2)$$
$$2r^2 = 125 - 27$$
$$2r^2 = 98$$
$$r^2 = 49$$
$$r = 7 \text{ cm}$$

Radius of cone is 7 cm.

(ii) Probability of an event E is given by

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$$= \frac{n(E)}{n(S)}$$

(a) If event E denotes occurrence of perfect square number, then $E = \{01, 04, 09, 16, 25\}$

$$n(E) = 5$$
$$P(E) = \frac{n(E)}{n(s)}$$
$$= \frac{5}{30} = \frac{1}{6}$$

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Probability of winning wall clock is $\frac{1}{6}$.

(b)If event E denotes occurrence of even number which is also a multiple of 3, is $E = \{06,12,18,24,30\}$

$$n(E) = 5$$
$$P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{5}{30} = \frac{1}{6}$$

Probability of winning water bottle is $\frac{1}{6}$

(c)If event E denotes occurrence of prime number $E = \{2,3,5,7,11,13, 17, 19, 23, 29\}$

$$n(E) = 10$$
$$P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{10}{30} = \frac{1}{3}$$

Probability of winning purse is $\frac{1}{3}$

(iii) $\angle OXB = 32°$

(a) In ΔOBX

A tangent to the circle is perpendicular to the radius at the point of contact.

Thus $\angle OBX = 90°$

Now in a triangle we have

$$\angle OXB + \angle XOB + \angle OBX = 180°$$
$$\angle BOX + 90° + 32° = 180°$$
$$\angle BOX + 122° = 180°$$
$$\angle BOX = 58°$$

(b) The angle subtended b an arc of a article at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$2\angle CYX = \angle COX$$
$$\angle CYX = \frac{\angle COX}{2}$$
$$= \frac{58°}{2}$$
$$= 29°$$

(c) Due to aternate segment Theorem

$$\angle AXZ = \angle ZYX$$
$$= 66°$$

(d) In ΔZXY we have $\angle XZY = \angle YXZ$, thus

$$\angle ZXY + \angle ZYX + XZY = 180°$$
$$2\angle XZY + 66° = 180°$$
$$2\angle XZY = 114°$$
$$\angle XZY = \angle ZXY$$
$$= 57°$$

A tangent to the circle is perpendicular to the radius at the point of contact.

Thus $\angle AXO = 90°$

$$\angle ZXO = 90° - 66°$$
$$= 24°$$
$$\angle OXY = \angle ZXY - \angle ZXO$$
$$= 57 - 24°$$
$$= 33°$$

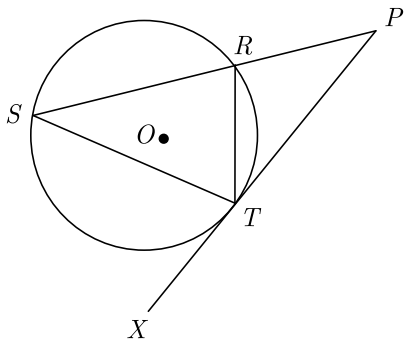
QUESTION 10.

(i) If 1701 is the n^{th} term of the Geometric Progression (G.P.) 7, 21, 63..., find:

(a) The value of 'n'

(b) Hence finds the sum of the 'n' terms of the G.P.

(ii) In the given diagram 'O' is the centre of the circle. Chord SR produced meets the tangent XTP at P.



(a) Prove that $\Delta PTR \sim \Delta PST$

(b) Prove that PT^2.

(c) If $PR = 4$ cm and $PS = 16$ cm, find the length of tangent PT

(iii) The given graph represents the monthly salaries (in ₹) of workers of a factory.

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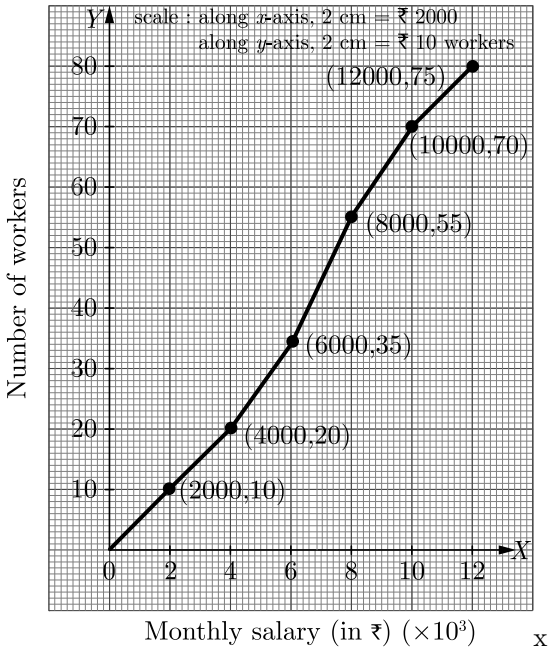


Using graph answer the following:

(a) The total number of workers.

(b) The median class

(c) The lower-quartile class

(d) Number of workers having monthly Salary more than or equal to ₹ 6,000 but less then 10,000.

Sol :

(i) (a) Here, first term is $a = 7$

Common ration $r = \frac{21}{7} = 3$

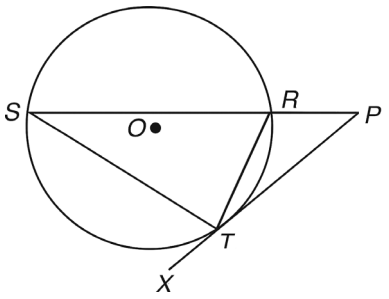
n^{th} term of given GP we have

$$a_n = ar^{n-1}$$
$$1701 = 7(3)^{n-1}$$
$$243 = (3)^{n-1}$$
$$(3)^5 = (3)^{n-1}$$
$$n - 1 = 5$$
$$n = 6$$

(b) Sum of n^{th} term

$$S_n = \frac{7(3^6 - 1)}{3 - 1}$$
$$= \frac{7(729 - 1)}{2}$$
$$= \frac{7 \times 728}{2}$$
$$= 2548$$

(ii)



(a) In ΔPTR and $\Delta\ PST$

Due to common angle

$$\angle RPT = \angle TPS$$

From aternate segment theorem we have

$$\angle RTP = \angle TSP$$

Due to AA similarity we get

$$\Delta\text{PTR} \sim \Delta PST$$

(b)As ΔPTR $\sim$ ΔPST we have

$$\frac{PT}{PS} = \frac{PR}{PT}$$
$$PT^2 = PS \times PR$$

(c) $PR = 4$ cm, $PS = 16$ cm,

$$PT^2 = PS \times PR$$
$$= 16 \times 4$$
$$= 64$$
$$PT = 8 \text{ cm}$$

(iii) (a)Total number of workers = 75

(b) Total number of workers (N) = 75

$$\frac{N+1}{2} = \frac{75+1}{2} = 38$$

The median class is 6000 – 8000

(c)

$$\frac{N+1}{4} = \frac{75+1}{4} = 19$$

Lower quratile class is $2000 - 4000$

(d) Number of workers having monthly salary more than or equal to ₹6000 but less than ₹ 10000 is $= 70 - 35 = 35$

END

NODIA

CHAPTER 1

GOODS AND SERVICES TAX

MULTIPLE CHOICE QUESTIONS

Main 2024

1. For an Intra-state sale, the CGST paid by a dealer to the Central government is ₹120. If the marked price of the article is ₹2000, the rate of GST is :
(a) 6% (b) 10%
(c) 12% (d) 16.67%

Sol :

$$\text{Marked Price} = ₹2000$$

$$\text{CGST} = ₹120$$

$$\text{Rate of CGST} = \frac{120}{2000} = 6\,\%$$

$$\text{Rate of GST} = 2 \times \text{Rate of CGST}$$

$$= 2 \times 6 = 12\ \%$$

Thus (c) is correct option.

SQP 2024

2. A retailer purchased an item for ₹1500 from a wholesaler and sells it to a customer at 10% profit. The sales are intra-state and the rate of GST is 10%. The amount of GST paid by the customer:
(a) ₹15 (b) ₹30
(c) ₹150 (d) ₹165

Sol :

$$\text{Selling Price} = ₹\,1,500 + 10\%\ \text{of}\ ₹\,1,500$$

$$= ₹\,1,500 + ₹\,150$$

$$= ₹1,650$$

$$\text{GST} = 10\,\%\ \text{of}\ ₹1,650$$

$$= ₹1,65$$

Thus (d) is correct option.

Main 2023

3. The printed price of an article is ₹3080. If the rate of GST is 10% then the GST charged is
(a) ₹154 (b) ₹308
(c) ₹30.80 (d) ₹15.40

Sol :

$$\text{GST} = \text{GST Rate} \times \text{Printed Price}$$

$$= 10\% \times ₹3080$$

$$= \frac{10}{100} \times 3080$$

$$= ₹308$$

Thus (b) is correct option.

SQP 2023

4. Rohan buys some goods worth ₹12, 000 from a local store. If the rate of GST is 5%, then the amount Rohan has to pay to buy the goods is:
(a) ₹13,200 (b) ₹12,300
(c) ₹13,500 (d) ₹12,600

Sol :

Selling price $\text{S.P.} = ₹12,000$

$$\text{GST} = 5\%\ \text{of}\ ₹12,000$$

$$= ₹600$$

Amount Ramesh has to pay

$$= \text{S.P+GST}$$

$$= 12,000 + 600$$

$$= ₹12,600$$

Thus (d) is correct option.

Comp 2023

5. If a dealer in supplies goods worth ₹30,000 to another local dealer with a rate of GST 28%, then the tax levied under CGST is:
(a) ₹5,600 (b) ₹1,400
(c) ₹4,200 (d) Nil

Sol :

It is a case of intra- state transaction, so 14 % will be SGST and 14 % CGST.

$$\text{CGST} = 14\%\ \text{of}\ 30,000$$

$$= ₹4,200$$

Thus (c) is correct option.

Main 2022 Sem I

6. The marked price of an article is ₹5,000. The shopkeeper gives a discount of 10%. If the rate of GST is 12%, then the amount paid by the customer including GST is
(a) ₹5,040 (b) ₹6,100

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(c) ₹6,272 (d) ₹6,160

Sol :

$$\begin{aligned}\text{Marked Price} &= ₹5{,}000\\ \text{Discount} &= 10\%\\ \text{Selling Price} &= ₹\,5{,}000 - 10\%\text{ of }₹\,5{,}000\\ &= ₹\,5{,}000 - ₹\,500\\ &= ₹4{,}500\\ \text{GST} &= 12\%\text{ of }₹4{,}500\\ &= ₹\,540\end{aligned}$$

Price paid by customer including GST

$$= 4{,}500 + 540$$

$$= ₹5{,}040$$

Thus (a) is correct option.

Main 2022 Sem I

7. The selling price of a shirt excluding GST is ₹800. If the rate of GST is 12% then the total price of the shirt is

(a) ₹704 (b) ₹960
(c) ₹896 (d) ₹848

Sol :

$$\begin{aligned}\text{S.P.} &= ₹800\\ \text{GST} &= 12\%\text{ of }₹800\\ &= ₹96\\ \text{Total Price} &= 800 + 96\\ &= ₹896\end{aligned}$$

Thus (c) is correct option.

Main 2022 Sem I

8. If a dealer in Mumbai supplies goods worth ₹30,000 to another dealer in Chennai with a rate of GST 12%, then the tax levied is:

(a) ₹3,600 IGST (b) ₹1,800 CGST
(c) ₹1,800 IGST (d) ₹1,800 SGST

Sol :

It is a case of inter-state transaction, so only IGST will be imposed.

$$\begin{aligned}\text{IGST} &= 12\%\text{ of }₹30{,}000\\ &= ₹3{,}600\end{aligned}$$

Thus (a) is correct option.

9. In intra-state transaction, if the rate of GST is 18%, then the share of central and state governments, respectively are:

(a) 10%, 8% (b) 8%, 10%
(c) 9%, 9% (d) 11%, 7%

Sol :

In intra-state transactions, GST is equally shared between the Central and State Governments.
Thus (c) is correct option.

10. If a dealer in Jaipur (Rajasthan) supplies articles to another dealer in Udaipur (Rajasthan) with a rate of GST 18%, then the tax levied under IGST is:

(a) 18% (b) 10%
(c) 9% (d) Nil

Sol :

It is a case of intra-state transactions, so CGST and SGST will be imposed only.
Thus (d) is correct option.

11. For an Intra-state sale, the SGST paid by a dealer to the State government is ₹270. If the marked price of the article is ₹3000, the rate of GST is :

(a) 5% (b) 12%
(c) 18% (d) 28%

Sol :

$$\begin{aligned}\text{Marked Price} &= ₹3000\\ \text{SGST} &= ₹270\\ \text{Rate of SGST} &= \frac{270}{3000} = 9\,\%\\ \text{Rate of GST} &= 2 \times \text{Rate of SGST}\\ &= 2 \times 9 = 18\ \%\end{aligned}$$

Thus (c) is correct option.

12. When goods and services are sold for ₹6,000 under intra-state transaction from dealers A to customer B and the rate of GST is 12%, then the cost price, including tax, for the customer B is:

(a) ₹6,720 (b) ₹5,900
(c) ₹5,640 (d) ₹5,450

Sol :

$$\text{Rate of GST} = 18\%$$

Cost price for customer

$$= 6{,}000 + 12\%\text{ of }6000$$

$$= ₹\,6{,}720$$

Thus (a) is correct option.

13. An imported pen is marked at ₹1,500. A dealer sells it at 5% profit. If the rate of GST is 12%,

NODIA

then the total amount paid by a consumer to buy it, is:
(a) ₹1,824 (b) ₹1,764
(c) ₹1,832 (d) ₹1,732

Sol :

$$\text{Marked price} = ₹1,500$$
$$\text{Profit} = 5\% \text{ of } ₹1,500$$
$$= ₹75$$
$$\text{Selling price} = ₹1,500 + ₹75 = ₹1,575$$
$$\text{GST rate} = 12\%$$
$$\text{GST} = 12\% \text{ of } ₹1,575 = ₹189$$

Total amount paid by consumer
$$= ₹1,575 + ₹189$$
$$= ₹1,764$$

Thus (b) is correct option.

14. Ramesh buys an article marked at ₹5,000 at a discount of 15%, the rate of tax being 18%. What is the amount of GST paid by Ramesh for this purchase ?
(a) ₹765 (b) ₹750
(c) ₹825 (d) ₹900

Sol :

Marked price = ₹5,000 and discount = 15%
Selling price is at 15% discount on marked price.
Thus
$$\text{S.P.} = \left(1 - \frac{15}{100}\right) \times 5,000$$
$$= \frac{85}{100} \times 5,000$$
$$= ₹4,250$$
$$\text{GST} = \text{GST Rate} \times \text{S.P.}$$
$$= 18\% \times 4,250$$
$$= \frac{18}{100} \times 4,250 = ₹765.$$

Thus (a) is correct option.

15. Raj Electricals gives a discount of 20% on the listed price. The list price of a washing machine is ₹20,000. The tax rate for it is 18%. The total amount a customer has to pay to purchase the article is:
(a) ₹22,600 (b) ₹17,000
(c) ₹18,880 (d) ₹25,600

Sol :

Customer has to pay the selling price and GST.
List Price = ₹20000, Discount = 20%, GST = 20%
$$\text{S.P.} = \left(1 - \frac{20}{100}\right) \times 20,000$$
$$= \frac{80}{100} \times 20,000$$
$$= ₹16,000$$

Now, total amount to be paid
$$= \text{S.P} + 18\ \%\text{Tax on S.P}$$
$$= ₹16,000 + 18\% \text{ of } ₹16,000$$
$$= ₹16,000 + ₹2,880$$
$$= ₹18,880.$$

Thus (c) is correct option.

16. Mahesh buys some goods worth ₹6,000 in the same city. If the rate of GST is 5%, then the amount Mahesh has to pay to buy the goods is:
(a) ₹6,150 (b) ₹6,000
(c) ₹6,600 (d) ₹6,300

Sol :

Selling price
$$\text{S.P.} = ₹6,000$$
$$\text{GST} = 5\% \text{ of } 6,000$$
$$= ₹300$$

Amount Ramesh has to pay
$$= \text{S.P+GST}$$
$$= ₹6,000 + ₹300$$
$$= ₹6,300$$

Thus (d) is correct option.

17. Fill in the missing entries for transaction within the same city.

Listed price	₹1,200
Discount	20%
Discount Price	a
GST	18%
CGST	b
SGST	c
IGST	d

(a) $a = ₹960,\ b = c = ₹86.40,\ d = 0$
(b) $a = ₹240,\ b = c = ₹21.60,\ d = ₹21.60$
(c) $a = ₹960,\ b = c = ₹172.80,\ d = ₹21.60$
(d) $a = ₹240,\ b = c = ₹43.20,\ d = 0$

Sol :

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It is a case of intra-state transaction so IGST will be nil.

Discounted Price

$$= ₹1,200 - 20\% \text{ of } ₹1,200$$

$$= ₹960$$

$$\text{CGST} = \text{SGST} = \tfrac{1}{2} \text{ GST}$$

$$= 9\% \text{ of } ₹960$$

$$= ₹86.40$$

$$\text{IGST} = \text{Nil}$$

So, $a = ₹960$,

$b = c = ₹86.40$, $d = 0$.

Thus (a) is correct option.

18. An article is marked at ₹1,000. A dealer sells it at 5% profit. If the rate of GST is 18%, then the total amount paid by a consumer to buy it, is:

(a) ₹1,324 (b) ₹1,239

(c) ₹1,120 (d) ₹1,180

Sol :

$$\text{Marked price} = ₹1,000$$

$$\text{Profit} = 5\% \text{ of } ₹1,000$$

$$= ₹50$$

$$\text{Selling price} = ₹1,000 + ₹50 = ₹1,050$$

$$\text{GST rate} = 18\%$$

$$\text{GST} = 18\% \text{ of } ₹1,050 = ₹189$$

Total amount paid by consumer

$$= ₹1,050 + ₹189$$

$$= ₹1,239$$

Thus (b) is correct option.

19. Rashi purchases an article for ₹5310 which includes a discount of 10% on the marked price and 18% GST on the selling price. The marked price of the article is:

(a) ₹4,200 (b) ₹5,000

(c) ₹5,500 (d) ₹5,900

Sol :

Let marked price of article be ₹ x.

$$(x - 10\% \text{ of } x) + (x - 10\% \text{ of } x)\frac{18}{100} = 5,310$$

$$\frac{9x}{10} + \frac{9x}{10} \times \frac{18}{100} = 5,310$$

$$\frac{900x + 162x}{1000} = 5,310$$

$$1,062x = 53,10,000$$

$$x = 5,000.$$

Thus (b) is correct option.

NODIA

20. The taxable value of a toy is ₹900. Rate of GST is 18%. What is the price of the toy ?

(a) ₹1,100 (b) ₹1,062

(c) ₹981 (d) ₹1,900

Sol :

$$\text{Taxable value of toy} = ₹900$$

$$\text{Rate of GST} = 18\%$$

$$\text{Amount of GST} = 18\% \text{ of } 900$$

$$= \frac{18}{100} \times 900$$

$$= ₹162$$

Thus, price of the toy for the customer

$$= \text{Taxable value} + \text{GST}$$

$$= 900 + 162$$

$$= ₹1062$$

Hence, the price of the toy for the customer is ₹1062.

Thus (b) is correct option.

21. Rakhi buy goods worth ₹7,750 from a grocery store. Since, she has a membership card, so she gets a discount of 15% on ₹5,000. If the GST charged is 18%, then the total amount she has to pay for the goods is:

(a) ₹8,260 (b) ₹7,890

(c) ₹6,306 (d) ₹7,375

Sol :

Total discounted price

$$= 2,750 + \left(1 - \frac{15}{100}\right) \times 5,000$$

$$= 2,750 + 4,250$$

$$= ₹7,000$$

$$\text{GST rate} = 18\% \text{ of } ₹7,000$$

$$= ₹1,260$$

Total amount Shashi has to pay

$$= 7000 + 1260$$

$$= ₹8,260.$$

Thus (a) is correct option.

22. Seema went to a shop to buy a sewing machines

worth ₹10,080. The rate of GST is 12%. He asked the shopkeeper to reduce the price of washing machine to an extent that she has to pay ₹10,080 inclusive GST. What is the reduced price of the machine ?

(a) ₹9,250 (b) ₹8,800
(c) ₹9,000 (d) ₹8,500

Sol :

Let the reduced price of the washing machine be ₹ x.

Then, ₹ x + 12% of x = ₹ 10,080

$$x + \frac{12x}{100} = 10{,}080$$

$$\frac{112x}{100} = 10{,}080$$

$$x = \frac{10{,}080 \times 100}{112}$$

$$= 9{,}000$$

The reduced price of the machine is ₹ 9,000
Thus (c) is correct option.

23. The manufacturing cost of an item is ₹800. The manufacturer marked up the price of his items by 60% and then sell them at a discount of 10%. Rate of GST rate on this item is 5%. If a consumer buy 10 such items, then the GST paid by him on the purchased is:

(a) ₹348 (b) ₹1,169
(c) ₹576 (d) ₹1,425

Sol :

C.P of an item = ₹ 800
Marked price of an item
= ₹ 800 + 60% of ₹800
= ₹ 1,280
Discounted price of an item
= ₹1,280 − 10% of ₹ 1,280
= ₹1,152
Discounted price of 10 such items
= ₹11,520
Rate of GST = 5%
So, GST = 5% of ₹11520
= ₹ 576
Thus (c) is correct option.

THREE MARKS QUESTIONS

Main 2023

24. Mrs. Arora bought the following articles from a departmental store:

S. No.	Item	Price	Rate of GST	Discount
1.	Hair oil	₹1200	18%	₹100
2.	Cashew nuts	₹600	12%	-

Find the:
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Hair Oil:

GST = Rate of GST × Discounted price
= 18% × (1200 − 100)
= $\frac{18}{100} \times 1100$
= ₹198

Cashew Nuts :

GST = Rate of GST × Price
= 12% × 600
= $\frac{12}{100} \times 600$
= ₹72

Total GST paid = GST on hair oil + GST on cashew nuts
= 198 + 72
= ₹270

Total bill amount including GST
= Discounted price of hair oil + Price of cashew nuts + Total GST paid
= (1200 − 100) + 600 + 270
= 1100 + 600 + 270
= ₹1970

(a) Total GST Paid is ₹270
(b) Total bill amount including GST is ₹1970

Comp 2023

25. Amarjit purchased a washing machine from Rimpy Electronic Goods. The discount of 8% was given on the printed price of ₹42000. Rate of GST charged was 28%. Find the purchase price of

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washing machine. Also, find the amount of CGST and SGST shown in the tax invoice.

Sol :

Marked price of washing machine = ₹42000

$$\text{Discount} = 8\% \text{ of } 42000$$
$$= \frac{8}{100} \times 42000$$
$$= ₹3360$$

Taxable value of washing machine
$$= 42000 - 3360$$
$$= ₹38640$$
$$\text{Rate of GST} = 28\%$$
$$\text{Amount of GST} = \frac{28}{100} \times 38640$$
$$= ₹10819.20$$

Thus, purchase price of the washing maching
$$= 38640 + 10819.20$$
$$= ₹49459.20$$

Now, amount of CGST
$$= 14\% \text{ of } 38640$$
$$= \frac{14}{100} \times 38640$$
$$= ₹5409.60$$
$$\text{Amount of SGST} = 14\ \%\ 38640$$
$$= ₹5409.60$$

Main 2020

26. Mr. Bedi visits the market and buys the following articles.
Medicines costing ₹ 950, GST @ 5%
A pair of shoes costing ₹ 3000, GST @ 18%
A laptop bag costing ₹ 1000 with a discount of 30%, GST @ 18%.
(i) Calculate the total amount of GST paid.
(ii) The total bill amount including GST paid by Mr. Bedi.

Sol :

$$\text{GST} = \text{Rate of GST} \times \text{Price}$$

GST amount on Medicines
$$= 5\% \text{ of } ₹950$$
$$= ₹47.5$$

GST amount on a pair of shoes
$$= 18\% \text{ of } ₹3000$$
$$= ₹540$$

Price of bag after 30 % discount
$$= 70\ \%\ \text{of } ₹1000$$
$$= ₹700$$

GST amount on a laptop bag
$$= 18\% \text{ of } ₹700$$
$$= ₹126$$

(i) Total amount of GST paid
$$= 47.5 + 540 + 126$$
$$= ₹\ 713.5$$

(ii) The total bill amount including GST
$$= \text{Cost of Articles} + \text{Total GST}$$
$$= 950 + 3000 + 700 + 713.5$$
$$= ₹\ 5363.5$$

27. Riya bought the following articles from a departmental store :

S. No.	Item	Price	Rate of GST
1.	Fruit Juice	₹300	12%
2.	Coffee	₹1200	5%

Find the :
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Fruit Juice :
$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 12\% \times 300$$
$$= \frac{12}{100} \times 300$$
$$= ₹36$$

Coffee :
$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 5\% \times 1200$$
$$= \frac{5}{100} \times 1200$$
$$= ₹60$$

Total GST paid = GST on Fruit Juice + GST on Coffee
$$= 36 + 60$$
$$= ₹96$$

Total bill amount including GST
$$= \text{Price of Fruit Juice} + \text{Price of Coffee}$$

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$$+ \text{ Total GST paid}$$
$$= 300 + 1200 + 96$$
$$= ₹1596$$

(a) Total GST Paid is ₹96
(b) Total bill amount including GST is ₹1596

28. Taxable value of a shirt is ₹850. Rate of GST is 12%. What is the price of the leather belt ?

Sol :

Taxable value of leather belt = ₹850

$$\text{Rate of GST} = 12\%$$
$$\text{Amount of GST} = 12\% \text{ of } 850$$
$$= \frac{12}{100} \times 850$$
$$= ₹102$$

Thus, price of the leather belt for the customer
$$= \text{Taxable value} + \text{GST}$$
$$= 850 + 102$$
$$= ₹952$$

Hence, the price of the leather belt for the customer is ₹952.

29. Mrs. Roy went to a departmental store and bought the following items from a departmental store :

S. No.	Item	Price	Quantity	Rate of GST
1.	Walnut	₹750 Per Pack	2	5%
2.	Unbranded Atta	₹40 Per Kg.	20 Kg.	0%
3.	Butter	₹200 Per Pack	5	12%

Find the :
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Walnut :

Total Amount of Walnut Purchased
$$= \text{Price Per Pack} \times \text{Quantity}$$
$$= 750 \times 2 = ₹1500$$
$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 5\% \times 1500$$
$$= \frac{5}{100} \times 1500$$
$$= ₹75$$

Unbranded Atta :

Total Amount of Unbranded Atta Purchased
$$= \text{Price Per Kg.} \times \text{Quantity}$$
$$= 40 \times 20 = ₹800$$
$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 0\% \times 800$$
$$= \text{Nil}$$

Note :
GST is not Applicable on Unbranded Atta.

Butter :

Total Amount of Butter Purchased
$$= \text{Price Per Pack} \times \text{Quantity}$$
$$= 200 \times 5 = ₹1000$$
$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 12\% \times 1000$$
$$= \frac{12}{100} \times 1000$$
$$= ₹120$$

Total GST paid = GST on Walnut
$$+ \text{ GST on Unbranded Atta}$$
$$+ \text{ GST on Butter}$$
$$= 75 + 0 + 120$$
$$= ₹195$$

Total bill amount including GST
$$= \text{Price of Walnut}$$
$$+ \text{ Price of Unbranded Atta}$$
$$+ \text{ Price of Butter}$$
$$+ \text{ Total GST paid}$$
$$= 1500 + 800 + 1000 + 195$$
$$= ₹3495$$

(a) Total GST Paid is ₹195
(b) Total bill amount including GST is ₹3495

2023

30. The following bill shows the GST rates and the marked Price of articles :

S. No.	Articles	Marked Price	Rate of GST
1.	Graphic Card	₹15500	18%

NODIA

2.	Laptop Adapter	₹1900	28%

Find the :
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Graphic Card :

$$\begin{aligned}\text{GST} &= \text{GST Rate} \times \text{Marked Price}\\ &= 18\% \times 15500\\ &= \frac{18}{100} \times 15500\\ &= ₹2790\end{aligned}$$

Laptop Adapter :

$$\begin{aligned}\text{GST} &= \text{GST Rate} \times \text{Marked Price}\\ &= 28\% \times 1900\\ &= \frac{28}{100} \times 1900\\ &= ₹532\end{aligned}$$

$$\begin{aligned}\text{Total GST paid} &= \text{GST on Graphic Card}\\ &\quad + \text{GST on Laptop Adapter}\\ &= 2790 + 532\\ &= ₹3322\end{aligned}$$

$$\begin{aligned}\text{Total bill amount including GST} &= \text{Price of Graphic Card}\\ &\quad + \text{Price of Laptop Adapter}\\ &\quad + \text{Total GST paid}\\ &= 15500 + 1900 + 3322\\ &= ₹20722\end{aligned}$$

(a) Total GST Paid is ₹3322
(b) Total bill amount including GST is ₹20722

31. The taxable value of a shirt is ₹1200. Rate of GST is 5%. What is the price of the shirt ?

Sol :

$$\begin{aligned}\text{Taxable value of shirt} &= ₹1200\\ \text{Rate of GST} &= 5\%\\ \text{Amount of GST} &= 5\% \text{ of } 900\\ &= \frac{5}{100} \times 1200\\ &= ₹60\end{aligned}$$

Thus, price of the shirt for the customer

$$\begin{aligned}&= \text{Taxable value} + \text{GST}\\ &= 1200 + 60\\ &= ₹1260\end{aligned}$$

Hence, the price of the shirt for the customer is ₹1260.

32. Kiran bought the following electronic items from an electronics store:

S. No.	Item	Price	Rate of GST	Discount
1.	Air Conditioner	₹65000	28%	₹5000
2.	Refrigerator	₹30000	28%	-

Find the:
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Air Conditioner:

$$\begin{aligned}\text{GST} &= \text{GST Rate} \times \text{Discounted price}\\ &= 28\% \times (65000 - 5000)\\ &= \frac{28}{100} \times 60000\\ &= ₹16800\end{aligned}$$

Refrigerator :

$$\begin{aligned}\text{GST} &= \text{GST Rate} \times \text{Price}\\ &= 28\% \times 30000\\ &= \frac{28}{100} \times 30000\\ &= ₹8400\end{aligned}$$

$$\begin{aligned}\text{Total GST paid} &= \text{GST on Air Conditioner}\\ &\quad + \text{GST on Refrigerator}\\ &= 16800 + 8400\\ &= ₹25200\end{aligned}$$

$$\begin{aligned}\text{Total bill amount including GST} &= \text{Discounted price of Air Conditioner}\\ &\quad + \text{Price of Refrigerator}\\ &\quad + \text{Total GST paid}\\ &= (65000 - 5000) + 30000 + 25200\\ &= 60000 + 30000 + 25200\\ &= ₹115200\end{aligned}$$

(a) Total GST Paid is ₹25200
(b) Total bill amount including GST is ₹115200

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33. Prakash bought the following articles from a departmental store:

S. No.	Item	Price	Rate of GST	Discount
1.	Article A	₹4000	18%	5%
2.	Article B	₹2400	5%	5%

Find the:
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Article A :

Discount on Article A

$$= \frac{5}{100} \times 4000$$

$$= ₹200$$

$$\text{GST} = \text{GST Rate} \times \text{Discounted price}$$

$$= 18\% \times (4000 - 200)$$

$$= \frac{18}{100} \times 3800$$

$$= ₹684$$

Article B :

Discount on Article B

$$= \frac{5}{100} \times 2400$$

$$= ₹120$$

$$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$$

$$= 5\% \times (2400 - 120)$$

$$= \frac{5}{100} \times 2280$$

$$= ₹114$$

Total GST paid

$$= \text{GST on Article A} + \text{GST on Article B}$$

$$= 684 + 114$$

$$= ₹798$$

Total bill amount including GST

$$= \text{Discounted price of Article A} + \text{Discounted Price of Article B} + \text{Total GST paid}$$

$$= (4000 - 200) + (2400 - 120) + 798$$

$$= 3800 + 2280 + 798$$

$$= ₹6878$$

(a) Total GST Paid is ₹798
(b) Total bill amount including GST is ₹6878

34. Sulekha bought the following Grocery Items from M/s Shri Ram Trading Company :

S. No.	Item	Price	Quantity	Rate of GST	Discount
1.	Sugar	₹40/Kg.	5 Kg.	5%	-
2.	Tea	₹240/kg.	1 Kg.	5%	-
3.	Ghee	₹750/pack	2 Pack	12%	10%
4.	Toothpaste	₹50/pack	2 Pack	18%	-

Find the:
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Sugar :

Total Amount of Sugar Purchased

$$= \text{Price Per Kg.} \times \text{Quantity}$$

$$= 40 \times 5 = ₹200$$

$$\text{GST} = \text{GST Rate} \times \text{Total Price}$$

$$= 5\% \times 200$$

$$= \frac{5}{100} \times 200$$

$$= ₹10$$

Tea :

Total Amount of Tea Purchased

$$= \text{Price Per Kg.} \times \text{Quantity}$$

$$= 240 \times 1 = ₹240$$

$$\text{GST} = \text{GST Rate} \times \text{Total Price}$$

$$= 5\% \times 240$$

$$= \frac{5}{100} \times 240$$

$$= ₹12$$

Ghee :

Total Amount of Ghee Purchased

$$= \text{Price Per Pack} \times \text{Quantity}$$

$$= 750 \times 2 = ₹1500$$

Discount on Ghee

$$= \frac{10}{100} \times 1500$$

$$= ₹150$$

$$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$$

NODIA

$= 12\% \times (1500-150)$

$= \frac{12}{100} \times 1350$

$= ₹162$

Toothpaste :

Total Amount of Toothpaste Purchased

= Price Per Pack × Quantity

$= 50 \times 2 = ₹100$

GST = GST Rate × Total Price

$= 18\% \times 100$

$= \frac{18}{100} \times 100$

$= ₹18$

Total GST paid

= GST on Sugar + GST on Tea + GST on Ghee + GST on Toothpaste

$= 10 + 12 + 162 + 18$

$= ₹202$

Total bill amount including GST

= Total Price of Sugar + Total Price of Tea + Discounted Price of Ghee + Total Price of Toothpaste + Total GST paid

$= 200 + 240 + (1500 - 150) + 100 + 202$

$= 200 + 240 + 1350 + 100 + 202$

$= ₹2092$

(a) Total GST Paid is ₹202

(b) Total bill amount including GST is ₹2092

35. A computer mechanic in Delhi charges repairing cost from three different persons Amar, Akbar, and Anthony with certain discounts. The repairing costs and the corresponding discounts are as given below :

S. No.	Name of the Person	Repairing Cost	Discount
1.	Amar	₹11000	30%
2.	Akbar	₹12000	40%
3.	Anthony	₹9000	30%

If the rate of GST is 18%, Find the :

(a) Total GST paid.

(b) Total bill amount including GST paid by Akbar.

Sol :

Amar :

Discount on Repair

$= \frac{30}{100} \times 11000 = ₹3300$

GST = GST Rate × Discounted Repairing Cost

$= 18\% \times (11000 - 3300)$

$= \frac{18}{100} \times 7700 = ₹1386$

Akbar :

Discount on Repair

$= \frac{40}{100} \times 12000 = ₹4800$

GST = GST Rate × Discounted Repairing Cost

$= 18\% \times (12000 - 4800)$

$= \frac{18}{100} \times 7200 = ₹1296$

Anthony :

Discount on Repair

$= \frac{30}{100} \times 9000 = ₹2700$

GST = GST Rate × Discounted Repairing Cost

$= 18\% \times (9000 - 2700)$

$= \frac{18}{100} \times 6300 = ₹1134$

Total GST paid

= GST Paid by Amar + GST Paid by Akbar + GST Paid by Anthony

$= 1386 + 1296 + 1134$

$= ₹3816$

Total bill amount including GST paid by Akbar

= Discounted Repairing Cost + GST paid by Akbar

$= 7200 + 1296$

$= ₹8496$

(a) Total GST Paid is ₹3816

(b) Total bill amount including GST paid by Akbar is ₹8496

36. Jatin bought a Desktop Computer and other computer accessories from Satyam Computers, Delhi as given below :

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S. No.	Item	Price	Rate of GST	Discount
1.	Desktop	₹40000	18%	₹4000
2.	Mouse	₹800	18%	-
3.	Key Board	₹2400	18%	₹600
4.	Pen Drive	₹900	18%	-

Find the :
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Desktop :

$$\text{GST} = \text{GST Rate} \times \text{Discounted price}$$
$$= 18\% \times (40000 - 4000)$$
$$= \frac{18}{100} \times 36000$$
$$= ₹6480$$

Mouse :

$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 18\% \times 800$$
$$= \frac{18}{100} \times 800$$
$$= ₹144$$

Key Board :

$$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$$
$$= 18\% \times (2400 - 600)$$
$$= \frac{18}{100} \times 1800$$
$$= ₹324$$

Pen Drive :

$$\text{GST} = \text{GST Rate} \times \text{Price}$$
$$= 18\% \times 900$$
$$= \frac{18}{100} \times 900$$
$$= ₹162$$

Total GST paid

$$= \text{GST on Desktop} + \text{GST on Mouse} + \text{GST on Key Board} + \text{GST on Pen Drive}$$
$$= 6480 + 144 + 324 + 162$$
$$= ₹7110$$

Total bill amount including GST

$$= \text{Discounted price of Desktop} + \text{Price of Mouse} + \text{Discounted Price of Key Board} + \text{Price of Pen Drive} + \text{Total GST Paid}$$
$$= 36000 + 800 + 1800 + 900 + 7110$$
$$= ₹46610$$

(a) Total GST Paid is ₹7110
(b) Total bill amount including GST is ₹46610

37. Rishika has arranged a party on her birthday in an orphanage. She bought three different types of ice creams- Vanilla, Butterscotch and Strawberry as given below :

S. No.	Ice-cream	Price (Per Litre)	Quantity	Discount
1.	Vanilla	₹100	10 Litre	5%
2.	Butterscotch	₹150	10 Litre	20%
3.	Strawberry	₹150	10 Litre	20%

The GST Rate is 18%. Find the:
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Vanilla Ice-cream :

Total Amount of Vanilla Ice-cream Purchased

$$= \text{Price Per Litre} \times \text{Quantity}$$
$$= 100 \times 10 = ₹1000$$

Discount on Vanilla Ice-cream

$$= \frac{5}{100} \times 1000$$
$$= ₹50$$

$$\text{GST} = \text{GST Rate} \times \text{Discounted price}$$
$$= 18\% \times (1000 - 50)$$
$$= \frac{18}{100} \times 950$$
$$= ₹171$$

Butterscotch Ice-cream :

Total Amount of Butterscotch Ice-cream Purchased

$$= \text{Price Per Litre} \times \text{Quantity}$$
$$= 150 \times 10 = ₹1500$$

Discount on Butterscotch Ice-cream

$$= \frac{20}{100} \times 1500$$

NODIA

$= ₹300$

GST = GST Rate × Discounted Price

$= 18\% \times (1500 - 300)$

$= \frac{18}{100} \times 1200$

$= ₹216$

Strawberry Ice-cream :

Total Amount of Strawberry Ice-cream Purchased

= Price Per Litre × Quantity

$= 150 \times 10 = ₹1500$

Discount on Strawberry Ice-cream

$= \frac{20}{100} \times 1500$

$= ₹300$

GST = GST Rate × Discounted Price

$= 18\% \times (1500 - 300)$

$= \frac{18}{100} \times 1200$

$= ₹216$

Total GST paid

= GST on Vanilla

+ GST on Butterscotch

+ GST on Strawberry

$= 171 + 216 + 216$

$= ₹603$

Total bill amount including GST

= Discounted price of Vanilla

+ Discounted Price of Butterscotch

+ Discounted Price of Strawberry

+ Total GST Paid

$= 950 + 1200 + 1200 + 603$

$= ₹3953$

(a) Total GST Paid is ₹603

(b) Total bill amount including GST is ₹3953

38. Rate of GST for different categories are given below :

Category	**Rate of GST**
Toiletries	18%
Automobiles	28%
Dry Fruits	5%
Edible Oils	5%
Processed food	12%
Printed material	18%

Anuradha bought the following items from a departmental store :

S. No.	Item	Price
1.	Coconut oil	₹2400
2.	Soap	₹900
3.	Cheese	₹500

Find the :

(a) Total GST paid.

(b) Total bill amount including GST.

Sol :

On the basis of first table given in question, we will identify the product category and determine rate of GST :

S. No.	Item	Price	Rate of GST
1.	Coconut oil	₹2400	5%
2.	Soap	₹150	18%
3.	Cheese	₹500	12%

Now,

Coconut Oil :

GST = GST Rate × Price

$= 5\% \times 2400$

$= \frac{5}{100} \times 2400$

$= ₹120$

Soap :

GST = GST Rate × Price

$= 18\% \times 150$

$= \frac{18}{100} \times 150$

$= ₹27$

Cheese :

GST = GST Rate × Price

$= 12\% \times 500$

$= \frac{12}{100} \times 500$

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$= ₹60$

Total GST paid

$= \text{GST on Coconut oil} + \text{GST on Soap} + \text{GST on Cheese}$

$= 120 + 27 + 60$

$= ₹207$

Total bill amount including GST

$= \text{Price of Coconut oil} + \text{Price of Soap} + \text{Price of Cheese} + \text{Total GST paid}$

$= 2400 + 150 + 500 + 207$

$= ₹3257$

(a) Total GST Paid is ₹207

(b) Total bill amount including GST is ₹3257

39. Mr. Saxena recently shifted from Jaipur to Delhi. He bought the following articles to make his life comfortable here in Delhi :

S. No.	Item	Price	Discount
1.	Iphone	₹140000	-
2.	Sofa	₹120000	10%
3.	Coir Mats, Matting and Floor Covering	₹80000	-
4.	Car	₹600000	5%

Rate of GST for different categories are given below :

Category	**Rate of GST**
High-end Motorcycles	28%
Automobiles	28%
Wooden Furniture	18%
Mobile Phones	12%
Luxury and Sin Items	28%
Fabric	12%

Find the :

(a) Total GST paid.

(b) Total bill amount including GST.

Sol :

On the basis of second table given in question, we will identify the product category and determine rate of GST :

S. No.	Item	Price	Rate of GST	Discount
1.	Iphone	₹140000	12%	-
2.	Sofa	₹120000	18%	10%
3.	Coir Mats, Matting and Floor Covering	₹80000	5%	-
4.	Car	₹600000	28%	5%

Now,

Iphone :

$\text{GST} = \text{GST Rate} \times \text{Price}$

$= 12\% \times 140000$

$= \frac{12}{100} \times 140000$

$= ₹16800$

Sofa :

$\text{Discount on Sofa} = 10\% \text{ of } ₹120000$

$= \frac{10}{100} \times 120000$

$= ₹12000$

$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$

$= 18\% \times (120000 - 12000)$

$= \frac{18}{100} \times 108000$

$= ₹19440$

Coir Mats, Matting and Floor Covering :

$\text{GST} = \text{GST Rate} \times \text{Price}$

$= 5\% \times 80000$

$= \frac{5}{100} \times 80000$

$= ₹4000$

Car :

$\text{Discount on Car} = 5\% \text{ of } 600000$

$= \frac{5}{100} \times 600000$

$= ₹30000$

$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$

$= 28\% \times (600000 - 30000)$

$= \frac{28}{100} \times 570000$

$= ₹159600$

NODIA

Total GST paid = GST on Iphoner
+ GST on Sofa
+ GST on Coir Mats, Matting and Floor Covering
+ GST on Car
= 16800 + 19440 + 4000 + 159600
= ₹199840

Total bill amount including GST
= Price of Iphone
+ Discounted Price of Sofa
+ Price of Coir Mats, Matting and Floor Covering
+ Discounted Price of Car
+ Total GST paid
= 140000 + (120000 − 12000) + 80000
+ (600000 − 30000) + 199840
= 140000 + 108000 + 80000 + 570000
+ 199840
= ₹1097840

(a) Total GST Paid is ₹199840
(b) Total bill amount including GST is ₹1097840

40. Rate of GST for different categories are given below :

Category	**Rate of GST**
Toiletries & Cosmetics	18%
Automobiles	28%
Dry Fruits	5%
Edible Oils	5%
Processed food	12%
Spices	5%

Fill in the Blanks :

S. No.	Item	Price	Amount of GST (₹)
1.	Olive oil	₹1200	.....
2.	Shampoo	₹300	.....
3.	Raisin	₹1500	.....
4.	Bay Leaves	₹500	.....
5.	Cheese	₹500	.....
6.	Face wash	₹400	.....
7.	Car	₹500000	.....
8.	Bread	₹50	.....
9.	Saffron	₹2000	.....
10.	Cashew	₹1500	.....

Sol :

S. No.	Item	Price	Amount of GST (₹)
1.	Olive oil	₹1200	₹60
2.	Shampoo	₹300	₹54
3.	Raisin	₹1500	₹75
4.	Bay Leaves	₹500	₹25
5.	Cheese	₹1000	₹120
6.	Face wash	₹400	₹72
7.	Car	₹500000	₹140000
8.	Bread	₹50	₹6
9.	Saffron	₹2000	₹100
10.	Cashew	₹1500	₹75

Calculation of Amount of GST for every product is given below :

Olive oil : GST = GST Rate × Price
= 5% × 1200
= $\frac{5}{100} \times 1200$
= ₹60

Shampoo : GST = GST Rate × Price
= 18% × 300
= $\frac{18}{100} \times 300$
= ₹54

Raisin : GST = GST Rate × Price
= 5% × 1500
= $\frac{5}{100} \times 1500$
= ₹75

Bay Leaves : GST = GST Rate × Price
= 5% × 500
= $\frac{5}{100} \times 500$
= ₹25

Cheese : GST = GST Rate × Price
= 12% × 1000

NODIA

$$= \frac{12}{100} \times 1000$$
$$= ₹120$$

Face wash : $\text{GST} = \text{GST Rate} \times \text{Price}$
$$= 18\% \times 400$$
$$= \frac{18}{100} \times 400$$
$$= ₹72$$

Car : $\text{GST} = \text{GST Rate} \times \text{Price}$
$$= 28\% \times 500000$$
$$= \frac{28}{100} \times 500000$$
$$= ₹140000$$

Bread : $\text{GST} = \text{GST Rate} \times \text{Price}$
$$= 12\% \times 50$$
$$= \frac{12}{100} \times 50$$
$$= ₹6$$

Saffron : $\text{GST} = \text{GST Rate} \times \text{Price}$
$$= 5\% \times 2000$$
$$= \frac{5}{100} \times 2000$$
$$= ₹100$$

Cashew : $\text{GST} = \text{GST Rate} \times \text{Price}$
$$= 5\% \times 1500$$
$$= \frac{5}{100} \times 1500$$
$$= ₹75$$

41. Ankit bought two items worth ₹4500 and ₹2000 at 20% and 10% discount respectively. If the rate of GST is 18%, then calculate the amount of bill for this intra-state transaction.

Sol :

Discounted price of first item at 20 % Discount
$$= \left(1 - \frac{20}{100}\right) \times 4500$$
$$= \frac{80}{100} \times 4500$$
$$= ₹3600$$

Discounted price of second item at 10 % Discount
$$= \left(1 - \frac{10}{100}\right) \times 2000$$
$$= \frac{90}{100} \times 2000$$
$$= ₹1800$$

Total price of two items
$$= 3600 + 1800$$
$$= ₹5400$$

$\text{Total Amount} = \text{Total Price} + \text{GST}$
$$= 5400 + 18\% \text{ of } ₹5400$$
$$= 5400 + 972$$
$$= ₹6372$$

42. Rate of GST for different categories are given below :

Category	**Rate of GST**
Cosmetics	18%
Automobiles	28%
Dry Fruits	5%
Edible Oils	5%
Processed food	12%
Spices	5%

(a) Mrs. Sharma bought a pack of face cream for ₹500 at 20% discount. Calculate amount of GST paid by Mrs. Sharma.

(b) On Jiya's birthday his father bought a cake and some biscuits for Rs.1800 at a discount of Rs 300. Calculate amount of GST paid by Jiya's father.

(c) Mrs. Mathur bought turmeric (curcuma) for Rs.1200 at 10% discount. Calculate amount of GST paid by Mrs. Mathur.

Sol :

(a) Face cream comes under the category of Cosmetics. Rate of GST on Cosmetics is 18%.
Discount on Pack of Face Cream
$$= 20\% \text{ of } 500$$
$$= \frac{20}{100} \times 500$$
$$= ₹100$$
$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$
$$= 18\% \times (500 - 100)$$
$$= \frac{18}{100} \times 400$$
$$= ₹72$$

(b) Cake and Biscuits comes under the category of Processed food. Rate of GST on Processed food is 12%.
Discount on Cake and Biscuits = ₹300
$\text{GST} = \text{GST Rate} \times \text{Discounted Price}$
$$= 12\% \times (1800 - 300)$$
$$= \frac{12}{100} \times 1500$$

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$= ₹180$

(c) Turmeric (curcuma) comes under the category of Spices. Rate of GST on Spices is 5%.
Discount on Turmeric (curcuma)

$= 10\%$ of 1200

$= \frac{10}{100} \times 1200$

$= ₹120$

GST $=$ GST Rate $\times$ Discounted Price

$= 5\% \times (1200 - 120)$

$= \frac{5}{100} \times 1080$

$= ₹54$

Note :
Turmeric (curcuma) other than fresh turmeric, ginger other than fresh ginger, saffron, thyme, bay leaves, curry and other spices attract 5% GST rate.

43. Rate of GST for different categories are given below :

Category	**Rate of GST**
Pharmaceutical Preparations	18%
Life Saving Drugs	5%
Toiletries	18%
Wooden Furniture	18%
Mobile Phones	12%
Consumer Durables	28%

Dr. Anurag is opening a new hospital. He bought the following items from different sellers given in table :

S. No.	Item	Price (Per Unit)	Quantity
1.	Insulin	₹600	50
2.	Hospital Beds	₹10000	10
3.	Air Conditioners	₹40000	10
4.	Reception Table	₹25000	1

Find the :
(a) Combined GST paid by Dr. Anurag.
(b) Combined bill amount for all items including GST.

Sol :

Insulin :

Insulin comes under the category of Life Saving Drugs. Rate of GST on Life Saving Drugs is 5%.

Total Amount of Insulin Purchased

$=$ Price Per Unit $\times$ Quantity

$= 600 \times 50 = ₹30000$

GST $=$ GST Rate $\times$ Price

$= 5\% \times 30000$

$= \frac{5}{100} \times 30000$

$= ₹1500$

Hospital Beds :

Hospital Beds comes under the category of Pharmaceutical Preparations. Rate of GST on Pharmaceutical Preparations is 18%.

Total Amount of Hospital Beds Purchased

$=$ Price Per Unit $\times$ Quantity

$= 10000 \times 10 = ₹100000$

GST $=$ GST Rate $\times$ Price

$= 18\% \times 100000$

$= \frac{18}{100} \times 100000$

$= ₹18000$

Air Conditioners :

Air Conditioners comes under the category of Consumer Durables. Rate of GST on Consumer Durables is 28%.

Total Amount of Air Conditioners Purchased

$=$ Price Per Unit $\times$ Quantity

$= 40000 \times 10 = ₹400000$

GST $=$ GST Rate $\times$ Price

$= 28\% \times 400000$

$= \frac{28}{100} \times 400000$

$= ₹112000$

Reception Table :

Reception Table comes under the category of Wooden Furniture. Rate of GST on Wooden Furniture is 18%.

GST $=$ GST Rate $\times$ Price

$= 18\% \times 25000$

$= \frac{18}{100} \times 25000$

$= ₹4500$

NODIA

Combined GST paid

= GST on Insulin
+ GST on Hospital Beds
+ GST on Air Conditioners
+ GST on Reception Table
= 1500 + 18000 + 112000 + 4500
= ₹136000

Combined bill amount including GST

= Price of Insulin
+ Price of Hospital Beds
+ Price of Air Conditioners
+ Price of Reception Table
+ Combined GST paid
= 30000 + 100000 + 400000 + 25000 + 136000
= ₹691000

(a) Total GST Paid is ₹136000
(b) Total bill amount including GST is ₹691000

44. Swaroop went to a mobile selling shop. He bought the following items given in table :

S. No.	Item	Price	Rate of GST	Discount
1.	Mobile	₹24000	12%	10%
2.	Memory Card	₹1000	18%	5%
3.	Charger	₹1200	18%	-

Find the :
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Mobile :

Discount on Mobile

= 10% of 24000
$= \frac{10}{100} \times 24000$
=₹2400

GST = GST Rate × Discounted Price
= 12% × (24000 − 2400)
$= \frac{12}{100} \times 21600$
= ₹2592

Memory Card :

Discount on Memory Card

= 5% of 1000
$= \frac{5}{100} \times 1000$
=₹50

GST = GST Rate × Discounted Price
= 18% × (1000 − 50)
$= \frac{18}{100} \times 950$
= ₹171

Charger :

GST = GST Rate × Price
= 18% × 1200
$= \frac{18}{100} \times 1200$
= ₹216

Total GST paid

= GST Paid on Mobile
+ GST Paid on Memory Card
+ GST Paid on Charger
= 2592 + 171 + 216
= ₹2979

Total bill amount including GST

= Discounted Price of Mobile
+ Discounted Price of Memory Card
+ Price of Charger
+ Total GST paid
= 21600 + 950 + 1200 + 2979
= ₹26729

(a) Total GST Paid is ₹2979
(b) Total bill amount including GST is ₹26729

45. Rate of GST for different categories are given below :

Category	**Rate of GST**
Cosmetics	18%
Fresh Vegetables	Nil
Dry Fruits	5%
Edible Oils	5%
Processed food	12%
Spices	5%

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Sudha bought the following items from market :

S. No.	Item	Price	Quantity
1.	Curry	₹120 Per Pack	2 Pack
2.	Potatoes	₹40 Per Kg.	10 Kg.
3.	Soft Drink	₹100 Per Bottle	5 Bottles

Find the :
(a) Total GST paid.
(b) Total bill amount including GST.

Sol :

Curry :

Curry comes under the category of Spices. Rate of GST on Spices is 5%.

Total Amount of Curry Purchased

$$= \text{Price Per Pack} \times \text{Quantity}$$

$$= 120 \times 2 = ₹240$$

$$\text{GST} = \text{GST Rate} \times \text{Price}$$

$$= 5\% \times 240$$

$$= \frac{5}{100} \times 240$$

$$= ₹12$$

Potatoes :

Potatoes comes under the category of Fresh Vegetables. Rate of GST on Fresh Vegetables is 0%.

Total Amount of Potatoes Purchased

$$= \text{Price Per Kg.} \times \text{Quantity}$$

$$= 40 \times 10 = ₹400$$

$$\text{GST} = \text{GST Rate} \times \text{Price}$$

$$= 0\% \times 400$$

$$= \text{Nil}$$

Soft Drink :

Soft Drink comes under the category of Processed Food. Rate of GST on Processed Food is 12%.

Total Amount of Soft Drink Purchased

$$= \text{Price Per Bottle} \times \text{Quantity}$$

$$= 100 \times 5 = ₹500$$

$$\text{GST} = \text{GST Rate} \times \text{Price}$$

$$= 12\% \times 500$$

$$= \frac{12}{100} \times 500$$

$$= ₹60$$

Total GST paid = GST on Curry + GST on Potatoes + GST on Soft Drink

$$= 12 + 0 + 60$$

$$= ₹72$$

Total bill amount including GST

= Price of Curry + Price of Potatoes + Price of Soft Drink + Total GST paid

$$= 240 + 400 + 500 + 72$$

$$= ₹1212$$

(a) Total GST Paid is ₹72
(b) Total bill amount including GST is ₹1212

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46. M/s Sunny Paints sold 4 tins of palm oils with taxable value of each tin ₹3400. If the rate of GST is 28%, then find the amount of CGST as well as SGST charged in the tax invoice.

Sol :

Here, total taxable value of 4 tins

$$= 3400 \times 4$$

$$= ₹13600$$

$$\text{Rate of GST} = 28\%$$

$$\text{Rate of CGST} = 14\%$$

$$\text{Rate of SGST} = 14\%$$

Amount of CGST charged,

$$= 14\% \text{ of } 13600$$

$$= \frac{14}{100} \times 13600$$

$$= ₹1904$$

Amount of SGST charged,

$$= 14\% \text{ of } 13600$$

$$= ₹1904$$

Hence, the amount of CGST charged is ₹1904 and the amount of SGST charged is ₹1904 in the tax invoice.

47. The taxable value of a leather belt is ₹900. Rate of GST is 18%. What is the price of the leather belt ?

Sol :

Taxable value of leather belt = ₹900

Rate of GST = 18%

Amount of GST = 18% of 900

$= \frac{18}{100} \times 900$

= ₹162

Thus, price of the leather belt for the customer

= Taxable value + GST

= 900 + 162

= ₹1062

Hence, the price of the leather belt for the customer is ₹1062.

48. The manufacturing cost of a wooden art piece is ₹1200. The manufacturer marked up the price of his items by 60% and then sell them at a discount of 10%. Rate of GST rate on this item is 5%. If a consumer buy 10 such art piece, then calculate the GST paid by consumer.

Sol :

C.P of an item = ₹ 1200

Marked price of an item

= ₹ 1200 + 60% of ₹1200

= ₹ 1,920

Discounted price of an item

= ₹1,920 − 10% of ₹ 1,920

= ₹1,728

Discounted price of 10 such items

= ₹17,280

Rate of GST = 5%

So, GST = 5% of ₹17280

= ₹ 864

49. RK Sharma purchased a electric bike from the same city worth ₹30,000 for his son at a discount of 20 %. Insurance premium cover of the bike is 10% of the discounted value. Accessories worth ₹8,000 are purchased at 10% discount. Rate of GST on the bike, insurance premium and the accessories are 28%, 18% and 12% respectively. What is the GST paid by Mr. Sharma to the Central Government?

Sol :

Marked price of bike = ₹ 30,000

Discount = 20%

Discounted price = 30,000 − 20% of 30,000

= ₹ 24,000

Insurance premium cover is 10 % of discounted price, thus

Premium Amount = 10% of 24,000

= ₹ 2,400

and Discounted price of accessories

= 8,000 − 10% of 8,000

= ₹ 7,200

Rate of GST on the bike = 28%

CGST = SGST = 14% of ₹ 24,000

= ₹ 3,360

Rate of GST on insurance premium = 18%

CGST = SGST = 9% of ₹ 2,400

= ₹ 216

Rate of GST on accessories = 12%

CGST = SGST = 6% of ₹ 7,200

= ₹ 432

So, total CGST paid by RK Sharma to the central Government

= ₹ 3,360 + ₹ 216 + ₹ 432

= ₹ 4,008

50. Manpreet Gas Agency in Union Territory purchased some gas cylinder for industrial use for ₹49,000 and sold them to the local customer for ₹54000. Find the GST to be paid at the rate of 5% and hence the CGST as well as UGST to be paid for this transaction.

Sol :

Tax paid at the time of purchase by Manpreet Gas Agency

= 5% of 49,000

$= \frac{5}{100} \times 49000$

= ₹2450

Tax paid at the time of sale

= 5% of 54,000

$= \frac{5}{100} \times 54000$

= ₹2700

Thus, GST payable = 2700 − 2450

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$= ₹250$

Hence, GST payable by Manpreet Gas Agency is ₹250.

$$\text{CGST payable} = \frac{250}{2} = ₹125$$

$$\text{UGST payable} = \frac{250}{2} = ₹125$$

NOTE : For Union Territories there is UGST instead of SGST.

51. M/s Earth Electrical marked up the price of an electrical equipment 20% above the cost price and offers two successive discounts of 10% and 5% on the marked price. If the rate of GST is 18% and a consumer pays CGST of ₹9,234, then calculate the cost price of the electrical equipment.

Sol :

Let x be the listed price of the goods.

$$\text{So, marked price} = ₹\, x + 20\% \text{ of } ₹\, x$$

$$= ₹\, \frac{6}{5}x$$

$$\text{Discounted price} = \frac{6}{5}x\left(1 - \frac{10}{100}\right)\left(1 - \frac{5}{100}\right)$$

$$= \frac{513}{500}x$$

$$\text{GST Rate} = 18\%$$

$$\text{Given} \quad \text{CGST} = 9\% \text{ of } \frac{513}{500}x$$

$$= \frac{9}{100} \times \frac{513}{500}x$$

But given CGST is ₹ 9,234, thus

$$\frac{9}{100} \times \frac{513}{500}x = 9,234$$

$$x = \frac{9,234 \times 100 \times 500}{9 \times 513}$$

$$= 100,000.$$

52. Ramawatar bought three items worth ₹400, ₹500 and ₹700 at discounts 12%, 15% and 20% respectively from a city shop. If the rate of GST is 18% on all item, calculate the amount of bill.

Sol :

Discounted price of first item at 12 % discount

$= ₹400 - 12\%$ of ₹400

$= ₹352$ at 15 % discount

Discounted price of second item

$= ₹500 - 15\%$ of ₹500

$= ₹425$

Discounted price of third item at 20 % discount

$= ₹700 - 20\%$ of ₹700

$= ₹560$

Total price of three items,

$= ₹(352 + 425 + 560)$

$= ₹1,337$

$\text{GST} = 18\%$ of ₹1337

$= ₹240.66$

Total bill = Total price +GST

$= ₹1,337 + ₹240.66$

$= ₹1,577.66$

END

NODIA

CHAPTER 2

BANKING

MULTIPLE CHOICE QUESTIONS

Main 2023

1. Naveen deposits ₹800 every month in a recurring deposit account for 6 months. If he receives ₹4884 at the time of maturity, then the interest he earns is:

(a) ₹84 (b) ₹42
(c) ₹24 (d) ₹284

Sol :

Monthly deposit $p = ₹\ 800$

Period of deposit $n = 6$ months

Maturity value $\text{MV} = ₹\ 4884$

Now $\text{MV} = P \times n + I$

$4884 = 800 \times 6 + I$

$I = 4884 - 4800$

$= ₹84$

Thus (a) is correct option.

Comp 2023

2. Manish deposited ₹1500 per month in a recurring deposit account for 2 years. What is the amount payable to him on maturity if the rate of interest is 8% per annum.

(a) ₹3,900 (b) ₹4,200
(c) ₹4,500 (d) ₹5,000

Sol :

Monthly deposit $p = ₹\ 1500$

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 2$ years

$= 24$ months

Maturity value

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 1500 \times 24 + 1500 \times \frac{24 \times 25}{24} \times \frac{8}{100}$$

$$= 36,000 + 3000$$

$$= ₹\ 39,000$$

Thus (a) is correct option.

Main 2021 Sem I

3. Mohit opened a recurring deposit account in a bank for 2 years. He deposit ₹1,000 every month and receives 25,500 on maturity. The interest he earned in 2 years is :

(a) ₹13,500 (b) ₹3,000
(c) ₹24,000 (d) ₹1,500

Sol :

Monthly deposit $p = ₹\ 1000$

Period of deposit $n = 2$ years

$= 24$ months

Maturity value $\text{MV} = ₹\ 25,500$

Now $\text{MV} = P \times n + I$

$25500 = 1000 \times 24 + I$

$I = 25500 - 24000$

$= ₹1,500$

Thus (d) is correct option.

Main 2021 Sem I

4. A man deposit ₹500 per month for 6 months and received ₹3,300 as the maturity value. The interest received by him is :

(a) ₹1,950 (b) ₹300
(c) ₹2,800 (d) None of these

Sol :

Monthly deposit $p = ₹\ 500$

Period of deposit $n = 6$ months

Maturity value $\text{MV} = ₹\ 3300$

Now $\text{MV} = P \times n + I$

$3300 = 500 \times 6 + I$

$I = 3300 - 3000$

$= ₹300$

Thus (b) is correct option.

5. Reena deposited ₹1000 per month for 12 months in a Paytm Bank's recurring deposit account. If

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the bank pays interest at a rate of 9% per annum, then the total amount deposited by Reena during this period is:
(a) ₹14,000 (b) ₹21,400
(c) ₹18,000 (d) ₹12,000

Sol :

Monthly deposit $p =$₹ 1000

Period of deposit $n = 2$ years
$= 24$ months

Total amount deposited $P = p \times n$
$= 1000 \times 12$
$=$ ₹ 1,200.

Thus (d) is correct option.

6. Naresh deposited ₹400 per month for 15 months in Federal bank's recurring deposit account. If the bank pays interest at a rate of 10% per annum, then the interest earned by Naresh during this period is:
(a) ₹350 (b) ₹250
(c) ₹400 (d) ₹300

Sol :

Monthly deposit $p =$₹ 400

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 15$ months

Interest $I = p \times \dfrac{n(n+1)}{2 \times 12} \times \dfrac{r}{100}$

$= 400 \times \dfrac{15 \times 16}{24} \times \dfrac{10}{100}$

$=$₹ 400

Thus (c) is correct option.

7. Gaurishakar opened a R.D. account in PNB Bank for 20 months. If the rate of interest is 9% per annum and he received ₹441 as interest at the end of maturity, then the monthly installment is
(a) ₹280 (b) ₹250
(c) ₹200 (d) ₹320

Sol :

Rate of simple interest $r = 9\,\%$ p.a.

Period of deposit $n = 20$ months

Total interest $I =$ ₹ 441

Let p be the monthly installment then we have

Interest $I = p \times \dfrac{n(n+1)}{2 \times 12} \times \dfrac{r}{100}$

$$441 = p \times \frac{20 \times 21}{24} \times \frac{9}{100}$$

$$441 = p \times \frac{1 \times 7}{8} \times \frac{9}{5}$$

$$441 = \frac{63p}{40}$$

$$p = \frac{441 \times 40}{63}$$

$$= ₹\,280$$

Thus (a) is correct option.

8. Eesha deposited ₹200 per month in a R.D. account of Canara Bank for 3 years. If the bank pays an interest of 11% per annum, then the matured value of this account is_____.
(a) ₹ 8,421 (b) ₹8,124
(c) ₹8,412 (d) ₹8,214

Sol :

(a) ₹8,421

Monthly deposit $p =$₹ 200

Period of deposit $n = 3$ years
$= 36$ months

Rate of simple interest $r = 11\,\%$ p.a.

Matured value (MV)

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 200 \times 36 + 200 \times \frac{36 \times 37}{24} \times \frac{11}{100}$$

$$= 7,200 + 2 \times \frac{3 \times 37}{2} \times \frac{11}{1}$$

$$= 7,200 + 3 \times 37 \times 11$$

$$= 7,200 + 1,221$$

$$= ₹\,8,421$$

Thus (a) is correct option.

9. Mr. Verma is having a R.D. account at Self Help Group Society. He deposited ₹2,500 per month for 2 years. At the time of maturity, he got ₹67,500. The total interest earned by him during this period is
(a) ₹8,500 (b) ₹8,000
(c) ₹7,000 (d) ₹7,500

Sol :

Monthly deposit $p =$₹ 2,500

Period of deposit $n = 2$ years

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$= 24$ months

Maturity value $\text{MV} = ₹\,67,500$

Total amount deposited $P = p \times n$

$= 2,500 \times 24$

$= ₹\,60,000.$

Interest received $I = \text{MV} - p \times n$

$= 67,500 - 60,000$

$= ₹\,7,500$

Thus (d) is correct option.

10. Raghav deposited ₹800 per month in a cumulative deposit account of Bank of India for 5 years. If the bank pays interest at a rate of 6% per annum, the amount payable to him at the time of maturity is
(a) ₹53,250 (b) ₹55,320
(c) ₹52,350 (d) ₹53,520

Sol :

Monthly deposit $p =₹\,800$

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 5$ years

$= 60$ months

Maturity value

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 800 \times 60 + 800 \times \frac{60 \times 61}{24} \times \frac{6}{100}$$

$$= 48,000 + 7320$$

$$= ₹\,55,320$$

Thus (b) is correct option.

11. Reena deposited ₹1000 per month for 12 months in a Paytm Bank's recurring deposit account. If the bank pays interest at a rate of 9% per annum, then the total amount deposited by Reena during this period is:
(a) ₹16,500 (b) ₹22,400
(c) ₹18,400 (d) ₹12,000

Sol :

Monthly deposit $p =₹\,1000$

Period of deposit $n = 2$ years

$= 24$ months

Total amount deposited $P = p \times n$

$= 1000 \times 12$

$= ₹\,1,200.$

Thus (d) is correct option.

12. Manish deposited ₹1500 per month in a recurring deposit account for 2 years. What is the amount payable to him on maturity if the rate of interest is 8% per annum.
(a) ₹4,600 (b) ₹3,900
(c) ₹4,500 (d) ₹4,200

Sol :

Monthly deposit $p =₹\,1500$

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 2$ years

$= 24$ months

Maturity value

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 1500 \times 24 + 1500 \times \frac{24 \times 25}{24} \times \frac{8}{100}$$

$$= 36,000 + 3000$$

$$= ₹\,39,000$$

Thus (b) is correct option.

13. Raghav deposited ₹400 per month in a recurring deposit account for 18 months. If the rate of interest is 9% per annum, then the interest earned by him during this period is:
(a) ₹3,856.50 (b) ₹3,343.50
(c) ₹330 (d) ₹256.50

Sol :

Monthly deposit $p =₹\,400$

Rate of simple interest $r = 9\,\%$ p.a.

Period of deposit $n = 18$ months

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 400 \times \frac{18 \times 19}{24} \times \frac{9}{100}$$

$$= 4 \times \frac{18 \times 19}{24} \times \frac{9}{1}$$

$$= 1 \times \frac{18 \times 19}{6} \times \frac{9}{1}$$

$$= 3 \times 19 \times 9$$

$$= ₹\,513$$

Thus (d) is correct option.

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14. Vimla had a recurring deposit account in ICICI bank and deposited ₹1000 per month for $2\frac{1}{2}$ years. If the rate of interest was 10% p.a., then the matured value of this account is:
(a) ₹32,775 (b) ₹33,875
(c) ₹23,775 (d) ₹34,975

Sol :

Monthly deposit $p =$₹ 1000

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 2\frac{1}{2}$ years
$= 30$ months

Maturity value,

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 1000 \times 30 + 1000 \times \frac{30 \times 31}{24} \times \frac{10}{100}$$
$$= 30{,}000 + 10 \times \frac{5 \times 31}{4} \times \frac{10}{1}$$
$$= 30{,}000 + 5 \times 5 \times 31 \times 5$$
$$= 30{,}000 + 3{,}875$$
$$= ₹\,33{,}875$$

Thus (b) is correct option.

15. Manish deposited ₹1500 per month in a recurring deposit account for 2 years. What is the amount payable to him on maturity if the rate of interest is 8% per annum.
(a) ₹4,500 (b) ₹4,200
(c) ₹3,800 (d) ₹5,000

Sol :

Monthly deposit $p =$₹ 1500

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 2$ years
$= 24$ months

Maturity value

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 1500 \times 24 + 1500 \times \frac{24 \times 25}{24} \times \frac{8}{100}$$
$$= 36{,}000 + 3000$$
$$= ₹\,39{,}000$$

Thus (c) is correct option.

16. Lavanya deposited ₹2000 per month for 24 months in HDFC bank's recurring deposit account. If the bank pays an interest of 10% p.a., then the amount she gets on maturity is:
(a) ₹48,000 (b) ₹45,000
(c) ₹53,000 (d) ₹5,000

Sol :

Monthly deposit $p =$₹ 1000

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 2$ years
$= 24$ months

Maturity value

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 2000 \times 24 + 2000 \times \frac{24 \times 25}{24} \times \frac{10}{100}$$
$$= 48{,}000 + 20 \times \frac{1 \times 25}{1} \times \frac{10}{1}$$
$$= 48{,}000 + 5000$$
$$= 53{,}000$$

Thus (d) is correct option.

17. Mrs. Agrawal deposits ₹1,000 every month in a recurring deposit account for 3 years at a nationalized bank. The rate of interest is 8% interest per annum. The matured value is
(a) ₹40,440 (b) ₹48,000
(c) ₹44,400 (d) ₹48,040

Sol :

(a) ₹40,440

Monthly deposit $p =$₹ 2000

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 3$ years
$= 36$ months

Maturity value

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 1000 \times 36 + 1000 \times \frac{36 \times 37}{24} \times \frac{8}{100}$$
$$= 36{,}000 + 10 \times \frac{36 \times 37}{24} \times \frac{8}{1}$$
$$= 36{,}000 + 10 \times \frac{36 \times 37}{3} \times \frac{1}{1}$$
$$= 36{,}000 + 10 \times \frac{12 \times 37}{1} \times \frac{1}{1}$$
$$= 36{,}000 + 4{,}440$$
$$= ₹\,40{,}440$$

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Thus () is correct option.

18. Lokesh deposits ₹800 per month in a cumulative deposit account for six years. What is the amount payable to him on maturity, if the rate of interest is 6% per annum.
(a) ₹6,1185 (b) ₹6,8182
(c) ₹6,8112 (d) ₹6,8185

Sol :

Monthly deposit $p = ₹\ 800$

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 6$ years
$= 72$ months

Maturity value

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 800 \times 72 + 800 \times \frac{72 \times 73}{24} \times \frac{6}{100}$$
$$= 57,600 + 8 \times \frac{3 \times 73}{1} \times \frac{6}{1}$$
$$= 57,600 + 10,512$$
$$= ₹\,68,112$$

Thus (c) is correct option.

19. Varsha deposited ₹360 per month in a cumulative time deposit account with PNB for 2 years. If the rate of interest is 7% per annum, then the amount she get at the time of maturity is:
(a) ₹2,790 (b) ₹9,720
(c) ₹9,270 (d) ₹7,290

Sol :

(c) ₹9,270

Monthly deposit $p = ₹\ 360$

Rate of simple interest $r = 7\,\%$ p.a.

Period of deposit $n = 2$ years
$= 24$ months

Maturity value

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 360 \times 24 + 360 \times \frac{24 \times 25}{24} \times \frac{7}{100}$$
$$= 8640 + 630$$
$$= ₹\,9,270$$

20. Mr. Nair get ₹6,455 at the end of one year at the rate of 14% per annum in a recurring deposit amount. The monthly installment is:
(a) ₹400 (b) ₹500
(c) ₹600 (d) ₹700

Sol :

(b) ₹500

Maturity value $MV = ₹\ 6,455$

Rate of simple interest $r = 14\,\%$ p.a.

Period of deposit $n = 1$ years
$= 12$ months

Let p be the monthly installment then we have

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$6,455 = p \times 12 + p \times \frac{12 \times 13}{24} \times \frac{14}{100}$$
$$6,455 = 12p + p \times \frac{1 \times 13}{2} \times \frac{14}{100}$$
$$6,455 = 12p + p \times \frac{13}{1} \times \frac{7}{100}$$
$$6,455 = 12p + 0.91p$$
$$p = \frac{6455}{12.91} = 500$$

The monthly installment is ₹ 500.
Thus (b) is correct option.

21. Tania deposited some money per month for $1\frac{1}{2}$ years at 9% per annum in Bajaj Finance Company. If she gets ₹15,426 at the time of maturity, then the amount invested by her per month is
(a) ₹1100 (b) ₹900
(c) ₹1,000 (d) ₹800

Sol :

Maturity value $MV = ₹\,15,426$

Rate of simple interest $r = 9\,\%$ p.a.

Period of deposit $n = 1\frac{1}{2}$ years
$= 18$ months

Let p be the monthly installment then we have

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$15,426 = p \times 18 + p \times \frac{18 \times 19}{24} \times \frac{9}{100}$$
$$857 = p + p \times \frac{19}{24} \times \frac{9}{100}$$
$$857 = p + p \times \frac{19}{8} \times \frac{3}{100}$$
$$857 = p + \frac{57p}{800}$$

NODIA

$$857 = p + \frac{57p}{800}$$

$$685600 = 800p + 57p$$

$$685600 = 857p$$

$$p = \frac{685600}{857} = 800$$

The amount invested per month is ₹ 800
Thus (d) is correct option.

22. SBI bank offered a scheme of investing ₹ x per month for 2 years. If the rate of interest offered by the bank is 10% p.a. and the total interest received will be ₹1,900, then the value of x is:
(a) ₹700 (b) ₹750
(c) ₹760 (d) ₹800

Sol :

Monthly deposit $p = x$

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 2$ years
$= 24$ months

Total interest $I =$₹ 1,900

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$1{,}900 = x \times \frac{24 \times 25}{24} \times \frac{10}{100}$$

$$1{,}900 = x \times \frac{1 \times 25}{1} \times \frac{1}{10}$$

$$1{,}900 = x \times \frac{1 \times 25}{1} \times \frac{1}{10}$$

$$1{,}900 = 2.5\,x$$

$$x = 760$$

Thus (c) is correct option.

23. David opened a Recurring Deposit Account in a Indusind bank and deposited ₹300 per month for two years. If he received ₹7,725 at the time of maturity, then the rate of interest per annum is:
(a) 8% (b) 7.5%
(c) 7% (d) 8.4%

Sol :

Monthly deposit $p =$₹ 300

Period of deposit $n = 2$ years
$= 24$ months

Maturity value MV = ₹ 7,725

Let r be the rate of interest then we have

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$7{,}725 = 300 \times 24 + 300 \times \frac{24 \times 25}{24} \times \frac{r}{100}$$

$$7{,}725 = 7{,}200 + 75r$$

$$525 = 75r$$

$$r = 7$$

The rate of interest is 7% p.a.
Thus (c) is correct option.

24. Leela deposited ₹350 per month in a R.D. account for $1\frac{1}{4}$ years at co-operative bank. If the matured value of this account is ₹5,565, then the interest received is:
(a) ₹350 (b) ₹305
(c) ₹385 (d) ₹315

Sol :

Monthly deposit $p =$₹ 350

Period of deposit $n = 1\frac{1}{4}$ years
$= 15$ months

Maturity value MV = ₹ 5,565

Interest received $I = \text{MV} - p \times n$
$= 5{,}565 - 350 \times 15$
$= 5{,}565 - 5{,}250$
$=$ ₹ 315

Thus (d) is correct option.

25. Karina opened a R.D. account with RBL bank for a period of two years. If the bank pays interest at the rate of 6% per annum and the monthly installment is ₹1,000, then the interest earned in one year is:
(a) ₹390 (b) ₹360
(c) ₹450 (d) ₹500

Sol :

Monthly deposit $p =$₹ 1000

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 2$ years
$= 24$ months

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 1{,}000 \times \frac{12 \times 13}{24} \times \frac{6}{100}$$

$=$ ₹ 390

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Thus (a) is correct option.

26. Mr. Khana has a four-year cumulative time deposit account in ICICI Bank and deposits ₹650 per month. If he receives ₹36,296 at the time of maturity, then the rate of interest is
(a) 11% (b) 8%
(c) 9% (d) 10%

Sol :

Monthly deposit $p = ₹\ 650$

Period of deposit $n = 4$ years
$= 48$ months

Maturity value $MV = ₹\ 36,296$

Let r be the rate of interest then we have

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$36,296 = 650 \times 48 + 650 \times \frac{48(48+1)}{24} \times \frac{r}{100}$$

$$1396 = 25 \times 48 + 25 \times \frac{2 \times 49}{1} \times \frac{r}{100}$$

$$1396 = 1200 + \frac{49r}{2}$$

$$2792 = 2400 + 49r$$

$$49r = 2792 - 2400 = 392$$

$$r = \frac{392}{49} = 8$$

The rate of interest is 8% p.a.
Thus (b) is correct option.

27. Neha has a R.D. account at Bank of Baroda. She deposited ₹600 per month for 4 years in this account. If she gets ₹5,880 as interest at the time of maturity, then the rate of interest is
(a) 10% (b) 9%
(c) 8% (d) 6%

Sol :

Monthly deposit $p = ₹\ 600$

Period of deposit $n = 4$ years
$= 48$ months

Total interest $I = ₹\ 5,880$

Let r be the rate of interest then we have

Interest $$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$5,880 = 600 \times \frac{48 \times 49}{24} \times \frac{r}{100}$$

$$5,880 = 6 \times \frac{2 \times 49}{1} \times \frac{r}{1}$$

$$5,880 = 12 \times 49 \times r$$

$$490 = 49r$$

$$r = 10$$

The rate of interest is 10% p.a.
Thus (a) is correct option.

28. Shahrukh opened a R.D. account in a Kotak bank and deposited ₹800 per month for $1\frac{1}{2}$ years. If he received ₹15,084 at the time of maturity, then the rate of interest per annum is:
(a) 6% (b) 6.5%
(c) 7% (d) 7.5%

Sol :

Monthly deposit $p = ₹\ 800$

Period of deposit $n = 1\frac{1}{2}$ years
$= 18$ months

Maturity value $MV = ₹\ 15,084$

Let r be the rate of interest then we have

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$15,084 = 800 \times 18 + 800 \times \frac{18(18+1)}{2 \times 12} \times \frac{r}{100}$$

$$838 = 800 + 8 \times \frac{19}{24} \times \frac{r}{1}$$

$$838 = 800 + \frac{19r}{3}$$

$$\frac{19r}{3} = 838 - 800$$

$$\frac{19r}{3} = 38$$

$$\frac{r}{3} = 2$$

$$r = 6$$

The rate of interest is 6% p.a.
Thus (a) is correct option.

NODIA

THREE MARKS QUESTIONS

Main 2020

29. Mr. Sonu has a recurring deposit account and deposits ₹ 750 per month for 2 yr. If he gets ₹ 19125 at the time of maturity, find the rate of interest.

Sol :

Monthly deposit $p = ₹\ 750$

Period of deposit $n = 2$ years

$= 24$ months

Maturity value $\text{MV} = ₹\ 19{,}125$

Now $\text{MV} = P \times n + I$

$19125 = 750 \times 24 + I$

$I = 19125 - 18000$

$= ₹1{,}125$

Let r per annum be the rate of interest then interest will be

$$I = \frac{P \times n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$1125 = \frac{750 \times 24(24+1)}{2 \times 12} \times \frac{r}{100}$$

$$1125 = \frac{750 \times 24 \times 25}{24} \times \frac{r}{100}$$

$$1125 = \frac{15 \times 25}{1} \times \frac{r}{2}$$

$$45 = \frac{15}{1} \times \frac{r}{2}$$

$$3 = \frac{1}{1} \times \frac{r}{2}$$

$$r = 6\ \%$$

Hence, the required rate of interest is 6% per annum.

Comp 2020

30. Mr Rajinder has a recurring deposit in a bank for 3 years at 9% p.a. simple interest. if he gets ₹ 3996 as interest at the time of maturity, find:
(i) the monthly installment.
(ii) the amount of maturity.

Sol :

Rate of simple interest $r = 9\,\%$ p.a.

Period of deposit $n = 3$ years

$= 36$ months

Total Interest $I = ₹\,3996$

Let Mr Rajinder deposited p per month.

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

Substituting all values we have

$$3996 = p \times \frac{36(36+1)}{2 \times 12} \times \frac{9}{100}$$

$$148 = p \times \frac{37}{2} \times \frac{1}{100}$$

$$4 = \frac{p}{200}$$

$$p = ₹\,800$$

Total amount deposited by Rajinder,

$$P = p \times n$$

$$= 800 \times 36$$

$$= ₹\,28800$$

Amount on maturity
Total amount of maturity,

$$\text{MV} = P + I$$

$$= ₹\,28800 + ₹\,3996$$

$$= ₹\,32796$$

(i) Monthly installment is ₹ 800
(ii) Amount of maturity is ₹ 32796

Main 2019

31. Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives ₹ 441 as interest at the time of maturity. Find the amount Rekha deposited each month.

Sol :

Rate of simple interest $r = 9\,\%$ p.a.

Period of deposit $n = 20$ months

Interest Amount $I = ₹\ 441$

Let p be the monthly deposit amount, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$441 = \frac{p \times 20(20+1)}{24} \times \frac{9}{100}$$

$$441 = \frac{p \times 20 \times 21}{24} \times \frac{9}{100}$$

$$441 = \frac{p \times 1 \times 21}{8} \times \frac{3}{5}$$

$$21 = \frac{p \times 1 \times 1}{8} \times \frac{3}{5}$$

$$8 = \frac{p \times 1 \times 1}{8} \times \frac{1}{5}$$

$$p = 8 \times 8 \times 5$$

$$= ₹320$$

Hence, the monthly installment is ₹ 320.

32. Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate of 6% per annum and the monthly installment is ₹ 1000, find the :
(i) Interest earned in 2 years.
(ii) Matured value.

Sol :

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Monthly deposit $p = ₹\ 1000$

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 2$ years

$= 24$ months

(i) Interest earned in 2 years

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$= 1000 \times \frac{24 \times 25}{2 \times 12} \times \frac{6}{100}$

$= ₹\ 1500$

(ii) Matured value

Matured value,

$MV = p \times n + I$

$= 1000 \times 24 + 1500$

$= 24000 + 1500$

$= ₹\ 25500$

33. Mohan deposits ₹ 80 per month in a cumulative deposit account for six years. Find the amount payable to him on maturity, if the rate of interest is 6% per annum.

Sol :

Monthly deposit $p = ₹\ 80$

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 6$ years

$= 72$ months

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$= 80 \times \frac{72 \times 73}{2 \times 12} \times \frac{6}{100}$

$= ₹\ 1051.20$

Matured value,

$MV = p \times n + I$

$= (80 \times 72) + 1051.20$

$= 5760 + 1051.20$

$= ₹\ 6811.20$

Hence, the amount payable to Mohan on maturity is ₹ 6811.20

Comp 2019

34. Swamy deposited ₹ 400 at the beginning of every month in a recurring deposit and received ₹ 16398 at the end of 3 years. Find the rate of interest given by the bank.

Sol :

Monthly deposit $p = ₹\ 400$

Period of deposit $n = 3$ years

$= 36$ months

Maturity value $MV = ₹\ 16398$

Let r per annum be the rate of interest then maturity value will be

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$16398 = 400 \times 36 + 400 \times \frac{36(36+1)}{2 \times 12} \times \frac{r}{100}$$

$$16398 = 400 \times 36 + 2 \times 3 \times 37 \times r$$

$$2733 = 400 \times 6 + 37r$$

$$37r = 2733 - 2400$$

$$= 333$$

$$r = \frac{333}{37} = 9$$

Thus, the rate of interest given by the bank is 9 % per annum.

Main 2018

35. Priyanka has a recurring deposit account of ₹1000 per month at 10% per annum. If she gets ₹5550 as interest at the time of maturity, find the total time for which the account was held.

Sol :

Monthly Deposit $p = ₹\ 1000$

Rate of simple interest $r = 10\,\%$ p.a.

Interest Amount $I = ₹\ 5000$

Let the account be held for n months. Therefore interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$5550 = 1000 \times \frac{n(n+1)}{24} \times \frac{10}{100}$$

$$5550 = 100 \times \frac{n(n+1)}{24}$$

$$55.5 \times 24 = n(n+1)$$

$$n^2 + n - 1332 = 0$$

$$n^2 + 37n - 36n - 1332 = 0$$

$$n(n+37) - 36(n+37) = 0$$

$$(n+37)(n-36) = 0$$

$$n = -37 \text{ or } 36$$

Since month cannot be negative, we have $n = 36$

NODIA

Hence account was held for 36 months,

Main 2018

36. Sonia has a Recurring Deposit Account in a bank and deposited ₹600 per month for $2\frac{1}{2}$ year. If the rate of intersect was 10% per annum, find the maturity value of this account.

Sol :

Monthly deposit $p =$₹ 600

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 2\frac{1}{2}$ years

$= 30$ months

Maturity Value,

$$\text{MV} = pn + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= np\left[1 + \frac{(n+1)r}{2400}\right]$$

$$= 18000\left[1 + \frac{31 \times 10}{2400}\right]$$

$$= 18000\left(1 + \frac{31}{240}\right)$$

$$= 18000 \times \frac{271}{240}$$

$$= Rs.\,20325$$

$=$₹ 20325

Comp 2018

37. Mr Rohit saves ₹ 20 daily and deposit ₹ 600 monthly in his R.D. account in S.B.P. for 3 years. If the rate of interest is $r\%$ per annum, then he can get ₹ 24430.50 on maturity. Find the value of r.

Sol :

Recurring deposit $p =$₹ 20 per day

$= 20 \times 30$

$=$₹ 600 per month

Period of deposit $n = 3$ years

$= 36$ months

Maturity value $\text{MV} =$₹ 24430.5

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$24430.5 = 600 \times 36 + 600 \times \frac{36(36+1)}{2 \times 12} \times \frac{r}{100}$$

$$24430.5 = 600 \times 36 + 3 \times 3 \times 37 \times r$$

$$2714.5 = 600 \times 4 + 37r$$

$$37r = 2714.5 - 2400$$

$$= 314.5$$

$$r = \frac{314.5}{37} = 8.5$$

Hence, rate of simple interest is 8.5 % p.a.

Main 2016

38. Mohan has a recurring deposit account in a bank for 2 years at 6% per annum simple interest. If the gets ₹1200 as interest at the time of maturity, then find

(i) the monthly installment.

(ii) the amount of maturity.

Sol :

Rate of simple interest $r = 6\,\%$ p.a.

Interest Amount $I =$₹ 1200

Period of deposit $n = 2$ years

$= 24$ months

Let p be the monthly deposit amount, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$1200 = \frac{p \times 24(24+1)}{24} \times \frac{6}{100}$$

$$1200 = \frac{p \times 25 \times 6}{100}$$

$$1200 = \frac{p \times 6}{4}$$

$$200 = \frac{p}{4}$$

$$p = 200 \times 4$$

$$= 800$$

Hence, the monthly installment is ₹ 800.

Amount of maturity,

$$\text{MV} = P \times n + I$$

$$= 800 \times 24 + 1200$$

$$= 19200 + 1200$$

$=$₹ 20400

(i) The monthly installment is ₹ 800.

(ii) The amount of maturity is ₹ 20,400.

Comp 2016

39. Reeta gets ₹ 48528 after 36 months. How much money she should invest per month in a R.D. account to get required amount when the bank pays the rate of interest 8% per annum?

Sol :

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 3$ years

$= 36$ months

Maturity value MV $=$₹ 48528

Let p be the amount required to deposit every month.

Maturity Value,

$$\text{MV} = pn + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$48528 = p \times 36 + p \times \frac{36(36+1)}{2 \times 12} \times \frac{8}{100}$$

$$1348 = p + p \times \frac{37}{3} \times \frac{1}{100}$$

$$404400 = 300p + 37p$$

$$404400 = 337p$$

$$p = \frac{404400}{37} = 1200$$

Hence, the required monthly deposit is ₹1200.

Main 2015

40. Katrina opened a recurring deposit account with a nationalized bank for a period of 2 yr. If the bank pays interest at the rate of 6% per annum and the monthly installment is ₹1000, find the:
(i) interest earned in 2 yr.
(ii) matured value.

Sol :

Monthly Deposit $p =$₹ 1000

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 2$ years

$= 24$ months

Interest earned in 2 years,

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$I = 1000 \times \frac{24 \times 25}{2 \times 12} \times \frac{6}{100}$$

$$= 10 \times 25 \times 6$$

$=$₹ 1500

Matured value

$$MV = p \times n + I$$

$$= 1000 \times 24 + 1500$$

$=$₹ 25500

Comp 2015

41. Mr. Johnson saves ₹ 800 every month and puts into a recurring deposit account paying interest at the rate of 8% par annum. In order to receive ₹ 57,760 at the maturity amount, for how many months should he deposit ₹ 800 ?

Sol :

Monthly deposit $p =$₹ 800

Rate of simple interest $r = 8\,\%$ p.a.

Maturity amount MV $=$₹ 57760

Let amount is deposit n months. Therefore maturity value be

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$57760 = 800 \times n + 800 \times \frac{n(n+1)}{2 \times 12} \times \frac{8}{100}$$

$$57760 = 800 \times n + 8 \times \frac{n(n+1)}{3}$$

$$21660 = 300n + n^2 + n$$

$$n^2 + 301n - 21,660 = 0$$

$$(n - 60)(n + 361) = 0$$

$$n = 60 \text{ or } n = -361,$$

But n is not negative, so rejecting negative value we get $n = 60$.

Thus, Mr. Johnson should deposit ₹ 800 every month for 60 months, i.e., 5 years.

Main 2014

42. Sharukh opened a recurring deposite in a bank and deposited ₹800 per month for $1\frac{1}{2}$ years. If the received ₹15084 at the time of maturity, find the rate of interest per annum.

Sol :

Monthly Deposit $p =$₹ 800

Period of deposit $n = 1\frac{1}{2}$ years

$= 18$ months

Maturity value MV $=$₹ 15084

Let r per annum be the rate of interest maturity value be

$$\text{MV} = np\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

Substituting all values we have

$$15084 = 18 \times 800\left(1 + \frac{(18+1)}{2 \times 12} \times \frac{r}{100}\right)$$

$$838 = 800\left(1 + \frac{19}{24} \times \frac{r}{100}\right)$$

$$838 = 800 + \frac{19}{3}r$$

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$$\frac{19}{3}r = 838 - 800 = 38$$

$$r = \frac{38 \times 3}{19} = 6$$

Hence, rate of interest is 6% per annum.

Comp 2014

43. Mr. Satish deposited ₹ 300 at the beginning of every month in a bank paying 8% S.I. per annum in a recurring deposit account. How much would he receive at the end of 4 years?

Sol :

Monthly deposit $p =$ ₹ 300

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 4$ years

$= 48$ months

Let r per annum be the rate of interest then maturity value will be

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 300 \times 48 + 300 \times \frac{48(48+1)}{2 \times 12} \times \frac{8}{100}$$

$$= 300 \times 48 + 3 \times 2 \times 49 \times 8$$

$$= 14400 + 2352$$

$$= 16752$$

7m Thus Mr. Satish will receive ₹ 16,752 at the end of 4 years.

Main 2013

44. Mr. Britto deposits a certain sum of money each month in a recurring deposit account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹8088 from the bank after 3 yr, find the value of his monthly installment.

Sol :

Rate of simple interest $r = 8\,\%$ p.a.

Maturity value MV = ₹ 8088

Period of deposit $n = 3$ years

$= 36$ months

Let p be the monthly deposit amount, then maturity value will be

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= pn\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

Substituting all values we have

$$8088 = p \times 36 \times \left(1 + \frac{(36+1)}{2 \times 12} \times \frac{8}{100}\right)$$

$$674 = 3p\left(1 + \frac{37}{3} \times \frac{1}{100}\right)$$

$$674 = 3p\left(\frac{300+37}{300}\right)$$

$$674 = 3p\left(\frac{337}{300}\right)$$

$$p = \frac{674 \times 300}{3 \times 337} = 200$$

Hence, the monthly installment is ₹ 200

Comp 2013

45. How much should Sunita deposit at the beginning of every month in recurring deposit account of a bank paying interest at the rate of 10%, if she wants to receive ₹ 6740 after 8 years?

Sol :

Rate of simple interest $r = 10\,\%$ p.a.

Period of deposit $n = 8$ years

$= 96$ months

Maturity value MV = ₹ 6740

Let p be the amount required to deposit every month.

Maturity Value,

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$6740 = p \times 96 + p \times \frac{96(96+1)}{2 \times 12} \times \frac{10}{100}$$

$$6740 = 96p + p \times 4 \times 97 \times \frac{1}{10}$$

$$67400 = 960p + 388p$$

$$67400 = 1348p$$

$$p = \frac{67400}{1348} = 50$$

Thus, Sunita should deposit ₹ 50 every month.

Main 2012

46. Kiran deposited ₹ 200 per month for 36 months in bank's recurring deposit account. If the bank pays interest at the rate of 11% per annum, find the amount she gets on maturity.

Sol :

Monthly deposit $p =$ ₹ 200

Rate of simple interest $r = 11\,\%$ p.a.

Period of deposit $n = 36$ months

Interest earned in 36 month,

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

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$$= 200 \times \frac{36 \times 37}{2 \times 12} \times \frac{11}{100}$$

$$= 3 \times 37 \times 11$$

$$= ₹\ 1221$$

Matured value,

$$\text{MV} = p \times n + I$$

$$= 200 \times 36 + 1221$$

$$= 24000 + 1500$$

$$= 8,421$$

Thus, Kiran will receive ₹ 8,421 on maturity.

Main 2011

47. Ahmed has a recurring deposit account in a bank. He deposits ₹2500 per month for 2 yr. If he gets ₹66250 at the time of maturity, find
(i) the interest paid by the bank.
(ii) the rate of interest.

Sol :

Monthly Deposit $p = ₹\ 2500$

Maturity value $MV = ₹\ 66250$

Period of deposit $n = 2$ years

$= 24$ months

Total money deposited $P = n \times p$

$= 24 \times 2500$

$= 60000$

Interest $I = MV - I$

$= 66250 - 60000$

$= ₹\ 6250$

Let r be the simple rate if interest, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$6250 = 2500 \times \frac{24 \times (24+1)}{2 \times 12} \times \frac{r}{100}$$

$$6250 = 25 \times 25 \times r$$

$$6250 = 625r$$

$$r = 10$$

Hence, rate of interest is 10% per annum.
(i) Interest paid by the bank is ₹ 6250
(ii) Rate of interest is 10 %

Main 2009

48. Mrs. Goswami deposits ₹1000 every month in a recurring deposite account for 3 yr at 8% interest per annum. Find the maturited value.

Sol :

Monthly deposit $p = ₹\ 1000$

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 3$ years

$= 36$ months

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 1000 \times \frac{36(36+1)}{2 \times 12} \times \frac{8}{100}$$

$$= 10 \times 12 \times 37$$

$$= ₹\ 4440$$

Matured value,

$$\text{MV} = p \times n + I$$

$$= 1000 \times 36 + 4440$$

$$= ₹\ 40440$$

Thus maturity value is ₹ 40440.

ALTERNATIVE :

Matured value,

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= pn\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

$$= 1000 \times 36\left(1 + \frac{(36+1)}{2 \times 12} \times \frac{8}{100}\right)$$

$$= 36000\left(1 + \frac{37}{3} \times \frac{1}{100}\right)$$

$$= 36000 + 120 \times 37$$

$$= 36000 + 4440$$

$$= ₹\ 40440$$

Main 2008

49. David opened a recurring deposit account in a bank and deposited ₹300 per month for 2 yr. If he received ₹7725 at the time of maturity, find the rate of interest per annum.

Sol :

Monthly deposit $p = ₹\ 300$

Period of deposit $n = 2$ years

$= 24$ months

Maturity value $\text{MV} = ₹\ 7725$

Let r be the rate of interest per annum then maturity value will be

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= p \times n\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

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Substituting all values we have

$$7725 = 300 \times 24\left(1 + \frac{(24+1)}{2 \times 12} \times \frac{r}{100}\right)$$

$$103 = 4 \times 24\left(1 + \frac{25}{24} \times \frac{r}{100}\right)$$

$$103 = 96\left(1 + \frac{1}{24} \times \frac{r}{4}\right)$$

$$103 = 96 + r$$

$$r = 103 - 96 = 7$$

Hence, the rate of interest is 7% per annum.

Main 2007, Main 2001

50. Solani deposited ₹150 per month in her bank for eight months under the recurring deposit scheme. What will be the maturity value of her deposit, if the rate of interest is 8% per anuum and the interest is calculated at the end of every month?

Sol :

Monthly deposit $p = ₹\,150$

Rate of simple interest $r = 8\,\%$ p.a.

Period of deposit $n = 8$ months

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 150 \times \frac{8(8+1)}{2 \times 12} \times \frac{8}{100}$$

$$= 3 \times \frac{8 \times 9}{24} \times \frac{1}{2}$$

$$= ₹\,36$$

Matured value,

$$MV = p \times n + I$$

$$= 150 \times 8 + 36$$

$$= ₹\,1236$$

Thus maturity value is ₹ 1236.

ALTERNATIVE :

Matured value,

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= pn\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

$$= 150 \times 8\left(1 + \frac{(8+1)}{2 \times 12} \times \frac{8}{100}\right)$$

$$= 1200\left(1 + \frac{9}{24} \times \frac{8}{100}\right)$$

$$= 1200\left(1 + \frac{3}{100}\right)$$

$$= 1200 + 36$$

$$= ₹\,1236$$

Main 2006

51. Mohan deposits ₹80 per month in a cumulative deposit account for 6 years. Find the amount payable to him on maturity, if rate of interest is 6% per annum.

Sol :

Monthly deposit $p = ₹\,80$

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 6$ years

$= 72$ month

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 80 \times \frac{72(72+1)}{2 \times 12} \times \frac{6}{100}$$

$$= 8 \times \frac{3 \times 73}{1} \times \frac{6}{10}$$

$$= ₹\,1051.20$$

Matured value,

$$MV = p \times n + I$$

$$= 80 \times 72 + 1051.20$$

$$= 5760 + 1051.20$$

$$= ₹\,6811.2$$

Hence, the amount payable to Mohan on maturity is ₹ 6811.2.

ALTERNATIVE :

Matured value,

$$MV = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= pn\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

$$= 80 \times 72\left(1 + \frac{(72+1)}{2 \times 12} \times \frac{6}{100}\right)$$

$$= 80 \times 72 \times \left(1 + \frac{73}{24} \times \frac{6}{100}\right)$$

$$= 80 \times 72 \times \left(1 + \frac{73}{400}\right)$$

$$= 80 \times 72 \times \frac{473}{400}$$

$$= \frac{72 \times 473}{5}$$

$$= ₹\,6811.2$$

Main 2005

52. Mr. RK Nair gets ₹6455 at the end of 1 yr a the rate of 14% per annum in a recurring deposit account. Find the monthly installment.

NODIA

Sol :

Rate of simple interest $r = 14\,\%$ p.a.

Maturity value $\text{MV} = ₹\,6455$

Period of deposit $n = 1$ years

$= 12$ months

Let p be the monthly deposit amount, maturity value will be

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= p \times n\left(1 + \frac{(n+1)}{2 \times 12} \times \frac{r}{100}\right)$$

Substituting all values we have

$$6455 = p \times 12\left(1 + \frac{(12+1)}{2 \times 12} \times \frac{14}{100}\right)$$

$$6455 = p\left(12 + \frac{13}{1} \times \frac{7}{100}\right)$$

$$6455 = p\left(\frac{1200+91}{100}\right)$$

$$6455 = p\left(\frac{1291}{100}\right)$$

$$p = \frac{645500}{1291} = 500$$

Hence, the monthly installment is ₹ 500.

Main 2003

53. Ritesh opened a R.D. account in a Bank for 20 months. If the rate of interest is 9% per annum and he received ₹882 as interest at the end of maturity, find the monthly installment.

Sol :

Rate of simple interest $r = 9\,\%$ p.a.

Period of deposit $n = 20$ months

Total interest $I = ₹\,882$

Let p be the monthly installment then we have

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$882 = p \times \frac{20 \times 21}{24} \times \frac{9}{100}$$

$$882 = p \times \frac{1 \times 7}{8} \times \frac{9}{5}$$

$$882 = \frac{63p}{40}$$

$$p = \frac{882 \times 40}{63}$$

$$= ₹560$$

Main 2002

54. Meena deposited ₹700 per month in a R.D. account for $1\frac{1}{4}$ years at bank. If the matured value of this account is ₹11,130, find the interest received.

Sol :

Monthly deposit $p = ₹\,700$

Period of deposit $n = 1\frac{1}{4}$ years

$= 15$ months

Maturity value $\text{MV} = ₹\,11,130$

Interest received $I = \text{MV} - p \times n$

$= 11,130 - 700 \times 15$

$= 11,130 - 10500$

$= ₹\,630$

FOUR MARK QUESTIONS

Main 2023

55. Salman deposits ₹1000 every month in a recurring deposit account for 2 years. If he receives ₹26000 on maturity, find :

(a) the total interest Salman earns.

(b) the rate of interest.

Sol :

Monthly deposit $p = ₹\,1000$

Period of deposit $n = 2$ years

$= 24$ months

Maturity value $\text{MV} = ₹\,26,000$

Now $\text{MV} = P \times n + I$

$$26000 = 1000 \times 24 + I$$

$$I = 26000 - 24000$$

$$= ₹2000$$

Hence, the total interest Salman earns ₹2000.

Let r per annum be the rate of interest then interest will be

Interest $I = \frac{P \times n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$2000 = \frac{1000 \times 24(24+1)}{2 \times 12} \times \frac{r}{100}$$

$$2 = \frac{25 \times r}{100}$$

$$2 = \frac{r}{4}$$

$$r = 8$$

Hence, the rate of interest is 8%

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(a) Salman earns ₹2000 as interest.
(b) Rate of interest is 8 %.

Comp 2023

56. Ram has a recurring deposit of ₹ 400 per month at 10% per annum. if he gets ₹ 16620 at the time of maturity, find total time for which the account was held.

Sol :

Monthly deposit $p = ₹\,400$

Rate of simple interest $r = 10\,\%$ p.a.

Maturity amount $\text{MV} = ₹\,16620$

Let the account be held for n months. Therefore interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 400 \times \frac{n \times (n+1)}{2 \times 12} \times \frac{10}{100}$$

$$= \frac{5(n^2+n)}{3}$$

Total amount deposited

$$P = p \times n$$

$$= 400n$$

Amount due on maturity,

$$\text{MV} = P + I$$

$$= 400n + \frac{5(n^2+n)}{3}$$

But given maturity value is ₹ 16620, therefore

$$16620 = 400n + \frac{5(n^2+n)}{3}$$

$$3324 = 80n + \frac{(n^2+n)}{3}$$

$$9972 = 240n + n^2 + n$$

$$n^2 + 241n - 9972 = 0$$

$$(n+277)(n-36) = 0$$

$$n = 36 \text{ or } n = -277,$$

But n is not negative, so rejecting negative value we get $n = 36$. Hence, number of months is 36.

$$n = 36$$

Main 2017

57. Mr. Richard has a recurring deposit account in a bank for 3 yr at 7.5% p.a. simple interest. If he gets ₹8325 as interest at the time of maturity, find
(i) the monthly deposit.
(ii) the maturity value.

Sol :

Rate of simple interest $r = 7.5\,\%$ p.a.

Interest Amount $I = ₹\,8325$

Period of deposit $n = 3$ years

$= 36$ months

Let p be the monthly deposit amount, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$8325 = \frac{p \times 36(36+1) \times 7.5}{2 \times 12 \times 100}$$

$$8325 = \frac{p \times 3 \times 37 \times 7.5}{200}$$

$$p = \frac{8325 \times 200}{3 \times 37 \times 7.5}$$

$$= \frac{225 \times 200}{3 \times 7.5}$$

$$= ₹\,2000$$

Hence, the monthly installment is ₹ 2000.
Amount of maturity

$$\text{MV} = P \times n + I$$

$$= 2000 \times 36 + 8325$$

$$= 72000 + 8325$$

$$= ₹\,80325$$

(i) Monthly deposit is ₹ 2000.
(ii) Maturity value is ₹ 80325.

Comp 2017

58. Shalini deposited ₹600 per month in a R.D. account at Axis bank. If the matured value of this account was ₹24,930 and the rate of interest was 10% per annum, find the time for which the account was held.

Sol :

Monthly deposit $p = ₹\,600$

Rate of simple interest $r = 10\,\%$ p.a.

Maturity value $\text{MV} = ₹\,24,930$

Let n be the month for which account was held.

Now maturity value,

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$24,930 = 600 \times n + 600 \times \frac{(n^2+n)}{24} \times \frac{10}{100}$$

$$24,930 = 600n + 2.5(n^2+n)$$

$$9970 = 240n + (n^2+n)$$

$$9970 = 241n + n^2$$

NODIA

$$n^2 + 241n - 9,970 = 0$$

Using quadratic formula,

$$n = \frac{-241 \pm \sqrt{(-241)^2 - 4 \times 1 \times (-9970)}}{2 \times 1}$$

$$= \frac{-241 \pm \sqrt{58081 + 39880}}{2}$$

$$= \frac{-241 \pm \sqrt{97961}}{2}$$

$$= \frac{-241 \pm 313}{2}$$

$$= \frac{-241 + 313}{2}, \frac{-241 - 313}{2}$$

$$n = \frac{72}{2}, \frac{-554}{2}$$

$$= 36, -277$$

Since, time can not be negative we take $n = 36$.

Thus account was held for 36 months or 3 years.

Main 2016

59. Rohan has a recurring deposit account in Bandhan Bank. He deposited some amount every month for 2 years at 6% per annum simple interest. If he gets ₹1800 as interest at the time of maturity, then find

(i) the monthly installment.

(ii) the amount of maturity.

Sol :

Rate of simple interest	$r = 6\,\%$ p.a.
Interest Amount	$I =$₹ 1800
Period of deposit	$n = 2$ years
	$= 24$ months

Let p be the monthly deposit amount, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$1800 = \frac{p \times 24(24+1)}{24} \times \frac{6}{100}$$

$$1800 = \frac{p \times 25 \times 6}{100}$$

$$1800 = \frac{p \times 6}{4}$$

$$300 = \frac{p}{4}$$

$$p = 300 \times 4$$

$$= 1200$$

Hence, the monthly installment is ₹ 1200.

Amount of maturity,

$$\text{MV} = P \times n + I$$

$$= 1200 \times 24 + 1800$$

$$= 28800 + 1800$$

$$=$₹ 30,600

(i) The monthly installment is ₹ 1200.

(ii) The amount of maturity is ₹ 30,600.

Main 2010

60. Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹2500 per month for 2 yr. At the time of maturity, he get ₹67500. Find

(i) the total interest earned by Mr. Gupta.

(ii) the rate of interest per annum.

Sol :

Monthly Deposit	$p =$₹ 2500
Maturity value	$MV =$₹ 67500
Period of deposit	$n = 2$ years
	$= 24$ months
Total money deposited	$P = n \times p$
	$= 24 \times 2500$
	$= 60000$
Interest	$I = MV - P$
	$= 67500 - 60000$
	$=$₹ 7500

Let r be the simple rate if interest, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$7500 = 2500 \times \frac{24 \times (24+1)}{2 \times 12} \times \frac{r}{100}$$

$$7500 = 25 \times 25 \times r$$

$$7500 = 625r$$

$$r = 12$$

Hence, rate of interest is 12% per annum.

(i) Total interest earned by Mr. Gupta is ₹ 7500

(ii) Rate of interest per annum is 12%.

Comp 2010

61. Mr Goel has a recurring deposit account in a bank. He deposits ₹ 5000 per month for 2 yr. If he gets ₹ 132,500 at the time of maturity, find

(i) the interest paid by the bank.

(ii) the rate of interest.

Sol :

Monthly Deposit	$p =$₹ 5000

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Maturity value $\text{MV} = ₹\ 132,500$

Period of deposit $n = 2$ years

$= 24$ months

Total money deposited $P = n \times p$

$= 24 \times 5000$

$= ₹\ 120,000$

Interest $I = MV - I$

$= 132,500 - 120,00$

$= ₹\ 12,500$

Let r be the simple rate if interest, then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$12,500 = 5000 \times \frac{24 \times (24+1)}{2 \times 12} \times \frac{r}{100}$$

$$12,500 = 50 \times 25 \times r$$

$$12,500 = 1250 \times r$$

$$r = 10$$

Hence, rate of interest is 10% per annum.

(i) Interest paid by the bank is ₹ 12,500

(ii) Rate of interest is 10 %

Main 2004

62. Mr Anwar deposits ₹ 300 per month for 27 months, in a recurring deposit account. If at the time of maturity he gets ₹ 8950.50, find the rate if interest.

Sol :

Monthly deposit $p = ₹\ 300$

Period of deposit $n = 27$ months

Maturity value $\text{MV} = ₹\ 8950.5$

Total amount deposited $P = p \times n$

$= 300 \times 27$

$= ₹\ 8100$

Let r per annum be the rate of interest then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 300 \times \frac{27(27+1)}{2 \times 12} \times \frac{r}{100}$$

$$= \frac{27 \times 28}{8} \times r$$

$$= \frac{189r}{2}$$

Total amount of maturity,

$$\text{MV} = P + I$$

$$= 8100 + \frac{189r}{2}$$

But given maturity value is ₹ 8950.5. Therefore

$$8950.5 = 8100 + \frac{189r}{2}$$

$$17901 = 16200 + 189r$$

$$189r = 17901 - 16200$$

$$= 1701$$

$$r = \frac{1701}{189} = 9$$

Hence, rate of simple interest is 9 % p.a.

Comp 2004

63. The matured value of a recurring deposit account in a Bank is ₹16,176. If the monthly installment is ₹400 and the rate of interest is 8% p.a., then find the time period of this deposit account.

Sol :

Monthly deposit $p = ₹\ 400$

Rate of simple interest $r = 8\ \%$ p.a.

Maturity value $\text{MV} = ₹\ 16,176$

Let n be the month for which account was held.

Now maturity value,

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$16,176 = 400 \times n + 400 \times \frac{n(n+1)}{24} \times \frac{8}{100}$$

$$16,176 = 400n + \frac{4(n^2+n)}{3}$$

$$4,044 = 100n + \frac{(n^2+n)}{3}$$

$$12,132 = 300n + n^2 + n$$

$$n^2 + 301n - 12,132 = 0$$

$$n^2 + 337n - 36n - 12,132 = 0$$

$$(n+337)(n-36) = 0$$

$$n = -337, 36$$

Since, time cannot be negative we take $n = 36$.

Thus $n = 36$ months or 3 years.

64. Ahmed has a recurring deposit account in a bank. He deposits ₹ 2500 per month for 2 years. If he get ₹ 66250 at the time of maturity, find

(i) the interest paid by the bank.

(ii) the rate of interest.

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Sol :

Monthly deposit $p = ₹\ 2500$

Period of deposit $n = 2$ years

$= 24$ months

Total money deposited $P = 2500 \times 24$

$= ₹\ 60000$

The money which Ahmed gets at the time of maturity i.e matured value,

$$\text{MV} = ₹\ 66250$$

The interest paid by the bank

$$I = \text{MV} - P$$
$$= 66250 - 60000$$
$$= ₹\ 6250$$

Let the rate of interest be r per annum, then by using the formula we have,

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$6250 = 2500 \times \frac{24(24+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 25 \times 25 \times r$$
$$6250 = 625r$$
$$r = 10$$

(i) Interest paid by bank is ₹ 6250.
(ii) Rate of interest is 10 % per annum.

65. Shobana has a cumulative time deposit account in State of India. She deposits ₹ 500 per month for a period of 4 years. If at the time of maturity she gets ₹ 28410, find the rate of (simple) interest.

Sol :

Monthly deposit $p = ₹\ 500$

Period of deposit $n = 4$ years

$= 48$ months

Let r per annum be the rate of interest then interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 500 \times \frac{48 \times 49}{2 \times 12} \times \frac{r}{100}$$
$$= ₹\ 490r$$

Total money deposited by Shobana

$$P = p \times n$$
$$= (500 \times 48)$$
$$= ₹\ 24000$$

Total amount of maturity,

$$\text{MV} = P + I$$
$$= 24000 + 490r$$

But given maturity value is ₹ 28410. Therefore

$$28410 = 24000 + 490r$$
$$490r = 28410 - 24000$$
$$490r = 4410$$
$$r = 9$$

Hence, rate of simple interest is 9 % p.a.

NODIA

66. Richard has a recurring deposit account in a post office for 3 years at 7.5% p.a. simple interest. If he gets ₹ 8325 as interest at the time of maturity, find
(i) the monthly installment
(ii) the amount of maturity.

Sol :

Rate of simple interest $r = 7.5\,\%$ p.a.

Period of deposit $n = 3$ years

$= 36$ months

Interest $I = ₹\ 8325$

Let p be the monthly deposit then we have

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$8325 = p \times \frac{36 \times 37}{2 \times 12} \times \frac{7.5}{100}$$
$$8325 = \frac{333p}{80}$$
$$p = 25 \times 80$$
$$= 2000$$

Total amount deposited by Richard

$$P = n \times p$$
$$= (2000 \times 36)$$
$$= ₹\ 72000$$

Amount of maturity

$$\text{MV} = P + I$$
$$= 72000 + 8325$$
$$= ₹\ 80325$$

67. Priyanka has a recurring deposit account of ₹ 1000 per month at 10% per annum. If she gets ₹ 5550 as interest at the time of maturity, find

the total time for which the account was held.

Sol :

Monthly deposit $p = ₹\ 1000$

Rate of simple interest $r = 10\,\%$ p.a.

Interest $I = ₹\ 5550$

Let the account be held for n months. Therefore interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$5550 = 1000 \times \frac{n(n+1)}{2 \times 12} \times \frac{10}{100}$$

$$5550 \times 24 = 100n(n+1)$$

$$111 \times 12 = n(n+1)$$

$$n(n+1) = 1332$$

$$n^2 + n - 1332 = 0$$

$$(n-36)(n+37) = 0$$

$$n = 36,\ -37$$

But n cannot be negative therefore $n = 36$. Hence, the account was held for 36 months i.e. 3 years.

68. Beena has a cumulative deposit account of ₹ 400 per month at 10% per annum simple interest. If she gets ₹ 30100 at the time of maturity, find the total time for which the account was held.

Sol :

Monthly deposit $p = ₹\ 400$

Rate of simple interest $r = 10\,\%$ p.a.

Maturity value $\text{MV} = ₹\ 5550$

Let the account be held for n months. Therefore interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 400 \times \frac{n(n+1)}{2 \times 12} \times \frac{10}{100}$$

$$= \frac{5n(n+1)}{3}$$

Total money deposited by Beena

$$P = p \times n$$

$$= 400 \times n$$

$$= 400n$$

The amount of maturity is ₹ 30100 thus,

$$\text{MV} = P + I$$

$$30100 = 400n + \frac{5n(n+1)}{3}$$

$$90300 = 1200n + 5n^2 + 5n$$

$$18060 = 241n + n^2$$

$$n^2 + 241n - 18060 = 0$$

$$(n-60)(n+301) = 0$$

$$n = 60,\ -301$$

But n cannot be negative thus $n = 60$. Hence, the account was held for 60 months i.e. 5 years.

69. Rahul deposited ₹ 500 per months in a bank's recurring deposit account for a period of 5 years. If the bank pays interest at the rate of 6% per annum, then find the amount, he will get on maturity.

Sol :

Monthly deposit $p = ₹500$

Rate of simple interest $r = 6\,\%$ p.a.

Period of deposit $n = 5$ years

$= 60$ months

Interest $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 500 \times \frac{60(60+1)}{24} \times \frac{6}{100}$$

$$= 75 \times 61$$

$$= ₹\ 4575$$

Matured value,

$$\text{MV} = p \times n + I$$

$$= 500 \times 60 + 4575$$

$$= 30000 + 4575$$

$$= ₹\ 34575$$

70. If a person deposit ₹ 3000 per month for $3\frac{1}{4}$ years in his recurring deposit account at 6.5% per annum, then find the amount he will get on maturity.

Sol :

Monthly deposit $p = ₹\ 240$

Rate of simple interest $r = 6.5\,\%$ p.a.

Period of deposit $n = 3\frac{1}{4}$ years

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$= 39$ months

Interest paid $I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$= 240 \times \frac{39 \times 40}{2 \times 12} \times \frac{6.5}{100}$

$= ₹\,1014$

Total amount deposited $P = p \times n$

$= 240 \times 39$

$= ₹\,9360$

Amount on maturity, $\text{MV} = P + I$

$= 9360 + 1014$

$= ₹\,10374$

71. Mr Shyam Dutt has a Recurring Bank Account of ₹ 2000 per month for a period of $2\frac{1}{2}$ years at a rate of 10% per annum. Find the amount he will receive at the time of maturity.

Sol :

Monthly deposit $= ₹\,2000$

Rate of interest $r = 10\%$ p.a.

Period of deposit $n = 2\frac{1}{2}$ years $=30$ months

Total amount deposited

$P = p \times n$

$= 2000 \times 30$

$= ₹\,60000$

Interest paid $I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$= 2000 \times \frac{30 \times 31}{2 \times 12} \times \frac{10}{100}$

$= ₹\,7750$

Amount due on maturity,

$\text{MV} = P + I$

$= ₹\,(60000 + 7750)$

$= ₹\,67750$

72. A man deposits ₹ 200 per month in a recurring deposit account at 9% p.a and earned a total interest of ₹ 1764. How many installments did he pay?

Sol :

Monthly deposit $p = ₹\,200$

Rate of simple interest $r = 9\%$ p.a.

Total Interest $I = ₹\,1764$

Let the account be held for n months. Therefore interest paid will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$1764 = 200 \times \frac{(n^2+n)}{24} \times \frac{9}{100}$$

$$1764 = \frac{3(n^2+n)}{4}$$

$$n^2 + 3n = 2352$$

$$n^2 + 3n - 2352 = 0$$

$$n^2 + 49n - 48n - 2352 = 0$$

$$(n+49)(n-48) = 0$$

$$n = -49 \text{ or } 48$$

Since -ve value is not possible, we have $n = 48$. Hence, number of months is 48 and number of years is 4.

73. Dajshi deposits ₹ 1000 per month for n months in a bank's R.D. account. If the bank pays the rate of interest 7% annually and the maturity value is ₹ 70675, then find the value of n.

Sol :

Monthly deposit $p = ₹\,1000$

Rate of simple interest $r = 7\,\%$ p.a.

Maturity value $\text{MV} = ₹\,70675$

Let the account be held for n months. Therefore interest will be

$$I = p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 1000 \times \frac{n \times (n+1)}{2 \times 12} \times \frac{7}{100}$$

$$= \frac{35}{12}(n^2+n)$$

Total amount deposited $P = p \times n$

$= 1000n$

Amount due on maturity,

$\text{MV} = P + I$

$= 1000n + \frac{35}{12}(n^2+n)$

But given maturity value is ₹ 70675, therefore

$$70675 = 1000n + \frac{35}{12}(n^2+n)$$

$$14135 = 200n + \frac{7}{12}(n^2+n)$$

$$169620 = 2400n + 7n^2 + 7n$$

NODIA

$$7n^2 + 2407n - 169620 = 0$$

$$(n - 60)(7n + 2827) = 0$$

$$n = 60 \text{ or } -\tfrac{2827}{7}$$

But n is not negative, so rejecting negative value we get $n = 60$. Hence, number of months is 36.

74. Zafarullah has a recurring deposit account in a bank which pays interest at the rate of 9.5% per annum for $3\frac{1}{2}$ years. If he gets ₹78,638 as the maturity value after $3\frac{1}{2}$ years, how much money should he deposit every month in this account?

Sol :

Rate of simple interest $r = 9.5\,\%$ p.a.

Period of deposit $n = 3\frac{1}{2}$ years

$= 42$ months

Maturity value MV = ₹78,638

Let p be the amount required to deposit every month.

Maturity Value,

$$\text{MV} = p \times n + p \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

Substituting all values we have

$$78638 = p \times 42 + p \times \frac{42(42+1)}{2 \times 12} \times \frac{9.5}{100}$$

$$78638 = p \times 42 + p \times \frac{7 \times 43}{4} \times \frac{9.5}{100}$$

$$78638 = 42p + \frac{5719p}{800}$$

$$78638 = \frac{39319p}{800}$$

$$p = \frac{78638 \times 800}{39319} = 1600$$

Thus, Zafarullah should deposit ₹1,600 every month.

END

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CHAPTER 3

SHARE AND DIVIDEND

MULTIPLE CHOICE QUESTIONS

Main 2024

1. The sum invested to purchase 15 shares of a company of nominal value ₹75 available at a discount of 20% is:
(a) ₹60 (b) ₹90
(c) ₹1350 (d) ₹900

Sol :

$$\text{Face Value} = ₹75$$
$$\text{Market Value} = ₹75 - 20\ \%\ \text{of}\ ₹75$$
$$= ₹60$$
$$\text{Investment} = 60 \times 15$$
$$= ₹900$$

Thus (d) is correct option.

SQP 2024

2. 1. Shares of company A, paying 12%, ₹100 shares are at ₹80.
2. Shares of company B, paying 12%, ₹100 shares at ₹100.
3. Shares of company C, paying 12%,₹100 shares are at ₹120.

Shares of which company are at premium?
(a) Company A
(b) Company B
(c) Company C
(d) Company A and C

Sol :

Share of company C are at premium because ₹100 is at ₹120.
Thus option (c) is correct.

3. Total capital of a company is divided into small units of equal value, known as :
(a) Fractions (b) Shares
(c) Units (d) Debentures

Sol :

Total capital of a company is divided into small units of equal value, known as shares.
Thus (b) is correct option.

4. Return earned by shareholder is called :
(a) Interest (b) Dividend
(c) Commission (d) Brokerage

Sol :

Return earned by shareholder is called dividend.
Thus (b) is correct option.

NODIA

Dividend is declared on :
(a) Market value (b) Cash value
(c) Face value (d) Fair value

Sol :

Dividend is declared on face value.
Thus (c) is correct option.

6. Total dividend from a company is also known as:
(a) Annual income
(b) No. of shares
(c) Half yearly income
(d) None of these

Sol :

Total dividend from a company is also known as annual income.
Thus (a) is correct option.

7. Face value is also known as :
(a) Fair value
(b) Nominal value
(c) Market value
(d) None of these

Sol :

Face value is also known as nominal value.
Thus (b) is correct option.

8. Shailja invests ₹9620 on ₹100 shares at ₹80. If the company pays her 18% dividend then her total dividend is :
(a) 2420 (b) 2200
(c) 2120 (d) 2160

Sol :

$$\text{Face value} = ₹100$$

$$\text{No. of shares} = \frac{\text{Investment}}{\text{Market value}} = \frac{9620}{80} = 120$$

Total Dividend

= Rate of dividend× no. of shares × Face Value

$$= \frac{18 \times 120 \times 100}{100}$$

$$= ₹2160$$

Thus (d) is correct option.

9. Sachin buys 1120 shares at ₹52 per share and gets dividend of ₹1456 then rate of return is :

(a) 1.5% (b) 5%
(c) 2.5% (d) 2%

Sol :

This investment is at face value.

$$\text{Investment} = \text{No. of share} \times \text{price of 1 share}$$

$$= 1120 \times 52$$

$$= ₹58240$$

$$\text{Return rate} = \frac{1456}{58240} \times 100 = 2.5\%$$

Thus (c) is correct option.

10. Amount of dividend on 1200 shares of ₹50 each of at the rate of 12% is :

(a) ₹6001 (b) ₹6000
(c) ₹6050 (d) ₹7200

Sol :

Total dividend

= Face value × No. of shares × Rate of dividend

$$= 50 \times 1200 \times \frac{12}{100}$$

$$= ₹7200$$

Thus (d) is correct option.

11. The money required to buy 100, ₹50 shares quoted at ₹48.50 is :

(a) ₹5000 (b) ₹2425
(c) ₹2425 (d) ₹4850

Sol :

$$\text{Quoted Price} = 48.50$$

$$\text{No. of Shares} = 100$$

$$\text{Investment} = 48.50 \times 100$$

$$= ₹4850$$

$$\text{Money requires} = ₹4850.$$

Thus (d) is correct option.

12. Market value of 300, ₹100 shares at a premium of ₹12 is :

(a) 33600 (b) 36300
(c) 36200 (d) 32600

Sol :

$$\text{Market value of 1 share} = 100 + 12 = ₹3112$$

$$\text{Market value of 300 shares} = 300 \times 112$$

$$= ₹33600$$

Thus (a) is correct option.

13. Romesh buys ₹100 shares at ₹15 premium in a company paying 15% dividend then market value of 700 shares is:

(a) ₹75,000 (b) ₹70,000
(c) ₹72,000 (d) ₹80,500

Sol :

$$\text{Face Value} = ₹100$$

$$\text{Market Value} = 100 + 15$$

$$= ₹115$$

Market value of 700 shares

$$= 700 \times 115$$

$$= ₹80500$$

Thus (d) is correct option.

14. Rahman invests ₹8400 on ₹100 shares at ₹70 if the company pays him 10% dividend then numbers of shares he buys is :

(a) 120 (b) 108
(c) 125 (d) 132

Sol :

$$\text{No. of shares} = \frac{\text{Investment}}{\text{Market value}}$$

$$= \frac{8400}{70}$$

$$= 120 \text{ shares.}$$

Thus (a) is correct option.

15. Ruhee invests ₹9620 on ₹100 shares at ₹80 if the company pays him 18% dividend then his percentage return on shares is :

(a) 22.0% (b) 22.5%

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(c) 20.5% (d) 20.6%

Sol :

$$\text{Face Value} = ₹100$$

$$\text{No. of shares} = \frac{\text{Investment}}{\text{Market value}} = \frac{9620}{80} = 120$$

Total Dividend

= Rate of dividend× no. of shares × Face Value

$$= \frac{18 \times 120 \times 100}{100}$$

$$= ₹2160$$

$$\text{Return \%} = \frac{\text{Annual Income}}{\text{Investment}} \times 100$$

$$= \frac{2160}{9620} \times 100$$

$$= 22.5\%$$

Thus (b) is correct option.

16. Rishi invests ₹61,600 in buying shares of nominal value ₹80 at 10% premium. The dividend on shares is 15% per annum then no. of shares he buys :

(a) 600 (b) 650
(c) 750 (d) 700

Sol :

$$\text{Face Value} = ₹80$$

$$\text{Market Value} = ₹80 + 10\ \%\ \text{of}\ 80$$

$$= ₹88$$

$$\text{No. of shares} = \frac{\text{Investment}}{\text{Market value}}$$

$$= \frac{61600}{88} = 700 \text{ shares.}$$

Thus (d) is correct option.

17. Market value of 200, ₹25 shares at a discount of ₹6 :

(a) ₹3000 (b) ₹3800
(c) ₹5000 (d) ₹3400

Sol :

$$\text{Market value of 1 share} = 25 - 6$$

$$= ₹19$$

$$\text{Market value of 160 shares} = 200 \times 19$$

$$= ₹3800$$

Thus (b) is correct option.

18. ₹100 shares of a company are available in the market at a premium of ₹20. What is the rate of dividend given by the company when a man's return on his investment is 15% ?

(a) 17% (b) 16%
(c) 18.5% (d) 18%

Sol :

$$\text{Nominal value} = ₹100$$

$$\text{Market value} = 100 + 20 = ₹120$$

$$\text{Return} = 120 \times \frac{15}{100} = ₹18$$

$$\text{Dividend Rate} = \frac{18}{100} \times 100 = 18\%$$

Thus (d) is correct option.

NODIA

19. Annual income from 850, ₹20 shares paying 8% dividend is :

(a) ₹1870 (b) ₹1360
(c) ₹1780 (d) ₹1630

Sol :

Total Dividend

= No. of shares × Face Value× Rate of dividend

$$= 850 \times 20 \times \frac{8}{100}$$

$$= ₹1360$$

Total dividend is his annual income.
Thus (b) is correct option.

20. By purchasing ₹25 shares from ₹40 each man gets 4% profit on his investment. What is the rate of dividend ?

(a) 6.8 % (b) 4.6 %
(c) 6.4 % (d) 4.8 %

Sol :

$$\text{Face Value} = ₹25$$

$$\text{Market value} = ₹40$$

$$\text{Income on one share} = \frac{4 \times 40}{100} = ₹1.6$$

$$\text{Dividend rate} = \frac{1.6}{25} \times 100 = 6.4\%$$

Thus (c) is correct option.

21. Lalita buys 340, ₹20 shares at a discount of 20% and receives a return of 12% on his money. Calculate the amount invested by her:

(a) ₹5440 (b) ₹6401

(c) ₹4540 (d) ₹6600

Sol :

$$\text{Discount} = 20\% \text{ of Face Value}$$
$$= \frac{20}{100} \times 20 = ₹4$$
$$\text{Market Value} = 20 - 4 = ₹16$$
$$\text{Investment} = \text{No. of shares} \times \text{Face value of 1 share}$$
$$= 340 \times 16$$
$$= ₹5440$$

Thus (a) is correct option.

22. ₹25 shares of a company are selling at ₹20. If the company is paying a dividend of 12% then rate of return is :

(a) 20% (b) 15%
(c) 19% (d) 12%

Sol :

$$\text{Dividend} = 12\% \text{ of } ₹25$$
$$= \frac{12}{100} \times 25 = ₹3$$
$$\text{Rate of return} = \frac{\text{Dividend}}{\text{Investment}} \times 100$$
$$= \frac{3}{20} \times 100 = 15\%$$

Thus (b) is correct option.

23. Rajesh possesses 600 shares of ₹30 of a NBFC. If company gives a dividend of 20% then his annual income is :

(a) ₹2464 (b) ₹3528
(c) ₹2644 (d) ₹3258

Sol :

Total Dividend

= No. of shares × Face Value ×Rate of dividend

$$= 640 \times 35 \times \frac{11}{100}$$
$$= ₹2464$$

Total dividend is his annual income from this share.

Thus (a) is correct option.

24. If Gautam invest ₹10965 on ₹100 shares at a discount of ₹15 then the no. of shares is :

(a) 130 (b) 126
(c) 120 (d) 129

Sol :

$$\text{Market Value} = 100 - 15$$
$$= ₹85$$
$$\text{No. of shares} = \frac{\text{Investment}}{\text{Market value}}$$
$$= \frac{10965}{85} = 129 \text{ shares}$$

Thus (d) is correct option.

25. If Nishant invest ₹17920 on ₹50 shares at premium of 12% then the no. of shares is :

(a) 420 (b) 430
(c) 340 (d) 320

$$\text{Market value} = ₹50 + 12\% \text{ of } ₹50$$
$$= 50 + 6$$
$$= ₹56$$
$$\text{No. of shares} = \frac{\text{investment}}{\text{market value}}$$
$$= \frac{17920}{56} = 320 \text{ shares}$$

Thus (d) is correct option.

26. ₹200 shares of a company are selling at ₹160. If the company is paying a dividend of 20% then rate of return is :

(a) 55% (b) 50%
(c) 60% (d) 25%

Sol :

$$\text{Dividend} = 200 \times \frac{20}{100} = ₹40$$
$$\text{Return Rate} = \frac{40}{160} \times 100 = 25\%$$

Thus (d) is correct option.

27. ₹40 shares of a company are selling at 20% premium. If Nitesh want to buy 480 shares then the investment is :

(a) ₹32400 (b) ₹32040
(c) ₹23040 (d) ₹23400

Sol :

$$\text{Market value} = 40 + 20\% \text{ of } 40$$
$$= 40 + 8$$
$$= ₹48$$
$$\text{Investment} = 480 \times 48$$
$$= ₹23040$$

Thus (c) is correct option.

NODIA

28. Nidhi possesses 550 shares of ₹25 of a finance company. If the company announces a dividend of 8% then her annual income is :
(a) ₹1300 (b) ₹1100
(c) ₹1150 (d) ₹1200

Sol :

Total Dividend

= No. of shares × Face Value ×Rate of dividend

$$= 550 \times 25 \times \frac{8}{100}$$

$$= ₹1100$$

Total dividend is her annual income from this share.
Thus (b) is correct option.

29. Leena invested ₹17100 on 15%, ₹100 shares at premium of 14% than her annual income :
(a) ₹2520 (b) ₹5250
(c) ₹2250 (d) ₹5520

Sol :

$$\text{Market Value} = 100 + 14$$

$$= ₹114$$

$$\text{No. of shares} = \frac{\text{Investment}}{\text{Market value}}$$

$$= \frac{17100}{114} = 150 \text{ shares}$$

Total Dividend

= No. of shares × Face Value ×Rate of dividend

$$= 150 \times 100 \times \frac{15}{100}$$

$$= ₹2250$$

Total dividend is her annual income.
Thus (c) is correct option.

30. Which of the following is the better investment
(I) 16%, ₹100 shares at ₹80
(II) 20%, ₹100 shares at ₹120?
(a) Only I
(b) Both I and II have equal values
(c) Only II
(d) Can't be determined

Sol :

(I) 16%, ₹100 share at ₹80

$$\text{Dividend} = 16\% \text{ of } 100$$

$$= ₹16$$

$$\text{Market Value} = ₹80$$

$$\text{Return\%} = \frac{16}{80} \times 100 = 20\%$$

(II) 20%, ₹100 at ₹120

$$\text{Dividend} = 20\% \text{ of } 100$$

$$= 20$$

$$\text{Market Value} = ₹120$$

$$\text{Rate of return} = \frac{\text{Dividend}}{\text{Investment}} \times 100$$

$$= \frac{20}{120} \times 100 = 16.67\%$$

Thus (a) is correct option.

31. Arjun owns 1120 shares of a company. Face value of each share is ₹35, and dividend is 9%. What is the annual income of Arjun from this share?
(a) ₹3582 (b) ₹3528
(c) ₹3825 (d) ₹3258

Sol :

Total Dividend

= No. of shares × Face Value ×Rate of dividend

$$= 1120 \times 35 \times \frac{9}{100}$$

$$= ₹3528$$

Total dividend is his annual income from this share.
Thus (b) is correct option.

32. Ritesh buys 580 shares at ₹30 per share and gets dividend of ₹870 then rate of return is :
(a) 5.5% (b) 4%
(c) 5% (d) 6%

Sol :

This investment is at face value.

$$\text{Investment} = \text{No. of share} \times \text{price of 1 share}$$

$$= 580 \times 30$$

$$= ₹17400$$

$$\text{Return rate} = \frac{870}{17400} \times 100 = 5\%$$

Thus (c) is correct option.

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THREE MARKS QUESTIONS

Main 2020

33. A company with 500 shares of nominal value ₹ 120 declares and annual dividend of 15%. Calculate.
(i) the total amount of dividend paid by the company.
(ii) annual income of Mr. Sharma who holds 80 shares of the company.
If the return percent of Mr. Sharma from his shares is 10%, find the market value of each share.

Sol :

Nominal value of one share = ₹ 120

Annual dividend = 15%

Total amount of dividend paid by company
= Number of Shares × Nominal Value of share × Rate of dividend

$= 500 \times 120 \times \frac{15}{100}$

= ₹ 9000

Dividend paid on one share $= \frac{9000}{500}$ = ₹ 18

Annual income of Mr. Sharma
= Number of Shares × Dividend on one Share

$= 80 \times 18$

= ₹ 1440

Let ₹ x be the market value of one share.
Since 10 % profit is on market value of share but this profit or dividend is ₹ 18.

Thus $x \times \frac{10}{100} = 18$

x = ₹ 180

(i) Total amount of dividend paid by the company is ₹ 9000
(ii) Annual income of Mr. Sharma is ₹ 1440.
(iii) Market value of each share is ₹ 180

Comp 2020

34. How much cash is obtained by selling :
(i) fifty, ₹20 shares at ₹2.50 premium ?
(ii) one hundred, ₹20 shares at ₹1.75 discount ?
(iii) two hundred fifty, ₹200 shares at par ?

Sol :

(i) fifty, ₹20 shares at ₹2.50 premium
Market value of 1 share = 20 + 2.50

= ₹22.50

Cash obtained by selling 50 shares

$= 22.50 \times 50$

= ₹1125

(ii) one hundred, ₹20 shares at ₹1.75 discount
Market value of 1 share = 20 − 1.75

= ₹18.25

Cash obtained by selling 100 shares

$= 18.25 \times 100$

= ₹1825

(iii) two hundred fifty, ₹200 shares at par
Cash obtained by selling 250 shares of ₹200

$= 250 \times 200$

= ₹50000

Main 2019

35. A man invests ₹4500 in shares of a company which is paying 7.5% dividend.
If ₹ 100 shares are available at a discount of 10%.
Find
(i) Number of shares he purchases.
(ii) His annual income.

Sol :

Total Investment = ₹4500

Rate of dividend = 75%

Nominal value of one share

= ₹ 100 − 10% of 100

= 100 − 10

= ₹ 90

Number of shares, he purchase

$= \frac{4500}{90} = 50$

Annual income
= Number of Shares × Nominal Value of share × Rate of dividend

$= 50 \times 90 \times \frac{7.5}{100}$

= ₹ 337.5

(i) He purchases 50 shares.
(ii) His annual income is ₹ 337.5

Main 2019

36. Sachin invests ₹ 8500 in 10%, ₹ 100 shares at ₹ 170. He sells the shares when the price of each share rises by ₹ 30. He invests the proceeds in 12% ₹ 100 shares at ₹ 125.
Find
(i) the sale proceeds.
(ii) the number of ₹ 125 shares he buys.
(iii) the change in his annual income.

Sol :

Money invested by Sachin = ₹ 8500

NODIA

Nominal value of one share $= ₹\,100$

Market value of one share $= ₹\,170$

Number of shares bought

$$= \frac{\text{Money invested}}{\text{Market value of one share}}$$

$$= ₹\,\frac{8500}{170} = 50$$

Dividend on one share

Dividend = 10% of Nominal Value

$$= ₹\,10$$

Total annual dividend

= Number of Shares × dividend on one share

$$= 50 \times 10$$

$$= ₹\,500$$

Thus his annual income is ₹500.

Market Price of share $= 170 + 300 = ₹200$

(i) Sale proceeds.

Now he sell 50 shares at market value ₹200.

Sale proceed is the amount received on selling 50 shares.

Sale Proceed = Number of shares × Market value

$$= 50 \times 200$$

$$= ₹\,10000$$

(ii) Number of ₹ 125 shares he buys.

Now he invests the ₹10000 in 12% ₹100 shares at ₹ 125.

Number of shares bought

$$= \frac{\text{Money invested}}{\text{Market value of one share}}$$

$$= \frac{10000}{125} = 80$$

(iii) Change in his annual income

Dividend on one share

Dividend = 12% of Nominal Value

$$= ₹\,12$$

Total annual dividend

= Number of Shares × dividend on one share

$$= 80 \times 12$$

$$= ₹\,960$$

Now his annual income is ₹960.

Change in his annual income

$$= 960 - 500$$

$$= ₹\,460$$

Comp 2019

37. Amit sold some ₹20 shares paying 8% dividend at ₹18 and invested the proceeds in ₹10 shares paying 12% dividend at 50% premium. If the change in his annual income is ₹360, find the number of shares sold by Mr Amit.

Sol :

Let x be the number of shares sold by Amit.

His annual income for x shares paying 8% dividend

$$= \frac{8}{100} \times 20 \times x$$

$$= \frac{8}{5}x$$

As he sold x shares at ₹18 each, his proceeds from share will be $18x$. Now, he invested $18x$ in ₹10 shares, paying 12% dividend, at 50% premium. Here ₹10 shares at 50% premium means market value of one share is ₹15.

Number of share Purchased $= \frac{18x}{15} = \frac{6}{5}x$

$$\text{His annual income} = \frac{12}{100} \times 10 \times \frac{6x}{5} = \frac{36}{25}x$$

Since change in income is ₹360, therefore

$$\frac{8}{5}x - \frac{36}{25}x = 360$$

$$40x - 36x = 360 \times 25$$

$$4x = 9000$$

$$x = 2250$$

Thus Amit hold 2250 shares.

Main 2018

38. A man invests ₹22500 in ₹50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate

(i) then number of shares purchased.

(ii) the annual dividend received.

(iii) The rate of return he gets on his investment. Given your answer correct to the nearest whole number.

Sol :

(i) Number of shares purchased.

Since the man invests in ₹50 shares available at 10% discount, the market value of one share,

$$= 90\ \%\ \text{of}\ 50$$

$$= ₹45$$

As the investment is ₹22500, the number of shares purchased

$$= \frac{22500}{45} = 500$$

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(ii) Annual dividend received

= number of shares × rate of dividend × face value of one share

$= 500 \times \frac{12}{100} \times 50$

$= 5 \times 12 \times 50 = ₹3000.$

(iii) Rate of return on his investment.

₹3000 can be considered as income on ₹22500,

$$\text{Rate of return} = \frac{\text{Dividend Amount}}{\text{Investment}} \times 100$$

$$= \frac{3000}{22500} \times 100$$

$$= \frac{40}{3}\% = 13\frac{1}{3}\%$$

Hence, the rate of return on his investment is 13% (nearest to the whole number).

Comp 2018

39. Mr. Sharma receives an annual income of ₹900 in buying 50 shares selling at ₹80. If the dividend declared is 20%, find the :

(a) amount invested by Mr. Sharma.

(b) percentage return on his investment.

Sol :

Annual income of Mr. Sharma = ₹900

No. of shares bought = 50

Face value of one share = ₹80

Dividend declared = 20%

(a) Amount invested by Mr. Sharma.

Total investment

= No. of Shares × Market Value of 1 share

$= 50 \times 80 = ₹4{,}000$

(b) Percentage return on investment

Rate of income on investment

$$= \frac{\text{dividend}}{\text{investment}} \times 100$$

$$= \frac{900}{4000} \times 100$$

$$= \frac{90}{4}$$

$$= 22.5\,\%$$

Main 2017

40. How much should a man invest in ₹50 shares selling at ₹60 to obtain an income of ₹450, if the rate of dividend declared is 10%. Also, find his yield percent, to the nearest whole number.

Sol :

Rate of dividend = 10%

Dividend on 1 share of ₹50 = 10% of ₹50

= ₹5

$$\text{Number of share bought} = \frac{\text{Total dividend}}{\text{Dividend on 1 Share}}$$

$$= \frac{450}{5} = 90$$

Market value of 1 share = ₹60

Total investment = 60 × 90

= ₹5400

Thus an investment of ₹5400 gives an income of ₹4500.

Now, $$\text{Yield} = \frac{450}{5400} \times 100$$

$$= 8.33\% \approx 8\%$$

Comp 2017

41. ₹ 100 shares of a company are sold at 15 % discount. If the return on investment is 20 %, find the rate of dividend declared.

Sol :

Let x be the rate of dividend declared.

Face value of a share = ₹ 100

Market value of a share = 100 − 85 = ₹ 85

Let x be the rate of dividend on share Thus dividend on a ₹ 100 share will be ₹ x.

Thus, on the investment of ₹ 85, dividend received is ₹ x.

$$\text{Return percentage} = \frac{x}{85} \times 100$$

But as per question the return percentage is 20%, thus

$$\frac{x}{85} \times 100 = 20 \Rightarrow x = 17$$

Hence, the rate of dividend declared is 17 %.

ALTERNATIVE :

Shares are sold at 15% discount and the return on investment is 20%.

Therefore, investment of ₹ 100 yields income of ₹ 20

Investment of ₹ 85 will yield income of $\frac{20}{100} \times 85$ = ₹ 17

Hence, the rate of dividend declared is 17 %.

Main 2016

42. Ashok invested ₹26400 on 12%, ₹25 shares of a company. If he receives a dividend of ₹2475, then find the

(i) number of shares he bought.

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(ii) market value of each share.

Sol :

$$\text{Annual income from 1 share} = 12\% \text{ of } ₹25$$

$$= \frac{12}{100} \times 25$$

$$= ₹3$$

$$\text{Total income} = ₹2575$$

(i) Number of share bought by Ashok

$$\text{Number of share} = \frac{\text{Total Income}}{\text{Income From 1 Share}}$$

$$= \frac{2475}{3} = 825$$

(ii) Market value of each share

$$\text{Market value} = \frac{\text{Total Investment}}{\text{Number of Share Bought}}$$

$$= \frac{26400}{825} = Rs.\,32$$

Comp 2016

43. Share of Vegii Foods is sold at premium of ₹ 25. The face value of a share is ₹ 100. Now answer the following :

(i) Find the amount required by Vedant to purchase 5000 shares.

(ii) What would be the gain of the original share holder from Vedant if he had bought each share at 10% discount?

Sol :

Share of face value ₹ 100 is sold at premium of ₹ 25, it means that market price of share is ₹ 125.

$$\text{Market value of 5000 shares} = 125 \times 5000$$

$$= ₹\,625000$$

The original share holder bought shares at 10% discount i.e. he had bought each share for ₹ 90.

Amount paid by the original share holder

$$= 90 \times 5000 = ₹\,450000$$

Gain to the original share holder

$$= 625000 - 450000$$

$$= ₹\,175000$$

Also, $$\text{Gain percent} = \frac{175000}{450000} \times 100$$

$$= 38.88$$

(i) Thus, amount paid by Vedant to buy 5000 shares is ₹ 156250.

(ii) Gain is 38.33 %.

Main 2014

44. Salman invest a sum of money in ₹50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is ₹600, calculate

(i) the number of shares, he bought

(ii) his total investment.

(ii) the rate of return on his investment

Sol :

$$\text{Nominal value of one share} = ₹50$$

$$\text{Dividend on one share} = \frac{15}{100} \times 50$$

$$= ₹750$$

Total dividend that Salman got from the company

$$= ₹600$$

(i) Number of shares Salman bought

$$\text{Number of shares} = \frac{\text{Total Dividend Amount}}{\text{Dividend on one Share}}$$

$$= \frac{600}{7.50} = 80$$

(ii) His total investment

$$\text{Premium on one share} = 20\,\%\ ₹50$$

$$= \frac{20}{100} \times 50$$

$$= ₹10$$

$$\text{Market value of one share} = 50 + 10$$

$$= ₹60$$

$$\text{Total investment for 80 shares} = 80 \times 60$$

$$= ₹4800$$

(iii) Rate of return on his investment

$$\text{Rate of Return} = \frac{\text{Total Dividend Amount}}{\text{Total Investment}} \times 100$$

$$= \frac{600}{4800} \times 100 = 12.5\%$$

Comp 2014

45. Ramesh receives an annual income of ₹1350 in buying 60 shares selling at ₹120. If the dividend declared is 20%, find the :

(a) amount invested by Ramesh.

(b) percentage return on his investment.

Sol :

$$\text{Annual income of Ramesh} = ₹1350$$

$$\text{No. of shares bought} = 60$$

$$\text{Face value of one share} = ₹120$$

$$\text{Dividend declared} = 15\%$$

(a) Amount invested by Ramesh.

Total investment

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$= \text{No. of Shares} \times \text{Market Value of 1 share}$

$= 60 \times 120 = ₹7{,}200$

(b) Percentage return on investment

Rate of income on investment

$$= \frac{\text{dividend}}{\text{investment}} \times 100$$

$$= \frac{1350}{7200} \times 100$$

$$= \frac{3}{16}$$

$$= 18.75\,\%$$

Main 2012

46. A man invests ₹9,600 ₹100 shares at ₹80. if the company pays him 18% dividend, find

(i) the number of shares, he buys,

(ii) his total dividend.

(iii) his percentage return on the shares.

Sol :

$$\text{Money invested} = ₹9{,}600$$

and $$\text{Nominal value} = ₹100$$

$$\text{Market value} = ₹80$$

and $$\text{rate of dividend} = 18\%$$

(i) Number of shares

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{9600}{80} = 120$$

(ii) Total dividend (annual income)

$= \text{Number of shares} \times \text{Rate of dividend} \times \text{Nominal value}$

$= 120 \times 18\% \times 100$

$$= 120 \times \frac{18}{100} \times 100$$

$= ₹2160$

(iii) His percentage return on the shares.

$$\text{Return percentage} = \frac{\text{Total Dividend}}{\text{Total Investment}} \times 100$$

$$= \frac{2160}{9600} \times 100$$

$$= 22.5\%$$

Comp 2012

47. Reena invested ₹12,000 and purchased 1000 shares of par value ₹ 10 each. Answer the following :

(i) How much above par is the market value of these shares ?

(ii) If the dividend per share is 15%, find Hina's income from these shares.

Sol :

Reenaa invested ₹12,000 and purchased 1000 shares of par value ₹ 10 each.

$$\text{Market value of 1000 shares} = ₹12000$$

$$\text{Market value of one share} = \frac{12000}{1000}$$

$$= ₹12$$

$$\text{Face value of one share} = ₹10$$

(i) Market value is ₹2 above par.

$$\text{Face value of 1000 shares} = ₹(1000 \times 10)$$

$$= ₹10000$$

Since the dividend is paid at the rate of 12%. Therefore,

$$\text{Dividend on 1000 shares} = \frac{12}{100} \times 10000$$

$$= ₹1200$$

(ii) Reena's income from 1000 shares is ₹1200.

Main 2011

48. Mr. Prakash invested ₹52,000 on ₹100 shares at a discount of ₹20 paying 8% dividend. At the end of year, he sell the shares at a premium of ₹20. Find

(i) the annual dividend.

(ii) the profit earned including his dividend.

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Sol :

$$\text{Money invested by Mr. Prakash} = ₹52{,}000$$

$$\text{Nominal value of one share} = ₹100$$

$$\text{Market value of one share} = 100 - 20$$

$$= ₹80$$

and $$\text{dividend} = 8\%$$

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{52000}{80} = 650$$

(i)Annual dividend

$= \text{Number of shares} \times \text{Rate of dividend} \times \text{Face value of one share}$

$$= \frac{8}{100} \times 650 \times 100$$

$= ₹5200$

(ii) Profit earned including his dividend

Since Mr. Prakash sells his shares at a premium of ₹20, selling price of one share

$= 100 + 20$

$= ₹120$

The selling value of his 650 shares,

$=$ Number of shares $\times$ Selling price of one share

$= 650 \times 120$

$= ₹78000$

Profit Earned,

$=$ Total Selling Price + Annual Dividend $-$ Money invested

$= 78000 + 5200 - 52000$

$= ₹31200$

Comp 2011

49. A man invests ₹3960 in shares of a company which pays 15% dividend at a time when a ₹25 share costs ₹33. Find :

(i) the number of shares he bought.

(ii) the annual income from his shares.

(iii) the rate of interest which he gets on his investment.

Sol :

(i) Number of shares he bought

Since the market value of one share is ₹33 and the money invested is ₹3960,

$$\text{the number of shares bought} = \frac{3960}{33} = 120.$$

(ii) Annual income

$=$ number of shares $\times$ rate of dividend $\times$ face value of one share

$$= 120 \times \frac{15}{100} \times 25 = ₹450$$

(iii) Rate of interest on investment

₹450 can be considered as the interest on ₹3960 for one year,

$$\text{the rate of interest} = \frac{450}{3960} \times 100 = \frac{125}{11}\% = 11\frac{4}{11}\%$$

Main 2009

50. Amit Kumar invests ₹36000 in buying ₹100 shares at ₹20 premium. The dividend is 15% per annum. Find

(i) the number of shares, he buys.

(ii) his yearly dividend.

(iii) the percentage return on his investment.

Given your answer correct to the nearest whole number.

Sol :

Nominal (face) value of a share $= ₹100$

At 20% premium, market value of a share,

$= 100 + 20$

$= ₹120$

Money invested $= ₹36{,}000$

and Rate of divided $= 15\%$

(i) Number of shares buys

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}} = \frac{36000}{120} = 300$$

(ii) Yearly dividend.

Divided on a share $= 15\%$ of 100

$$= \frac{15}{100} \times 100 = ₹15$$

Divided on 300 shares $= 15 \times 300 = ₹4500$

(ii)Return percentage

$$= \frac{\text{Total Dividend Amount}}{\text{Total Investment}} \times 100 = \frac{4500}{36000} \times 100 = 12.5\%$$

Comp 2009

51. How much should a man invest in ₹50 shares selling at ₹60 to obtain an income off ₹450, if the rate of dividend declared is 10%. Also find his yield percent, to the nearest whole number.

Sol :

Dividend on one share of ₹50,

$= 10\%$ of 50

$$= \frac{10}{100} \times 50 = ₹5$$

Since the total income is ₹450, the number of shares bought

$$= \frac{\text{annual income}}{\text{income on one share}} = \frac{450}{5} = 90$$

Since the market value of one share $= ₹60$,

Investment $=$ Number of shares $\times$ Market value of one share

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$= 60 \times 90 = ₹5400$

Yield percent i.e. rate of return

$$= \frac{\text{Annual Income}}{\text{Investment}} \times 100$$

$$= \frac{450}{5400} \times 100 = \frac{450}{54}\%$$

$$= \frac{25}{3}\% = 8\frac{1}{3}\%$$

$= 8\%$, nearest to the whole number.

Main 2007

52. Ajay owns 560 shares of a company. The face value of each share is ₹25. The company declares a dividend of 9%. Calculate

(i) the dividend that Ajay will get.

(ii) the rate interest on his investment, if Ajay had paid ₹30 for each share.

Sol :

Number of shares owns by Ajay $= 560$

Face (nominal) value of a share $= ₹25$

Rate of divided $= 9\%$

$$\text{Dividend on a share} = \frac{9}{100} \times 25$$

$$= ₹2.25$$

(i) Dividend that Ajay will get

$$\text{Dividend on 560 shares} = 2.25 \times 560$$

$$= ₹1260$$

(ii)Rate interest on his investment

Since Ajay paid ₹30 for each share, then

$$\text{Total investment} = 560 \times 30$$

$$= ₹16800$$

$$\text{Rate of interest} = \frac{\text{Total Dividend Amount}}{\text{Total Investment}} \times 100$$

$$= \frac{1260}{16800} \times 100$$

$$= 7.5\%$$

Comp 2007

53. A dividend of 9% was declared on ₹100 shares selling at a certain price. If the rate of return is $7\frac{1}{2}\%$, calculate :

(i) the market value of the share.

(ii) the amount to be invested to obtain an annual dividend of ₹630.

Sol :

Dividend on one share of ₹100 = ₹9

(i) the market value of the share.

Let x be the market value of one share.

$$\text{Profit on one share} = 7\tfrac{1}{2}\% \text{ of } x$$

$$= \frac{15}{2} \times \frac{1}{100} \times x$$

$$= \frac{3x}{40}$$

Since the dividend paid on one share is ₹9,

$$\frac{3x}{40} = 9$$

$$x = 120$$

Thus market value of each share is ₹120.

(ii) Amount to be invested to obtain an annual dividend of ₹630

As the total income is ₹630, the number of shares bought

$$= \frac{630}{9} = 70$$

Market value of each share = ₹120,

Amount to be invested $= 120 \times 70 = ₹8400$

Main 2005

54. Mr. Tiwari invested ₹29040 in 15% ₹100 shares quoted at a premium of 20%. Calculate

(i) the number of shares bought by Mr. Tiwari.

(ii) Mr. Tiwari's income from the investment.

(iii) the percentage return on his investment.

Sol :

Money invested by Mr. Tiwari = ₹29,040

Rate of dividend = 15%

and nominal valu of a share = ₹100

Rate of premium of a share = 20%

(i) Number of shares bought by Mr. Tiwari.

$$\text{Market value of one share} = 100 + \frac{20}{100} \times 100$$

$$= ₹120$$

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{29040}{120} = 242$$

(ii) Mr. Tiwari's income from the investment.

Total dividend

$$= \text{Number of shares} \times \text{Rate of dividend} \times \text{Nominal value}$$

$$= 242 \times 15\% \times 100$$

$$= \frac{242 \times 15 \times 100}{100}$$

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$= ₹3630$

(iii)Percentage return on his investment

$$\text{Return percentage} = \frac{\text{Total Dividend}}{\text{Total Investment}} \times 100$$

$$= \frac{3630}{29040} \times 100$$

$$= 12.5\%$$

Comp 2005

55. A man sold 400 ₹20 shares paying 5% at ₹18 and invested the proceeds in ₹10 shares, paying 7% at ₹12. How many ₹10 shares did he buy and what was the change in income?

Sol :

Selling price of 400 (₹20) shares at ₹18

$$= 18 \times 400$$

$$= ₹7200$$

Market price of ₹10 share = ₹12,
the number of ₹10 shares purchased

$$= \frac{7200}{12} = 600$$

Annual income from original (₹20) shares

= Number of shares × Rate of dividend × Face value of one share

$$= 400 \times \frac{5}{100} \times 20 = ₹400$$

Annual income from new (₹10) shares

= Number of shares × Rate of dividend × Face value of one share

$$= 600 \times \frac{7}{100} \times 10 = ₹420$$

Change in annual income = 420 − 400 = ₹20
Thus ₹20 is increase in income.

Comp 2002

56. Rajat invests ₹90,000 in 8%, ₹100 shares at ₹120. He sell the shares, when the price rises to ₹150 and invests the proceeds in 12% ₹100 shares at ₹125. Calculate

(i) the sale proceeds.

(ii) the number of ₹125 shares, he buys.

(iii) the change in his annual income from dividend.

Sol :

Money invested by Vivek = ₹ 90,000

Nominal value of a share = ₹ 100

Market value of one share = ₹120

Number of shares he bought,

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{90000}{120} = 750$$

(i) Sale proceeds

Sale proceed is the amount received on selling 750 shares = 750 × 150 = ₹112,500

(ii) Number of ₹ 125 shares, he buys.

Since he bought a share at ₹ 125, therefore number of shares he bought $= \frac{112500}{125} = 900$

(iii) Change in annual income

$$= \{12\% \text{ of } (900 \times 100)\} - \{8\% \text{ of } (750 \times 100)\}$$

$$= 10800 - 6000$$

$$= ₹4800$$

Main 2001

57. A man invests ₹8800 on buying shares of face value of 100 each at a premium of 10% in a company. If he earns ₹1200 at the end of the year as dividend, find

(i) the number of shares, he has in the company.

(ii) the dividend percentage per share.

Sol :

Money invested = ₹8,800

Nominal (face) value of share = ₹100

Rate of premium of a share = 10%

and dividend earned in a year = ₹1200

Market value of a share $= (1 + \frac{10}{100}) \times 100$

= ₹110

(i) Number of shares, he has in the company.

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{8800}{110} = 80$$

(ii) Dividend percentage per share.

Total nominal Value

= Number of shares × Face price of one share

$= 100 \times 80 = 8000$

$$\text{Dividend } \% = \frac{\text{Dividend}}{\text{Total Nominal value}} \times 100$$

$$= \frac{1200}{8000} \times 100 = 15\%$$

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FOUR MARK QUESTIONS

Main 2015

58. Rohit invested ₹9600 on ₹100 shares at ₹20 premium paying 8% dividend. Rohit sold the shares when the price rose to ₹160. He invested the proceeds (excluding dividend) in 10% ₹50 shares at ₹40. Find the :

(i) original number of shares.
(ii) sale proceeds.
(iii) new number of shares.
(iv) change in the two dividends.

Sol :

(i) Original number of shares

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{9600}{120} = 80$$

(ii) Sale proceeds

= Number of shares × Selling price of one share

$= 80 \times 160$

= ₹12800

(iii) New number of shares

$$\text{New number of shares} = \frac{12800}{40}$$

$$= 320$$

(iv) Change in the two dividends

Changes in two dividends

$= \{10\% \text{ of } (320 \times 50)\} - \{8\% \text{ of } (80 \times 100)\}$

$= 1600 - 640$

= ₹960

Comp 2015

59. Mr. Lohia invests ₹26680 in buying ₹50 shares at a discount of 8%. He sells shares worth ₹15000 at a premium of 6% and the rest at a discount of 10%. Find his total gain or loss from the transaction.

Sol :

Since Mr. Lohia buys shares at a discount of 8%,
Market value of one share,

$$= \left(1 - \frac{8}{100}\right) \text{ of } 50 = ₹46.$$

Number of shares purchased

$$= \frac{26680}{46} = 580$$

Number of shares worth (face value) ₹15000

$$= \frac{15000}{50} = 300$$

He sold 300 shares at a premium of 6%,

Market value of one share

$$= \left(1 + \frac{6}{100}\right) \text{ of } 50 = ₹53$$

Selling value of 300 shares at ₹53 each

$$= 300 \times 53 = ₹15900$$

The number of remaining shares

$$= 580 - 300 = 280$$

Lohia sold 280 shares at discount of 10%,
Market value of one share

$$= \left(1 - \frac{10}{100}\right) \text{ of } 50 = ₹45$$

The selling value of 280 shares at ₹45 each

$$= 280 \times 45 = ₹12600$$

$$\text{Total selling value} = 15900 + 12600$$

$$= ₹28500$$

$$\text{Lohia's total gain} = 28500 - 26680$$

$$= ₹1820$$

Main 2013

60. Salman buys 50 shares of face value ₹100 available at 132.

(i) What is his investment?
(ii) If the dividend is 7.5%, what will be his annual income?
(iii) If he wants to increases his annual income by ₹150, how many extra shares should he buy?

Sol :

$$\text{Number of shares} = 50$$

$$\text{Face value of each share} = ₹100$$

and $$\text{market value of each share} = ₹132$$

(i) What is his investment?

$$\text{Salman's investments} = 132 \times 50$$

$$= ₹6600$$

(ii) Salman's annual income

$$\text{Rate of dividend} = 7.5\%$$

Annual Income

= Number of shares × Rate of Dividend × face value of one share

$= 50 \times \frac{7.5}{100} \times 100$

= ₹375

(iii) How many extra shares should he buy?

Annual income from one share = 7.5% = ₹7.5

Shares required for extra ₹150 income $= \frac{150}{7.5} = 20$

Hence, the number of extra shares, he buys, is 20.

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Comp 2013

61. Neelkamal has invested ₹54000 partly in 6% ₹100 shares at ₹140 in a automobile company and partly in 5% ₹100 shares at ₹125 in a food sector company. If his total income is ₹2240, how much has he invested in each?

Sol :

Let x be the investment of the Neelkamal in 6% ₹100 shares at ₹140 and y be his investment in 5% ₹100 shares at ₹125.

Income on one share of ₹140 = 6% of 100

$= ₹6$

Income on $x = \frac{6}{140}x = \frac{3}{70}x$

Income on one share of ₹125 = 5% of 100

$= ₹5$

Income on $y = \frac{5}{125}y$

$= \frac{1}{25}y$

But the total income of the man is ₹2240, thus

$$\frac{3}{70}x + \frac{1}{25}y = 2240$$

$$15x + 14y = 350 \times 2240$$

$$15x + 14y = 784{,}000 \qquad (1)$$

His total investment is ₹54000, thus

$$x + y = 54000 \qquad (2)$$

$$14x + 14y = 756{,}000 \qquad (3)$$

Subtracting eq (3) from eq (1) we have

$x = 28{,}000$

Substituting this result in eq (2) we have

$y = 54000 - x$

$= 54000 - 28000$

$= ₹26000$

Investment in 6% shares at ₹140 is ₹28000 and investment in 5% shares at ₹125 is ₹26000

Main 2010

62. Vivek invests ₹4500 in 8%, ₹10 shares at ₹15. He sell the shares, when the price rises to ₹30 and invests the proceeds in 12% ₹100 shares at ₹125. Calculate

(i) the sale proceeds.

(ii) the number of ₹125 shares, he buys.

(iii) the change in his annual income from dividend.

Sol :

Money invested by Vivek = ₹ 4500

Nominal value of a share = ₹ 10

Market value of one share = ₹15

Number of shares he bought,

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{4500}{15} = 300$$

(i) Sale proceeds

Sale proceed is the amount received on selling 300 shares $= 300 \times 30 =$ ₹ 9000

(ii) Number of ₹ 125 shares, he buys.

Since he bought a share at ₹ 125, therefore number of shares he bought $= \frac{9000}{125} = 72$

(iii) Change in annual income

$= \{12\% \text{ of } (72 \times 100)\} - \{8\% \text{ of } (300 \times 10)\}$

$= 864 - 240$

$= ₹ 624$

Comp 2010

63. A man invests ₹22500 in ₹50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate:

(i) the number of shares purchased.

(ii) the annual dividend received.

(iii) the rate of return he gets on his investment. Give your answer correct to the whole number.

Sol :

(i) Number of shares purchased.

Since the man invests in ₹50 shares available at 10% discount, the market value of one share

$= \left(1 - \frac{10}{100}\right)$ of ₹50

$= \left(50 \times \frac{9}{10}\right) = ₹45$

As the investment is ₹22500, the number of shares purchased

$= \frac{22500}{45} = 500$

(ii) Annual dividend received

= number of shares × rate of dividend × face value of one share

$= 500 \times \frac{12}{100} \times 50$

$= 5 \times 12 \times 50 = ₹3000.$

(iii) Rate of return on his investment.

₹3000 can be considered as income on ₹22500,

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$$\text{Rate of return} = \frac{3000}{22500} \times 100$$

$$= \frac{40}{3}\% = 13\frac{1}{3}\%$$

Hence, the rate of return on his investment
= 13% (nearest to the whole number).

Main 2008

64. A company with 4000 shares of nominal value of ₹110 each declares on annual dividend of 15%. Calculate

(i) the total amount of dividend paid by the company.

(ii) the annual income of Shahrukh, who holds 88 shares in the company.

(iii) if he received only 10% on his investment, find the price Shahrukh paid for each share.

Sol :

Given, Number of shares = 4000

Nominal value of share = ₹ 110

and Rate of dividend = 15%

(i) Total amount of dividend paid by the company.

Total dividend

= Number of shares × Rate of dividend × Face value of one share

$$= 4000 \times \frac{15}{100} \times 110$$

= ₹ 6600

(ii) Annual income of Shahrukh,
Annual income of Shahrukh on 88 shares,

$$= 88 \times \frac{15}{100} \times 110$$

= ₹ 1452

(iii) Price Shahrukh paid for each share.
Let x be the Shahrukh's investment. According to the question, 10% of money invested is the annual income of Sharhrukh, thus

$$10\% \text{ of } x = 1452$$

$$\frac{10x}{100} = 1452$$

$$x = 1452 \times 10$$

= ₹14520

The price Shahrukh paid for each share,

$$= \frac{14520}{88} = ₹165$$

Comp 2008

65. A man invests ₹13500 partly in 6% ₹100 shares at ₹140 and partly in 5% ₹100 shares at ₹125. If his total income is ₹560, how much has he invested in each?

Sol :

Let the investment of the man in 6% ₹100 shares at ₹140 be ₹x, then his investment in 5% ₹100 shares at ₹125 = 13500 − x

Income on one share of ₹140 = 6% of 100

= ₹6

$$\text{Income on } x = \frac{6}{140}x = \frac{3}{70}x$$

Income on one share of ₹125 = 5% of 100

= ₹5

$$\text{Income on } (13500 - x) = \frac{5}{125}(13500 - x)$$

$$= \frac{1}{25}(13500 - x)$$

But the total income of the man is ₹560, thus

$$\frac{3}{70}x + \frac{1}{25}(13500 - x) = 560$$

$$15x + 14\,(13500 - x) = 350 \times 560$$

$$15x - 14x = 350 \times 560 - 14 \times 13500$$

$$x = 7000$$

$$13500 - x = 13500 - 7000 = 6500$$

Investment in 6% shares at ₹140 is ₹7000 and investment in 5% shares at ₹125 is ₹6500

Main 2006

66. Mr. Ram Gopal invested ₹8000 in 7% ₹100 shares at ₹80. After a year, he sold these shares at ₹75 each and invested the proceeds in 18% at ₹25 shares at ₹41. Find

(i) his dividend for the first year.

(ii) his annual income from the second investment.

(iii) the percentage of increases in return on his original investment.

Sol :

Money invested by Mr. Ram Gopal = ₹8000

Market value of one share = ₹80

Rate of dividend = 7%

(i) His dividend for the first year.
Number of shares, he buys

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{8000}{80}$$

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$= 100$

Dividend of one share $= 7\%$ of ₹100

$= ₹7$

Hence, Mr. Ram Gopal got a dividend $100 \times 7 =$ ₹700 in the first year

(ii) His annual income from the second investment.

Selling Price of one Share $= ₹75$

Selling price of 100 shares $= 75 \times 100$

$= ₹7500$

Total amount, he gets $= 7500 + 700$

$= ₹8200$

Now, further he invests this money again.

Total amount of money invested $= ₹8200$

Market value $= ₹41$

Nominal value $= ₹25$

Rate of dividend $= 18\%$

Number of shares, he bought,

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{8200}{41} = 200$$

Total nominal value $= 200 \times 25$

$= ₹5000$

Dividend he get

Dividend $= 18\% \times$ Total nominal value

$$= \frac{18}{100} \times 5000$$

$= ₹900$

Hence, annual income in second year is ₹900.

(iii) Percentage of increases in return on his original investment.

Increase in amount from first to second year

$= 900 - 700$

$= ₹200$

Percentage increase in return

$$= \frac{\text{Increases Amount}}{\text{Initial investment}} \times 100$$

$$= \frac{200}{8000} \times 100$$

$= 2.5\%$

Comp 2006

67. A person invested 20%, 30% and 25% of his savings in buying shares of three different companies A, B and C which declared dividends of 10%, 12% and 15% respectively. If his total income on account of dividends be ₹2337.50, find his savings and the amount which he invested in buying share of each company.

Sol :

Let total savings be ₹100.

Money invested in company $A = ₹20$

Money invested in company $B = ₹30$

Money invested in company $C = ₹25$

Then,

$$\text{Income from company } A = \frac{10}{100} \times 20 = ₹2$$

$$\text{Income from company } B = \frac{12}{100} \times 30 = ₹3.60$$

$$\text{Income from company } C = \frac{15}{100} \times 25 = ₹3.75$$

Total income on saving of ₹100,

$= 2 + 3.60 + 3.75$

$= ₹9.35$

Now, if income is ₹9.35, then savings = ₹100

If income is ₹2337.50, then savings

$$= \frac{100 \times 2337.50}{9.35}$$

$= ₹25000$

$$\text{Money invested in company } A = \frac{25000 \times 20}{100}$$

$= ₹5000$

$$\text{Money invested in company } B = \frac{25000 \times 30}{100}$$

$= ₹7500$

$$\text{Money invested in company } C = \frac{25000 \times 25}{100}$$

$= ₹6250$

Thus, amount invested in company A, B and C respectively are ₹5000, ₹7500 and ₹6250.

Main 2004

68. A man invested ₹45000 in 15% ₹100 shares quoted at ₹125. When the market value of these shares rose to ₹140, he sold some shares, just enough to raise ₹8400. Calculate

(i) the number of shares, he still holds.

(ii) the dividend due to him on these remaining shares.

Sol :

Money invested by man $= ₹45000$

Rate of divided $= 15\%$

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Nominal value of a share = ₹100

Market value of a share = ₹124

Number of shares, he bought,

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{45000}{125} = 360$$

(i) Number of shares, he still holds.

When the market value of a share rose to ₹140, he sold some shares upto the amount of ₹8400.

$$\text{Number o shares he hold} = \frac{8400}{140} = 60$$

$$\text{Number of shares, he still holds} = 360 - 60$$

$$= 300$$

(ii) Dividend on these remaining shares.

Dividend on the remaining shares

$$= 300 \times \frac{15}{100} \times 100$$

$$= ₹4500$$

Comp 2004

69. Suresh has a choice to invest in shares of two companies A and B. ₹100 shares of company A are available at 10% premium and it pays 8% dividend whereas ₹50 shares of company B are available at 12% discount and it pays 7% dividend. If he invests equally in both the companies and the sum of his annual incomes from them is ₹1340, find how much, in all, does he invest?

Sol :

Let Suresh invest ₹x in each company.

For company A

Face value of each share = ₹100,

$$\text{Market value of each share} = \left(1 + \frac{10}{100}\right) \text{ of } ₹100$$

$$= ₹110$$

$$\text{Number of shares bought} = \frac{x}{110}$$

Annual income= No. of shares × Rate of dividend × Face Value of one share

$$= \frac{x}{110} \times \frac{8}{100} \times 100 = ₹\frac{4x}{55}$$

For company B

Face value of each share = ₹50

$$\text{Market value of each share} = \left(1 - \frac{12}{100}\right) \text{ of } 50$$

$$= ₹44$$

$$\text{The number of shares bought} = \frac{x}{44}$$

Annual income = Number of shares × Rate of dividend × F.V. of one share

$$= \frac{x}{44} \times \frac{7}{100} \times 50 = \frac{7x}{88}$$

Sum of annual income from both companies

$$= \left(\frac{4x}{55} + \frac{7x}{88}\right) = \left(\frac{4}{5} + \frac{7}{8}\right) \times \frac{x}{11}$$

$$= \frac{32 + 35}{40} \times \frac{x}{11} = \frac{67x}{440}$$

According to given, we have

$$\frac{67x}{440} = 1340$$

$$x = 8800$$

Suresh invests ₹8800 in each company.

Total investment of Suresh = 8800 + 8800

= ₹17600

Main 2002

70. A man wants to buy 62 shares available at ₹132 (par value of ₹100).

(i) How much should he invest?

(ii) If the dividend is 7.5%, what will be his annual income?

(iii) If he wants to increase his annual income by ₹150, how many extra shares should he buy?

Sol :

Number of shares = 62

Face value of each share = ₹100

and Market value of each share = ₹132

(i) How much should he invest?

Investments Requirement = 132 × 62

= ₹8184

(ii) Annual income

Rate of dividend = 7.5%

Annual Income

= Number of shares × Rate of Dividend × face value of one share

$= 62 \times \frac{7.5}{100} \times 100$

= ₹465

(iii) How many extra shares should he buy?

Let x be extra share that he buys.

Then, Total number of shares = $62 + x$

Total face value = $100 \times (62 + x)$

$$\text{Annual income} = \frac{(62 + x) \times 7.5 \times 100}{100}$$

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$$= (62 + x) \times 7.5$$

According to the question, Salman's annual income increased by ₹ 150. Therefore

$$(62 + x) \times 7.5 = 465 + 150$$

$$(62 + x) \times 7.5 = 615$$

$$62 + x = \frac{615}{7.5} = 82$$

$$x = 82 - 62 = 20$$

Hence, the number of extra shares, he buys, is 20.

FIVE MARK QUESTIONS

Main 2003

71. A man invests ₹20020 in buying shares of nominal value ₹26 at 10% premium. The dividend on the share is 15% per annum. Calculate
(i) the number shares, he buys.
(ii) the dividend, he receives annually.
(iii) the rate of interest, he gets on his money.

Sol :

Money invested = ₹20020

nominal value = ₹26

rate of premium = 10%

and rate of dividend = 15%

Market value of one share

= Nominal value + 10% of nominal value

= 26 + 10% of 26

= 26 + 2.6

= ₹28.6

(i) The number of shares he buys

$$\text{Number of shares} = \frac{\text{Money Invested}}{\text{Market Value of One Share}}$$

$$= \frac{20020}{28.60} = 700$$

(ii) the annually dividend he receives

= 15% of nominal value × Number of shares

$$= \frac{15}{100} \times 26 \times 700$$

= ₹2730

(iii) the rate of interest, he gets on his money.

$$\text{Rate of Interest} = \frac{\text{Annually Dividend}}{\text{Money Invested}} \times 100$$

$$= \frac{2730}{20020} \times 100 = 13.363\%$$

= 13.64% (aproox.)

Comp 2003

72. Amit and Richa invest ₹12000 each in buying shares of two companies. Amit buys 15% ₹100 shares at a discount of ₹20, while Richa buys ₹25 shares at a premium of 20%. If both receive equal dividends at the end of the year, find the rate percent of the dividend declared by Richa's company.

Sol :

Market value of one share purchased by Amit

= 100 − 20 = ₹80

Investment by Amit = ₹12000

Number of shares purchase by Amit

$$= \frac{12000}{80} = 150$$

Annual dividend received by Amit

= Number of shares held × Rate of dividend × Face value of one share

$$= 150 \times \frac{15}{100} \times 100 = ₹2250.$$

Annual dividend received by Richa = ₹2250.
Here both Amit and Richa get equal dividends.
Richa purchased ₹25 shares at premium of 20%.
Market value of one share purchased by Richa,

$$= \left(1 + \frac{20}{100}\right) \text{ of } 25$$

$$= \frac{6}{5} \times 25 = ₹30$$

Investment by Richa = ₹12000

Number of shares purchased by Richa

$$= \frac{12000}{30} = 400$$

Let r% be the rate of dividend declared by Richa's company, then annual dividend of Richa

= Number of shares × rate of dividend × Face value of one share

$$2250 = 400 \times \frac{r}{100} \times 25$$

$$2250 = 100r$$

$$r = 22.5$$

Hence, the rate percent of the dividend declared by Richa's company is 22.5%.

END

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CHAPTER 4

LINEAR INEQUATIONS IN ONE VARIABLE

MULTIPLE CHOICE QUESTIONS

Main 2023

1. The solution set for the inequation $2x+4 \le 14$, $x \in W$ is :

(a) $\{1, 2, 3, 4, 5\}$ (b) $\{0, 1, 2, 3, 4, 5\}$
(c) $\{1, 2, 3, 4\}$ (d) $\{0, 1, 2, 3, 4\}$

Sol :

$$2x+4 \le 14,\ x \in W$$
$$2x \le 14-4,\ x \in W$$
$$2x \le 10,\ x \in W$$
$$x \le 5,\ x \in W$$
$$x \in \{0,1,2,3,4,5\}$$

Thus (b) is correct option.

2. Solution set of $12 - x \ge 3x - 2$, given that $x \in W$ is:

(a) $\{0, 1, 2\}$ (b) $\{1, 2, 3\}$
(c) $\{1, 2\}$ (d) $\{0, 1, 2, 3\}$

Sol :

(d) $\{0, 1, 2, 3\}$

Since, $12 - x \ge 3x - 2 \qquad x \in W$

$$12 + 2 \ge 3x + x$$
$$14 \ge 4x$$
$$\frac{14}{4} \ge \frac{4x}{4}$$
$$\tfrac{7}{2} \ge x$$
$$x \le \tfrac{7}{2}$$
$$x \le 3\tfrac{1}{2} \qquad x \in W$$
$$x = \{0,1,2,3\}$$

Thus (d) is correct option.

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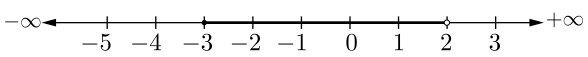
3. The solution set representing the following number line is :



(a) $[x : x \in R, -3 \le x < 2]$
(b) $[x : x \in R, -3 < x < 2]$
(c) $[x : x \in R, -3 \le x \le 2]$
(d) $[x : x \in R, -3 \le x \le 2]$

Sol :

From the figure, the solution set of the following number line is :

$$[x : -3 \le x < 2, x \in R]$$

Thus (a) is correct option.

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4. The solution set of the inequation :
$x - 3 \ge -5,\ x \in R$ is :

(a) $[x : x > -2, x \in R]$
(b) $[x : x \le -2, x \in R]$
(c) $[x : x \ge -2, x \in R]$
(d) $[-2, -1, 0, 1, 2]$

Sol :

We have $x - 3 \ge -5$

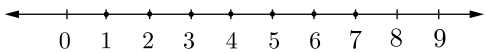
Adding 3 on both sides we have

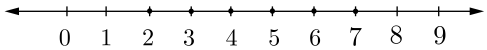
$$x - 3 + 3 \ge -5 + 3$$
$$x \ge -2,\ x \in R$$

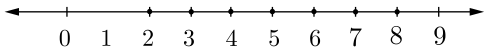
Thus (c) is correct option.

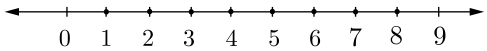
Main 2022 Sem I

5. The solution set on the number line of the linear inequation : $2y - 6 < y + 2 \le 2y$ $y \in N$ is :

(a) 

(b) 

(c) 

(d) 

Sol :

We have $2y - 6 < y + 2 \le 2y,\ y \in N$

Subtracting $2y$ throughout the inequality, we get

$$2y - 6 - 2y < y + 2 - 2y \le 2y - 2y$$
$$-6 < -y + 2 \le 0$$

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Subtracting 2, throughout the inequality, we get

$$-6-2<-y+2-2\le 0-2$$

$$-8<-y\le -2$$

$$8>y\ge 2$$

But $y\in N$, thus solution set is

$$y=\{2,3,4,5,6,7\}$$

Thus (b) is correct option.

Main 2022 Sem I

6. The solution set for the linear inequation $-8\le x-7<-4,\ x\in I$ is :
(a) $\{x:x\in R,-1\le x<3\}$
(b) $\{0,1,2,3\}$
(c) $\{-1,0,1,2,3\}$
(d) $\{-1,0,1,2\}$

Sol :

We have $-8\le x-7<-4,\ x\in I$

Adding 7, throughout the inequality, we get

$$-8+7\le x-7+7<-4+7$$

$$-1\le x<3,\ x\in I$$

Hence, the solution set is

$$x\in\{-1,0,1,2\}$$

Thus (d) is correct option.

7. In which of the following cases, the sign of inequality does not reverse in a linear inequation ?
(a) On dividing a negative term both sides
(b) On adding a negative term to both sides
(c) On multiplying a negative term both sides
(d) On changing the sign of each term on both sides

Sol :

When a negative term is added to both sides of a linear inequation, the sign of inequality remains the same.
Thus (b) is correct option.

8. If $\{x:4\le x<\infty\}$ is a solution set of a linear inequation in x, then the solution set of x must be_____.
(a) N (b) Z
(c) W (d) R

Sol :

Since, $4\le x<\infty$, so all values from 4 including rational and irrational numbers are the solution of x.
We know that rational and irrational numbers constitute real numbers (R).
The solution set of x must be R.
Thus (d) is correct option.

9. Multiplying -3 on both sides of a linear inequation $\frac{-2x+7}{3}\le\frac{-4x+5}{3}$ become_____.
(a) $-2x+7\ge -4x+5$
(b) $2x-7\ge 4x-5$
(c) $-2x+7\le 4x-5$
(d) $2x-7\le 4x-5$

Sol :

Multiplying a negative term on both sides of a linear inequation, the sign of inequality get reversed.

So, $(-3)\left(\frac{-2x+7}{3}\right)\ge(-3)\left(\frac{-4x+5}{3}\right)$

$$(-1)(-2x+7)\ge(-1)(-4x+5)$$

$$2x-7\ge 4x-5$$

Thus (b) is correct option.

10. To solve the linear inequation $2x+6<21,\ x\in I$ we add (–6) to both sides. With this operation, the sign of inequality_____.
(a) reverses (b) remain same
(c) data insufficient (d) none of these

Sol :

(b) remain same
Adding a negative term or on subtracting a term from both sides of a linear inequation, the sign of inequality remains the same.
Thus (b) is correct option.

11. If $x\ge y$ and we take their reciprocals, then $\frac{1}{x}$ _____ $\frac{1}{y}$.
(a) $\ge$ (b) $\le$
(c) $>$ (d) $<$

Sol :

If we take reciprocal of a linear inequality, then the sign of inequality get reversed.
Thus (b) is correct option.

12. If $x\in N$ then the solution of $x\le 3$ is:
(a) $\{0,1,2\}$ (b) $\{1,2,3\}$
(c) $\{1,2\}$ (d) $\{0,1,2,3\}$

Sol :

Since, $x\le 3, x\in N$ (Natural numbers)

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$x = 1,2,3.$

Thus (b) is correct option.

13. If $x \in W$ then the solution of $x \leq 3$ is:

(a) {0, 1, 2} (b) {1, 2, 3}

(c) {1, 2} (d) {0, 1, 2, 3}

Sol :

Since, $x \leq 3, x \in W$ (Whole numbers)

$x = 0,1,2,3.$

Thus (d) is correct option.

14. The solution set for the inequation $2x + 7 \leq 17$, $x \in W$ is :

(a) {1, 2, 3, 4, 5} (b) {0, 1, 2, 3, 4, 5}

(c) {1, 2, 3, 4} (d) {0, 1, 2, 3, 4}

Sol :

We have $2x + 7 \leq 17,\ x \in W$

$2x \leq 17 - 7$

$2x \leq 10$

$x \leq 5$

Since $x \in W$ we have

$x \in \{0,1,2,3,4,5\}$

Thus (b) is correct option.

15. The solution set for the inequation $2x + 7 \leq 17$, $x \in N$ is :

(a) {1, 2, 3, 4, 5} (b) {0, 1, 2, 3, 4, 5}

(c) {1, 2, 3, 4} (d) {0, 1, 2, 3, 4}

Sol :

We have $2x + 7 \leq 17,\ x \in N$

$2x \leq 17 - 7$

$2x \leq 10$

$x \leq 5,$

Since $x \in N$ we have

$x \in \{1,2,3,4,5\}$

Thus (b) is correct option.

16. Solution set of inequation $5 - 2x \geq x - 10$, given that $x \in N$, will be

(a) {0, 1, 2, 3, 4, 5} (b) {1, 2, 3, 4}

(c) {1, 2, 3, 4, 5} (d) {0, 1, 2, 3, 4}

Sol :

We have $5 - 2x \geq x - 10$

$5 - 2x - 5 \geq x - 10 - 5$ (Subtract 5)

$-2x \geq x - 15$

$-2x - x \geq x - 15 - x$ (Subtract x)

$-3x \geq -15$

$-x \geq -5$ (Divide by 3)

$\left(-\frac{1}{3}\right)(-3x) \leq \left(-\frac{1}{3}\right)(-15)$ [Multiply by $-\frac{1}{3}$]

$x \leq 5$

but $x \in N$ thus $x \in \{1,2,3,4,5,...\}$.

Hence solution set is {1, 2, 3, 4, 5}.

Thus (b) is correct option.

17. If $x \in R$, the solution set of $8 - 3x \geq 28 + 2x$ is:

(a) All real numbers less than (– 4)

(b) All real numbers less than or equal to (– 4)

(c) All real numbers greater than (– 4)

(d) All real numbers greater than or equal to (-4)

Sol :

We have, $8 - 3x \geq 28 + 2x \qquad x \in R$

$8 - 28 \geq 2x + 3x$

$-20 \geq 5x$

$\frac{-20}{5} \geq \frac{5x}{5}$

$-4 \geq x$

$x \leq -4$

Thus (b) is correct option.

18. If $-2 \leq \frac{1}{2} - \frac{2x}{3}$, $x \in R$, then the largest value of x is:

(a) $\frac{15}{4}$ (b) 4

(c) 3 (d) None of these

Sol :

We have $-2 < \frac{1}{2} - \frac{2x}{3}, x \in R$

$-2 < \frac{3 - 4x}{6}$

$-12 \leq 3 - 4x$

$-15 \leq -4x$

$-\frac{15}{-4} \geq \frac{-4x}{-4}$

$\frac{15}{4} \geq x$ or $x \leq \frac{15}{4}$

Since, the solution set of x is R (real numbers).

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The largest value of x is $\frac{15}{4}$
Thus (a) is correct option.

19. If $\frac{x}{2}-5\le\frac{x}{3}-4$ and x is a natural even number, then the solution set of x is:
(a) $\{-6,-4,-2\}$ (b) $\{6,-4,-2,2,4,6\}$
(c) $\{2,4,6\}$ (d) $\{2,4,6,8\}$

Sol :

We have, $\frac{x}{2}-5\le\frac{x}{3}-4$

$$\frac{x}{2}-\frac{x}{3}\le-4+5$$

$$\frac{x}{6}\le 1$$

$$x\le 6$$

Since, x is a set of natural even numbers,

$$x=\{2,4,6\}$$

Thus (c) is correct option.

20. If $\frac{x}{2}-5\le\frac{x}{3}-4$ and x is a natural odd number, then the solution set of x is:
(a) $\{-6,-4,-2\}$ (b) $\{6,-4,-2,2,4,6\}$
(c) $\{0,1,3,5\}$ (d) $\{1,3,5\}$

Sol :

We have, $\frac{x}{2}-5\le\frac{x}{3}-4$

$$\frac{x}{2}-\frac{x}{3}\le-4+5$$

$$\frac{x}{6}\le 1$$

$$x\le 6$$

Since, x is a set of natural even numbers,

$$x=\{1,3,5\}$$

Thus (d) is correct option.

21. If $-3x+17<-13$, then
(a) $x>10$ (b) $x<10$
(c) $x\le 10$ (d) $x\ge 10$

Sol :

We have $-3x+17<-13$
Subtracting 17 from both sides,

$$-3x<-13-17$$

$$-3x<-30$$

Dividing both sides by 3 we get

$$-x<-10$$

Dividing both sides by (-1) we have

$$x>10$$

Thus (a) is correct option.

22. If $\frac{5-2x}{3}<\frac{x}{6}-5$ then
(a) $x>8$ (b) $x<8$
(c) $x\le 8$ (d) $x\ge 8$

Sol :

We have, $\frac{5-2x}{3}\le\frac{x}{6}-5$

$$2(5-2x)\le x-30$$

$$10-4x\le x-30$$

$$-5x\le-40$$

i.e., $x\ge 8$

Thus (d) is correct option.

23. If $\frac{3x-4}{2}\ge\frac{x+1}{4}-1$ then
(a) $x>1$ (b) $x<1$
(c) $x\le 1$ (d) $x\ge 1$

Sol :

We have, $\frac{3x-4}{2}\ge\frac{x+1}{4}-1$

$$\frac{3x-4}{2}\ge\frac{x-3}{4}$$

$$2(3x-4)\ge(x-3)$$

$$6x-8\ge x-3$$

$$5x\ge 5 \text{ or } x\ge 1$$

Thus (d) is correct option.

24. If $4x+3<5x+7\ \forall\ x\in R$ then
(a) $x>-4$ (b) $x<4$
(c) $x>4$ (d) $x<-4$

Sol :

We have, $4x+3<5x+7$

$$4x-5x<7-3$$

$$-x<4$$

$$x>-4$$

Thus (a) is correct option.

25. If $3x+8>2$ then which of the following is true?
(a) $x\in\{-1,0,1,2,\ldots\}$, when x is an integer
(b) $x\in\{-2,\infty\}$, when x is a real number
(c) Both (a) and (b)

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(d) None of the above

Sol :

We have, $3x+8>2$

Adding –8 on both sides,

$$3x+8-8>2-8$$

$$3x>-6$$

Dividing by 3 on both sides,

$$\frac{3x}{3}>\frac{-6}{3}$$

$$x>-2$$

Dividing by 3 on both sides,

(i) When x is an integer, the solution of the given inequality is {–1, 0, 1, 2 ....}.

(ii) When x is a real number, the solution of the given inequality is $(-2,\infty)$. i.e., all the numbers lying between -2 and ∞ but -2 and co are not included.

Thus (a) is correct option.

26. If $3(x-1)\geq 2(x-3)$ then

(a) $x>-3$ (b) $x<-3$

(c) $x\leq -3$ (d) $x\geq -3$

Sol :

We have, $3x-3\leq 2x-6$

Transferring the term $2x$ to LHS and the term -3 to RHS,

$$3x-2x\leq -6+3$$

$$x\leq -3$$

Thus (c) is correct option.

27. If $3x-7>5x-1\,\forall\, x\in R$ then

(a) $x>-3$ (b) $x<-3$

(c) $x\leq -3$ (d) $x\geq -3$

Sol :

We have, $3x-7>5x-1$

$$3x-5x>-1+7$$

$$-2x>6$$

$$-x>3$$

$$x<-3$$

Thus (c) is correct option.

28. If $4x+3<6x+7$ then

(a) $x>-2$ (b) $x<-2$

(c) $x\leq -2$ (d) $x\geq -2$

Sol :

We have, $4x+3<6x+7$

$$4x-6x<6x+4-6x$$

$$-2x<4 \text{ or } x>-2$$

Thus (a) is correct option.

29. If $3(2-x)\geq 2(1-x)$ then

(a) $x\geq 4$ (b) $x<4$

(c) $x>4$ (d) $x\leq 4$

Sol :

We have, $3(2-x)\geq 2(1-x)=6-3x\geq 2-2x$

Transferring the term 2 to LHS and the term $-3x$ to RHS,

$$6-2\geq -2x+3x$$

$$4\geq x$$

$$x\leq 4$$

Thus (d) is correct option.

30. If $3\leq 3t-18\leq 18$, then which one of the following is true?

(a) $8<t\leq 12$

(b) $8\leq t<12$

(c) $8<t+1\leq 13$

(d) $8\leq t+1\leq 13$

Sol :

We have $3<3t-18\leq 18$

Adding 18 to each term,

$$3+18<3t-18+18\leq 18+18$$

$$21\leq 3t\leq 36$$

Dividing by 3 to each term,

$$\frac{21}{3}\leq\frac{3t}{3}\leq\frac{36}{3}$$

$$7\leq t\leq 12$$

Adding 1 to each term,

$$7+1\leq t+1\leq 12+1$$

$$8<t+1\leq 13$$

Thus (c) is correct option.

31. The solution set of the inequalities $6\leq -3(2x-4)<12$ is

(a) $(-\infty,1]$ (b) $(0,1]$

(c) $(0,1]\cup[1,\infty)$ (d) $[1,\infty)$

Sol :

The given inequality

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$$6 \leq -3(2x-4) < 2$$

$$6 \leq -6x+12 < 12$$

Adding (-12) to each term,

$$6-12 \leq -6x+12-12 < 12-12$$

$$-6 \leq -6x < 0$$

Dividing by (-6) to each term,

$$\frac{-6}{-6} \geq \frac{-6x}{-6} > \frac{0}{-6}$$

$$1 \geq x > 0$$

$$0 < x \leq 1$$

Solution set is $(0, 1]$

Thus (b) is correct option.

32. Ravi obtained 70 and 75 marks in first two unit tests. Then, the minimum marks he should get in the third test to have an average of atleast 60 marks, are

(a) 45 (b) 35

(c) 25 (d) None of these

Sol :

Let Ravi got x marks in third unit test

Average marks obtained by Ravi

$$= \frac{\text{Sum of marks in all tests}}{\text{Number of tests}}$$

$$= \frac{70+75+x}{3} = \frac{145+x}{3}$$

Now, it is given that he wants to obtain an average of atleast 60 marks.

Atleast 60 marks means that the marks should be greater than or equal to 60.

i.e., $\frac{145+x}{3} \geq 60$

$$145+x \geq 60 \times 3$$

$$145+x \geq 180$$

Now, transferring the term 145 to HHS,

$$x \geq 180-145$$

$$x \geq 35$$

i.e., Ravi should get greater than or equal to 35 marks in third unit test to get an average of atleast 60 marks.

Minimum marks Ravi should get $= 35$

Thus (b) is correct option.

33. To receive grade A in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, the minimum marks that Sunita must obtain in fifth examination to get grade A in the course, are

(a) 72 (b) 82

(c) 90 (d) None of these

Sol :

Let Sunita got x marks in the fifth exam.

Average marks obtained by Sunita

$$= \frac{\text{Sum of marks in all exams}}{\text{Number of exams}}$$

$$= \frac{87+92+94+95+x}{5} = \frac{368+x}{5}$$

Now, it is given that Sunita wants to obtain grade A for that her average marks should be greater than or equal to 90.

i.e. $\frac{368+x}{5} \geq 90$

$$368+x \geq 450$$

Transferring the term 368 to RHS,

$$x \geq 450-368$$

$$x \geq 82$$

i.e., Sunita should get greater than or equal to 82 marks in fifth exam to get grade A.

Minimum marks $= 82$

Thus (b) is correct option.

34. Ravi goes to market with ₹200 to buy rice, which is available in packets of 1 kg. The price of one packet of rice is ₹30. If x denotes the number of packets of rice which he buys, then the total amount spent by him is ₹$30x$. The mathematical formulation of the given problem is

(a) $30x > 200$ (b) $30x < 200$

(c) $30x \leq 200$ (d) $30x \geq 200$

Sol :

Since, Ravi has to buy rice in packets only, he may not be able to spend the entire amount of ₹200 because 200 is not a multiple of 30. Hence,

$$30x < 200$$

Thus (b) is correct option.

35. Reshma has ₹120 and wants to buy some registers and pens. The cost of one register is ₹40 and that of a pen is ₹20. The mathematical formulation of the given problem is

(a) $40x+20y = 120$

(b) $40x+20y > 120$

(c) $40x+20y \leq 120$

(d) $40x+20y \geq 120$

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Sol :

Let x denotes the number of registers and y, the number of pens which Reshma buys, then the total amount spent by her is ₹ $(40x+20y)$ In this case, the total amount spent may be ₹120, then

$$40x+20y \le 120$$

Note :

The statement $40x+20y \le 120$ consists of two statements $40x+20y < 120$ and $40x+20y = 120$ where $40x+20y < 120$ is an inequality and $40x+20y = 120$ is an equation.

36. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm, then minimum length of the shortest side is

(a) 7 cm (b) 8 cm
(c) 9 cm (d) 10 cm

Sol :

Let the shortest side be x cm.
Then, according to the given condition,

$$\text{Longest side} = 3x \text{ cm}$$

and $$\text{third side} = (3x-2) \text{ cm}$$

Now, $$\text{perimeter of triangle} \ge 61$$

i.e., $$\text{sum of all sides} \ge 61$$

$$x+3x+3x-2 \ge 61$$

$$2+7x-2 \ge 61+2$$

$$7x \ge 63$$

$$\frac{7x}{7} \ge \frac{63}{7}$$

$$x \ge 9$$

Minimum length of the shortest side is 9 cm.
Thus (c) is correct option.

37. The marks obtained by a student of class XI in first and second terminal examinations are 62 and 48, respectively. The minimum marks he should get in, the annual examination to have an average of atleast 60 marks, are

(a) 70 (b) 50
(c) 74 (d) 48

Sol :

Let x be the marks obtained by student in the annual examination. Then,

$$\frac{62+48+x}{3} \ge 60$$

$$110+x \ge 180$$

$$x \ge 70$$

Thus, the student must obtain a minimum of 70 marks to get an average of atleast 60 marks.
Thus (a) is correct option.

38. The number of pairs of consecutive odd natural numbers both of which are larger than 10, such that their sum is less than 40, is

(a) 8 (b) 6
(c) 4 (d) 3

Sol :

Let x be the smaller of the two consecutive odd natural number, so that the other one is $x+2$. Then, we should have

$$x > 10 \quad \ldots(1)$$

and $$x+(x+2) < 40 \quad \ldots(2)$$

Solving (2), we get

$$2x+2 < 40$$

i.e., $$x < 19 \quad \ldots(3)$$

From (1) and (3), we get

$$10 < x < 19$$

Since x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be (11,13), (13, 15), (15, 17), (17, 19)
Thus (c) is correct option.

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39. Match the statements given in Column I with the sets in Column II and choose the correct option from the codes given below.

	Column I		**Column II**
A.	$3x-5 < x+7$, x is a natural number.	1.	$\{0, 1, 2, 3, 4, 5\}$
B.	$3x-5 < x+7$, x is an integer	2.	$\{1, 2, 3, 4, 5\}$
C.	$3x-5 < x+7$, x is a whole number.	3.	$\{x : x \in R \text{ and } x < 6\}$
D.	$3x-5 < x+7$, x is a real number.	4.	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

Codes

	A	B	C	D
(a)	2	1	3	4
(b)	4	1	2	3

(c)	2	4	1	3
(d)	2	1	4	3

Sol :

We have, $3x-5<x+7$

Adding 5 on both sides,

$$3x-5+5<x+7+5$$

$$3x<x+12$$

Subtracting x from both sides,

$$3x-x<x+12-x$$

$$2x<12$$

Dividing by 2 both sides,

$$\frac{2x}{2}<\frac{12}{2}$$

$$x<6$$

A. If x is a natural number,
solution set $=\{1, 2, 3, 4, 5\}$

B. If x is an integer, solution set
$=\{\ldots-3,-2,-1,0,1,2,3,4,5\}$

C. If x is a whole number,
solution set $=\{0,1,2,3,4, 5\}$

D. It x is a real number,
solution set $=\{x:x\in R, x<6\}$

Thus (c) is correct option.

40. Consider the following statements
Statement 1 : The solution set of $5x-3<7$, when x is an integer, is $\{\ldots, -3, -2, -1\}$.
Statement 2 : The solution of $5x-3<7$, when x is a real number, is $(-\infty, 2)$.
Choose the correct option.
(a) Statement 1 is true
(b) Statement 2 is true
(c) Both are true
(d) Both are false

Sol :

We have, $5x-3<7$

Adding 3 on both sides,

$$5x-3+3<7+3$$

$$5x<10$$

Dividing both sides by 5,

$$\frac{5x}{5}<\frac{10}{5}$$

$$x<2$$

(i) When x is an integer, the solution of the given inequality is $\{\ldots., -1, 0, 1\}$.
(ii) When x is a real number, the solution of given inequality is $(-\infty, 2)$ i.e., all the numbers lying between $-\infty$ and 2 but ∞ and 2 are not included as $x<2$.
Thus (b) is correct option.

41. Which of the following is/are true?
Statement 1 : If $x\geq -3$, then $x+5\geq 2$
Statement 2 : If $-x\leq -4$, then $2x<8$
Statement 3 : If $\frac{1}{x-2}<0$, then $x<2$
(a) Only 1 is true (b) 1 and 2 are ture
(c) 1 and 2 are true (d) All are true

Sol :

(1) Given, $x\geq -3$
Adding 5 on both sides,

$$x+5\geq -3+5$$

$$x+5\geq 2$$

(2) We have, $-x\leq -4$
Multiply (–2) on both sides,

$$2x\geq 8$$

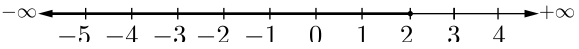
(3) Given, $\frac{1}{x-2}<0$

$$x-2<0$$

$$x<2$$

Thus (c) is correct option.

42. Consider the following statements
Statement 1 The solution set of $7x+3<5x+9$ is $(-\infty, 3)$.
Statement 2 The graph of the solution of above inequality is represented by



Choose the correct option.
(a) Statement 1 is true
(b) Statement 2 is true
(c) Both are true
(d) Both are false

Sol :

We have, $7x+3<5x+9$

$$2x<6 \text{ or } x<3$$

$$x\in(-\infty, 3)$$

The graphical representation of the solutions are given in figure.

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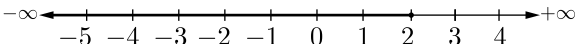


Thus both statement are correct.
This (c) is correct option.

THREE MARKS QUESTIONS

Main 2023

43. Solve the following inequation. Write down the solution set and represent it on the real number line.
$-5(x-9) \geq 17-9x > x+2,\ x \in R$

Sol :

We have,

$$-5(x-9) \geq 17-9x > x+2,\ x \in R$$

Splitting into two inequations we have

$$-5(x-9) \geq 17-9x \quad (1)$$

and $$17-9x > x+2 \quad (2)$$

Now taking first inequation

$$-5(x-9) \geq 17-9x$$
$$-5x+45 \geq 17-9x$$
$$-5x+9x \geq 17-45$$
$$4x \geq -28$$
$$x \geq -7 \ \ x \in R \quad (3)$$

Now taking second inequation

$$17-9x > x+2$$
$$-9x-x > 2-17$$
$$-10x > -15$$

Multiplying by -1 throughout we have

$$-x > -1.5$$
$$x < 1.5,\ x \in R \quad (4)$$

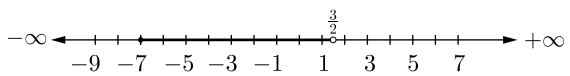
From (3) and (4) we get

$$-7 \leq x < 1.5,\ x \in R$$

Hence, the solution set is

$$\{x : -7 \leq x < 1.5, x \in R\}$$

The graphical representation of the solution set on a number line is as follows:



Comp 2023

44. Solve the following inequation, and graph the solution on the number line:
$2x-5 \leq 5x+4 < 11,\ x \in R.$

Sol :

We have $2x-5 \leq 5x+4 < 11,\ x \in R$

Splitting into two inequations we have

$$2x-5 \leq 5x+4 \text{ and } 5x+4 < 11$$

Now
$$2x-5 \leq 5x+4$$
$$2x-5-5x \leq 5x+4-5x$$
$$-3x-5 \leq 4$$
$$-3x-5+5 \leq 4+5$$
$$-3x \leq 9$$
$$-x \leq 3$$

Multiplying by -1 throughout we have

$$x \geq -3$$

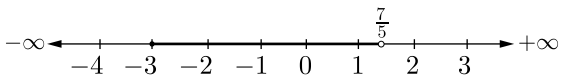
Now
$$5x+4 < 11$$
$$5x+4-4 < 11-4$$
$$5x < 7$$
$$x < \frac{7}{5}$$

Thus $-3 \leq x$ and $x < \frac{7}{5}$

or $-3 \leq x < \frac{7}{5}$ where $x \in R$

Hence solution set is $\{x : -3 \leq x < \frac{7}{5}, x \in R,\}$

The graph of the solution set is shown by the thick portion of the number line. The solid circle at -3 indicates that the number -3 is included among the solutions and the open circle at $\frac{7}{5}$ indicates that the number $\frac{7}{5}$ is not included among the solutions.



Main 2020

45. Solve the following inequation and represent the solution set on the number tine.
$\frac{3x}{5}+2 < x+4 \leq \frac{x}{2}+5,\ x \in R$

Sol :

We have $\frac{3x}{5}+2 < x+4 \leq \frac{x}{2}+5,\ x \in R$

Splitting into two inequations we have

$$\frac{3x}{5}+2 < x+4 \ \ x \in R$$

and $$x+4 \leq \frac{x}{2}+5,\ x \in R$$

Taking first inequation we have

$$\frac{3x}{5}+2 < x+4$$

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$$\frac{3x}{5} - x < 4 - 2$$

$$\frac{3x - 5x}{5} < 2$$

$$-\frac{2x}{5} < 2$$

$$-\frac{x}{5} < 1$$

$$-x < 5$$

Multiplying by -1 throughout we have

$$x > -5,\ x \in R \qquad (1)$$

Now taking second inequation we get

$$x + 4 \le \frac{x}{2} + 5$$

$$x - \frac{x}{2} \le 5 - 4$$

$$\frac{x}{2} \le 1$$

$$x \le 2,\ x \in R \qquad (2)$$

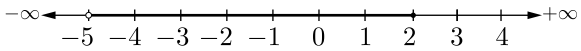
From Eqs. (1) and (2), we get

$$-5 < x \le 2,\ x \in R$$

Hence, the solution set is

$$\{x : -5 < x \le 2, x \varepsilon R\}$$

The graph of the solution set is shown by thick portion of the number line in the following figure.



Main 2019

46. Solve the following inequation and write down the solution set.

$11x - 4 < 15x + 4 \le 13x + 14,\ x \in W$

Represent the solution on a real number line.

Sol :

We have, $11x - 4 < 15x + 4 \le 13x + 14,\ x \in W$

We split above inequation in two inequation,

$$11x - 4 < 15x + 4$$

and $$15x + 4 \le 13x + 14$$

Taking first inequation we have

$$11x - 4 < 15x + 4$$

$$11x - 15x < 4 + 4$$

$$-4x < 8$$

$$-x < 2$$

Multiplying by -1 throughout we have

$$x > -2\ x \in W \qquad (1)$$

Now taking second inequation we get

$$15x + 4 \le 13x + 14$$

$$15x - 13x \le 14 - 4$$

$$2x \le 10$$

$$x \le 5\ x \in W \qquad .....(2)$$

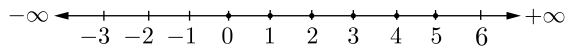
From Eqs. (1) and (2), we get

$$-2 < x \le 5\ x \in W$$

Since, $x \varepsilon W$

Hence, the solution set is $\{0, 1, 2, 3, 4, 5\}$

The graphical representation of the solution set on a number line is as follows:



Comp 2019

47. Solve the following inequation and graph the solution set on the number line $2x - 3 < x + 2 \le 3x + 5\ x \in R$

Sol :

We have, $2x - 3 < x + 2 \le 3x + 5$

Splitting into two inequations we have

$$2x - 3 < x + 2 \text{ and } x + 2 \le 3x + 5$$

$$2x - x < 3 + 2 \text{ and } 2 - 5 \le 3x - x$$

$$x < 5 \quad \text{and} \quad -3 \le 2x$$

$$x < 5 \quad \text{and} \quad \frac{-3}{2} \le x$$

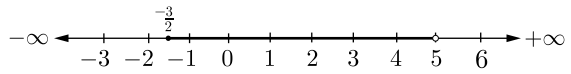
$$\frac{-3}{2} \le x \quad \text{and} \quad x < 5$$

$$\frac{-3}{2} \le x < 5$$

But $x \in R$, thus solution set is given by

$$\left\{x : -\tfrac{3}{2} \le x < 5, x \in R\right\}$$

The graph of the solution set is shown by thick portion of the number line in the following figure.



Main 2018

48. Solve the following equation, write down the solution set and represent it on the real number line.

$-2 + 10x \le 13x + 10 < 24 + 10x,\ x \in Z$

Sol :

We have,

$$-2 + 10x \le 13x + 10 < 24 + 10x,\ x \in Z$$

We break above inequation in two inequation,

$-2 + 10x \le 13x + 10$ and $13x + 10 < 24 + 10x$

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Taking first inequation we have

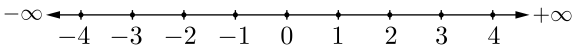
$$-2+10x \le 13x+10$$
$$10x-13x \le 10+2$$
$$-3x \le 12$$
$$-x \le 4$$
$$x \ge -4 \quad ...(1)$$

Now taking second inequation we get

$$13x+10 < 24+10x$$
$$13x-10x < 24-10 \Rightarrow 3x < 14$$
$$x < \frac{14}{3}$$
$$x < 4\frac{2}{3} \quad ...(2)$$

$$-4 \le x < 4\frac{2}{3} \quad \text{[Eq. (1) and (2)]}$$

Since, $x \in Z$, required solution set is
$[-4,-3,-2,-1,0,-1,2,3,4]$
The graphical representation of the solution set on a number line is as follows:



Comp 2018

49. Find the set of values of x satisfying $3x-5 \le 7x+3$ and $5-x \ge \frac{x}{4}-\frac{5}{4}$ Where $x \in N$. Also the graph the solution set on the number line.

Sol :

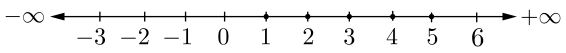
We have,

$$3x-5 \le 7x+3 \text{ and } 5-x \ge \frac{x}{4}-\frac{5}{4}$$
$$-5-3 \le 7x-3x \text{ and } -x-\frac{x}{4} \ge -\frac{5}{4}-5$$
$$-8 \le 4x \text{ and } \frac{-5}{4}x \ge \frac{-25}{4}$$
$$-2 \le x \text{ and } x \le \frac{-25}{4}\times\frac{-4}{5}$$
$$-2 \le x \text{ and } x \le 5$$
$$-2 \le x \le 5$$

But $x \in N$ i.e. thus solution set is given by

$$\{x:-2 \le x < 5, x \in N\}$$

So, the solution set is {1, 2, 3, 4, 5}.
The graph of the solution set is shown by thick dots on the following number line.



Main 2017

50. If $x \in Z$, solve the inequation $2+5x < 3x-5 \le 4x$. Also, represent its solution set on a number line.

Sol :

We have $2+5x < 3x-5 \le 4x$
Splitting into two inequations we have

$$2+5x < 3x-5 \text{ and } 3x-5 \le 4x$$

Now

$$2+5x < 3x-5$$
$$5x-3x < -5-2$$
$$2x < -7$$
$$x < -\frac{7}{2}$$

Now

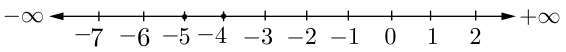
$$3x-5 \le 4x$$
$$-5 \le 4x-3x$$
$$-5 \le x$$

Thus we get $-5 \le x$ and $x < -\frac{7}{2}$

or $$-5 \le x < -\frac{7}{2}$$

But, $x \in Z$ i.e. $\{x:-5 \le x < -\frac{7}{2}, x \in Z\}$

which gives $x = -5, -4$
Hence, the solution set is $\{-5, -4\}$.
The graphical representation of the solution set on a number line is as follows:



Comp 2017

51. If $x \in Z$, solve the inequation $3+5x < 3x-5 \le 4x+1$. Also, represent its solution set on a number line.

Sol :

We have $3+5x < 3x-5 \le 4x+1$, $x \in Z$
Splitting into two inequations we have

$$3+5x < 3x-5 \quad (1)$$

and

$$3x-5 \le 4x+1 \quad (2)$$

Now

$$3+5x < 3x-5$$
$$5x-3x < -5-3$$
$$2x < -8$$
$$x < -4 \quad (3)$$

Now

$$3x-5 \le 4x+1$$
$$-5-1 \le 4x-3x$$
$$-6 \le x \quad (4)$$

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From Eqs. (3) and (4), we get $-6 \le x$ and $x < -4$

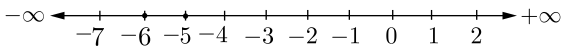
or $-6 \le x < -4$

But, $x \in Z$ i.e. $\{x : -6 \le x < -4, x \in Z\}$

which gives $x = -6, -5$

Hence, the solution set is $\{-6, -5\}$.

The graphical representation of the solution set on a number line is as follows:



Main 2016

52. Solve the following inequation, write the solution set and represent it on the number line.

$$-3(x-7) \ge 15 - 7x > \frac{x+1}{3},\ x \in R$$

Sol :

We have $-3(x-7) \ge 15 - 7x > \frac{x+1}{3},\ x \in R$

Splitting into two inequations we have

$$-3(x-7) \ge 15 - 7x \qquad (1)$$

and $$15 - 7x > \frac{x+1}{3} \qquad (2)$$

Taking first inequation we have

$$-3(x-7) \ge 15 - 7x$$
$$-3x + 21 \ge 15 - 7x$$
$$-3x + 7x \le 15 - 21$$
$$4x \ge -6$$
$$x \ge -\tfrac{6}{4}$$
$$x \ge -\tfrac{3}{2}$$
$$x \ge -1.5 \qquad ...(3)$$

Now taking second inequation we get

$$15 - 7x > \frac{x+1}{3}$$
$$3(15 - 7x) > x + 1$$
$$45 - 21x > x + 1$$
$$-21x - x > 1 - 45$$
$$-22x > -44$$
$$-x > 2$$

Multiplying by -1 throughout we have

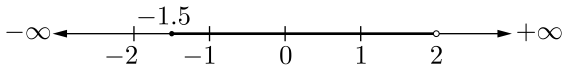
$$x < 2 \qquad ...(4)$$

From Eqs. (3) and (4), we get

$$-1.5 \le x < 2$$

i.e. $\{x : -1.5 \le x < 2,\ x \in R\}$

The graph of the solution set on the number line is shown by dark line.



Comp 2016

53. Solve the following system of inequations and graph the solution set:

$x + 5 \le 4(x-1)$ and $7 - 2x > -3,\ x \in R$

Sol :

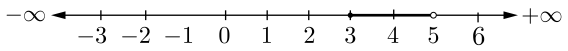
We have

$$x + 5 \le 4(x-1) \text{ and } 7 - 2x > -3$$
$$x + 5 \le 4x - 4 \text{ and } 7 - 2x > -3$$
$$5 + 4 \le 4x - x \text{ and } 7 + 3 > 2x$$
$$9 \le 3x \text{ and } 10 > 2x$$
$$3 \le x \text{ and } 5 > x$$
$$3 \le x \text{ and } x < 5$$
$$3 \le x < 5$$

But $x \in R$ thus solution set is given by

$$\{x : 3 \le x < 5, x \in R\}$$

The graph of the solution set is as given below:



Main 2015

54. Solve the following inequation and write the solution set:

$$13x - 5 < 15x + 4 < 7x + 12,\ x \in R$$

Represent the solution on a real number line.

Sol :

We have $13x - 5 < 15x + 4 < 7x + 12,\ x \in R$

Splitting into two inequations we have

$$13x - 5 < 15x + 4 \qquad ...(1)$$

and $$15x + 4 < 7x + 12 \qquad ...(2)$$

Taking eq (1) we have

$$13x - 5 < 15x + 4$$
$$13x - 15x < 4 + 5$$
$$-2x < 9$$

Multiplying by -1 throughout we have

$$x > -\tfrac{9}{2} \qquad ...(3)$$

Taking eq (2) we have

Here, $15x + 4 < 7x + 12$

$$8x < 8$$

NODIA

$$x < 1 \qquad \ldots(4)$$

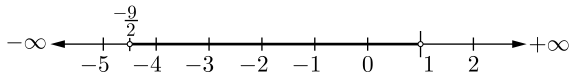
From Eqs. (3) and (4), we get

$$-\tfrac{9}{2} < x < 1 \text{ i.e.}$$

Since $x \in R$, required solution set is

$$[x : -\tfrac{9}{2} < x < 1, x \in R]$$

The graph of the solution set on the number line is shown by dark line



Comp 2015

55. Solve the following inequation and graph the solution set on the number line
$2x-4 < x+2 \le 3x+7 \;\; x \in R$

Sol :

We have, $\quad 2x-4 < x+2 \le 3x+7$

Splitting into two inequations we have

$$2x-4 < x+2 \text{ and } x+2 \le 3x+7$$

$$2x-x < 2+4 \text{ and } 2-7 \le 3x-x$$

$$x < 6 \quad \text{and} \quad -5 \le 2x$$

$$x < 5 \quad \text{and} \quad \frac{-5}{2} \le x$$

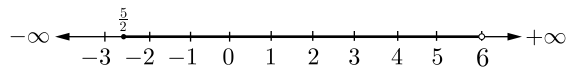
$$\frac{-5}{2} \le x \quad \text{and} \quad x < 6$$

$$\frac{-5}{2} \le x < 6$$

But $x \in R$, thus solution set is given by

$$\{x : -\tfrac{5}{2} \le x < 6, x \in R\}$$

The graph of the solution set is shown by thick portion of the number line in the following figure.



Main 2014

56. Find the value of x, which satisfies the inequation
$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, \; x \in W.$
Graph the solution set on the number line.

Sol :

We have $\quad -2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, \; x \in W$

Making denominator 6 throughout we have

$$\frac{-17}{6} < \frac{3-4x}{6} \le \frac{12}{6}$$

Multiplying throughout by 6, we get

$$-17 < 3-4x \le 12$$

Splitting into two inequations we have

$$-17 < 3-4x \text{ and } 3-4x \le 12$$

$$4x < 3+17 \text{ and } 3-12 \le 4x$$

$$4x < 20 \text{ and } -9 \le 4x$$

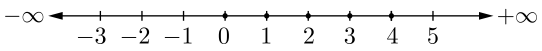
$$x < \frac{20}{4} \text{ and } \frac{-9}{4} \le x$$

$$x < 5 \text{ and } x \ge -\tfrac{9}{4}$$

Thus $-\frac{9}{4} \le x < 5$.
Since $x \in W$, required solution set is

$$\{x : -\tfrac{9}{4} \le x < 5, \; x \in W\}.$$

Required set is $\{0,1,2,3,4\}$. [as $x \in W$]
The graph of the solution set on the number line is shown by dotted points.



Comp 2014

57. Solve the following equation, write down the solution set and represent it on the real number line.
$10x-5 \le 13x+10 < 25+10x, \quad x \in Z$

Sol :

We have,

$$10x-5 \le 13x+10 < 25+10x, \; x \in Z$$

We split above inequation in two inequation,

$$10x-5 \le 13x+10 \qquad (1)$$

and $\quad 13x+10 < 25+10x \qquad (2)$

Taking first inequation we have

$$10x-5 \le 13x+10 \;\; x \in Z$$

$$10x-13x \le 10+5$$

$$-3x \le 15$$

$$-x \le 5$$

Multiplying throughout by -1, we get

$$x \ge -5 \qquad \ldots(3)$$

Now taking second inequation we get

Also, $\quad 13x+10 < 25+10x \;\; x \in Z$

$$13x-10x < 25-10$$

$$3x < 15$$

$$x < 5 \qquad \ldots(4)$$

From Eqs. (3) and (4), we get

$$-5 \le x < 5$$

Since, $x \in Z$, required solution set is
$[-5,-4,-3,-2,-1,0,-1,2,3,4]$

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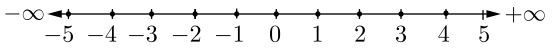


Main 2013

58. Solve the following inequation and represent the solution set on the number line

$$\frac{-x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in R.$$

Sol :

We have $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in R$

$$-\frac{x}{3} \le \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

$$-\frac{x}{3} \le \frac{3x-8}{6} < \frac{1}{6}$$

Multiplying throughout by 6, we get

$$-\frac{x}{3} \times 6 \le \left(\frac{3x-8}{6}\right) \times 6 < \frac{1}{6} \times 6$$

$$-2x \le 3x - 8 < 1$$

Splitting into two inequations we have

$$-2x \le 3x - 8 \text{ and } 3x - 8 < 1$$

Now, $-2x \le 3x - 8$

$$-2x - 3x \le -8$$

$$-5x \le -8$$

Multiplying by -1 throughout we have

$$5x \ge 8$$

$$x \ge \tfrac{8}{5} \qquad (1)$$

and $3x - 8 < 1$

$$3x < 1 + 8$$

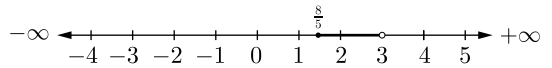
$$3x < 9$$

$$x < 3 \qquad ...(2)$$

From Eqs. (1) and (2), we get $\frac{8}{5} \le x < 3$.
Since $x \in R$, required solution set is

$$\{x : \tfrac{8}{5} \le x < 3 \text{ and } x \in R\}$$

The graph of the solution set on the number line is shown by the dark line.



Comp 2013

59. Find the set of values of x satisfying
$5x - 3 \le 9x + 5$ and $5 - \frac{3}{4}x \ge \frac{1}{2}x - \frac{5}{4}$ where $x \in N$.
Also the graph the solution set on the number line.

Sol :

We have,

$$5x - 3 \le 9x + 5 \text{ and } 5 - \tfrac{3}{4}x \ge \tfrac{1}{2}x - \tfrac{5}{4}$$

$$-5 - 3 \le 9x - 5x \text{ and } -\tfrac{3}{4}x - \tfrac{1}{2}x \ge -\tfrac{5}{4} - 5$$

$$-8 \le 4x \text{ and } -\tfrac{5}{4}x \ge -\tfrac{25}{4}$$

$$-2 \le x \text{ and } -x \ge -5$$

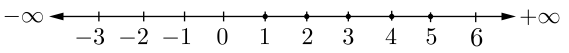
$$-2 \le x \text{ and } x \le 5$$

$$-2 \le x \le 5$$

But $x \in N$ i.e. thus solution set is given by

$$\{x : -2 \le x < 5, x \in N\}$$

So, the solution set is $\{1, 2, 3, 4, 5\}$.
The graph of the solution set is shown by thick dots on the following number line.



Main 2012

60. Solve the following inequation and represent the solution set on the number line

$$4x - 19 < \frac{3x}{5} - 2 \le \frac{-2}{5} + x,\ x \in R.$$

Sol :

We have $4x - 19 < \frac{3x}{5} - 2 \le -\frac{2}{5} + x,\ x \in R$

Splitting into two inequations we have

$$4x - 19 < \frac{3x}{5} - 2 \qquad (1)$$

and $\frac{3x}{5} - 2 \le -\frac{2}{5} + x \qquad (2)$

Taking eq (1) we have

$$4x - 19 < \frac{3x}{5} - 2$$

$$4x - \frac{3x}{5} < -2 + 19$$

$$\frac{20x - 3x}{5} < 17$$

$$\frac{17x}{5} < 17$$

$$x < 5 \qquad (3)$$

Taking eq (2) we have

$$\frac{3x}{5} - 2 \le -\frac{2}{5} + x$$

$$\frac{3x}{5} - x \le \frac{-2}{5} + 2$$

$$\frac{3x - 5x}{5} \le \frac{-2 + 10}{5}$$

$$\frac{-2x}{5} \le \frac{8}{5}$$

NODIA

$$-x \le 4$$

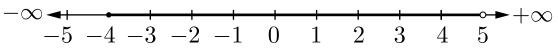
Multiplying by -1 throughout we have

$$x \ge -4 \qquad ...(4)$$

From Eqs. (3) and 4), we get $-4 \le x < 5$.
Since $x \in R$, required solution set is

$$\{x : -4 \le x < 5 \text{ and } x \in R\}$$

The graph of the solution set on the number line is shown by dark line.



Comp 2011

61. Solve the following inequation, write the solution set and represent it on the number line.
$27 - 8x \ge 15 - 6x > \frac{4}{3}x + \frac{1}{3}$, $x \in R$

Sol :

We have $27 - 8x \ge 15 - 6x > \frac{4}{3}x + \frac{1}{3}$, $x \in R$

Splitting into two inequations we have

$$27 - 8x \ge 15 - 6x \qquad (1)$$

and $$15 - 6x > \frac{4}{3}x + \frac{1}{3} \qquad (2)$$

Taking first inequation we have

$$27 - 8x \ge 15 - 6x$$
$$-8x + 6x \le 15 - 27$$
$$-2x \ge -12$$
$$-x \ge -6$$

Multiplying throughout by -1, we get

$$x \le 6 \qquad ...(3)$$

Taking eq (2) we have

$$15 - 6x > \frac{4}{3}x + \frac{1}{3}$$
$$3(15 - 6x) > 4x + 1$$
$$45 - 18x > 4x + 1$$
$$-18x - 4x > 1 - 45$$
$$-22x > -44$$
$$-x > -2$$

Multiplying throughout by -1, we get

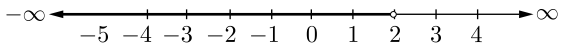
$$x < 2 \qquad ...(4)$$

From Eqs. (3) and (4), we get $x < 2$ and $x \le 6$

Thus $$x < 2$$

i.e. $$\{x : x < 2, x \in R\}$$

The graph of the solution set on the number line is shown by dark line.



Main 2011, Main 2002

62. Solve the following inequation and represent the solution set on the number line
$2x - 5 \le 5x + 4 < 11$, where $x \in I$.

Sol :

We have $2x - 5 \le 5x + 4 < 11$, where $x \in I$
Splitting into two inequations we have

$$2x - 5 \le 5x + 4 \qquad (1)$$

and $$5x + 4 < 11 \qquad (2)$$

Taking eq (1) we have

$$2x - 5 \le 5x + 4$$
$$2x - 5x \le 4 + 5$$
$$-3x \le 9$$
$$-x \le 3$$

Multiplying by -1 throughout we have

$$x \ge -3 \qquad (3)$$

Taking eq (1) we have

$$5x + 4 < 11$$
$$5x < 11 - 4$$
$$5x < 7$$
$$x < \frac{7}{5} \qquad (4)$$

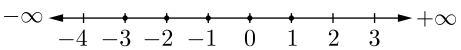
From Eqs. (3) and (4), we get $-3 \le x < \frac{7}{5}$.
Since $x \in I$, required solution set is

$$\{x : -3 \le x < \tfrac{7}{5}, x \in I\}$$

Thus required solution set is

$$\{-3, -2, -1, 0, 1\}.$$

The graph of the solution set on the number line is shown by dotted points.



Comp 2011

63. Solve the following system of inequations and graph the solution set:
$3 - x \le 2(x - 3)$ and $5 - x > x - 7$, $x \in R$

Sol :

We have

$$3 - x \le 2(x - 3) \text{ and } 5 - x > x - 7$$
$$3 - x \le 2x - 6 \text{ and } 5 + 7 > x + x$$
$$3 + 6 \le 2x + x \text{ and } 12 > 2x$$
$$9 \le 3x \text{ and } 6 > x$$

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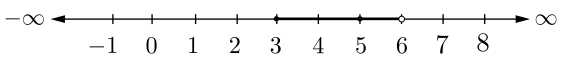
$3 \le x$ and $x < 6$

$3 \le x < 6$

But $x \in R$ thus solution set is given by

$$\{x : 3 \le x < 6, x \in R\}$$

The graph of the solution set is as given below:



Main 2010

64. Solve the following inequation and represent the solution set on the number line

$$-3 < \frac{-1}{2} - \frac{2x}{3} \le \frac{5}{6},\ x \in R.$$

Sol :

We have $-3 < -\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6},\ x \in R$

Splitting into two inequations we have

$$-3 < -\frac{1}{2} - \frac{2x}{3} \qquad (1)$$

and $$-\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6} \qquad (2)$$

Taking eq (1) we have

$$-3 < \frac{-3-4x}{6}$$

$$-18 < -3 - 4x$$

$$-18 + 3 < -4x$$

$$-15 < -4x$$

Multiplying by -1 throughout we have

$$15 > 4x$$

$$x < \tfrac{15}{4} \qquad (3)$$

Taking eq (1) we have

$$-\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6}$$

$$\frac{-3-4x}{6} \le \frac{5}{6}$$

$$-3 - 4x \le 5$$

$$-4x \le 5 + 3$$

$$-4x \le 8$$

$$-x \le 2$$

Multiplying by -1 throughout we have

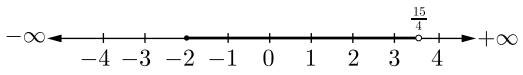
$$x \ge -2 \qquad (4)$$

From Eqs. (3) and (4), we get $-2 \le x < \frac{15}{4}$

Since $x \in R$, required solution set is

i.e. $\{x : -2 \le x < \frac{15}{4},\ x \in R\}$

The graph of the solution set on the number line is shown by dark line.



Comp 2010

65. Solve the following inequation and write the solution set:

$$11x - 8 < 15x + 4 < 7x + 20,\ x \in R$$

Represent the solution on a real number line.

Sol :

We have $11x - 8 < 15x + 4 < 7x + 20,\ x \in R$

Splitting into two inequations we have

$$11x - 8 < 15x + 4 \qquad ...(1)$$

and $$15x + 4 < 7x + 20 \qquad ...(2)$$

Taking eq (1) we have

$$11x - 8 < 15x + 4$$

$$11x - 15x < 4 + 8$$

$$-4x < 12$$

$$-x < 3$$

Multiplying throughout by -1, we get

$$x > -3 \qquad ...(3)$$

Taking eq (2) we have

Here, $$15x + 4 < 7x + 20$$

$$15x - 7x < 20 - 4$$

$$8x < 16$$

$$x < 2 \qquad ...(4)$$

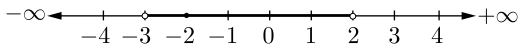
From Eqs. (3) and (4), we get

$$-3 < x < 2 \text{ i.e.}$$

Since $x \in R$, required solution set is

$$[x : -3 < x < 2, x \in R]$$

The graph of the solution set on the number line is shown by dark line



Main 2009

66. Solve the inequation and represent the solution set on the number line

$$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, \text{ where } x \in I.$$

Sol :

We have $-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x,\ x \in I$

Splitting into two inequations we have

$$-3 + x \le \frac{8x}{3} + 2 \qquad (1)$$

NODIA

and $\frac{8x}{3}+2 \le \frac{14}{3}+2x$ (2)

Taking eq (1) we have

$$-3+x \le \frac{8x+6}{3}$$
$$-9+3x \le 8x+6$$
$$-9-6 \le 8x-3x$$
$$-15 \le 5x$$
$$-3 \le x$$

Taking eq (2) we have

$$\frac{8x}{3}+2 \le \frac{14}{3}+2x$$
$$\frac{8x+6}{3} \le \frac{14+6x}{3}$$
$$8x+6 \le 14+6x$$
$$8x-6x \le 14-6$$
$$2x \le 8$$
$$x \le 4 \quad ...(4)$$

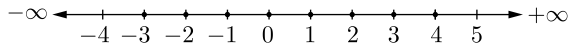
From Eqs. (3) and (4), we get $-3 \le x \le 4$
Since $x \in I$, required solution set is

$$\{x : -3 \le x \le 4;\ x \in I\}$$

Thus required solution set is

$$\{-3,2,-1,0,1,2,3,4\}.$$

The graph of the solution set on the number line is shown by dotted points.



Comp 2009

67. Find the value of x, which satisfies the inequation

$$-1\frac{5}{6} < \frac{3}{2}-\frac{2x}{3} \le 3,\ x \in W.$$

Graph the solution set on the number line.

Sol :

We have $-1\frac{5}{6} < \frac{3}{2}-\frac{2x}{3} \le 3,\ x \in W$

$$\frac{-11}{6} < \frac{9-4x}{6} \le \frac{18}{6}$$

Multiplying throughout by 6, we get

$$-11 < 9-4x \le 18$$

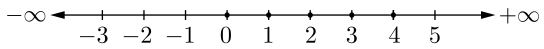
Splitting into two inequations we have

$$-11 < 9-4x \text{ and } 9-4x \le 18$$
$$4x < 9+11 \text{ and } 9-18 \le 4x$$
$$4x < 20 \text{ and } -9 \le 4x$$
$$x < \frac{20}{4} \text{ and } \frac{-9}{4} \le x$$
$$x < 5 \text{ and } x \ge -\tfrac{9}{4}$$

Thus $-\frac{9}{4} \le x < 5$.
Since $x \in W$, required solution set is

$$\{x : -\tfrac{9}{4} \le x < 5,\ x \in W\}.$$

Required set is $\{0,1,2,3,4\}$. [as $x \in W$]
The graph of the solution set on the number line is shown by dotted points.



Main 2008

68. Solve the given inequation and graph of the solution set on the number line

$$2y-3 < y+1 \le 4y+7,\ y \in R.$$

Sol :

We have $2y-3 < y+1 \le 4y+7,\ y \in R$
Splitting into two inequations we have

$$2y-3 < y+1 \quad (1)$$

and $y+1 \le 4y+7$ (2)

Taking eq (1) we have

$$2y-3 < y+1$$
$$2y-y < 1+3$$
$$y < 4 \quad ...(3)$$

Taking eq (2) we have

$$y+1 \le 4y+7$$
$$y-4y \le 7-1$$
$$-3y \le 6$$
$$-y \le 2$$

Multiplying by -1 throughout we have

$$y \ge -2 \quad ...(4)$$

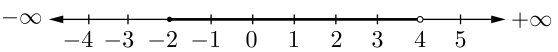
From Eqs. (3) and (4), we get

$$-2 \le y < 4$$

Since $x \in R$, required solution set is

$$\{y : -2 \le y < 4,\ y \in R\}.$$

The graph of the solution set on the number line is shown by dark line.



Main 2007

69. Solve the following inequation and graph the solution set on the number line

$$-2\frac{2}{3} \le x+\frac{1}{3} < 3\frac{1}{3},\ x \in R.$$

Sol :

We have $-2\frac{2}{3} \le x+\frac{1}{3} < 3\frac{1}{3},\ x \in R$

NODIA

$$-\frac{8}{3} \le \frac{3x+1}{3} < \frac{10}{3}$$

$$-8 \le 3x+1 < 10$$

$$-8-1 \le 3x+1-1 < 10-1$$

$$-9 \le 3x < 9$$

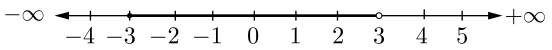
$$-\frac{9}{3} \le \frac{3x}{3} < \frac{9}{3}$$

$$-3 \le x < 3$$

Since $x \in R$, required solution set is

$$\{x : -3 \le x < 3,\ x \in R\}$$

The graph of solution set on the number line is shown by dark line.



Comp 2007

70. Solve the following inequation and represent the solution set on the number line

$$5x-18 < \frac{8x}{5}-1 \le \frac{3}{5}+2x,\ x \in R.$$

Sol :

We have $5x-18 < \frac{8x}{5}-1 \le \frac{3}{5}+2x,\ x \in R.$

Splitting into two inequations we have

$$5x-18 < \frac{8x}{5}-1 \qquad (1)$$

and $$\frac{8x}{5}-1 \le \frac{3}{5}+2x \qquad (2)$$

Taking eq (1) we have

$$5x-18 < \frac{8x}{5}-1$$

$$5x-\frac{8x}{5} < -1+18$$

$$\frac{25x-8x}{5} < 17$$

$$\frac{17x}{5} < 17$$

$$x < 5 \qquad (3)$$

Taking eq (2) we have

$$\frac{8x}{5}-1 \le \frac{3}{5}+2x$$

$$\frac{8x}{5}-2x \le \frac{3}{5}+1$$

$$\frac{8x-10x}{5} \le \frac{3+5}{5}$$

$$\frac{-2x}{5} \le \frac{8}{5}$$

$$-2x \le 8$$

$$-x \le 4$$

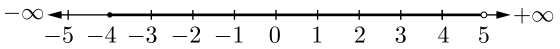
Multiplying throughout by -1, we get

$$x \ge -4 \qquad ...(4)$$

From Eqs. (3) and 4), we get $-4 \le x < 5$.
Since $x \in R$, required solution set is

$$\{x : -4 \le x < 5 \text{ and } x \in R\}$$

The graph of the solution set on the number line is shown by dark line.



Comp 2007

71. Solve the following inequation and represent the solution set on the number line

$3x-4 \le 6x+5 < 12+x$, where $x \in I$.

Sol :

We have $3x-4 \le 6x+5 < 12+x$, where $x \in I$
Splitting into two inequations we have

$$3x-4 \le 6x+5 \qquad (1)$$

and $$6x+5 < 12+x \qquad (2)$$

Taking eq (1) we have

$$3x-4 \le 6x+5$$

$$3x-6x \le 5+4$$

$$-3x \le 9$$

$$-x \le 3$$

Multiplying throughout by -1, we get

$$x \ge -3 \qquad (3)$$

Taking eq (1) we have

$$6x+5 < 12+x$$

$$6x-x < 12-5$$

$$5x < 7$$

$$x < \tfrac{7}{5} \qquad (4)$$

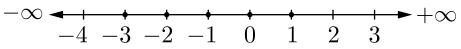
From Eqs. (3) and (4), we get $-3 \le x < \frac{7}{5}$.
Since $x \in I$, required solution set is

$$\{x : -3 \le x < \tfrac{7}{5}, x \in I\}$$

Thus required solution set is

$$\{-3,-2,-1,0,1\}.$$

The graph of the solution set on the number line is shown by dotted points.



Main 2005

72. If $A = \{x : 11x-5 > 7x+3, x \in R\}$ and
$B = \{x : 18x-9 \ge 15+12x, x \in R\}$,

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then find the range of set $A \cap B$ and represent it on the number line.

Sol :

We have

$$A = \{x : 11x - 5 > 7x + 3, x \in R\} \quad (1)$$

$$B = \{x : 18x - 9 \geq 15 + 12x, x \in R\} \quad (2)$$

From (1) we have

$$11x - 5 > 7x + 3$$

$$11x - 7x > 3 + 5$$

$$4x > 8$$

$$x > 2$$

From (2) we have

$$18x - 9 \geq 15 + 12x$$

$$18x - 12x \geq 15 + 9$$

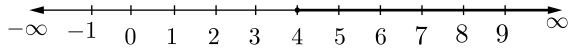
$$6x \geq 24$$

$$x \geq 4$$

$$A \cap B = \{x : x > 2 \text{ and } x \geq 4\}$$

$$A \cap B = \{x : x \geq 4\}$$

Hence, the range of the set is $x \geq 4$.
The graph of the set $A \cap B$ on the number line is shown by dark line.



NOTE : Here, range of set $A \cap B$ means the common solution of inequations represented by A and B.

Main 2004

73. Given that $x \in I$, solve the inequation and graph the solution on the number line

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2.$$

Sol :

We have $3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$

Making denominator 6 throughout we have

$$\frac{18}{6} \geq \frac{3(x-4)}{6} + \frac{2x}{6} \geq \frac{12}{6}$$

$$18 \geq 3x - 12 + 2x \geq 12$$

$$18 \geq 5x - 12 \geq 12$$

Adding 12 throughout we get

$$18 + 12 \geq 5x - 12 + 12 \geq 12 + 12$$

$$30 \geq 5x \geq 24$$

Dividing throughout by 5, we get

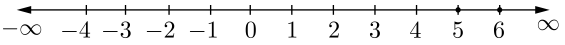
$$6 \geq x \geq \tfrac{24}{5}$$

or $4\frac{4}{5} \leq x \leq 6$

Since $x \in I$, required solution set is

$$\{x : 4\tfrac{4}{5} \leq x \leq 6, x \in I\}$$

Thus solution set is $x = \{5, 6\}$
The graph of solution set on the number line is shown by dark point.



Comp 2004

74. Solve the given inequation and graph of the solution set on the number line

$$3y < 2y + 4 \leq 5y + 10, \; y \in R.$$

Sol :

We have $3y < 2y + 4 \leq 5y + 10, \; y \in R$
Splitting into two inequations we have

$$3y < 2y + 4 \quad (1)$$

and $2y + 4 \leq 5y + 10 \quad (2)$

Taking eq (1) we have

$$3y < 2y + 4$$

$$3y - 2y < 4$$

$$y < 4 \quad \ldots(3)$$

Taking eq (2) we have

$$2y + 4 \leq 5y + 10$$

$$4 - 10 \leq 5y - 2y$$

$$-6 \leq 3y$$

$$-2 \leq y \quad \ldots(4)$$

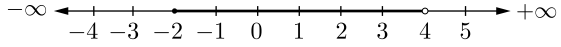
From Eqs. (3) and (4), we get

$$-2 \leq y < 4$$

Since $x \in R$, required solution set is

$$\{y : -2 \leq y < 4, \; y \in R\}.$$

The graph of the solution set on the number line is shown by dark line.



Main 2003

75. Solve $2 \leq 2x - 3 \leq 5$, $x \in R$ and mark it on the number line.

Sol :

We have, $2 \leq 2x - 3 \leq 5 \qquad x \in R$
Adding 3 throughout we get

$$2 + 3 \leq 2x - 3 + 3 \leq 5 + 3$$

NODIA

$$5 \le 2x \le 8$$

Dividing throughout by 2, we get

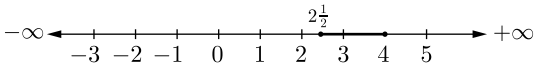
$$\tfrac{5}{2} \le x \le 4$$

$$2\tfrac{1}{2} \le x \le 4$$

Since $x \in R$, required solution set is

$$\{x : 2\tfrac{1}{2} \le x \le 4,\ x \in R\}$$

The graph of solution set on the number line is shown by dark line.



Comp 2003

76. Solve the following inequation and graph the solution set on the number line

$$-3\frac{2}{3} \le x + 1\frac{1}{3} < 4\frac{1}{3},\ x \in R.$$

Sol :

We have $$-3\frac{2}{3} \le x + 1\frac{1}{3} < 4\frac{1}{3}$$

$$-\frac{11}{3} \le x + \frac{4}{3} < \frac{13}{3}$$

$$-11 \le 3x + 4 < 13$$

$$-11 - 4 \le 3x + 4 - 4 < 13 - 4$$

$$-15 \le 3x < 9$$

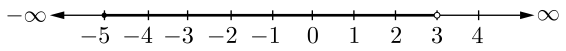
$$-\frac{15}{3} \le \frac{3x}{3} < \frac{9}{3}$$

$$-5 \le x < 3$$

Since $x \in R$, required solution set is

$$\{x : -5 \le x < 3,\ x \in R\}$$

The graph of solution set on the number line is shown by dark line.



Main 2000

77. Solve the inequation $-3 \le 3 - 2x < 9,\ x \in R$ and represent your solution on the number line.

Sol :

We have, $$-3 \le 3 - 2x < 9 \qquad x \in R$$

Adding -3 throughout we get

$$-3 - 3 \le 3 - 2x - 3 < 9 - 3$$

$$-6 \le -2x < 6$$

Dividing throughout by 2, we get

$$-3 \le -x < 3$$

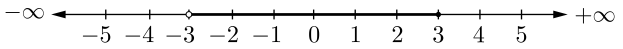
Multiplying throughout by -1, we get

$$3 \ge x > -3$$

$$-3 < x \le 3$$

Since $x \in R$, then solution set is

$$\{x : -2 < x \le 3, x \in R\}$$



FOUR MARK QUESTIONS

Main 2018

78. Solve the following inequation and represent the solution set on a number line.

$$-8\tfrac{1}{2} < -\tfrac{1}{2} - 4x \le 7\tfrac{1}{2},\ x \in I$$

Sol :

We have, $$-8\tfrac{1}{2} < -\tfrac{1}{2} - 4x \le 7\tfrac{1}{2} \qquad x \in I$$

Adding $\frac{1}{2}$ throughout we get

$$-8\tfrac{1}{2} + \tfrac{1}{2} < -\tfrac{1}{2} - 4x + \tfrac{1}{2} \le 7\tfrac{1}{2} + \tfrac{1}{2}$$

$$-8 < -4x \le 8$$

Dividing throughout by 4, we get

$$-2 < -x \le 2$$

Multiplying throughout by -1, we get

$$2 > x \ge -2$$

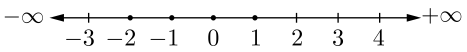
or $$-2 \le x < 2$$

Since $x \in I$, required solution set is

$$\{x : -2 \le x < 2,\ x \in I\}$$

Thus required solution set is $\{-2, -1, 0, 1\}$

The graphical representation of the solution set is as follows :



Comp 2008

79. Solve the following inequation and represent the solution set on the number line

$$\frac{4}{3} - \frac{x}{3} \le \frac{x}{2} < \frac{3}{2},\ x \in R.$$

Sol :

We have $$\frac{4}{3} - \frac{x}{3} \le \frac{x}{2} < \frac{3}{2},\ x \in R$$

Making denominator 6 throughout we have

$$\frac{8}{6} - \frac{2x}{6} \le \frac{3x}{6} < \frac{9}{6}$$

Multiplying throughout by 6, we get

$$\frac{8 - 2x}{6} \le \frac{3x}{6} < \frac{9}{6}$$

$$8 - 2x \le 3x < 9$$

Splitting into two inequations we have

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$$8-2x \le 3x \text{ and } 3x < 9$$

Now, $8-2x \le 3x$

$$8 \le 3x+2x$$

$$8 \le 5x$$

$$5x \ge 8$$

$$x \ge \tfrac{8}{5} \quad (1)$$

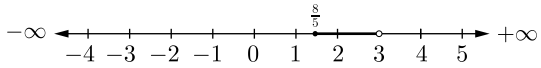
and $3x < 9$

$$x < 3 \quad ...(2)$$

From Eqs. (1) and (2), we get $\frac{8}{5} \le x < 3$.
Since $x \in R$, required solution set is

$$\{x : \tfrac{8}{5} \le x < 3 \text{ and } x \in R\}$$

The graph of the solution set on the number line is shown by the dark line.



Comp 2006

80. Solve the following inequation and represent the solution set on the number line

$$-2 < \frac{1}{2} - \frac{2x}{3} \le \frac{11}{6},\ x \in R.$$

Sol :

We have $-2 < \frac{1}{2} - \frac{2x}{3} \le \frac{11}{6},\ x \in R.$

Splitting into two inequations we have

$$-2 < \frac{1}{2} - \frac{2x}{3} \quad (1)$$

and $\frac{1}{2} - \frac{2x}{3} \le \frac{11}{6} \quad (2)$

Taking eq (1) we have

$$-2 < \frac{1}{2} - \frac{2x}{3}$$

$$-2 < \frac{3-4x}{6}$$

$$-12 < 3-4x$$

$$-12-3 < -4x$$

$$-15 < -4x$$

Multiplying throughout by -1, we get

$$15 > 4x$$

$$x < \tfrac{15}{4} \quad (3)$$

Taking eq (1) we have

$$\frac{1}{2} - \frac{2x}{3} \le \frac{11}{6}$$

$$\frac{3-4x}{6} \le \frac{11}{6}$$

$$3-4x \le 11$$

$$-4x \le 11-3$$

$$-4x \le 8$$

$$-x \le 2$$

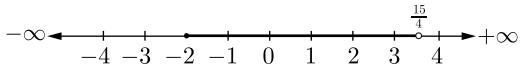
Multiplying throughout by -1, we get

$$x \ge -2 \quad (4)$$

From Eqs. (3) and (4), we get $-2 \le x < \frac{15}{4}$
Since $x \in R$, required solution set is

i.e. $\{x : -2 \le x < \frac{15}{4},\ x \in R\}$

The graph of the solution set on the number line is shown by dark line.



Comp 2005

81. Solve the following inequation and write down the solution set.

$$11x-8 < 15x+4 \le 13x+12,\ \ x \in W$$

Represent the solution on a real number line.

Sol :

We have, $11x-8 < 15x+4 \le 13x+12,\ x \in W$
Splitting into two inequations we have

$$11x-8 < 15x+4 \quad (1)$$

and $15x+4 \le 13x+12 \quad (2)$

Taking first inequation we have

$$11x-8 < 15x+4$$

$$11x-15x < 4+8$$

$$-4x < 12$$

$$-x < 3$$

Multiplying throughout by -1, we get

$$x > -3,\ x \in W \quad (3)$$

Now taking second inequation we get

$$15x+4 \le 13x+12$$

$$15x-13x \le 12-4$$

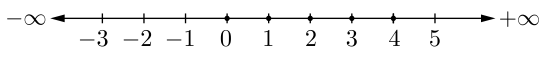
$$2x \le 8$$

$$x \le 4,\ x \in W \quad .....(4)$$

From Eqs. (3) and (4), we get

$$-3 < x \le 4,\ x \in W$$

Since, $x \in W$
Hence, the solution set is {0, 1, 2, 3, 4}
The graphical representation of the solution set on a number line is as follows:



NODIA

Main 2006

82. Given that $x \in R$, solve the following inequality and graph the solution set on the number line
$-1 \le 3+4x < 23$.

Sol :

We have $-1 \le 3+4x < 23,\ x \in R.$
Splitting into two inequations we have

$$-1 \le 3+4x \qquad (1)$$

and $$3+4x < 23 \qquad (2)$$

Taking eq (1) we have

$$-1 \le 3+4x$$
$$-1-3 \le 4x$$
$$-4 \le 4x$$
$$-1 \le x$$

Taking eq (2) we have

$$3+4x < 23$$
$$4x < 23-3$$
$$4x < 20$$
$$x < 5 \qquad (4)$$

From Eqs. (3) and (4), we get $-1 \le x < 5$
Since $x \in R$, required solution set is

$$\{x : -1 \le x < 5,\ x \in R\}$$

The graph of solution set on the number line is shown by dark line.

ALTERNATIVE :

We have, $-1 \le 3+4x < 23 \qquad x \in R$
Adding -3 throughout we get

$$-1-3 \le 3+4x-3 < 23-3$$
$$-4 \le 4x < 20$$

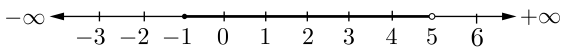
Dividing throughout by 4, we get

$$-1 \le x < 5$$

Since $x \in R$, required solution set is

$$\{x : -1 \le x < 5,\ x \in R\}$$

The graph of solution set on the number line is shown by dark line.



Main 2002

83. Solve $4 \le 2x-1 \le 7,\ x \in R$ and mark it on the number line.

Sol :

We have, $4 \le 2x-1 \le 7 \qquad x \in R$
Adding 1 throughout we get

$$4+1 \le 2x-1+1 \le 7+1$$
$$5 \le 2x \le 8$$

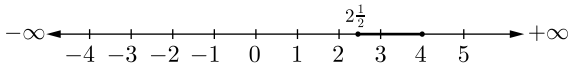
Dividing throughout by 2, we get

$$\tfrac{5}{2} \le x \le 4$$
$$2\tfrac{1}{2} \le x \le 4$$

Since $x \in R$, required solution set is

$$\{x : 2\tfrac{1}{2} \le x \le 4,\ x \in R\}$$

The graph of solution set on the number line is shown by dark line.



Comp 2003

84. Solve the inequation $-3+2x < 3 \le 9+2x$, $x \in R$ and represent your solution on the number line.

Sol :

We have, $-3+2x < 3 \le 9+2x \qquad x \in R$
Adding $-3-2x$ throughout we get

$$-3+2x-3-2x < 3-2x-3 \le 9+2x-3-2x$$
$$-6 < -2x \le 6$$

Dividing throughout by 2, we get

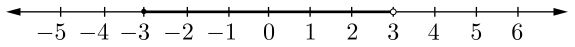
$$-3 < -x \le 3$$

Multiplying throughout by -1, we get

$$3 > x \ge -3$$
$$-3 \le x < 3$$

Since $x \in R$, then solution set is

$$\{x : -3 \le x < 3, x \in R\}$$



Main 2001

85. Find the value of x, which satisfies the inequation
$-2 \le \frac{1}{2} - \frac{2x}{3} \le 1\frac{5}{6},\ x \in N.$

Sol :

We have $-2 \le \frac{1}{2} - \frac{2x}{3} \le 1\frac{5}{6},\ x \in N$

We have $-2 \le \frac{1}{2} - \frac{2x}{3} \le 1\frac{5}{6},\ x \in N$

Making denominator 6 throughout we have

$$-\frac{12}{6} \le \frac{3-4x}{6} \le \frac{11}{6}$$
$$-12 \le 3-4x \le 11$$

Subtracting -3 throughout we get

$$-12-3 \le 3-4x-3 \le 11-3$$

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$$-15 \le -4x \le 8$$

Dividing throughout by 4, we get

$$-\frac{15}{4} \le -\frac{4x}{4} \le \frac{8}{4}$$

$$-3\tfrac{3}{4} \le -x \le 2$$

Multiplying throughout by -1, we get

$$3\tfrac{3}{4} \geqslant x \ge -2$$

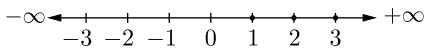
$$-2 \le x \le 3\tfrac{3}{4}$$

Since $x \in N$, then solution set is

$$\{x : -2 \le x \le 3\tfrac{3}{4};\ x \in N\}$$

Thus required solution set is $\{1,2,3\}$.

The graph of the set on the number line is shown by dotted points.



Comp 2001

86. Solve the following inequation and represent the solution set on a number line.

$-7\frac{1}{2} < \frac{1}{2} - 4x \le 8\frac{1}{2},\ x \in I$

Sol :

We have, $-7\frac{1}{2} < \frac{1}{2} - 4x \le 8\frac{1}{2} \qquad x \in I$

Adding $-\frac{1}{2}$ throughout we get

$$-7\tfrac{1}{2} - \tfrac{1}{2} < \tfrac{1}{2} - 4x - \tfrac{1}{2} \le 8\tfrac{1}{2} - \tfrac{1}{2}$$

$$-8 < -4x \le 8$$

Dividing throughout by 4, we get

$$-2 < -x \le 2$$

Multiplying throughout by -1, we get

$$2 > x \ge -2$$

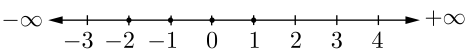
or $\qquad -2 \le x < 2$

Since $x \in I$, required solution set is

$$\{x : -2 \le x < 2,\ x \in I\}$$

Thus required solution set is $\{-2,-1,0,1\}$

The graphical representation of the solution set is as follows :



Comp 2000

87. Find the value of x, which satisfies the inequation

$-1 \le \frac{3}{2} - \frac{2x}{3} \le 2\frac{5}{6},\ x \in N.$

Sol :

We have $\qquad -1 \le \frac{3}{2} - \frac{2x}{3} \le 2\frac{5}{6},\ x \in N.$

Making denominator 6 throughout we have

$$-\frac{6}{6} \le \frac{9-4x}{6} \le \frac{17}{6}$$

$$-6 \le 9 - 4x \le 17$$

Subtracting -9 throughout we get

$$-6-9 \le 9-4x-9 \le 17-9$$

$$-15 \le -4x \le 8$$

Dividing throughout by 4, we get

$$-\frac{15}{4} \le -\frac{4x}{4} \le \frac{8}{4}$$

$$-3\tfrac{3}{4} \le -x \le 2$$

Multiplying throughout by -1, we get

$$3\tfrac{3}{4} \geqslant x \ge -2$$

$$-2 \le x \le 3\tfrac{3}{4}$$

Since $x \in N$, then solution set is

$$\{x : -2 \le x \le 3\tfrac{3}{4};\ x \in N\}$$

Thus required solution set is $\{1,2,3\}$.

The graph of the set on the number line is shown by dotted points.

NODIA

END

CHAPTER 5

QUADRATIC EQUATION IN ONE VARIABLE

MULTIPLE CHOICE QUESTIONS

Main 2024

1. The roots of the quadratic equation $px^2 - qx + r = 0$ are real and equal if:

(a) $p^2 = 4qr$ (b) $q^2 = 4pr$

(c) $-q^2 = 4pr$ (d) $p^2 > 4qr$

Sol :

Roots of quadratic equation $ax^2 + bx + c = 0$ are real and equal if

$$b^2 = 4ac$$

Substituting values from $px^2 - qx + r = 0$ we get

$$(-q)^2 = 4pr$$

$$q^2 = 4pr$$

Thus (b) is correct option.

SQP 2024

2. If the roots of equation $x^2 - 6x + k = 0$ are real and distinct, then value of k is:

(a) > -9 (b) > -6

(c) < 6 (d) < 9

Roots of quadratic equation $ax^2 + bx + c = 0$ are real and distinct if

$$b^2 > 4ac$$

Substituting values from $x^2 - 6x + k = 0$ we get

$$6^2 > 4k$$

$$36 > 4k$$

$$9 > k$$

Thus (d) is correct option.

Main 2023

3. If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then p is equal to :

(a) 4 (b) 3

(c) 5 (d) 2

Sol :

Substituting $x = 3$ in $x^2 - px + 3 = 0$ we get

$$3^2 - p(3) + 3 = 0$$

$$9 - 3p + 3 = 0$$

$$3p = 12$$

$$p = 4$$

Thus (a) is correct option.

Main 2022 Sem I

4. The roots of the quadratic equation $4x^2 - 7x + 2 = 0$ are 1.390 and 0.359. The roots correct to 2 significant digits are :

(a) 1.39 and 0.36 (b) 1.3 and 0.35

(c) 1.4 and 0.36 (d) 1.390 and 0.360

Sol :

All non-zero digits are significant the leftmost non-zero digit is the first or most significant figure. If there is a decimal point, the rightmost digit is the last or least significant figure.

If the is no decimal point explicitly shown, the rightmost non-zero digit is the least significant figure.

All digits between the most significant figure and the least significant figure are significant 1.390 correct to two significant figure = 1.40

0.359 correct to two significant figure = 0.36

Thus (c) is correct option.

Note :

Significant digits are different from rounding off to certain decimal places e.g. 1.390 can be written as 1.39 by

1. rounding off upto 2 decimal places
2. having 3 significant digits 1, 3 and 9.

Main 2022 Sem I

5. Which of the following quadratic equation has 2 and 3 as its roots?

(a) $x^2 - 5x + 6 = 0$

(b) $x^2 + 5x + 6 = 0$

(c) $x^2 - 5x - 6 = 0$

(d) $x^2 + 5x - 6 = 0$

Sol :

If $x = 2$ and $x = 3$ are roots of the quadratic equation, $x - 2 = 0$ and $x - 3 = 0$ are factors of quadratic equation. Thus

$$(x - 2)(x - 3) = 0$$

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$$x(x-3)-2(x-3)=0$$
$$x^2-3x-2x+6=0$$
$$x^2-5x+6=0$$

Thus (a) is correct option.

Main 2022 Sem I

6. The roots of $100x^2-20x+1=0$ is :

(a) $\frac{1}{20}$ and $\frac{1}{20}$ (b) $\frac{1}{10}$ and $\frac{1}{20}$

(c) $\frac{1}{10}$ and $\frac{1}{10}$ (d) none of the above

Sol :

We have $100x^2-20x+1=0$

$$100x^2-10x-10x+1=0$$
$$10x(10x-1)-1(10x-1)=0$$
$$(10x-1)^2=0$$
$$100\left(x-\tfrac{1}{10}\right)^2=0$$
$$\left(x-\tfrac{1}{10}\right)^2=0$$
$$x=\frac{1}{10} \text{ and } \frac{1}{10}$$

Thus (c) is correct option.

7. The equation $x^2+2x+1=(4-kx)^2+3$ will be quadratic, if the value of k is:

(a) $k=1$ (b) $k\neq 1$

(c) Any number (d) Insufficient data

Sol :

We have, $x^2+2x+1=(4-kx)^2+3$

$$x^2+2x+1=16-8kx+kx^2+3$$
$$x^2+2x+1=kx^2-8kx+19$$
$$(k-1)x^2-2(4k+1)x+18=0$$

For this equation to be quadratic, coefficient of x^2 should not be zero.

i.e., $k-1\neq 0$

$$k\neq 1$$

Thus (a) is correct option.

8. If $x=2$ is a solution of the quadratic equation $kx^2+2x-3=0$, then the value of k is:

(a) -1 (b) -4

(c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

Sol :

Since, $x=2$ is a solution of $kx^2+2x-3=0$,

$$k(2)^2+2(2)-3=0$$
$$4k+4-3=0$$
$$4k+1=0$$
$$k=-\tfrac{1}{4}$$

Thus (d) is correct option.

9. If one root of a quadratic equation $6x^2-x-k=0$ is $\frac{2}{3}$, then the value of k is:

(a) 2 (b) 1/9

(c) 8/3 (d) 1

Sol :

Since, $\frac{2}{3}$ is a root equation $6x^2-x-k=0$,

$$6\left(\frac{2}{3}\right)^2-\frac{2}{3}-k=0$$
$$\frac{8}{3}-\frac{2}{3}-k=0$$
$$2-k=0$$
$$k=2$$

Thus (a) is correct option.

10. If the equation $3x^2-6x+k=0$ has real and distinct roots, then the value of k is:

(a) $k\leq 3$ (b) $k=3$

(c) $k>3$ (d) $k<3$

Sol :

Since, the given equation has real and distinct (unequal) roots,

$$D>0$$
$$b^2-4ac>0$$
$$(-6)^2-4\times 3\times k>0$$
$$36-12k>0$$
$$36>12k$$
$$3>k \text{ or } k<3$$

Thus (d) is correct option.

11. Which of the following is a root of the equation $x^2-7x+12=0$?

(a) 5 (b) -1

(c) 3 (d) -4

Sol :

We have, $x^2-7x+12=0$

$$x^2-4x-3x+12=0$$

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$$x(x-4)-3(x-4)=0$$
$$(x-3)(x-4)=0$$
$$x=3,4$$

Thus (c) is correct option.

12. One root of the quadratic equation $3x^2-4x-4=0$ is:

(a) 3/2 (b) 2
(c) 2/3 (d) 6

Sol :

We have, $3x^2-4x-4=0$
$$3x^2-6x+2x-4=0$$
$$3x(x-2)+2(x-2)=0$$
$$(3x+2)(x-2)=0$$
$$x=-\tfrac{2}{3},2$$

Thus (b) is correct option.

13. If a is a natural number and one of the roots of the equation $3x^2-14x+8=0$, then the value of a is:

(a) 4 (b) 2/3
(c) 8 (d) 6

Sol :

We have, $3x^2-14x+8=0$
$$3x^2-12x-2x+8=0$$
$$3x(x-4)-2(x-4)=0$$
$$(3x-2)(x-4)=0$$
$$x=\tfrac{2}{3},4$$

Since, a is a natural number.
$$a=4$$

Thus (a) is correct option.

14. The value of k, for which the roots of $x^2+kx-1=0$ are real and unequal is:

(a) 0 (b) – 1
(c) 2 (d) Any number

Sol :

Since, the roots of given quadratic equation are real and unequal,

Discriminant $D>0$
$$b^2-4ac>0$$
$$(k)^2-4\times1\times(-1)>0$$
$$k^2+4>0$$

Now, for any positive or negative of k, k^2 is always positive.
So, k^2+4 is always greater than zero.
Hence, for any value of k, the roots of the given equation are real and unequal.

Thus (d) is correct option.

15. The roots of the quadratic equation $x^2-0.04=0$ are

(a) ±0.2 (b) ±0.02
(c) 0.4 (d) 2

Sol :

We have $x^2-0.04=0$
$$x^2=0.04$$
$$x=\pm\sqrt{0.04}$$
$$x=\pm\,0.2.$$

Thus (a) is correct option.

16. If ½ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then the value of k is

(a) 2 (b) -2
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Sol :

We have $x^2+kx-\frac{5}{4}=0$

Since, ½ is a root of the given quadratic equation, it must satisfy it.

Thus $\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-\frac{5}{4}=0$
$$\frac{1}{4}+\frac{k}{2}-\frac{5}{4}=0$$
$$\frac{1+2k-5}{4}=0$$
$$2k-4=0\ \Rightarrow\ k=2$$

Thus (a) is correct option.

17. Value(s) of k for which the quadratic equation $2x^2-kx+k=0$ has equal roots is/are

(a) 0 (b) 4
(c) 8 (d) 0, 8

Sol :

We have $2x^2-kx+k=0$
Comparing with $ax^2+bx+c=0$ we $a=2$,

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$b = -k$ and $c = k$.
For equal roots, the discriminant must be zero.
Thus $b^2 - 4ac = 0$
$(-k)^2 - 4(2)k = 0$
$k^2 - 8k = 0$
$k(k-8) = 0 \Rightarrow k = 0, 8$
Hence, the required values of k are 0 and 8.
Thus (d) is correct option.

18. If the equation $(m^2+n^2)x^2 - 2(mp+nq)x + p^2 + q^2 = 0$ has equal roots, then
(a) $mp = nq$ (b) $mq = np$
(c) $mn = pq$ (d) $mq = \sqrt{np}$

Sol :

For equal roots, $b^2 = 4ac$
$4(mp+nq)^2 = 4(m^2+n^2)(p^2+q^2)$
$m^2q^2 + n^2p^2 - 2mnpq = 0$
$(mq-np)^2 = 0$
$mq - np = 0$
$mq = np$
Thus (b) is correct option.

19. The linear factors of the quadratic equation $x^2 + kx + 1 = 0$ are
(a) $k \geq 2$ (b) $k \leq 2$
(c) $k \geq -2$ (d) $2 \leq k \leq -2$

Sol :

We have, $x^2 + kx + 1 = 0$
Comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = k$ and $c = 1$.
For linear factors, $b^2 - 4ac \geq 0$
$k^2 - 4 \times 1 \times 1 \geq 0$
$(k^2 - 2^2) \geq 0$
$(k-2)(k+2) \geq 0$
$k \geq 2$ and $k \leq -2$
Thus (d) is correct option.

20. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Sol :

We have $2x^2 - \sqrt{5x} + 1 = 0$
Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,
Now $b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1)$
$= 5 - 8 = -3 < 0$
Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has no real roots i.e., imaginary roots.
Thus (c) is correct option.

21. The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are
(a) 1, 8 (b) $-1, -8$
(c) $-1, 8$ (d) $1, -8$

Sol :

We have $x^{2/3} + x^{1/3} - 2 = 0$
Substituting $x^{1/3} = y$ we obtain,
$y^2 + y - 2 = 0$
$(y-1)(y+2) = 0 \Rightarrow y = 1$ or $y = -2$
Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$
or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$
Hence, the real roots of the given equations are 1, -8.
Thus (d) is correct option.

22. $(x^2+1)^2 - x^2 = 0$ has
(a) four real roots (b) two real roots
(c) no real roots (d) one real root

Sol :

We have $(x^2+1)^2 - x^2 = 0$
$x^4 + 1 + 2x^2 - x^2 = 0$
$x^4 + x^2 + 1 = 0$
$(x^2)^2 + x^2 + 1 = 0$
Let $x^2 = y$ then we have
$y^2 + y + 1 = 0$
Comparing with $ay^2 + by + c = 0$ we get $a = 1$, $b = 1$ and $c = 1$
Discriminant, $D = b^2 - 4ac$
$= (1)^2 - 4(1)(1) = -3$
Since, $D < 0$, $y^2 + y + 1 = 0$ has no real roots.
i.e. $x^4 + x^2 + 1 = 0$ or $(x^2+1)^2 - x^2 = 0$ has no real roots.

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Thus (c) is correct option.

23. The equation $2x^2+2(p+1)x+p=0$, where p is real, always has roots that are
(a) Equal
(b) Equal in magnitude but opposite in sign
(c) Irrational
(d) Real

Sol :

We have $2x^2+2(p+1)x+p=0$,
Comparing with $ax^2+bx+c=0$ we get $a=2$, $b=2(p+1)$ and $c=p$.

Now $b^2-4ac=[2(p+1)]^2-4(2p)$
$=4(p+1)^2-8p$
$=4p^2+8p+4-8p$
$=4(p^2+1)$

For any real value of p, $4(p^2+1)$ will always be positive as p^2 cannot be negative for real p. Hence, the discriminant b^2-4ac will always be positive.
When the discriminant is greater than 0 or is positive, then the roots of a quadratic equation are real.
Thus (d) is correct option.

24. If $x^2+y^2=25$, $xy=12$, then x is
(a) $(3,\ 4)$ (b) $(3,-3)$
(c) $(3,4,-3,-4)$ (d) $(3,-3)$

Sol :

We have $x^2+y^2=25$
and $xy=12$
$x^2+\left(\frac{12}{x}\right)^2=25$
$x^4+144-25x^2=0$
$(x^2-16)(x^2-9)=0$
Hence, $x^2=16 \Rightarrow x=\pm 4$
and $x^2=9 \Rightarrow x=\pm 3$
Thus (c) is correct option.

25. The quadratic equation $2x^2-3\sqrt{2}\,x+\frac{9}{4}=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Sol :

We have $2x^2-3\sqrt{2}\,x+\frac{9}{4}=0$
Here $a=2,\ b=-3\sqrt{2},\ c=\frac{9}{4}$
Discriminant $D=b^2-4ac$
$=(-3\sqrt{2})^2-4\times 2\times\frac{9}{4}$
$=18-18=0$
Thus, $2x^2-3\sqrt{2}\,x+\frac{9}{4}=0$ has real and equal roots.
Thus (b) is correct option.

26. The quadratic equation $x^2+x-5=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Sol :

We have $x^2+x-5=0$
Here, $a=1,\ b=1,\ c=-5$
Now, $D=b^2-4ac$
$=(1)^2-4\times 1\times(-5)$
$=21>0$
So $x^2+x-5=0$ has two distinct real roots.
Thus (a) is correct option.

27. The quadratic equation $x^2+3x+2\sqrt{2}=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Sol :

We have $x^2+3x+2\sqrt{2}=0$
Here, $a=1,\ b=3$ and $c=2\sqrt{2}$
Now, $D=b^2-4ac$
$=(3)^2-4(1)(2\sqrt{2})$
$=9-8\sqrt{2}<0$
Hence, roots of the equation are not real.
Thus (c) is correct option.

28. The quadratic equation $5x^2-3x+1=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Sol :

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We have $5x^2 - 3x + 1 = 0$

Here $a = 5,\ b = -3,\ c = 1$

Now, $D = b^2 - 4ac$

$= (-3)^2 - 4(5)(1)$

$= 9 - 20 < 0$

Hence, roots of the equation are not real.
Thus (c) is correct option.

29. The quadratic equation $x^2 - 4x + 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Sol :

We have $x^2 - 4x + 3\sqrt{2} = 0$

Here $a = 1,\ b = -4$ and $c = 3\sqrt{2}$

Now $D = b^2 - 4ac = (-4)^2 - 4(1)(3\sqrt{2})$

$= 16 - 12\sqrt{2}$

$= 16 - 12 \times (1.41)$

$= 16 - 16.92 = -0.92$

$b^2 - 4ac < 0$

Hence, the given equation has no real roots.
Thus (c) is correct option.

30. **Assertion (A):** If $5 + \sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5 - \sqrt{7}$.
Reason (R): Surd roots of quadratic equation with rational coefficients occur in conjugate pairs.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Sol :

In quadratic equation with rational coefficient, irrational roots occur in conjugate pairs. If one root is $5 + \sqrt{7}$ then second root will be $5 - \sqrt{7}$.
Hence, assertion is true and reason is also true and correct explanation.
Thus (a) is correct option.

31. **Assertion :** The values of x are $-\frac{a}{2}, a$ for a quadratic equation $2x^2 + ax - a^2 = 0$.
Reason : For quadratic equation $ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $2x^2 + ax - a^2 = 0$

$x = \dfrac{-a \pm \sqrt{a^2 + 8a^2}}{4}$

$= \dfrac{-a + 3a}{4}$

$= \dfrac{2a}{4}, \dfrac{-4a}{4}$

$x = \dfrac{a}{2}, -a$

Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

32. **Assertion :** $4x^2 - 12x + 9 = 0$ has repeated roots.
Reason : The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Reason is false because if $D = 0$, equation has repeated roots.

Assertion $4x^2 - 12x + 9 = 0$

$D = b^2 - 4ac$

$= (-12)^2 - 4(4)(9)$

$= 144 - 144 = 0$

Roots are repeated.
Assertion (A) is true but reason (R) is false.
Thus (c) is correct option.

33. **Assertion :** The roots of the quadratic equation

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$x^2+2x+2=0$ are imaginary.

Reason : If discriminant $D=b^2-4ac<0$ then the roots of quadratic equation $ax^2+bx+c=0$ are imaginary.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $x^2+2x+2=0$

Discriminant, $D=b^2-4ac$

$=(2)^2-4\times1\times2$

$=4-8=-4<0$

Roots are imaginary.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

34. **Assertion :** The equation $x^2+3x+1=(x-2)^2$ is a quadratic equation.

Reason : Any equation of the form $ax^2+bx+c=0$ where $a\neq0$, is called a quadratic equation.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have, $x^2+3x+1=(x-2)^2=x^2-4x+4$

$x^2+3x+1=x^2-4x+4$

$7x-3=0$

It is not of the form $ax^2+bx+c=0$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

35. **Assertion :** The equation $8x^2+3kx+2=0$ has equal roots then the value of k is $\pm\frac{8}{3}$.

Reason : The equation $ax^2+bx+c=0$ has equal roots if $D=b^2-4ac=0$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $8x^2+3kx+2=0$

Discriminant, $D=b^2-4ac$

$=(3k)^2-4\times8\times2$

$=9k^2-64$

For equal roots, $D=0$

$9k^2-64=0$

$9k^2=64$

$k^2=\frac{64}{9}\Rightarrow k=\pm\frac{8}{3}$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

THREE MARKS QUESTIONS

Main 2023

36. A man covers a distance of 100 km, travelling with a uniform speed of x km/hr. Had the speed been 5 km/hr more it would have taken 1 hour less. Find x the original speed.

Sol :

Let x k,/hr be the original speed of man.

Time taken to cover 100 km,

$$\text{Time}=\frac{\text{Distance}}{\text{Speed}}=\frac{100}{x}$$

Time taken to cover 100 km with $x+5$ speed

$$\text{Time}=\frac{\text{Distance}}{\text{Speed}}=\frac{100}{x+5}$$

Had the speed been 5 km/hr more it would have taken 1 hour less.

Thus $\frac{100}{x+5}=\frac{100}{x}-1$

$\frac{100}{x}-\frac{100}{x+5}=1$

$\frac{100(x+5)-100x}{x(x+5)}=1$

$\frac{100x+500-100x}{x(x+5)}=1$

$$\frac{500}{x^2+5x} = 1$$
$$500 = x^2+5x$$
$$x^2+5x-500 = 0$$
$$x^2+25x-20x-500 = 0$$
$$x(x+25)-20(x+25) = 0$$
$$(x-20)(x+25) = 0$$
$$x = 20, -25$$

Since speed cannot be negative, $x = 20$.
Hence, the original speed x is 20 km/hr.

Comp 2023

37. Find the value(s) of k if the quadratic equation $3x^2 - k\sqrt{3}\,x + 4 = 0$ has real roots.

Sol :

If discriminant $D = b^2 - 4ac$ of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $3x^2 - k\sqrt{3}\,x + 4 = 0$

Comparing with $ax^2 + bx + c = 0 = 0$ we get

$$a = 3, b = -k\sqrt{3} \text{ and } c = 4$$

For real roots $b^2 - 4ac \geq 0$

$$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$$
$$3k^2 - 48 \geq 0$$
$$k^2 - 16 \geq 0$$
$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

Main 2020

38. Solve the following quadratic equation.

$$x^2 - 7x + 3 = 0$$

Give your answer correct to two decimal places.

Sol :

We have $x^2 - 7x + 3 = 0$

Comparing above equation with $ax^2 + bx + c = 0$ we have $a = 1$, $b = -7$ and $c = 3$

Using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{7 \pm \sqrt{(-7)^2 - 4 \times 1 \times 3}}{2 \times 1}$$
$$= \frac{7 \pm \sqrt{49-12}}{2}$$
$$= \frac{7 \pm \sqrt{37}}{2}$$
$$= \frac{7 \pm 6.08}{2}$$
$$= \frac{7+6.08}{2}, \frac{7-6.08}{2}$$
$$= \frac{13.08}{2}, \frac{0.92}{2}$$
$$x = 6.54, 0.46$$

Comp 2020

39. For the equation $\sqrt{4-x} + \sqrt{x+9} = 5$, find the values of x.

Sol :

We have, $\sqrt{4-x} + \sqrt{x+9} = 5$

$$\sqrt{4-x} = 5 - \sqrt{x+9}$$
$$4 - x = (5 - \sqrt{x+9})^2$$
$$4 - x = 25 - 10\sqrt{x+9} + (x+9)$$
$$2x + 30 = 10\sqrt{x+9}$$
$$x + 15 = 5\sqrt{x+9}$$
$$(x+15)^2 = 25(x+9)$$
$$x^2 + 30x + 225 = 25x + 225$$
$$x^2 + 5x = 0$$
$$x(x+5) = 0$$
$$x = 0, -5$$

Main 2020

40. The difference of two natural number is 7 and their product is 450. Find the numbers.

Sol :

Let two natural numbers are respectively x and y such that $x > y$.
According to the given condition we have

$$x - y = 7$$
$$x = 7 + y \quad \text{.....(1)}$$

and $xy = 450$

$$(7+y)\,y = 450 \quad \text{[using Eq. (1)]}$$
$$7y + y^2 = 450$$
$$y^2 + 7y - 450 = 0$$
$$y^2 + 25y - 18y - 450 = 0$$
$$y(y+25) - 18(y+25) = 0$$
$$(y-18)(y+25) = 0$$
$$y = 18 \quad [y \neq -25]$$
$$x = 7 + 18 = 25$$

Required natural number are 25 and 18.

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Main 2019

41. Solve for x the quadratic equation $x^2-4x-8=0$. Give your answer correct to three significant figures.

Sol :

We have $x^2-4x-8=0$

Comparing above equation with $ax^2+bx+c=0$ we have $a=1,\ b=-4$ and $c=-8$

Using quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{+4\pm\sqrt{(-4)^2-4\times1\times(-8)}}{2\times1}$$

$$=\frac{4\pm\sqrt{16+32}}{2}$$

$$=\frac{4\pm\sqrt{48}}{2}$$

$$=\frac{4\pm4\sqrt{3}}{2}=\frac{2(2\pm2\sqrt{3})}{2}$$

$$=2\pm2\sqrt{3}$$

$$=2+2\sqrt{3},2-2\sqrt{3}$$

$$=2+2(1.732),2-2(1.732)$$

$$=2+3.464,2-3.464$$

$$=5.646,-1.464$$

$$=5.46,-1.46$$

[correct to three significant figures]

Comp 2019

42. Solve for x : $9x^2-6ax+(a^2-b^2)=0$

Sol :

We have $9x^2-6ax+a^2-b^2=0$

Comparing with $Ax^2+Bx+C=0$ we have

$$A=9,B=-6a,C=(a^2-b^2)$$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{6a\pm\sqrt{(-6a)^2-4\times9k(a^2-b^2)}}{2\times9}$$

$$=\frac{6a\pm\sqrt{36a^2-36a^2+36b^2}}{18}$$

$$=\frac{6a\pm\sqrt{36b^2}}{18}=\frac{6a\pm6b}{18}$$

$$=\frac{a\pm b}{3}$$

$$x=\frac{(a+b)}{3},\frac{(a-b)}{3}$$

Thus $x=\frac{a+b}{3},x=\frac{a-b}{3}$

Main 2019

43. The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers.

Sol :

Let two consecutive natural numbers having multiples of 3 are $3x$ and $3x+3$.

According to the given condition,

$$3x(3x+3)=810$$

$$9(x^2+x)=810$$

$$x^2+x=90$$

$$x^2+x-90=0$$

$$x^2+10x-9x-90=0$$

$$x(x+10)-9(x+10)=0$$

$$(x-9)(x+10)=0$$

$$x=9,\ x=-10$$

Since, numbers are natural, so we discard negative value of x. Thus $x=9$ and required numbers are

$$3x=3\times9=27$$

and $3x+3=27+3=30.$

Comp 2019

44. In a class test Raveena got a total of 30 mark in English and Mathematics. Had she got 2 more marks in Mathematics and 3 marks less in English then the product of her marks obtained would have be 210. Find the individual marks obtained in two subjects.

Sol :

Let Raveena got marks in English be x.

Marks in Mathematics $=(30-x)$

Marks in English $=(x-3)$

Now $(x-3)(30-x+2)=210$

$$35x-96-x^2=210$$

$$x^2-35x+306=0$$

$$x^2-18x-17x+306=0$$

$$x(x-18)-17(x-18)=0$$

$$(x-18)(x-17)=0$$

$$x=18,17$$

Hence, if she got 18 marks in English, then she got 12 in mathematics. If she got 17 marks in English, then she got 13 marks in mathematics.

NODIA

Main 2018

45. Find the value of k for which the following equation has equal roots.

$x^2+4kx+(k^2-k+2)=0$

Sol :

We have $x^2+4kx+(k^2-k+2)=0$ ...(i)

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=1,\ b=4k \text{ and } c=k^2-k+2$$

Since, the given equation has equal roots,

$$b^2-4ac=0$$
$$(4k)^2-4\times1\times(k^2-k+2)=0$$
$$16k^2-4k^2+4k-8=0$$
$$12k^2+4k-8=0$$
$$3k^2+k-2=0$$
$$3k^2+3k-2k-2=0$$
$$3k(k+1)-2(k+1)=0$$
$$(3k-2)(k+1)=0$$
$$k=\tfrac{2}{3},\ -1$$

Comp 2018

46. Determine the positive value of k for which the equation $x^2+kx+64=0$ and $x^2-8x+k=0$ will both have real and equal roots.

Sol :

We have $x^2+kx+64=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=1, b=k, c=64$$

For real and equal roots, $b^2-4ac=0$

Thus $k^2-4\times1\times64=0$

$$k^2-256=0 \Rightarrow k=\pm16 \quad (1)$$

Now for equation $x^2-8x+k=0$ we have

$$b^2-4ac=0$$
$$(-8)^2-4\times1\times k=0$$
$$64=4k$$
$$k=\frac{64}{4}=16 \quad (2)$$

From (1) and (2), we get $k=16$. Thus for $k=16$, given equations have equal roots.

Main 2017

47. Solve the equation $4x^2-5x-3=0$ and give your answer correct to two decimal places.

Sol :

We have, $4x^2-5x-3=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=4,\ b=-5 \text{ and } c=-3$$

Now $D^2=b^4-4ac$

$$=(-5)^2-4(4)(-3)$$
$$=25+48=73$$

Since $D^2>0$, the given equation has real roots.

Now, roots are

$$\alpha=\frac{-b+\sqrt{D}}{2a}$$
$$=\frac{-(-5)+\sqrt{73}}{2\times4}$$
$$=\frac{5+\sqrt{73}}{8}$$
$$=\frac{5+8.54}{8}$$
$$=1.69$$

and $\beta=\frac{-b-\sqrt{D}}{2a}$

$$=\frac{-(-5)-\sqrt{73}}{2\times4}$$
$$=\frac{5-8.54}{8}$$
$$=-0.44$$

NODIA

Comp 2017

48. Find that non-zero value of k, for which the quadratic equation $kx^2+1-2(k-1)x+x^2=0$ has equal roots. Hence find the roots of the equation.

Sol :

We have $kx^2+1-2(k-1)x+x^2=0$

$$(k+1)x^2-2(k-1)x+1=0$$

Comparing with $ax^2+bx+c=0$ we get

$$a=k+1, b=-2(k-1), c=1$$

For real and equal roots, $b^2-4ac=0$

$$4(k-1)^2-4(k+1)\times1=0$$
$$4k^2-8k+4-4k-4=0$$
$$4k^2-12k=0$$
$$4k(k-3)=0$$

As k can't be zero, thus $k=3$.

Main 2016

49. Solve the quadratic equation $x^2-3(x+3)=0$. Give your answer correct to two significant figures.

Sol :

We have $x^2-3(x+3)=0$

$$x^2-3x-9=0$$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=1,\ b=-3 \text{ and } c=-9$$

Now $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-3)\pm\sqrt{(-3)^2-4\times1\times(-9)}}{2}$$

$$=\frac{3\pm\sqrt{9+36}}{2}=\frac{3\pm\sqrt{45}}{2}$$

$$=\frac{3\pm6.71}{2} \qquad [\sqrt{45}=6.7082=6.71]$$

$$=4.85,\ -1.85$$

Main 2016

50. A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10km/h and as such it takes two hours longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation adn solve it to evaluate x.

Sol :

Let x in km/h be the uniform speed of bus.
Time taken by bus to cover a distance of 240 km

$$T_1=\frac{240}{x}h$$

Due to heavy rain, speed of bus is reduced by 10 km/h. Thus reduced speed is $(x-10)$ km/h.
Now, time taken by bus to cover a distance of 240 km with speed $(x-10)$ km/h

$$T_2=\left(\frac{240}{x-10}\right)\text{ h}$$

According to the equation, we have

$$T_2-T_1=2$$

$$\frac{240}{x-10}-\frac{240}{x}=2$$

$$240\left[\frac{x-x+10}{x(x-10)}\right]=2$$

$$\frac{2400}{x(x-10)}=2$$

$$2x(x-10)=2400$$

$$x(x-10)=1200$$

$$x^2-10x-1200=0$$

which is the required equation.
Using quadratic formula, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{10\pm\sqrt{100+4800}}{2}$$

$$=\frac{10\pm\sqrt{4900}}{2}$$

$$=\frac{10\pm70}{2}=40,\ -30$$

$$=40 \qquad [x=-30 \text{ (not possible)}]$$

Hence, the speed of bus is 40 km/h.

Comp 2016

51. If the roots of the quadratic equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal, prove that $2a=b+c$.

Sol :

We have $(a-b)x^2+(b-c)x+(c-a)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=(a-b,),\ b=(b-c),\ c=c-a$$

For real and equal roots, $b^2-4ac=0$

$$(b-c)^2-4(a-b)(c-a)=0$$

$$b^2+c^2-2bc-4(ac-a^2-bc+ab)=0$$

$$b^2+c^2-2bc-4ac+4a^2+4bc-4ab=0$$

$$4a^2+b^2+c^2+2bc-4ab-4ac=0$$

Using $a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2$,

$$(-2a+b+c)^2=0$$

$$-2a+b+c=0$$

Hence, $$b+c=2a$$

Main 2015

52. Find the value k for which $x=3$ is a solution of the quadratic equation, $(k+2)x^2-kx+6=0$. Thus, find the other root of the equation.

Sol :

We have $(k+2)x^2-kx+6=0$
If $x=3$, then

$$(k+2)(3)^2-k(3)+6=0$$

$$9(k+2)-3k+6=0$$

$$9k+18-3k+6=0$$

$$6k+24=0$$

$$k+4=0$$

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$$k = -4$$

Main 2015

53. Sum of two natural numbers is 8 and the difference of their reciprocals is $\frac{2}{15}$. Find the numbers.

Sol :

Let the required natural numbers be x and $8-x$, then, according to question.

$$\frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$

$$\frac{8-x-x}{x(8-x)} = \frac{2}{15}$$

$$\frac{8-2x}{8x-x^2} = \frac{2}{15}$$

$$\frac{4-x}{8x-x^2} = \frac{1}{15}$$

$$8x - x^2 = 60 - 15x$$

$$-x^2 + 8x + 15x - 60 = 0$$

$$x^2 - 23x + 60 = 0$$

$$x^2 - 20x - 3x + 60 = 0$$

$$x(x-20) - 3(x-20) = 0$$

$$(x-3)(x-20) = 0$$

$$x = 3,\ 20$$

Since $x = 20$ not possible, $x = 3$.
Required natural number are x and $8-x$ i.e. 3 and 5.

Comp 2015

54. Solve for x : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$ $x \neq -1, -2, -4$

Sol :

We have $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$\frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$$

$$(3x+4)(x+4) = 4(x^2+3x+2)$$

$$3x^2 + 16x + 16 = 4x^2 + 12x + 8$$

$$x^2 - 4x - 8 = 0$$

Now $x = \frac{-b \pm \sqrt{b^2+4ac}}{2a}$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16+32}}{2}$$

$$= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$$= 2 \pm 2\sqrt{3}$$

Hence, $x = 2 + 2\sqrt{3}$ and $2 - 2\sqrt{3}$

Main 2014

55. Solve for x using the quadratic formula. Write your answer correct to two significant figures.

$$(x-1)^2 - 3x + 4 = 0$$

Sol :

We have $(x-1)^2 - 3x + 4 = 0$

Using identity $(a-b)^2 = a^2 + b^2 - 2ab$ we get

$$x^2 + 1^2 - 2x - 3x + 4 = 0$$

$$x^2 - 5x + 5 = 0$$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 1,\ b = -5 \text{ and } c = 5$$

Using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25-20}}{2} = \frac{5 \pm \sqrt{5}}{2}$$

$$= \frac{5 \pm 2.236}{2}$$

Taking + sign, we get

$$x = \frac{5 - 2.236}{2} = \frac{7.236}{2}$$

$$= 3.618 = 3.62$$

Taking − sign, we get

$$x = \frac{5 - 2.236}{2} = \frac{2.764}{2}$$

$$= 1.382 = 1.38$$

Hence, the values of x are 3.62 and 1.38.

Comp 2014

56. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Sol :

Let the odd number be $2x+1$, then consecutive odd number will be $2x+1+2 = 2x+3$.

Now, according to question we have

$$(2x+1)^2 + (2x+3)^2 = 394$$

$$4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 394$$

$$8x^2 + 16x - 384 = 0$$

$$x^2 + 2x - 48 = 0$$

$$x^2 + 8x - 6x - 48 = 0$$

NODIA

$$x(x+8)-6(x+8)=0$$
$$x=-8,6$$

Rejecting the negative value,

$$\text{Ist number}=2\times 6+1=13$$

and second odd number $=15$

Main 2013

57. Solve the following and calculate the answer correct to two decimal places.

$$x^2-5x-10=0$$

Sol :

We have $x^2-5x-10=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$a=1,\ b=-5$ and $c=-10$

Using quadratic formula, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-5)\pm\sqrt{(-5)^2-4\times 1\times(-10)}}{2\times 1}$$
$$=\frac{5\pm\sqrt{25+40}}{2}=\frac{5\pm\sqrt{65}}{2}$$
$$=\frac{5\pm 8.062}{2} \qquad [\sqrt{65}=8.062]$$
$$x=\frac{5+8.062}{2}=\frac{13.062}{2} \qquad \text{Taking + sign}$$
$$=6.531=6.53$$
$$x=\frac{5-8.062}{2}=\frac{-3.062}{2} \qquad \text{Taking − sign}$$
$$=-1.531=-1.53$$

Hence, the solutions are 6.53 and -1.53.

Comp 2013

58. Find the value of p for which the quadratic equation $(p+1)x^2-6(p+1)x+3(p+9)=0$, $p\neq -1$ has equal roots. Hence find the roots of the equation.

Sol :

We have $(p+1)x^2-6(p+1)x+3(p+9)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=p+1, b=-6(p+1), c=3(p+9)$$

For real and equal roots, $b^2-4ac=0$

$$36(p+1)^2-4(p+1)\times 3(p+9)=0$$
$$3(p^2+2p+1)-(p+1)(p+9)=0$$
$$3p^2+6p+3-(p^2+9p+p+9)=0$$
$$2p^2-4p-6=0$$
$$p^2-2p-3=0$$
$$p^2-3p+p-3=0$$
$$p(p-3)+1(p-3)=0$$
$$(p-3)(p+1)=0$$
$$p=-1,3$$

Neglecting $p\neq -1$ we get $p=3$

Now the equation becomes

$$4x^2-24x+36=0$$
$$x^2-6x+9=0$$
$$(x-3)(x-3)=0$$
$$x=3,3$$

Thus roots are 3 and 3.

Main 2012

59. Without solving the following quadratic equation, find the value of m for which the given equation has real and equal roots.

$$x^2+2(m-1)x+(m+5)=0$$

Sol :

We have $x^2+2(m-1)x+(m+5)=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=1,\ b=2(m-1) \text{ and } c=m+5$$

Since, given equation has real and equal roots. So, discriminant $D=0$

$$b^2-4ac=0$$
$$[2(m-1)]^2-4\times 1\times(m+5)=0$$
$$4(m-1)^2-4(m+5)=0$$
$$(m-1)^2-(m+5)=0$$
$$m^2+1^2-2m-m-5=0$$
$$m^2-3m-4=0$$
$$m^2-4m+m-4=0$$
$$m(m-4)+1(m-4)=0$$
$$(m+1)(m-4)=0$$
$$m=-1 \text{ or } m=4$$

Hence, the values of m are -1 and 4.

Main 2012

60. Solve the following equation and give your answer correct to three significant figures.

$$5x^2-3x-4=0$$

Sol :

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We have $5x^2-3x-4=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$a=5,\ b=-3$ and $c=-4$

Using quadratic formula, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{(-3)^2-4\times5\times(-4)}}{2\times5}$$
$$=\frac{3\pm\sqrt{9+80}}{10}$$
$$=\frac{3\pm\sqrt{89}}{10}$$
$$=\frac{3\pm9.433}{10}$$

Now $x=\frac{3+9.433}{10}$ Taking + sign

$$=\frac{12.433}{10}=1.243$$

and $x=\frac{3-9.433}{10}$ Taking − sign

$$=\frac{-6.433}{10}=-0.643$$

Hence, the solutions are 1.243 and -0.643.

Comp 2012

61. Find the values of k for which the equation $(3k+1)x^2+2(k+1)x+1$ has equal roots. Also find the roots.

Sol :

We have $(3k+1)x^2+2(k+1)x+1$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(3k+1), B=2(k+1), C=1$

If roots are equal, $B^2-4AC=0$

$$[2(k+1)]^2-4(3k+1)(1)=0$$
$$4(k^2+2k+1)-(12k+4)=0$$
$$4k^2+8k+4-12k-4=0$$
$$4k^2-4k=0$$
$$4k(k-1)=0$$
$$k=0,1.$$

Substituting $k=0$, in the given equation,

$$x^2+2x+1=0$$
$$(x+1)^2=0$$
$$x=-1$$

Again substituting $k=1$, in the given equation,

$$4x^2+4x+1=0$$
$$(2x+1)^2=0$$

or, $x=-\frac{1}{2}$

Hence, roots are $-1, -\frac{1}{2}$.

Main 2011

62. Rs. 480 is divided equally among x children. If the number of children was 20 more, then each would have got Rs. 12 less. Find the value of x.

Sol :

Given, number of children $=x$

Total money = Rs 480

Money received by each children $=\frac{480}{x}$

If number of children was 20 more, i.e. total number of children $=x+20$,

Money received by each children $=\frac{480}{x+20}$

According to the question, we have

$$\frac{480}{x}-\frac{480}{x+20}=12$$
$$\frac{480(x+20)-480x}{x(x+20)}=12$$
$$\frac{480(x+20-x)}{x(x+20)}=12$$
$$\frac{40(20)}{x(x+20)}=1$$
$$x(x+20)=800$$
$$x^2+20x-800=0$$
$$x^2+40x-20x-800=0$$
$$x(x+40)-20(x+40)=0$$
$$(x-20)(x+40)=0$$
$$x=20 \text{ or } x=-40$$

Since number of student cant be negative, $x=20$

Hence, the number of children is 20.

Main 2011

63. Solve the equation $x-\frac{18}{x}=6$. Give your answer to two significant figures.

Sol :

We have $x-\frac{18}{x}=6$

$$\frac{x^2-18}{x}=6$$
$$x^2-18=6x$$
$$x^2-6x-18=0$$

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Comparing above equation with $ax^2+bx+c=0$ we have

$$a=1,\ b=-6 \text{ and } c=-18$$

Using quadratic formula, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-6)\pm\sqrt{(-6)^2-4\times1\times(-18)}}{2\times1}$$
$$=\frac{6\pm\sqrt{36+72}}{2}=\frac{6\pm\sqrt{108}}{2}$$
$$=\frac{6\pm\sqrt{36\times3}}{2}=\frac{6\pm6\sqrt{3}}{2}$$
$$=\frac{2(3\pm3\sqrt{3})}{2}=3\pm3\sqrt{3}$$
$$=3\pm3\times1.732 \qquad [\sqrt{3}=1.732]$$
$$=3\pm5.196$$

Now $x=3+5.196$ Taking + sign

$=8.196=8.20$ (approx)

and $x=3-5.196$ Taking − sign

$=-2.196=-2.20$ (approx)

Hence, the solution are 8.20 and -2.20.

Comp 2011

64. Solve the following equation: $\frac{1}{x}-\frac{1}{x-2}=3$, $x\neq0,2$

Sol :

We have $\frac{1}{x}-\frac{1}{x-2}=3 \qquad (x\neq0,2)$

$$\frac{x-2-x}{x(x-2)}=3$$
$$\frac{-2}{x(x-2)}=3$$
$$3x(x-2)=-2$$
$$3x^2-6x+2=0$$

Comparing it by ax^2+bx+c, we get $a=3$, $b=-6$ and $c=2$.

Now, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(2)}}{2(3)}$$
$$=\frac{6\pm\sqrt{36-24}}{6}=\frac{6\pm\sqrt{12}}{6}$$
$$=\frac{6\pm2\sqrt{3}}{6}$$
$$=\frac{3+\sqrt{3}}{3},\ \frac{3-\sqrt{3}}{3}$$

Main 2010

65. Without solving the following quadratic equation, find the value of p for which the roots are equal.

$$px^2-4x+3=0$$

Sol :

Given quadratic equation is

We have $px^2-4x+3=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=p,\ b=-4 \text{ and } c=3$$

Since, given equation has equal roots. So, discriminant D will be zero.

i.e. $D=b^2-4ac=0$

$$(-4)^2-4\times p\times3=0$$
$$16-12p=0$$
$$12p=16$$
$$p=\tfrac{16}{12}=\tfrac{4}{3}$$

Hence, the value of p is $\frac{4}{3}$.

Comp 2010

66. Solve for x : $\frac{1}{x+4}-\frac{1}{x+7}=\frac{11}{30}$ $x\neq-4,\ -7$.

Sol :

We have $\frac{1}{x+4}-\frac{1}{x+7}=\frac{11}{30}$

$$\frac{x+7-x-4}{(x+4)(x+7)}=\frac{11}{30}$$
$$\frac{3}{x^2+4x+7x+28}=\frac{11}{30}$$
$$\frac{3}{x^2+11x+28}=\frac{11}{30}$$
$$11x^2+121x+308=90$$
$$11x^2+121x+218=0$$

Comparing with $ax^2+bx+c=0$, we get $a=11$, $b=121$ and $a=218$ we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-121\pm\sqrt{14641-9592}}{22}$$
$$x=\frac{-121\pm\sqrt{5049}}{22}$$
$$=\frac{-121\pm71.06}{22}$$
$$x=\frac{-49.94}{22},\ \frac{-192.06}{22}$$
$$x=-2.27,\ -8.73.$$

Main 2009

67. Solve the following quadratic equation and give the answer correct to two significant figures.

$$4x^2-7x+2=0$$

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Sol :

We have $4x^2 - 7x + 2 = 0$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 4,\ b = -7 \text{ and } c = 2$$

Using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 4 \times 2}}{2 \times 4}$$

$$= \frac{7 \pm \sqrt{49 - 32}}{8} = \frac{7 \pm \sqrt{17}}{8}$$

$$= \frac{7 \pm 4.123}{8} \qquad [\sqrt{17} = 4.123]$$

Now $x = \frac{7 + 4.123}{8} = \frac{11.123}{8}$ Taking + sign

$$= 1.39$$

and $x = \frac{7 - 4.123}{8}$ Taking − sign

$$= \frac{2.877}{8}$$

$$= 0.359 = 0.36 \text{ (approx)}$$

Hence, the solutions are 1.39 and 0.36.

Comp 2009'

68. Solve for x : $x^2 + 5x - (a^2 + a - 6) = 0$

Sol :

We have $x^3 + 5x - (a^2 + a - 6) = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus $x = \frac{-5 \pm \sqrt{25 + 4(a^2 + a - 6)}}{2}$

$$= \frac{-5 \pm \sqrt{25 + 4a^2 + 4a - 24}}{2}$$

$$= \frac{-5 \pm \sqrt{4a^2 + 4a + 1}}{2}$$

$$= \frac{-5 \pm (2a + 1)}{2}$$

$$= \frac{2a - 4}{2}, \frac{-2a - 6}{2}$$

Thus $x = a - 2,\ x = -(a + 3)$

Alternative :

We have $x^2 + 5x - (a^2 + a - 6) = 0$

$$x^2 + 5x - [a^2 + 3a - 2a - 6] = 0$$

$$x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$$

$$x^2 + 5x - (a + 3)(a - 2) = 0$$

$$x^2 + [a + 3 - (a - 2)]x - (a + 3)(a - 2) = 0$$

$$x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$$

$$x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$$

$$[x + (a + 3)][x - (a - 2)] = 0$$

Thus $x = -(a + 3)$ and $x = (a - 2)$

Hence, roots of given equations are $x = -(a + 3)$ and $x = a - 2$.

Main 2008

69. Solve the quadratic equation $5x(x + 2) = 3$ and give your answer correct to two decimal places.

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Sol :

We have $5x(x + 2) = 3$

$$5x^2 + 10x - 3 = 0$$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 5,\ b = 10 \text{ and } c = -3$$

Using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{(10)^2 - 4 \times 5 \times (-3)}}{2 \times 5}$$

$$= \frac{-10 \pm \sqrt{100 + 60}}{10}$$

$$= \frac{-10 \pm \sqrt{160}}{10}$$

$$= \frac{-10 \pm 12.649}{10} \qquad [\sqrt{160} = 12.649]$$

$x = \frac{-10 + 12.649}{10} = \frac{2.649}{10}$ Taking + sign

$$= 0.2649 = 0.26 \text{ (approx)}$$

$x = \frac{-10 - 12.649}{10}$ Taking − sign

$$= \frac{-22.649}{10} = -2.2649$$

$$= 2.26 \text{ (approx)}$$

Hence, the solutions are 0.26 and -2.26.

Comp 2008

70. Solve the following quadratic equation for x :

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Sol :

We have $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p^2, b = p^2 - q^2,\ c = -q^2$$

The roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(p^2-q^2) - \sqrt{(p^2-q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$
$$= \frac{-(p^2-q^2) - \sqrt{p^4+q^4-2p^2q^2+4p^2q^2}}{2p^2}$$
$$= \frac{-(p^2-q^2) - \sqrt{p^4+q^4+2p^2q^2}}{2p^2}$$
$$= \frac{-(p^2-q^2) - \sqrt{(p^2+q^2)^2}}{2p^2}$$
$$= \frac{-(p^2-q^2) \pm (p^2+q^2)}{2p^2}$$

Thus $x = \dfrac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \dfrac{2q^2}{2p^2} = \dfrac{q^2}{p^2}$

and $x = \dfrac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \dfrac{-2p^2}{2p^2} = -1$

Hence, roots are $\dfrac{q^2}{p^2}$ and -1.

Main 2007

71. Solve the quadratic equation $x^2-3x-9=0$ for x and give your answer correct to two decimal places.

Sol :

We have $x^2-3x-9=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=1,\ b=-3 \text{ and } c=-9$$

Now $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4\times 1\times(-9)}}{2}$$
$$= \frac{3 \pm \sqrt{9+36}}{2} = \frac{3 \pm \sqrt{45}}{2}$$
$$= \frac{3 \pm 6.71}{2} \qquad [\sqrt{45} = 6.7082 = 6.71]$$
$$= 4.85,\ -1.85$$

Comp 2007

72. Find the value of k for which the quadratic equation $(k-2)x^2+2(2k-3)x+(5k-6)=0$ has equal roots.

Sol :

We have $(k-2)x^2+2(2k-3)x+(5k-6)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=k-2, b=2(2k-3), c=(5k-6)$$

For real and equal roots, $b^2-4ac=0$

$$\{2(2k-3)\}^2 - 4(k-2)(5k-6) = 0$$
$$4(4k^2-12k+9) - 4(k-2)(5k-6) = 0$$
$$4k^2-12k+9-5k^2+6k+10k-12 = 0$$
$$k^2-4k+3=0$$
$$k^2-3k-k+3=0$$
$$k(k-3)-1(k-3)=0$$
$$(k-3)(k-1)=0$$

Thus $k=1,3$

Comp 2006

73. If (-5) is a root of the quadratic equation $2x^2+px+15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, then find the values of p and k.

Sol :

We have $2x^2+px-15=0$

Since $x=-5$ is the root of above equation. It must satisfy it.

$$2(-5)^2+p(-5)-15=0$$
$$50-5p-15=0$$
$$5p=35 \Rightarrow p=7$$

Now $p(x^2+x)+k=0$ has equal roots

or $7x^2+7x+k=0$

Taking $b^2-4ac=0$ we have

$$7^2-4\times 7\times k=0$$
$$7-4k=0$$
$$k=\frac{7}{4}$$

Hence $p=7$ and $k=\dfrac{7}{4}$.

Main 2005

74. Solve the equation $2x-\dfrac{1}{x}=7$. Write answer correct to two decimal places.

Sol :

We have $2x-\dfrac{1}{x}=7$

$$\frac{2x^2-1}{x}=6$$
$$2x^2-1=7x$$
$$2x^2-7x-1=0$$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=2,\ b=-7 \text{ and } c=-1$$

Using quadratic formula, we get

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$
$$= \frac{7 \pm \sqrt{49 + 8}}{4}$$
$$= \frac{7 \pm \sqrt{57}}{4}$$
$$= \frac{7 \pm 7.55}{4}$$
$$= 1.75 \pm 1.89$$
$$= 3.64, -0.14$$

Comp 2005

75. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

Sol :

We have $x^2 + px + 16 = 0$ ...(1)

If this equation has equal roots, then discriminant $b^2 - 4ac$ must be zero.

i.e., $b^2 - 4ac = 0$ ...(2)

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = p$ and $c = 16$

Substituting above in equation (2) we have

$$p^2 - 4 \times 1 \times 16 = 0$$
$$p^2 = 64 \Rightarrow p = \pm 8$$

When $p = 8$, from equation (1) we have

$$x^2 + 8x + 16 = 0$$
$$x^2 + 2 \times 4x + 4^2 = 0$$
$$(x + 4)^2 = 0$$
$$x = -4, -4$$

Hence, roots are -4 and -4.

When $p = -8$ from equation (1) we have

$$x^2 - 8x + 16 = 0$$
$$x^2 - 2 \times 4x + 4^2 = 0$$
$$(x - 4)^2 = 0 \Rightarrow x = 4, 4$$

Hence, the required roots are either -4, -4 or 4, 4

Main 2004

76. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places.

Sol :

We have $3x^2 - x - 7 = 0$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 3, b = -1 \text{ and } c = -7$$

Using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(1)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$
$$= \frac{1 \pm \sqrt{1 + 84}}{6}$$
$$= \frac{1 \pm \sqrt{85}}{6}$$
$$= \frac{1 \pm 9.22}{6}$$
$$= \frac{10.22}{6}, \frac{-8.22}{6}$$
$$x = 1.70, -1.37$$

Comp 2004

77. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Sol :

Let the usual speed of train be x km/hr.

According to question we have

$$\frac{300}{x} - \frac{300}{x+5} = 2$$
$$x^2 + 5x - 750 = 0$$
$$x^2 + 30x - 25x - 750 = 0$$
$$(x + 30)(x - 25) = 0$$
$$x = -30 \text{ or } x = 25$$

Since, speed cannot be negative $x \neq -30$.

Thus speed of train is 25 km/hr.

Comp 2003

78. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol :

Let the speed of stream be x km/h.

Then the speed of boat upstream $= (24 - x)$ km/h.

Speed of boat downstream $= (24 + x)$ km/h.

According to the question, we have

$$\frac{32}{24 - x} - \frac{32}{24 + x} = 1$$
$$32\left[\frac{1}{24 - x} - \frac{1}{24 + x}\right] = 1$$

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$$32\left[\frac{24+x-24+x}{576-x^2}\right]=1$$
$$32(24+x-24+x)=576-x^2$$
$$64x=576-x^2$$
$$x^2+64x-576=0$$
$$x^2+72x-8x-576=0$$
$$x(x+72)-8(x+72)=0$$
$$(x-8)(x+72)=0$$
$$x=8,-72$$

Since speed cannot be negative, we reject $x=-72$.
The speed of stream is 8 km/h.

Comp 2002

79. Solve the quadratic equation, $2x^2+ax-a^2=0$ for x.

Sol :

We have $2x^2+ax-a^2=0$

Comparing with $Ax^2+Bx+C=0$ we have

$$A=2, B=a, C=-a^2$$

Now
$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$
$$=\frac{-a\pm\sqrt{a^2-4\times2\times(-a^2)}}{2\times2}$$
$$=\frac{-a\pm\sqrt{a^2+8a^2}}{4}$$
$$=\frac{-a\pm\sqrt{9a^2}}{4}=\frac{-a\pm3a}{4}$$
$$x=\frac{-a+3a}{4},\frac{-a-3a}{4}$$

Thus $x=\frac{a}{2},-a$

Main 2001

80. Solve the quadratic equation $x^2-10x+6=0$ for x and give your answer correct to two decimal places. (2001)

Sol :

We have $x^2-10x+6=0$

Comparing above equation with $ax^2+bx+c=0$ we have

$$a=1, b=-10 \text{ and } c=6$$

Using quadratic formula, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-10)\pm\sqrt{(-10)^2-4\times1\times6}}{2\times1}$$
$$=\frac{-10\pm\sqrt{100-24}}{2}$$
$$=\frac{-10\pm\sqrt{76}}{2}$$
$$=\frac{-10\pm2\sqrt{19}}{2}$$
$$=-5\pm\sqrt{19}$$
$$=-5\pm4.36$$
$$x=9.36, 0.64$$

Comp 2001

81. For what value of k, the roots of the quadratic equation $kx(x-2\sqrt{5})+10=0$ are equal ?

Sol :

We have $kx(x-2\sqrt{5})+10=0$

or, $kx^2-2\sqrt{5}\,kx+10=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=k, b=-2\sqrt{5}\,k \text{ and } c=10$$

Since, roots are equal, $D=b^2-4ac=0$

$$(-2\sqrt{5}\,k)^2-4\times k\times10=0$$
$$20k^2-40k=0$$
$$20k(k-2)=0$$
$$k(k-2)=0$$

Since $k\neq0$, we get $k=2$

Comp 2000

82. If $x=-2$ is a root of the equation $3x^2+7x+p=0$, find the value of k so that the roots of the equation $x^2+k(4x+k-1)+p=0$ are equal.

Sol :

We have $3x^2+7x+p=0$

Since $x=-2$ is the root of above equation, it must satisfy it.

Thus $3(-2)+7(-2)+p=0$

$$p=2$$

Since roots of the equation $x^2+4kx+k^2-k+2=0$ are equal,

$$16k^2-4(k^2-k+2)=0$$
$$16k^2-4k^2+4k-8=0$$
$$12k^2+4k-8=0$$
$$3k^2+k-2=0$$
$$(3k-2)(k+1)=0$$
$$k=\frac{2}{3},-1$$

Hence, roots $=\frac{2}{3},-1$

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FOUR MARK QUESTIONS

Main 2023

83. Solve the following quadratic equation:

$$x^2 + 4x - 8 = 0$$

Give your answer correct to one decimal place. (Use mathematical tables if necessary.)

Sol :

We have $x^2 + 4x - 8 = 0$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 1,\ b = 4,\ c = -8$$

Using quadratic formula we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-8)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{16 + 32}}{2}$$

$$= \frac{-4 \pm \sqrt{48}}{2}$$

$$= \frac{-4 \pm 4\sqrt{3}}{2}$$

$$= \frac{-4 \pm 4 \times 1.73}{2}$$

$$= \frac{-4 \pm 6.92}{2}$$

$$= \frac{-4 + 6.92}{2} \cdot \frac{-4 - 6.92}{2}$$

$$= \frac{2.92}{2} \cdot \frac{-10.92}{2}$$

$$= 1.46 - 5.46$$

Hence, $x = -5.46,\ 1.46$

Comp 2023

84. Two taps running together can fill a cistern in $2\frac{8}{11}$ minutes. If one tap takes 1 minute more than the other to fill the cistern, find the time in which each tap separately can fill the cistern.

Sol :

Two taps together fill the cistern in $2\frac{8}{11}$ minutes

$$= \frac{30}{11} \text{ minutes}$$

Thus It will fill in 1 minute $= \frac{11}{30}$ cistern

Let first tap fills the same cistern in x minutes

and 2nd tap will take $= (x + 1)$ minutes

Thus $$\frac{1}{x} + \frac{1}{(x+1)} = \frac{11}{30}$$

$$\frac{x + 1 + x}{x(x+1)} = \frac{11}{30}$$

$$60x + 30 = 11x^2 + 11x$$

$$11x^2 - 49x - 30 = 0$$

$$11x^2 - (55 - 6)x - 30 = 0$$

$$11x^2 - 55x + 6x - 30 = 0$$

$$11x(x - 5) + 6(x - 5) = 0$$

$$(x - 5)(11x + 6) = 0$$

$$x = 5,\ x = -\frac{6}{11}$$

Here $x = -\frac{6}{11}$ is not possible. Hence, 1st tap takes 5 minute and 2nd tap takes 6 minute.

Comp 2020

85. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

Sol :

Let the average speed of the truck be x km/hr. for first 150 km and for next 200 km, speed be $(x + 20)$ km/hr.

Now $$\frac{150}{x} + \frac{200}{x + 20} = 5$$

$$150(x + 20) + 200x = 5x(x + 20)$$

$$150x + 3000 + 200x = 5x(x + 20)$$

$$x^2 - 50x - 600 = 0$$

$$x^2 - 60x + 10x - 600 = 0$$

$$x(x - 60) + 10(x - 60) = 0$$

$$(x - 60)(x + 10) = 0$$

Negative value is rejected, thus first speed of truck is 60 km/h.

Comp 2019

86. Find the value(s) of k such that the quadratic equation $5x^2 - kx + 3 = 0$ has equal roots.

Sol :

We have $5x^2 - kx + 3 = 0$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 5,\ b = -k \text{ and } c = 3$$

Since, given equation has real and equal roots. So, discriminant D should be zero.

i.e. $D = b^2 - 4ac = 0$

$$(-k)^2 - 4 \times 5 \times 3 = 0$$

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$$k^2 - 60 = 0$$
$$k^2 = 60 = \pm\sqrt{60}$$
$$= \pm\sqrt{4 \times 15}$$
$$= \pm 2\sqrt{15}$$

Hence, the values of k are $\pm 2\sqrt{15}$.

Main 2018

87. Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places.

Sol :

We have $x^2 + 7x = 7$

$$x^2 + 7x - 7 = 0$$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 1,\ b = 7 \text{ and } c = -7$$

Using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{(7)^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$
$$= \frac{-7 \pm \sqrt{49 + 28}}{2} = \frac{-7 \pm \sqrt{77}}{2}$$
$$= \frac{-7 \pm 8.77}{2}$$
$$x = \frac{-7 + 8.77}{2} \qquad \text{Taking + sign}$$
$$= \frac{1.77}{2} = 0.88$$
$$x = \frac{-7 - 8.77}{2} \qquad \text{Taking − sign}$$
$$= -\frac{15.77}{2}$$
$$x = -7.88$$

Comp 2018

88. The sum of squares of two consecutive even numbers is 340. Find the numbers.

Sol :

Let the number be x and $x + 2$.

Now $(x)^2 + (x+2)^2 = 340$

$$x^2 + x^2 + 4x + 4 = 340$$
$$2x^2 + 4x - 336 = 0$$
$$x^2 + 2x - 168 = 0$$
$$(x+14)(x-12) = 0$$
$$x = 12$$

Thus numbers are 12 and 14.

Main 2017

89. The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.

Sol :

Let x be the age of Vivek and $(47 - x)$ be the age of younger brother Amit.

Now according to question,

$$x \times (47 - x) = 550$$
$$47x - x^2 = 550$$
$$x^2 - 47x + 550 = 0$$
$$x^2 - 25x - 22x + 550 = 0$$
$$x(x-25) - 22(x-25) = 0$$
$$(x-25)(x-22) = 0$$
$$x = 22,\ 25$$

Thus age of Vivek is 25 years and age of his younger brother is 22 yr.

Comp 2017

90. A student scored a total of 32 marks in class tests in mathematics ans science. Had he scored 2 marks less in science and 4 more in mathematics, the product of his marks would have been 253. Find his marks in two subjects.

Sol :

Let marks in mathematics be x, then marks in science will be $32 - x$.

According to question,

$$(32 - x - 2)(x + 4) = 253$$
$$(30 - x)(x + 4) = 253$$
$$26x - x^2 + 120 = 253$$
$$x^2 - 26x + 133 = 0$$
$$x^2 - 19x - 7x + 133 = 0$$
$$x(x-19) - 7(x-19) = 0$$
$$x = 7 \text{ or } x = 19$$

If $x = 7$ then marks in mathematics $= 7$, and marks in science $= 25$

If $x = 19$, then marks in mathematics $= 19$ and marks in science $= 13$.

Main 2015

91. Three consecutive natural number are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the number.

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Sol :

Let the three consecutive natural numbers be $x, x+1$ and $x+2$.

Now $(x+1)^2 = (x+2)^2 - (x)^2 + 60$

$x^2 + 2x + 1 = x^2 + 4x + 4 - x^2 + 60$

$x^2 - 2x - 63 = 0$

$x^2 - 9x + 7x - 63 = 0$

$x(x-9) + 7(x-9) = 0$

$(x-9)(x+7) = 0$

$x = 9$ or $x = -7$

As x can't be negative, $x = 9$.
Hence three numbers are 9, 10, 11.

Comp 2015

92. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol :

Let the speed of stream be x km/h.
Then the speed of boat upstream $= (18 - x)$ km/h
Speed of boat downstream $= (18 + x)$ km/h
According to the question, we have

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\frac{24[(18+x) - (18-x)]}{18^2 - x^2} = 1$$

$48x = 324 - x^2$

$x^2 + 48x - 324 = 0$

$x^2 + 54x - 6x - 324 = 0$

$x(x+54) - 6(x+54) = 0$

$(x+54)(x-6) = 0$

$x + 54 = 0, x - 6 = 0$

$x = -54, x = 6$

Since speed cannot be negative, we reject $x = -54$.
The speed of stream is 6 km/h.

Main 2014

93. A two-digit positive number, such that the product of its digits is 6. If 9 is added to the number, then the digits interchange their places. Find the number.

Sol :

Let $10x + y$ be the required two-digit number where y unit place and x is tenth place.
According to the question,

$xy = 6$ and $10x + y + 9 = 10y + x$

$y = \frac{6}{x}$ and $9x - 9y + 9 = 0$

Now, $9(x - y) = -9$

$x - y = -1$ ...(1)

$x - \frac{6}{x} = -1$ $\left[y = \frac{6}{x}\right]$

$\frac{x^2 - 6}{x} = -1$

$x^2 - 6 = -x$

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x+3) - 2(x+3) = 0$

$(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

Since digit cannot be negative, $x = 2$
Substituting $x = 2$ in Eq. (1), we get

$2 - y = -1$

$-y = -1 - 2$

$-y = -3 \Rightarrow y = 3$

The required two-digit number

$10x + y = 10 \times 2 + 3 = 23$

Hence, the required number is 23.

Comp 2014

94. Solve for x : $4x^2 + 4bx - (a^2 - b^2) = 0$

Sol :

We have $4x^2 + 4bx - (a^2 - b^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = 4, B = 4b$ and $C = b^2 - a^2$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-4b \pm \sqrt{(4b)^2 - 4.4(b^2 - a^2)}}{2.4}$$

$$= \frac{-4b \pm \sqrt{16b^2 - 16b^2 + 16a^2}}{8}$$

$$= \frac{-4b \pm 4a}{8}$$

$$= -\frac{(a+b)}{2}, \frac{(a-b)}{2}$$

Hence the roots are $-\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$

Main 2013

95. A shopkeeper purchases a certain number of books for Rs. 960. If the cost per book was Rs. 8 less, the umber of books that could be purchased

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for Rs. 960 would be 4 more. Write an equation, taking the original cost of each book to be Rs. x and solve it to find the original cost of the books.

Sol :

Let x be the cost of one book purchase.
The total cost of books purchase $= Rs.\,960$
The total number of books purchase $= \dfrac{960}{x}$
If cost of 1 book is Rs. 8 less, then cost of one book purchase is $Rs.(x-8)$
The total number of books purchase $= \dfrac{960}{(x-8)}$
According to the question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960\left(\frac{1}{x-8} - \frac{1}{x}\right) = 4$$

$$\frac{960(x-x+8)}{x(x-8)} = 4$$

$$\frac{240 \times 8}{x(x-8)} = 1$$

$$x^2 - 8x = 1920$$

$$x^2 - 8x - 1920 = 0$$

$$x^2 - 48x + 40x - 1920 = 0$$

$$x(x-48) + 40(x-48) = 0$$

$$(x-48)(x+40) = 0$$

$$x = 48,\ -40$$

Since cost cannot be negative, we have

$$x = 48$$

Hence, the original cost of each book is Rs. 48.

Main 2013

96. Without solving the following quadratic equation, find the value of p for which the given equation has real and equal roots.

$$x^2 + (p-3)x + p = 0$$

Sol :

The condition for real and equal roots, the discriminant should be zero i.e. $b^2 - 4ac = 0$. Use this result and find the value of p.
Given quadratic equation is
We have $x^2 + (p-3)x + p = 0$
Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 1,\ b = p-3 \text{ and } c = p$$

For real and equal roots, discriminant should be zero.

i.e. $D = b^2 - 4ac = 0$

$$(p-3)^2 - 4 \times 1 \times p = 0$$

$$p^2 + 9 - 6p - 4p = 0$$

$$p^2 - 10p + 9 = 0$$

$$p^2 - 9p - p + 9 = 0$$

$$p(p-9) - 1(p-9) = 0$$

$$(p-1)(p-9) = 0$$

$$p = 1 \text{ or } p = 9$$

Hence, the values of p are 1 and 9.

Comp 2013

97. The denominator of a fraction is two more than its numerator. If the sum of the fraction and its reciprocal is $\dfrac{34}{15}$, find the fraction.

Sol :

Let numerator be x, then denominator will be $x+2$.
and $\text{fraction} = \dfrac{x}{x+2}$
Now $\dfrac{x}{x+2} + \dfrac{x+2}{x} = \dfrac{34}{15}$

$$15(x^2 + x^2 + 4x + 4) = 34(x^2 + 2x)$$

$$30x^2 + 60x + 60 = 34x^2 + 68x$$

$$4x^2 + 8x - 60 = 0$$

$$x^2 + 2x - 15 = 0$$

$$x^2 + 5x - 3x - 15 = 0$$

$$x(x+5) - 3(x+5) = 0$$

$$(x+5)(x-3) = 0$$

We reject the $x = -5$. Thus $x = 3$ and fraction $= \dfrac{3}{5}$

Main 2012

98. A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the tme taken for the journey would have been 1 h 40 min les? Find the original speed of the car.

Sol :

Let x be the original speed of the car in km/h
Distance cover by a car $= 400$ km
Origin time taken, to travel distance 400 km

$$T_1 = \frac{400}{x} \qquad \left[\text{time} = \frac{\text{distance}}{\text{speed}}\right]$$

Further, speed is increased by 12 km/h i.e new speed is $= (x+12)$ km/h
Now time taken, to travel distance 400 km with new speed

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$$T_2 = \frac{400}{x+12}h$$

But new taken for the journey is 1 h 40 min less i.e $\frac{100}{60} = \frac{5}{3}$ hours less

Thus $\frac{400}{x} - \frac{400}{x+12} = \frac{5}{3}$

$$400\left[\frac{1}{x} - \frac{1}{x+12}\right] = \frac{5}{3}$$

$$400\left[\frac{x+12-x}{x(x+12)}\right] = \frac{5}{3}$$

$$80(12) \times 3 = x(x+12)$$

$$2880 = x^2 + 12x$$

$$x^2 + 12x - 2880 = 0$$

$$x^2 + 60x - 48x - 2880 = 0$$

$$x(x+60) - 48(x+60) = 0$$

$$(x+60)(x-48) = 0$$

$$x = 48 \text{ or } x = -60$$

Since speed cannot be negative, $x = 48$ km/h
Hence, the original speed of the car is 48 km/h.

Comp 2012

99. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Sol :

Let x be the original speed of the train.
Total distance covered by the train,

$$d = 480 \text{ km}$$

Time taken to cover the distance 480 km,

$$t = \frac{480}{x} \text{ hr}$$

If the speed has been increased 8 km/hr, i.e., $(x+8)$,then time taken to cover the distance 480 km,

$$t_1 = \frac{480}{x-8}\text{hr}$$

According to question, we have

$$t_1 - t = 3$$

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left[\frac{x-x+8}{x(x-8)}\right] = 3$$

$$\frac{8}{x^2-8x} = \frac{3}{480} = \frac{1}{160}$$

$$x^2 - 8x - 1280 = 0$$

$$x^2 - 40x + 32x - 1280 = 0$$

$$x(x-40) + 32(x-40) = 0$$

$$(x-40)(x-32) = 0$$

$$x = 40, -32$$

Since, negative speed is not possible, hence the original speed of the train is 40 km/hr.

Comp 2011

100. If the price of a book is reduced by Rs. 5, a person can buy 5 more book for Rs. 300. Find the original list price of the book.

Sol :

Let the original list price be Rs. x.

No. of books bought for Rs. 300 $= \frac{300}{x}$

Reduced list price of the book $= (x-5)$

No. of books bought for Rs. 300 $= \frac{300}{x-5}$

According to questions, we have

$$\frac{300}{x-5} - \frac{300}{x} = 5$$

$$x^2 - 5x - 300 = 0$$

$$(x-20)(x+15) = 0$$

Since price cannot be negative, $x = 20$

Thus original list price is 20 rs.

Main 2010

101. A positive number is divided into two parts, such that the sum of the squares of the two parts is 208. The square of the larger parts is 18 times the smaller part. Taking x as the smaller part of the two parts, find the number.

Sol :

Let x be the smaller and y be the larger part of positive number
According to the question,
Since square of the larger part (y) is 18 times the smaller part (x), we have

$$y^2 = 18x$$

$$y = \sqrt{18x}$$

Since sum of the squares of the two parts is 208,

$$x^2 + y^2 = 208$$

$$x^2 + 18x = 208 \quad [y^2 = 18x]$$

$$x^2 + 18x - 208 = 0$$

$$x^2 + 26x - 8x - 208 = 0$$

$$x(x+26) - 8(x+26) = 0$$

$$(x-8)(x+26) = 0$$

$$x = 8 \text{ or } x = -26$$

Since number is positive, $x = 8$
Then, smaller part of a number is $= 8$ and larger

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part of a number,

$$y = \sqrt{18x}$$
$$= \sqrt{18 \times 8}$$
$$= \sqrt{144} = 12$$

Hence, the required number is $8 + 12 = 20$.

Comp 2010

102. In a class test, the sum of the marks obtained by a student in mathematics and science is 28. Had he got 3 marks more in mathematics and 4 marks less in science, the product of the marks would have been 180. Find his marks in two subjects.

Sol :

Let marks obtained in maths be x, the marks obtained in science will be $28 - x$.

Now $(x+3)(28-x-4) = 180$

$$(x+3)(24-x) = 180$$
$$24x - x^2 + 72 - 3x = 180$$
$$x^2 - 21x + 108 = 0$$
$$(x-9)(x-12) = 0$$
$$x = 9 \text{ or } x = 12$$

Case I : $x = 9$

Marks obtained in maths $= 9$

Marks obtained in science $= 28 - 9 = 19$

Case II : $x = 12$

Marks obtained in maths $= 12$

Marks obtained in science $= 28 - 12 = 16$

Main 2009

103. The speed of an express train is x km/h and the speed of an ordinary train is 12 km/h less than that of of the express train. If the ordinary train takes one hour more than the express train to cover a distance of 240 km, then find the speed of the express train.

Sol :

Given, speed of an express train is x km/h.

Then, speed of an ordinary train $= (x-12)$ km/h

But distance cover both trains $= 240$ km

Time taken by express train,

$$t_1 = \frac{240}{x} \qquad \left[\text{time} = \frac{\text{distance}}{\text{speed}}\right]$$

and time taken by ordinary train,

$$t_2 = \frac{240}{x-12}$$

According to the question,

$$t_1 = t_2 - 1$$
$$\frac{240}{x} = \frac{240}{x-12} - 1$$
$$\frac{240}{x} = \frac{240 - x + 12}{x-12}$$
$$240(x-12) = x(252 - x)$$
$$240x - 2880 = 252x - x^2$$
$$x^2 - 12x - 2880 = 0$$
$$x^2 - 60x + 48x - 2880 = 0$$
$$x(x-60) + 48(x-60) = 0$$
$$(x-60)(x+48) = 0$$
$$x = 60 \text{ or } x = -48$$

Since speed is only positive, $x = 60$

Hence, the speed of the express train is 60 km/h.

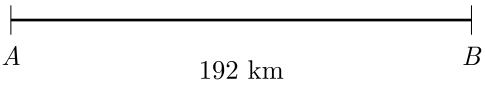
Comp 2009

104. A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passenger train.

Sol :

Let the speed of ordinary passenger train be x.

Thus speed of super-fast train $= (x+16)$ km/h.



Now $T_{passenger} = \frac{192}{x}$

and $T_{\sup erfast} = \frac{192}{(x+16)}$

As per question we have

$$\frac{192}{x} - \frac{192}{x+16} = 2$$
$$192(x+16) - 192x = 2(x^2 + 16x)$$
$$192x + 192 \times 16 - 192x = 2(x^2 + 16x)$$
$$x^2 + 16x - 1536 = 0$$
$$x^2 + 48x - 32x - 1536 = 0$$
$$x(x+48) - 32(x+48) = 0$$
$$(x-32)(x+48) = 0$$
$$x = 32 \text{ or } -48$$

Since speed can't be negative, therefore -48 is not possible. Speed of passenger train is 32 km/h.

Main 2008

105. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed

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to join the party, the cost of the food for each member is increased by Rs. 10. Find how many students went for the picnic?

Sol :

Let x be the number of students, who planned for picnic. Now $x-8$ is the number of students, who went for picnic.

Total budget for the food $= Rs.\,480$

Budget for each student planned for picnic $= \frac{480}{x}$

Budget for each student went for picnic $= \frac{480}{x-8}$

According to the question,

$$\frac{480}{x-8} - \frac{480}{x} = 10$$

$$480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 10$$

$$\frac{480(x-x+8)}{x(x-8)} = 10$$

$$48(8) = x(x-8)$$

$$384 = x^2 - 8x$$

$$x^2 - 8x - 384 = 0$$

$$x^2 - 24x + 16x - 384 = 0$$

$$x(x-24) + 16(x-24) = 0$$

$$(x-24)(x+16) = 0$$

$$x = 24 \text{ or } x = -16$$

Since x is only positive, $x = 24$

Hence, the number of students who went for the picnic, is 24.

Comp 2008

106. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank. Find the time in which each tap can separately fill the tank.

Sol :

Let the tap with smaller diameter fills the tank in x hours, then the other tap fills the tank in $(x-8)$ hours

In one hour small tap will fill $\frac{1}{x}$ tank.

In one hour large tap will fill $\frac{1}{x-8}$ tank.

Thus both tap will fill $\frac{1}{x} + \frac{1}{x-8}$ tank in one hour.

9 hours 36 minutes $= 9 + \frac{36}{60} = 9 + \frac{3}{5} = \frac{48}{5}$

Since two water taps together can fill a tank in $\frac{48}{5}$ hour, tank fill by both pipe in one hour is $\frac{1}{\frac{48}{5}} = \frac{5}{48}$.

Thus $\frac{1}{x} + \frac{1}{x-8} = \frac{5}{48}$

$$\frac{x-8+x}{x(x-8)} = \frac{5}{48}$$

$$5x(x-8) = (2x-8)48$$

$$5x^2 - 136x + 384 = 0$$

$$x = \frac{136 \pm \sqrt{(136)^2 - 4 \times 5 \times 384}}{2.5}$$

$$= \frac{136 \pm \sqrt{18496 - 7680}}{10}$$

$$x = \frac{136 \pm 104}{10} = 24, \frac{16}{5}$$

There is no possibility of $x = \frac{16}{5}$ because it is less than 8 Hours.

Thus smaller tap can fill the tank in 24 hours and larger tap can fill in 16 hrs.

Main 2007

107. Five years ago, a woman's age was the square of her son's age. After ten years, her age will be twice that of her son's age. Find

(i) the age of the son, five years ago.

(ii) the present age of the woman.

Sol :

Let x years be the present age of woman and y year be the age of her son.

Five years ago, woman's age $= (x-5)$ years

and son's age $= (y-5)$ ears

According to the question,

$$(x-5) = (y-5)^2 \quad \ldots(1)$$

After ten years, woman's age $= (x+10)$ years

and son's age $= (y+10)$ years

According to the question,

$$(x+10) = 2(y+10)$$

$$x + 10 = 2y + 20$$

$$x = 2y + 20 - 10$$

$$x = 2y + 10 \quad \ldots(2)$$

Substituting the value of x in Eq. (i), we get

$$2y + 10 - 5 = y^2 + 5^2 - 10y$$

$$y^2 - 10y - 2y + 25 - 5 = 0$$

$$y^2 - 12y + 20 = 0$$

$$y^2 - 10y - 2y + 20 = 0$$

$$y(y-10) - 2(y-10) = 0$$

$$(y-2)(y-10) = 0$$

$$y = 2 \text{ or } y = 10$$

NODIA

Here $y=2$ is not possible because $2-5=-3$, which is not positive.

Thus $y=10$

Substituting $y=10$ in Eq. (2), we get

(i) Five years ago, the age of the son

$$=y-5=10-5=5 \text{ yr}$$

(ii) The present age of the woman

$$=x=30 \text{ yr}$$

Comp 2007

108. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately.

Sol :

Let time taken by pipe A be x minutes and time taken by pipe B be $x+5$ minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank.

In one minute pipe B will fill $\frac{1}{x+5}$ tank.

Thus pipes $A+B$ will fill $\frac{1}{x}+\frac{1}{x+5}$ tank in one minute.

As per question, two pipes running together can fill a tank in $11\frac{1}{9}=\frac{100}{9}$ minutes, in one minute $\frac{9}{100}$ tank will be filled.

Now according to the question we have

$$\frac{1}{x}+\frac{1}{x+5}=\frac{9}{100}$$
$$\frac{x+5+x}{x(x+5)}=\frac{9}{100}$$
$$100(2x+5)=9x(x+5)$$
$$200x+500=9x^2+45x$$
$$9x^2-155x-500=0$$
$$9x^2-180x+25x-500=0$$
$$9x(x-20)+25(x-20)=0$$
$$(x-20)(9x+25)=0$$
$$x=20,\frac{-25}{9}$$

As time can't be negative we take $x=20$ minutes

and $x+5=25$ minutes

Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

Main 2006

109. A plane left 30 minutes late than its scheduled time and in order to reach destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

Sol :

Let x be the usual speed of plane.

Time taken to cover 1500 km with speed x,

$$t_1=\frac{\text{Distance}}{\text{Speed}}=\frac{1500}{x}\text{hr}$$

Time taken to cover 1500 km with increased speed,

$$t_2=\frac{1500}{x+100}\text{hr}$$

Now we have,

$$t_1-t_2=30 \text{ Minutes}=\frac{1}{2}\text{hr}$$

Substituting values of of t_1 and t_2 in above we obtain

$$\frac{1500}{x}-\frac{1500}{x+100}=\frac{1}{2}$$
$$\frac{1}{x}-\frac{1}{x+100}=\frac{1}{2\times 1500}=\frac{1}{3000}$$
$$\frac{x+100-x}{x(x+100)}=\frac{1}{3000}$$
$$\frac{100}{x^2+100x}=\frac{1}{3000}$$
$$x^2+100x=300000$$
$$x^2+100x-300000=0$$
$$x^2+600x-500x-300000=0$$
$$x(x+600)-500(x+600)=0$$
$$(x+600)(x-500)=0$$

Thus $x=-600,\ 500$

Speed can not be negative. So we consider only positive value $x=500$ km/hr

Comp 2006

110. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

Sol :

Suppose B alone finish the work in x days and A alone takes $(x-6)$ days.

$$\text{B's one day work}=\frac{1}{x}$$
$$\text{A' one day work}=\frac{1}{x-6}$$
$$\text{and (A+B)'s one day work}=\frac{1}{4}$$

According to the question, we have

$$\frac{1}{x}+\frac{1}{x-6}=\frac{1}{4}$$
$$x^2-14x+24=0$$
$$x^2-12x-2x+24=0$$
$$x(x-12)-2(x-12)=0$$
$$(x-12)(x-2)=0$$

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, $x = 12$ or $x = 2$

But x cannot be less than 6. So $x = 12$

Hence B can finish the work in 12 days.

Main 2005

111. By increasing the speed of a car by 10 km/h, the time of journey for a distance of 72 km is reduced by 36 min. Find the original speed of the car.

Sol :

Let x be the original speed of car in km/h.

Time taken be this speed to cover 72 km

$$t_1 = \frac{72}{x} \qquad \left[\text{time} = \frac{\text{distance}}{\text{speed}}\right]$$

Time taken to cover 72 km distance with new speed $(x+10)$ km/h

$$t_2 = \frac{72}{x+10}$$

Time with new tipped is reduces by 36 min i.e. $\frac{36}{60} = \frac{3}{5}$ hours.

Thus $t_1 - t_2 \ \frac{3}{5}$

$$\frac{72}{x} - \frac{72}{x+10} = \frac{3}{5}$$

$$72\left[\frac{1}{x} - \frac{1}{x+10}\right] = \frac{3}{5}$$

$$\frac{72(x+10-x)}{x(x+10)} = \frac{3}{5}$$

$$\frac{72(10)}{x(x+10)} = \frac{3}{5}$$

$$3x(x+10) = 5 \times 72 \times 10$$

$$x(x+10) = 1200$$

$$x^2 + 10x - 1200 = 0$$

$$x^2 + 40x - 30x - 1200 = 0$$

$$x(x+40) - 20(x+40) = 0$$

$$(x+40)(x-30) = 0$$

$$x = 30, -40$$

Here speed of car cannot be negative. Hence, the speed of the car is 30 km/h.

Comp 2005

112. A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original average speed?

Sol :

Let x be the original average speed of train.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken to travel to complete 63 km,

$$t_1 = \frac{63}{x}$$

Time taken to travel to complete next 72 km,

$$t_2 = \frac{72}{x+6}$$

Now $t_1 + t_2 = 3$

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$$\frac{21}{x} + \frac{24}{x+6} = 1$$

$$\frac{21(x+6)+24x}{x(x+6)} = 1$$

$$21x + 126 + 24x = x^2 + 6x$$

$$x^2 - 39x - 126 = 0$$

$$x^2 - 42x + 3x - 126 = 0$$

$$(x-42)\ (x+3) = 0 \ \Rightarrow \ x = 42, -3$$

Since speed cant be negative, $x = 42$ and original speed of train is 42 km/hr.

Comp 2004

113. Solve for x : $x^2 - (2b-1)x + (b^2 - b - 20) = 0$

Sol :

We have $x^2 - (2b-1)x + (b^2 - b - 20) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$A = 1, B = -(2b-1), C = (b^2 - b - 20)$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{(2b-1) \pm \sqrt{(2b-1)^2 - 4(b^2 - b - 20)}}{2}$$

$$= \frac{(2b-1) \pm \sqrt{4b^2 - 4b + 1 - 4b^2 + 4b + 80}}{2}$$

$$= \frac{(2b-1) \pm \sqrt{81}}{2} = \frac{(2b-1) \pm 9}{2}$$

$$= \frac{2b+8}{2}, \frac{2b-10}{2}$$

$$= b+4, \ b-5$$

Thus $x = b+4$ and $x = b-5$

NODIA

Comp 2003

114. Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$

Sol :

We have $4x^2 - 4px + (p^2 - q^2) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 4, b = -4p, c = (p^2 - q^2)$$

The roots are given by the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4p \pm \sqrt{16p^2 - 4 \times 4 \times (p^2 - q^2)}}{2 \times 4}$$

$$= \frac{4p \pm \sqrt{16p^2 - 16p^2 + 16q^2}}{8}$$

$$= \frac{4p \pm 4q}{8}$$

Thus roots are $\frac{p+q}{2}, \frac{p-q}{2}$.

Alternative :

We have $4x^2 - 4px + p^2 - q^2 = 0$

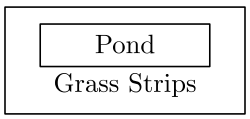
$$(2x - p)^2 - q^2 = 0$$

$$(2x - p + q)(2x - p - q) = 0$$

Thus $x = \frac{(p-q)}{2}$ and $x = \frac{p+q}{2}$

Comp 2002

115. In a rectangular part of dimension $50\text{ m} \times 40\text{ m}$ a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.



Sol :

Let width of grass strip be x.

Length of pond $= (50 - 2x)$ m.

and breadth of pond $= (40 - 2x)$ m.

and area of park – area of pond = area of grass strip

$$(50 \times 40) - (50 - 2x)(40 - 2x) = 1184$$

$$2000 - 2000 + 180x - 4x^2 = 1184$$

$$x^2 - 45x + 296 = 0$$

$$x^2 - 37x - 8x + 296 = 0$$

$$x(x - 37) - 8(x - 37) = 0$$

$$x = 8, 37$$

Here 37 is not possible, thus $x = 8$

Length of pond $= 50 - 2 \times 8$

$= 34$ m

and breadth of pond $= 40 - 2 \times 8 = 24$ m.

Comp 2001

116. In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

Sol :

Let x be the original speed of flight.

Flight distance $d = 600$ km

Original duration, $t = \frac{600}{x}$ $\left(t = \frac{d}{s}\right)$

Duration with reduced speed $x - 200$,

$$t_1 = \frac{600}{x - 200}$$

Flight time increased by 30 minute i.e $\frac{1}{2}$ hour with reduced speed, thus

$$t_1 - t = \frac{1}{2}$$

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2} \left(t = \frac{d}{s}\right)$$

$$600\left[\frac{1}{x-200} - \frac{1}{x}\right] = \frac{1}{2}$$

$$\frac{x - x + 200}{x(x-200)} = \frac{1}{2 \times 600}$$

$$\frac{200}{x^2 - 200x} = \frac{1}{1200}$$

$$x^2 - 200x = 240000$$

$$x^2 - 200x - 240000 = 0$$

$$x^2 - 600x + 400x - 240000 = 0$$

$$x(x - 600) + 400(x - 600) = 0$$

$$(x - 600)(x + 400) = 0 \Rightarrow x = 400,\ 600$$

Since, speed cannot be negative, therefore original speed is 600 km/hr.

Original distance $d = 600$ km

Time $t = \frac{600\text{ km}}{600\text{ km/hr}} = 1$ hr

Hence, the original duration of flight is 1 hr.

FIVE MARK QUESTIONS

Main 2003

117. In an auditorium, seats were arranged in rows and columns. The number of rows was equal to the number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find

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(i) the number of rows in the original arrangement.
(ii) the number of seats in the auditorium after re-arrangement.

Sol :

Let x be the number of rows then number of seats in each row is x.

Thus, total number of seats $= x \times x = x^2$

Now, if number of rows was double i.e. $2x$ and number of seats in each row was reduced by 10 i.e. $x-10$.

Thus, total number of seats is $2x \times (x-10)$.

According to the question,

$$2x(x-10) = x^2 + 300$$
$$2x^2 - 20x - x^2 - 300 = 0$$
$$x^2 - 20x - 300 = 0$$
$$x^2 - 30x + 10x - 300 = 0$$
$$x(x-30) + 10(x-30) = 0$$
$$(x-30)(x+10) = 0$$
$$x = -10 \text{ or } x = 30$$

But $x = 30$ because number of rows is positive.

(i) The number of rows in the original arrangement is 30.
(ii) Number of seats in the auditorium after re-arrangement

$$= x^2 + 300 = (30)^2 + 300$$
$$= 900 + 300 = 1200 \text{ seats}$$

Main 2002

118. An aeroplane travelled a distance of 400 km at an average speed of x km/h. On the return journey, the speed was increased by 40 km/h. Write down an expression for the time taken for
(i) the onward journey.
(ii) the return journey.
If the return journey 30 min less than the onward journey, then write down an equation in x and find its value.

Sol :

Let t in hours be the time taken by aeroplane.

The average speed of aeroplane $= x$ km/h

According to the question, we have

$$400 = x \times t \quad [\text{distance} = \text{speed} \times \text{time}]$$
$$t = \frac{400}{x} \quad ...(1)$$

On the return journey, speed increased by 40 km/h i.e $(x+40)$ km/h and time taken is 30 minute less i.e $\frac{1}{2}$ hours less time

and Time taken $= t - \frac{1}{2}$ hours

According to the question,

$$400 = (x+40)\left(t - \frac{1}{2}\right)$$
$$\left(t - \frac{1}{2}\right) = \frac{400}{x+40}$$
$$t = \frac{400}{x+40} + \frac{1}{2} \quad ...(2)$$

From Eq. (2), substituting the value of t in Eq. (2), we get

$$\frac{400}{x} = \frac{400}{x+40} + \frac{1}{2}$$
$$\frac{400}{x} - \frac{400}{x+40} = \frac{1}{2}$$
$$400\left[\frac{1}{x} - \frac{1}{x+40}\right] = \frac{1}{2}$$
$$\frac{400(x+40-x)}{x(x+40)} = \frac{1}{2}$$
$$800(40) = x^2 + 40x$$
$$x^2 + 40x - 32000 = 0$$
$$x^2 + 200x - 160x - 32000 = 0$$
$$x(x+200) - 160(x+200) = 0$$
$$(x+200)(x-160) = 0$$
$$x = 160 \text{ or } -120$$

But $x = 160$ because speed is positive.

(i) The speed of the onward journey is 160 km/h.
(ii) The speed of the return journey is $160 + 40$ i.e. 200 km/h.

Main 2001

119. The hotel bill for a number of people for overnight stay is Rs. 4800. If there were 4 more, then the bill each had to pay would have reduced Rs. 200. Find the number of people staying overnight.

Sol :

Let x be the number of people staying overnight.

The total amount for overnight stay = Rs 4800

Amount to be paid by each person $= \frac{4800}{x}$

If there are 4 more persons i.e. $x+4$, then the amount paid by each person is $= \frac{4800}{x+4}$

According to the question, we have

$$\frac{4800}{x} - \frac{4800}{x+4} = 200$$
$$4800\left[\frac{1}{x} - \frac{1}{x+4}\right] = 200$$
$$\frac{4800(x+4-x)}{x(x+4)} = 200$$

NODIA

$= 6:1$

Thus (c) is correct option.

6. The simplest form of the ratio 27 : 81 is:
(a) 1 : 3 (B) 3 : 9
(c) 27 : 81 (d) 9 : 27

Sol :

$$\frac{27}{81} = \frac{9}{27} = \frac{1}{3}$$

Thus (a) is correct option.

7. Which of the following ratio is same as 2 : 5?
(a) 6 : 18 (b) 14 : 49
(c) 10 : 25 (d) 9 : 24

Sol :

$$\frac{6}{18} = \frac{1}{3}; \quad \frac{14}{49} = \frac{2}{7}; \quad \frac{10}{25} = \frac{2}{5}$$

Thus (c) is correct option.

8. The ratio between 3.6 m and 75 cm is:
(a) 18 : 275 (b) 24 : 5
(c) 6 : 125 (d) 4 : 5

Sol :

To find the ratio between two numbers, their units must be same.

$$3.6 \text{ m} = 3.6 \times 100 \text{ cm} = 360 \text{ cm}$$

Now, ratio between 360 cm and 75 cm is

$$\frac{360}{75} = \frac{72}{15} = \frac{24}{5}$$

Thus (b) is correct option.

9. If $x:y = 2:3$, then $(7x-4y):(5x+2y)$ is equal to
(a) 1 : 8 (b) 2 : 3
(c) 4 : 9 (d) 5 : 7

Sol :

We have, $\dfrac{7x-4y}{5x+2y} = \dfrac{7(\frac{x}{y})-4}{5(\frac{x}{y})+2}$

$$= \frac{7(\frac{2}{3})-4}{5(\frac{2}{3})+2} \qquad x:y = 2:3$$

$$= \frac{3}{3} \times \frac{7(\frac{2}{3})-4}{5(\frac{2}{3})+2}$$

$$= \frac{14-12}{10+6} = \frac{2}{16} = \frac{1}{8}$$

Thus $(7x-4y):(5x+2y) = 1:8$

Thus (a) is correct option.

10. If $x:y = 5:3$, then the value of $(8x-5y)$ $(8x-5y):(6x+7y)$ is:
(a) 24 : 27 (b) 35 : 37
(c) 25 : 9 (d) 25 : 17

Sol :

$$\frac{8x-5y}{6x+7y} = \frac{8(\frac{x}{y})-5}{6(\frac{x}{y})+7}$$

$$= \frac{8(\frac{5}{3})-5}{6(\frac{5}{3})+7} \qquad x:y = 5:3$$

$$= \frac{3}{3} \times \frac{8(\frac{5}{3})-5}{6(\frac{5}{3})+7}$$

$$= \frac{40-15}{30+21} = \frac{25}{51}$$

Thus (d) is correct option.

11. If $2x+3y:3x+5y = 18:29$, then the ratio $x:y$ is:
(a) 2 : 3 (b) 3 : 5
(c) 3 : 4 (d) 5 : 29

Sol :

$$\frac{2x+3y}{3x+5y} = \frac{18}{29}$$

$$29(2x+3y) = 18(3x+5y)$$

$$58x+87y = 54x+90y$$

$$4x = 3y$$

$$\frac{x}{y} = \frac{3}{4}$$

Thus (c) is correct option.

12. If $a:b = \frac{1}{2}:\frac{3}{8}$ and $b:c = \frac{1}{3}:\frac{5}{9}$ then $a:c$ is:
(a) 1 : 9 (b) 1 : 4
(c) 2 : 3 (d) 4 : 5

Sol :

We have, $a:b = \dfrac{1}{2}:\dfrac{3}{8}$

$$\frac{a}{b} = \frac{\frac{1}{2}}{\frac{3}{8}} = \frac{\frac{1}{2} \times 8}{\frac{3}{8} \times 8} = \frac{4}{3}$$

and $b:c = \dfrac{1}{3}:\dfrac{5}{9}$

and $$\frac{b}{c} = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{\frac{1}{3} \times 9}{\frac{5}{9} \times 9} = \frac{3}{5}$$

Now, $$\frac{a}{c} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c}$$

$$= \frac{4}{3} \times \frac{3}{5} = \frac{4}{3}$$

Thus (d) is correct option.

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CHAPTER 6

RATIO AND PROPORTION

MULTIPLE CHOICE QUESTIONS

Main 2024

1. The given table shows the distance covered and the time taken by a train moving at a uniform speed along a straight track.

Distance (in m)	60	90	y
Time (in sec)	2	x	5

The values of x and y are :

(a) $x = 4, y = 150$
(b) $x = 3, y = 100$
(c) $x = 4, y = 100$
(d) $x = 3, y = 150$

Sol :

We have $$\frac{60}{2} = \frac{90}{x} = \frac{y}{5}$$

$$30 = \frac{90}{x} = \frac{y}{5}$$

Now $$x = \frac{90}{30} = 3 \text{ sec}$$

$$y = 30 \times 5 = 150 \text{ m}$$

Thus (d) is correct option.

SQP 2024

2. The table shows the values of x and y, where x is proportional to y

x	6	12	N
y	M	18	6

What are the values of M and N ?

(a) $M = 4, N = 9$
(b) $M = 9, N = 3$
(c) $M = 9, N = 4$
(d) $M = 12, N = 0$

Sol :

We have $$\frac{6}{M} = \frac{12}{18} = \frac{N}{6}$$

$$\frac{6}{M} = \frac{2}{3} = \frac{N}{6}$$

Now $$M = \frac{6}{\frac{2}{3}} = 9$$

$$N = \frac{2}{3} \times 6 = 4 \text{ m}$$

Thus (c) is correct option.

Main 2023

3. The mean proportional between 4 and 9 is:

(a) 4 (b) 6
(c) 9 (d) 36

Sol :

Let the mean proportional between 4 and 9 be x.

Then, $4 : x = x : 9$

$$\frac{4}{x} = \frac{x}{9}$$

$$x^2 = 36$$

$$x = \sqrt{36} = 6$$

Thus (b) is correct option.

Main 2022 Sem I

4. The mean proportional between 9 and 16 is :

(a) 25 (b) 144
(c) 7 (d) 12

Sol :

Let the mean proportional between 9 and 16 be x.

Now $$\frac{9}{x} = \frac{x}{16}$$

$$x^2 = 9 \times 16$$

$$x^2 = 144$$

$$x = \sqrt{144}$$

Hence, the required mean proportional is 12.
Thus (d) is correct option.

5. The ratio $\frac{2}{3} : \frac{1}{9}$ in its simplest form is:

(a) 1 : 3 (b) 2 : 1
(c) 6 : 1 (d) 2 : 9

Sol :

Multiplying each term by the LCM of 3 and 9 *i.e.* 9 we have

$$\frac{2}{3} : \frac{1}{9} = \frac{2}{3} \times 9 : \frac{1}{9} \times 9$$

NODIA

$$\frac{24(4)}{x(x+4)} = 1$$

$$x^2 + 4x = 96$$

$$x^2 + 4x - 96 = 0$$

$$x^2 + 12x - 8x - 96 = 0$$

$$x(x+12) - 8(x+12) = 0$$

$$(x-8)(x+12) = 0$$

$$x = 8 \text{ or } x = -12$$

Number of people cannot be negative. Hence, the number of people staying overnight is 8.

Main 2000

120. Find the value(s) of m for which the quadratic equation $(1+3m)x^2 - 2(1+m)x + m = 0$ has equal roots.

We have $(1+3m)x^2 - 2(1+m)x + m = 0$

Comparing above equation with $ax^2 + bx + c = 0$ we have

$$a = 1+3m,\ b = -2(1+m) \text{ and } c = m$$

Since, given equation has real roots. So, discriminant $D = 0$.

$$b^2 - 4ac = 0$$

$$[-2(1+m)]^2 - 4 \times (1+3m) \times m = 0$$

$$4[1 + m^2 + 2m] - 4[m + 3m^2] = 0$$

$$4\left[(1 + m^2 + 2m) - (m + 3m^2)\right] = 0$$

$$1 + m^2 + 2m - m - 3m^2 = 0$$

$$2m^2 - m - 1 = 0$$

$$2m^2 - 2m + m - 1 = 0$$

$$2m(m-1) + 1(m-1) = 0$$

$$(2m+1)(m-1) = 0$$

$$m = 1 \text{ or } 2m + 1 = 0 \Rightarrow m = -\tfrac{1}{2}$$

Hence, the values of m are 1 and $-\frac{1}{2}$.

END

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13. If $\frac{3x+5y}{3x-5y}=\frac{7}{3}$ then $x:y$ is:
(a) 25 : 6 (b) 7 : 3
(c) 9 : 49 (d) 10 : 4

Sol :

$$\frac{3x+5y}{3x-5y}=\frac{7}{3}$$
$$3(3x+5y)=7(3x-5y)$$
$$9x+15y=21x-35y$$
$$12x=50y$$
$$\frac{x}{y}=\frac{50}{12}=\frac{25}{6}$$

Thus (a) is correct option.

14. If $x:50=3:2$, then value of x is_____.
(a) 75 (b) 65
(c) 55 (d) 50

Sol :

$$x:50=3:2$$
$$\frac{x}{50}=\frac{3}{2}$$
$$2x=150$$
$$x=75$$

Thus (a) is correct option.

15. If $4x=7y=9z$, then $x:y:z=$
(a) 24 : 31 : 19 (b) 63 : 36 : 28
(c) 4 : 7 : 9 (d) 16 : 49 : 81

Sol :

$$4x=7y=9z=k \text{ (say)}$$
$$x=\frac{k}{4};\ y=\frac{k}{7};\ z=\frac{k}{9}$$

Now, $x:y:z=\frac{k}{4}:\frac{k}{7}:\frac{k}{9}$

$$=\frac{k}{4}\times 252:\frac{k}{7}\times 252:\frac{k}{9}\times 252$$
$$=63k:36k:28k=63:36:28$$

16. If $x:y=3:2$, then $(3x^2+2y^2):(3x^2-2y^2)$ is:
(a) 17 : 13 (b) 45 : 23
(c) 27 : 8 (d) 35 : 19

Sol :

$$\frac{x}{y}=\frac{3}{2}$$
$$\frac{x^2}{y^2}=\frac{9}{4}$$
$$\frac{3x^2}{2y^2}=\frac{3\times 9}{2\times 4}=\frac{27}{8}$$

Applying componendo and dividendo, we get
$$\frac{3x^2+2y^2}{3x^2-2y^2}=\frac{27+8}{27-8}=\frac{35}{19}$$

Thus (d) is correct option.

17. If $x:y=2:9$ and $y:z=3:5$, then $x:z$ is:
(a) 2 : 15 (b) 4 : 15
(c) 6 : 5 (d) 2 : 5

Sol :

We have $x:y=2:9$ and $y:z=3:5$

Now $\frac{x}{z}=\frac{\frac{x}{y}}{\frac{z}{y}}=\frac{x}{y}\times\frac{y}{z}$

$$=\frac{2}{9}\times\frac{3}{5}$$
$$=\frac{2}{15}$$

Thus (a) is correct option.

18. If $a:b=3:2$, then $(a+b)^2:(a-b)^2$ is:
(a) 9 : 4 (b) 1 : 25
(c) 25 : 1 (d) 4 : 9

Sol :

We have $\frac{a}{b}=\frac{3}{2}$

Applying componendo and dividendo, we get
$$\frac{a+b}{a-b}=\frac{3+2}{3-2}=\frac{5}{1}$$
$$\left(\frac{a+b}{a-b}\right)^2=\left(\frac{5}{1}\right)^2$$
$$\frac{(a+b)^2}{(a-b)^2}=\frac{25}{1}$$

Thus (c) is correct option.

19. If $x^2+4y^2=4xy$, then $x:y$ is:
(a) 1 : 4 (b) 4 : 1
(c) 2 : 1 (d) 1 : 2

Sol :

We have, $x^2+4y^2=4xy$
$$x^2+4y^2-4xy=0$$
$$(x-2y)^2=0$$
$$x-2y=0$$
$$x=2y$$
$$\frac{x}{y}=\frac{2}{1}$$

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Now, required compounded ratio

$$= (49 \times 5 \times 36) : (36 \times 7 \times 35)$$

$$= 1 : 1$$

Thus (a) is correct option.

35. If $(2x + 5) : (8x + 9)$ is the duplicate ratio of 3 : 5, then the value of x is:

(a) 4 (b) 1

(c) 2 (d) 3

Sol :

Since, $(2x+5):(8x+9)$ is the duplicate ratio of 3 : 5

$$\frac{2x+5}{8x+9} = \frac{(3)^2}{(5)^2} = \frac{9}{25}$$

$$25(2x+5) = 9(8x+9)$$

$$50x + 125 = 72x + 81$$

$$125 - 81 = 72x - 50x$$

$$44 = 22x$$

$$x = 2$$

Thus (c) is correct option.

36. If $x : y = 3 : 4$ and $y : z = 5 : 2$, then $x : y : z$ is:

(a) 6 : 20 : 6 (b) 3 : 9 : 2

(c) 3 : 20 : 2 (d) 15 : 20 : 8

Sol :

We have, $\frac{x}{y} = \frac{3}{4}$

and $\frac{y}{z} = \frac{5}{2}$

To find the ratio $x : y : z$, we have to make y in both the ratios equal.

$$\frac{x}{y} = \frac{3}{4} \times \frac{5}{3} = \frac{15}{20}$$

and $\frac{y}{z} = \frac{5}{2} \times \frac{4}{4} = \frac{20}{8}$

$$x : y : z = 15 : 20 : 8$$

Thus (d) is correct option.

37. If $x+3$ is the mean proportion between $x+2$ and $x+9$, then the value of x is.

(a) $-\frac{9}{5}$ (b) $\frac{9}{5}$

(c) $\frac{5}{9}$ (d) $-\frac{5}{9}$

Sol :

Since $x+3$ is the mean proportion $x+2$ and $x+9$, we have

$$(x+3)^2 = (x+2)(x+9)$$

$$x^2 + 6x + 9 = x^2 + 11x + 18$$

$$-5x = 9$$

$$x = -\frac{9}{5}$$

Thus (d) is correct option.

38. If the numbers $p + 3$, $p + 2$, $3p - 7$ and $2p - 3$ are in proportion, then the value of p is:

(a) – 1 (b) 1

(c) 5 (d) – 5

Sol :

Since the numbers $p+3, p+2, 3p-7$ and $2p-3$ are in proportion.

$$\frac{p+3}{p+2} = \frac{3p-7}{2p-3}$$

$$(p+3)(2p-3) = 3p^2 - p - 14$$

$$p^2 - 4p - 5 = 0$$

$$p(p-5) + 1(p-5) = 0$$

$$(p+1)(p-5) = 0$$

$$p = -1, 5$$

For $p = -1$, $3p-7$ and $2p-3$ are negative, which is not possible.

$$p = 5$$

Thus (c) is correct option.

39. The number that must be added to each of the number 6, 15, 20 and 43 to make them proportional is

(a) 0 (b) 1

(c) 2 (d) 3

Sol :

Let x be the required number.

Then, according to the question $(6 + x)$, $(15 + x)$, $(20 + x)$ and $(43 + x)$ are in proportion. Thus

$$\frac{6+x}{15+x} = \frac{20+x}{43+x}$$

$$(6+x)(43+x) = (15+x)(20+x)$$

$$x^2 + 49x + 258 = x^2 + 35x + 300$$

$$14x = 42$$

$$x = 3$$

Thus (d) is correct option.

40. If $(x-2), (x+2), (2x+1)$ and $(2x+19)$ are in proportion, then the value of x is

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(a) $\frac{3}{10}$ (b) $\frac{3}{20}$

(c) $\frac{3}{25}$ (d) 10

Sol :

Let x be the fourth proportional.

Now $\frac{\frac{1}{3}}{\frac{1}{4}} = \frac{\frac{1}{5}}{x}$

$$\frac{x}{3} = \frac{1}{20}$$

$$x = \frac{3}{20}$$

Thus (b) is correct option.

26. The fourth proportional to 0.4, 2 and 10 is:

(a) 50 (b) 12.5

(c) 2 (d) 15

Sol :

Let x be the fourth proportional.

Now 0.4, 2, 10 and x are in proportion, so we have

$$\frac{0.4}{2} = \frac{10}{x}$$

$$0.4x = 20$$

$$x = 50$$

Thus (a) is correct option.

27. The duplicate ratio of $\sqrt{3} : 5$ is:

(a) 3 : 25 (b) 7 : $\sqrt{3}$

(c) $2\sqrt{3}$: 14 (d) $(\sqrt{3}+2)$: 9

Sol :

Duplicate ratio of $\sqrt{3} : 5$ is $\frac{(\sqrt{3})^2}{(5)^2} = \frac{3}{25}$

Thus (a) is correct option.

28. The triplicate ratio of 1 : 2 is :

(a) 3 : 6 (b) 1 : 8

(c) 3 : 9 (d) 3 : 1

Sol :

Triplicate ratio of 1 : 3 is $\frac{(1)^3}{(2)^3} = \frac{1}{8}$

Thus (b) is correct option.

29. The sub-duplicate ratio of 9 : 16 is:

(a) 7 : 14 (b) 4.5 : 8

(c) 81 : 256 (d) 3 : 4

Sol :

Sub-duplicate ratio of 9 : 16 is $\frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

30. The sub-triplicate ratio of 64 : 125 is:

(a) 16 : 17 (b) 61 : 340

(c) 4 : 5 (d) $\frac{64}{3} : \frac{125}{3}$

Sol :

Sub-triplicate ratio of 64 : 125 is $\frac{\sqrt[3]{64}}{\sqrt[3]{125}} = \frac{4}{5}$

Thus (c) is correct option.

31. The compounded ratio of 2 : 3 and 5 : 9 is:

(a) 10 : 27 (b) 6 : 45

(c) 18 : 15 (d) 7 : 12

Sol :

Compounded ratio of 2 : 3 and 5 : 9 is

$(2 \times 5) : (3 \times 9) = 10 : 27$

32. The reciprocal ratio of $\frac{1}{9}$: 2 is:

(a) 9 : $\frac{1}{2}$ (b) 18 : 1

(c) 9 : 2 (d) 2 : 18

Sol :

We have, $\frac{1}{9} : 2 = \frac{1}{9} \times 9 : 2 \times 9$

$$= 1 : 18$$

Now, reciprocal ratio of 1 : 18 = 18 : 1

Thus (b) is correct option.

33. The mean proportional between 4 and 9 is :

(a) $\frac{5}{2}$ (b) $\frac{13}{2}$

(c) 6 (d) 6.5

Sol :

Mean proportional of 4 and 9 is $\sqrt{4 \times 9} = \sqrt{36} = 6$

Thus (c) is correct option.

34. The ratio compounded of the duplicate of 7 : 6, sub-duplicate of 25 : 49 and the reciprocal of 35 : 36 is :

(a) 1 : 1 (b) 2 : 3

(c) 4 : 7 (d) 5 : 9

Sol :

Duplicate of 7 : 6 = 49 : 36

Sub-duplicate of 25 : 49 = 5 : 7

and Reciprocal of 35 : 36 = 36 : 35

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Substituting $y=12$ in (1), we get

$$x=\frac{36}{12}$$

$$x=3$$

Hence, the numbers are $x=3$ and $y=12$.

Main 2020

45. Using properties of proportion find $x:y$, given

$$\frac{x^2+2x}{2x+4}=\frac{y^3+3y}{3y+9}$$

Sol :

We have $$\frac{x^2+2x}{2x+4}=\frac{y^2+3y}{3y+9}$$

Using componendo and dividendo properties,

$$\frac{x^2+2x+2x+4}{x^2+2x-2x-4}=\frac{y^2+3y+3y+9}{y^2+3y-3y-9}$$

$$\frac{x^2+4x+4}{x^2-4}=\frac{y^2+6y+9}{y^2-9}$$

$$\frac{(x+2)^2}{(x+2)(x-2)}=\frac{(y+3)^2}{(y+3)(y-3)}$$

$$\frac{x+2}{x-2}=\frac{y+3}{y-3}$$

Again, applying componendo and dividendo property, we get

$$\frac{x+2+x-2}{x+2-x+2}=\frac{y+3+y-3}{y+3-y+3}$$

$$\frac{2x}{4}=\frac{2y}{6}$$

$$\frac{x}{2}=\frac{y}{3}$$

$$\frac{x}{y}=\frac{2}{3}$$

$$x:y=2:3$$

Main 2020

46. If $x=\dfrac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$, prove that

$$x^2-4ax+1=0$$

Sol :

We have $$x=\frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$$

$$\frac{x}{1}=\frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$$

Applying componendo and dividendo property, we get,

$$\frac{x+1}{x-1}=\frac{\sqrt{2a+1}+\sqrt{2a-1}+\sqrt{2a+1}-\sqrt{2a-1}}{\sqrt{2a+1}+\sqrt{2a-1}-\sqrt{2a-1}+\sqrt{2a-1}}$$

$$\frac{x+1}{x-1}=\frac{\sqrt{2a+1}}{\sqrt{2a-1}}$$

Squaring both sides, we get,

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{2a+1}{2a-1}$$

Again applying componendo and dividendo property, we get,

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1}=\frac{2a+1+2a-1}{2a+1-2a+1}$$

$$\frac{2x^2+2}{4x}=\frac{4a}{2}$$

$$\frac{x^2+1}{2x}=2a$$

$$x^2+1=4ax$$

$$x^2-4ax+1=0 \qquad \text{Hence proved}$$

Comp 2020

47. If b is the mean proportional between a and c, prove that

$$\frac{a^2-b^2+c^2}{a^{-2}-b^{-2}+c^{-2}}=b^4$$

Sol :

Since b is the mean proportion between a and c, a, b, c are in continued proportion.

$$\frac{a}{b}=\frac{b}{c}=k(\text{say})$$

$$a=bk \text{ and } b=ck$$

$$a=(ck)\,k=ck^2 \text{ and } b=ck$$

Now $$\frac{a^2-b^2+c^2}{a^{-2}-b^{-2}+c^{-2}}=b^4$$

$$\text{LHS}=\frac{a^2-b^2+c^2}{a^{-2}-b^{-2}+c^{-2}}$$

$$=\frac{(ck^2)^2-(ck)^2+c^2}{(ck^2)^{-2}-(ck)^{-2}+c^{-2}}$$

$$=\frac{c^2k^4-c^2k^2+c^2}{c^{-2}k^{-4}-c^{-2}k^{-2}+c^{-2}}$$

$$=\frac{c^2(k^4-k^2+1)}{c^{-2}k^{-4}(1-k^2+k^4)}$$

$$=\frac{c^4k^4(k^4-k^2+1)}{(k^4-k^2+1)}=c^4k^4$$

$$=(ck)^4=b^4$$

$$=\text{RHS}$$

Main 2019

48. The numbers $K+3$, $K+2$, $3K-7$ and $2K-3$ are in proportion. Find K.

Sol :

Given numbers $K+3$, $K+2$, $3K-7$ and $2K-3$ are in proportion. Therefore

$$(K+3):(K+2)::(3K-7):(2K-3)$$

$$\frac{K+3}{K+2}=\frac{3K-7}{2K-3}$$

NODIA

(a) 5 (b) 6
(c) 3 (d) 4

Sol :

Since, $(x-2),(x+2),(2x+1)$ and $(2x+19)$ are in proportion.

$$\frac{x-2}{x+2}=\frac{2x+1}{2x+19}$$

$$(x-2)(2x+19)=(x+2)(2x+1)$$

$$2x^2+15x-38=2x^2+5x+2$$

$$10x=40$$

$$x=4$$

Thus (d) is correct option.

41. If a, b, c and d are in proportional, then $\sqrt{\frac{3a^2+8b^2}{3c^2+8d^2}}$ is equal to:

(a) $\frac{c}{a}$ (b) $\frac{b}{d}$
(c) $\frac{a}{b}$ (d) $\frac{c}{d}$

Sol :

Since, a,b, c and d are in proportion.

$$\frac{a}{b}=\frac{c}{d}=k \text{ (say)}$$

$$a=bk \text{ and } c=dk$$

Now, $$\sqrt{\frac{3a^2+8b^2}{3c^2+8d^2}}=\sqrt{\frac{3(bk)^2+8b^2}{3(dk)^2+8d^2}}$$

$$=\sqrt{\frac{b^2(3k^2+8)}{d^2(3k^2+8)}}$$

$$=\frac{\sqrt{b^2}}{d^2}=\frac{b}{d}$$

Thus (b) is correct option.

THREE MARKS QUESTIONS

Main 2023

42. What number must be added to each of the numbers 4, 6, 8, 11 in order to get the four numbers in proportion?

Sol :

Let the number to be added be x.
According to the question, we have

$$(4+x):(6+x)::(8+x):(1+x)$$

$$\frac{4+x}{6+x}=\frac{8+x}{11+x}$$

$$(4+x)(11+x)=(8+x)(6+x)$$

$$x^2+15x+44=x^2+14x+48$$

$$x=4$$

Hence, the required number to be added is 4.

Main 2023

43. Using componendo and dividendo solve for x:

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+1}-\sqrt{2x-1}}=3$$

Sol :

We have $$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}}=3$$

Using componendo and dividendo, we have

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}+(\sqrt{2x+2}-\sqrt{2x-1})}{\sqrt{2x+2}-\sqrt{2x-1}-(2x+2)-\sqrt{2x-1}}=\frac{3+1}{3-1}$$

$$\frac{2\sqrt{2x+2}}{2\sqrt{2x-1}}=\frac{4}{2}$$

$$\frac{\sqrt{2x+2}}{\sqrt{2x-1}}=2$$

Squaring both sides, we get

$$\frac{2x+2}{2x-1}=4$$

$$2x+2=4(2x-1)$$

$$2x+2=8x-4$$

$$6x=6$$

$$x=1$$

Comp 2023

44. If 6 is the mean proportion between two numbers x and y and 48 is the third proportional to x and y, find the numbers.

Sol :

Since 6 is the mean proportion between two numbers x and y, we have

$$\frac{x}{6}=\frac{6}{y}$$

$$xy=36$$

$$x=\frac{36}{y} \quad ...(1)$$

Since 48 is the third proportional to x and y, therefore,

$$\frac{x}{y}=\frac{y}{48}$$

$$y^2=48x$$

$$y^2=48\times\frac{36}{y} \quad \text{Using (1)}$$

$$y^3=48\times 36$$

$$y^3=12^3 \Rightarrow y=12$$

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Thus (c) is correct option.

20. The ratios 2 : 3, 8 : 15, 11 : 12, 7 : 16 in their ascending order of magnitude are:

(a) $\frac{2}{3} < \frac{7}{16} < \frac{8}{15} < \frac{11}{12}$

(b) $\frac{7}{16} < \frac{8}{15} < \frac{2}{3} < \frac{11}{12}$

(c) $\frac{7}{16} < \frac{8}{15} < \frac{7}{12} < \frac{2}{3}$

(d) $\frac{2}{3} < \frac{11}{12} < \frac{8}{15} < \frac{7}{16}$

Sol :

LCM of $3, 15, 12, 16 = 240$

$$\frac{2}{3} = \frac{2}{3} \times \frac{240}{240} = \frac{160}{240}$$

$$\frac{8}{15} = \frac{8}{15} \times \frac{240}{240} = \frac{128}{240}$$

$$\frac{11}{12} = \frac{11}{12} \times \frac{240}{240} = \frac{220}{240}$$

and $$\frac{7}{16} = \frac{7}{16} \times \frac{240}{240} = \frac{105}{240}$$

Now, since $105 < 128 < 160 < 220$

$$\frac{105}{240} < \frac{128}{240} < \frac{160}{240} < \frac{220}{240}$$

$$\frac{7}{16} < \frac{8}{15} < \frac{2}{3} < \frac{11}{12}$$

which is the ascending order of the given ratios.
Thus (b) is correct option.

21. If $(2x^2 - 5y^2) : xy = 1 : 3$, then $x : y$ is:

(a) 3 : 2 (b) – 5 : 3

(c) 5 : 3 (d) – 3 : 2

Sol :

We have $$\frac{2x^2 - 5y^2}{xy} = \frac{1}{3}$$

$$3(2x^2 - 5y^2) = xy$$

$$6x^2 - xy - 15y^2 = 0$$

$$6x^2 - 10xy + 9xy - 15y^2 = 0$$

$$2x(3x - 5y) + 3y(3x - 5y) = 0$$

$$(2x + 3y)(3x - 5y) = 0$$

$$2x = -3y \text{ or } 3x = 5y$$

$$\frac{x}{y} = -\frac{3}{2} \text{ or } \frac{x}{y} = \frac{5}{3}$$

But, ratio is always taken between positive quantities,

$$\frac{x}{y} = \frac{5}{3}$$

Thus (c) is correct option.

22. If $m : n = 5 : 3$, then $(m^2 + n^2) : (m^2 - n^2)$ is:

(a) 17 : 8 (b) 15 : 7

(c) 8 : 17 (d) 7 : 15

Sol :

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We have $$\frac{m}{n} = \frac{5}{3}$$

$$\frac{m^2}{n^2} = \frac{25}{9}$$

Applying componendo and dividendo,

$$\frac{m^2 + n^2}{m^2 - n^2} = \frac{25 + 9}{25 - 9} = \frac{34}{16} = \frac{17}{8}$$

23. If $(x^2 + y^2) : (x^2 - y^2) = 17 : 8$, then $x : y$ is:

(a) 3 : 7 (b) 25 : 9

(c) 8 : 17 (d) 5 : 3

Sol :

We have $$\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$$

Applying componendo and dividendo, we have

$$\frac{(x^2 + y^2) + (x^2 - y^2)}{(x^2 + y^2) - (x^2 - y^2)} = \frac{17 + 8}{17 - 8}$$

$$\frac{2x^2}{2y^2} = \frac{25}{9}$$

$$\frac{x^2}{y^2} = \frac{25}{9}$$

Taking sub-duplicate ratio, we get

$$\frac{x}{y} = \frac{5}{3}$$

24. The third proportional to 2 and 7 is:

(a) $\sqrt{14}$ (b) 24.5

(c) 14 (d) 9

Sol :

Let x be the third proportional.
Since 2, 7 and x are in proportion, we have

$$\frac{2}{7} = \frac{7}{x}$$

$$2x = 49$$

$$x = 24.5$$

Thus (b) is correct option.

25. The fourth proportional to $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ is _____.

$$(K+3)(2K-3) = (K+2)(3K-7)$$
$$2K^2+3K-9 = 3K^2-K-14$$
$$K^2-4K-5 = 0$$
$$K^2-5K+K-5 = 0$$
$$K(K-5)+1(K-5) = 0$$
$$(K-5)(K+1) = 0$$
$$K = -1, 5$$

Main 2019

49. Using properties of proportion solve for x, given $\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} = 4$.

Sol :

We have $\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} = \frac{4}{1}$

Applying componendo and dividendo rule,

$$\frac{(\sqrt{5x}+\sqrt{2x-6})+(\sqrt{5x}-\sqrt{2x-6})}{(\sqrt{5x}+\sqrt{2x-6})-(\sqrt{5x}-\sqrt{2x-6})} = \frac{4+1}{4-1}$$

$$\frac{2\sqrt{5x}}{2\sqrt{2x-6}} = \frac{5}{3}$$
$$3\sqrt{5x} = 5\sqrt{2x-6}$$

Squaring both sides, we get

$$9(5x) = 25(2x-6)$$
$$45x = 50x-150$$
$$5x = 150$$
$$x = 30$$

Comp 2019

50. If b is the mean proportion between a and c, show that :

$$\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}$$

Sol :

Since b is the mean proportion between a and c, a, b, c are in continued proportion.

Thus $\frac{a}{b} = \frac{b}{c} = k$(say)

$$a = bk \text{ and } b = ck$$
$$a = (ck)k = ck^2 \text{ and } b = ck$$

Now $\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}$

$$\text{LHS} = \frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4}$$
$$= \frac{(ck^2)^4+(ck^2)^2(ck)^2+(ck)^4}{(ck)^4+(ck)^2c^2+c^4}$$
$$= \frac{c^4k^8+c^4k^6+c^4k^4}{c^4k^4+c^4k^2+c^4}$$
$$= \frac{c^4k^4(k^4+k^2+1)}{c^4(k^4+k^2+1)}$$
$$= k^4$$

$$\text{RHS} = \frac{a^2}{c^2}$$
$$= \frac{(ck^2)^2}{c^2}$$
$$= \frac{c^2k^4}{c^2} = k^4$$

LHS = R.H.S. Hence Proved

Main 2018

51. Using properties of proportion, solve for x. Given that x is positive.

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$$

Sol :

We have $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = \frac{4}{1}$

Using componendo and dividendo rule we have

$$\frac{2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1}}{2x+\sqrt{4x^2-1}-2x+\sqrt{4x^2-1}} = \frac{4+1}{4-1}$$
$$\frac{4x}{2\sqrt{4x^2-1}} = \frac{5}{3}$$
$$\frac{2x}{\sqrt{4x^2-1}} = \frac{5}{3}$$

Squaring on both sides we get

$$\frac{4x^2}{4x^2-1} = \frac{25}{9}$$
$$36x^2 = 100x^2-25$$
$$100x^2-36x^2 = 25$$
$$64x^2 = 25$$
$$x^2 = \frac{25}{64}$$
$$x = \pm\sqrt{\frac{25}{64}} = \pm\frac{5}{8}$$

Thus $x = \frac{5}{8}$.

Comp 2018

52. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, show that each ratio is equal to $\frac{1}{2}$ or -1.

Sol :

We have $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$

$$\text{Each ratio} = \frac{\text{sum of antecedents}}{\text{sum of consequents}}$$
$$= \frac{a+b+c}{b+c+c+a+a+b}$$
$$= \frac{a+b+c}{2(a+b+c)}$$

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$$= 2^3 \times 6^3$$
$$= 2 \times 6$$
$$= 12$$

Now, $x = \frac{36}{y} = \frac{36}{12} = 3 \qquad [y = 12]$

Hence, required numbers are $x = 3$ and $y = 12$.

Comp 2011

69. If a, b, c, d are in continued proportion, prove that:

$$\sqrt{ab} - \sqrt{bc} + \sqrt{cd} = \sqrt{(a-b+c)(b-c+d)}$$

Sol :

As a, b, c, d are in continued proportion, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$. Then

$a = bk,\ b = ck,\ c = dk$

$b = ck = dk \cdot k = dk^2$ and

$a = bk = dk^2 \cdot k = dk^3$

$\sqrt{ab} - \sqrt{bc} + \sqrt{cd} = \sqrt{(a-b+c)(b-c+d)}$

Substituting $a = dk^3$, $b = dk^2$ and $c = dk$ on each side of the required equality, we obtain

$$\text{LHS} = \sqrt{ab} - \sqrt{bc} + \sqrt{cd}$$
$$= \sqrt{dk^3 \cdot dk^2} - \sqrt{dk^2 \cdot dk} + \sqrt{dk \cdot d}$$
$$= \sqrt{d^2k^5} - \sqrt{d^2k^3} + \sqrt{d^2k}$$
$$= d\sqrt{k}\,(k^2 - k + 1)$$
$$\text{RHS} = \sqrt{(a-b+c)(b-c+d)}$$
$$= \sqrt{(dk^3 - dk^2 + dk)(dk^2 - dk + d)}$$
$$= \sqrt{dk(d^2 - k + 1) \cdot d(k^2 - k + 1)}$$
$$= d\sqrt{k}\,(k^2 - k + 1)$$

LHS = RHS.

Main 2010

70. If x, y and z are in continued proportion, then prove that $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$.

Sol :

Since x, y and z are in continued proportion,

$$\frac{x}{y} = \frac{y}{z}$$
$$y^2 = xz$$

We have $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$

$$\text{LHS} = \frac{(x+y)^2}{(y+z)^2}$$
$$= \frac{x^2 + y^2 + 2xy}{y^2 + z^2 + 2yz}$$
$$= \frac{x^2 + xz + 2xy}{xz + z^2 + 2yz} \qquad [y^2 = xz]$$
$$= \frac{x(x+z+2y)}{z(x+z+2y)}$$
$$= \frac{x}{z}$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

Main 2010

71. If $x = \frac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}}$, show that $b^2x^2 - 2a^2x + b^2 = 0$.

Sol :

We have $x = \frac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}}$

$$\frac{x}{1} = \frac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}}$$
$$\frac{x+1}{x-1} = \frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2-b^2}}$$

Applying componendo and dividendo we have

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2-b^2}}$$
$$\frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2} \qquad \text{[Squaring both sides]}$$
$$\frac{(x+1)^2 + (x-1)^2}{(x+1)^2 - (x-1)^2} = \frac{(a^2+b^2) + (a^2-b^2)}{(a^2+b^2) - (a^2-b^2)}$$

Applying componendo and dividendo we have

$$\frac{2(x^2+1)}{4x} = \frac{2a^2}{2b^2}$$
$$\frac{x^2+1}{2x} = \frac{a^2}{b^2}$$
$$b^2x^2 + b^2 = 2a^2x$$
$$b^2x^2 - 2a^2x + b^2 = 0$$

Comp 2010

72. If a, b, c, d are in continued proportion, prove that:

$$\sqrt{(a+b+c)(b+c+d)} = \sqrt{ab} + \sqrt{bc} + \sqrt{cd}$$

Sol :

As a, b, c, d are in continued proportion, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$. Then

$a = bk,\ b = ck,\ c = dk$

$b = ck = dk \cdot k = dk^2$ and

$a = bk = dk^2 \cdot k = dk^3$

$\sqrt{(a+b+c)(b+c+d)} = \sqrt{ab} + \sqrt{bc} + \sqrt{cd}$

NODIA

$$= 3k^3$$

Thus, LHS = RHS Hence proved.

Comp 2016

57. What quantity must be added to each term of the ratio $(a+2b) : (a-2b)$ to make it equal to the duplicate ratio of $(a+b) : (a-b)$?

Sol :

Let p be added to each term of the ratio $(a+2b) : (a-2b)$, to make it equal to the duplicate ratio of $(a+b) : (a-b)$.

Thus $$\frac{a+2b+p}{a-2b+p} = \frac{(a+b)^2}{(a-b)^2}$$

$$\frac{(a+2b+p)+(a-2b+p)}{(a+2b+p)-(a-2b+p)} = \frac{(a+b)^2+(a-b)^2}{(a+b)^2-(a-b)^2}$$

(By componendo and dividendo)

$$\frac{2a+2p}{4b} = \frac{2a^2+2b^2}{4ab}$$

$$a+p = \frac{a^2+b^2}{a}$$

$$a+p = a+\frac{b^2}{a}$$

$$p = \frac{b^2}{a}$$

Main 2015

58. If a, b and c are in continued proportion, prove that

$$(a+b+c)(a-b+c) = a^2+b^2+c^2$$

Sol :

Here a, b and c are in continued proportion, then

Thus $$\frac{a}{b} = \frac{b}{c}$$

$$ac = b^2$$

$$\begin{aligned} \text{LHS} &= (a+b+c)(a-b+c) \\ &= a^2+ab+ac-ab-b^2-bc+ac+bc+c^2 \\ &= a^2-b^2+c^2+2ac \\ &= a^2-b^2+c^2+2b^2 \qquad [\because\ ac = b^2] \\ &= a^2+b^2+c^2 \\ &= \text{RHS} \end{aligned}$$

Hence proved.

Main 2015

59. Given $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$, using componendo and dividendo, find $x : y$.

Sol :

We have $$\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$$

Using componendo and dividendo property we get

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

$$\frac{x+2}{x-2} = \frac{y+3}{y-3}$$

Again applying componendo and dividendo, we get

$$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

$$\frac{2x}{4} = \frac{2y}{6}$$

$$\frac{x}{2} = \frac{y}{3}$$

$$\frac{x}{y} = \frac{2}{3}$$

Hence, $x : y = 2 : 3$

Main 2014

60. If $(x-9) : (3x+6)$ is duplicate ratio of $4 : 9$, then find the value of x.

Sol :

Using the definition, the duplicate ratio of $a : b$ is $a^2 : b^2$.

Since $(x-9) : (3x+6)$ is the duplicate ratio of $4 : 9$ we have

$$\frac{x-9}{3x+6} = \left(\frac{4}{9}\right)^2$$

$$\frac{x-9}{3x+6} = \frac{16}{81}$$

$$81x-729 = 48x+96$$

$$81x-48x = 729+96$$

$$33x = 825$$

$$x = \frac{825}{33}$$

$$= 25$$

Main 2014

61. If $\frac{x^2+y^2}{x^2-y^2} = \frac{17}{8}$, then find the value of

(i) $x : y$ (ii) $\frac{x^3+y^3}{x^3-y^3}$.

Sol :

We have $$\frac{x^2+y^2}{x^2-y^2} = \frac{17}{8}$$

(i) $x : y$

Applying componendo and dividendo, we get

$$\frac{(x^2+y^2)+(x^2-y^2)}{(x^2+y^2)-(x^2-y^2)} = \frac{17+8}{17-8}$$

NODIA

$$\frac{2x^2}{2y^2}=\frac{25}{9}$$

$$\frac{x^2}{y^2}=\left(\frac{5}{3}\right)^2$$

Taking square root both sides, we get

$$\frac{x}{y}=\frac{5}{3} \qquad ...(i)$$

$$x : y = 5 : 3$$

(ii) $\frac{x^3+y^3}{x^3-y^3}$

From Eq. (i), we have

$$\frac{x}{y}=\frac{5}{3}$$

On cubing both sides, we get

$$\frac{x^3}{y^3}=\frac{(5)^3}{(3)^3}$$

$$\frac{x^3}{y^3}=\frac{125}{27}$$

Applying componendo and dividendo, we get

$$\frac{x^3+y^3}{x^3-y^3}=\frac{125+27}{125-27}=\frac{152}{98}$$

Thus $$\frac{x^3+y^3}{x^3-y^3}=\frac{76}{49}$$

Main 2013, Main 2005

62. What number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional?

Sol :

Let x be number added to each of the numbers 6, 15, 20 and 43 to make them in proportion.
If x is added to each number, then they become proportional. Thus

$$(6+x) : (15+x) :: (20+x) : (43+x)$$

$$\frac{6+x}{15+x}=\frac{20+x}{43+x}$$

$$(6+x)(43+x)=(20+x)(15+x)$$

$$258+6x+43x+x^2=300+20x+15x+x^2$$

$$258+49x+x^2=300+35x+x^2$$

$$258-35x=300-258$$

$$14x=42$$

$$x=\frac{42}{14}=3$$

Hence, the added number is 3.

Main 2013

63. Using the properties of proportion, solve for x, given $\frac{x^4+1}{2x^2}=\frac{17}{8}$.

Sol :

We have $$\frac{x^4+1}{2x^2}=\frac{17}{8}$$

Applying componendo and dividendo, we get

$$\frac{x^4+1+2x^2}{x^4+1-2x^2}=\frac{17+8}{17-8}$$

$$\frac{x^4+1+2x^2}{x^4+1-2x^2}=\frac{25}{9}$$

$$\frac{(x^2+1)^2}{(x^2-1)^2}=\left(\frac{5}{3}\right)^2$$

Applying componendo and dividendo, we get

$$\frac{x^2+1+x^2-1}{x^2+1-(x^2-1)}=\frac{5+3}{5-3}$$

$$\frac{2x^2}{2}=\frac{8}{2}\Rightarrow\frac{x^2}{1}=4$$

$$x^2=4$$

$$x=\pm 2$$

Comp 2013

64. Solve the following equation for x, using properties of proportion: $\frac{x^4+1}{2x^2}=\frac{17}{8}$

Sol :

We have $$\frac{x^4+1}{2x^2}=\frac{17}{8}$$

Applying componendo and dividendo, we obtain

$$\frac{x^4+1+2x^2}{x^4+1-2x^2}=\frac{17+8}{17-8}$$

$$\frac{(x^2+1)^2}{(x^2-1)^2}=\frac{25}{9}$$

$$\frac{x^2+1}{x^2-1}=\frac{5}{3}$$

Applying componendo and dividendo, we get

$$\frac{(x^2+1)+(x^2-1)}{(x^2+1)-(x^2-1)}=\frac{5+3}{5-3}$$

$$\frac{2x^2}{2}=\frac{8}{2}$$

$$x^2=4$$

$$x=\pm 2$$

Main 2012

65. The monthly pocket money of Ravi and Sanjeev are in the ratio $5 : 7$. Their expenditures are in the ratio $3 : 5$. If each saves Rs. 80 every month, then find their monthly pocket money.

Sol :

As the pocket money of Ravi and Sanjeev are in ratio of $5 : 7$, let it be $5x$ and $7x$ respectively.

Since each saves money Rs 80 and

Expenditure = Pocket money − Saving money

Expenditure of Ravi $= 5x-80$

and Expenditure of Sanjeev $= 7x-80$

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According to the question, their expenditures are in the ratio 3 : 5. Therefore

$$(5x-80) : (7x-80) :: 3 : 5$$

$$\frac{5x-80}{7x-80} = \frac{3}{5}$$

$$5(5x-80) = 3(7x-80)$$

$$25x-400 = 21x-240$$

$$25x-21x = 400-240$$

$$4x = 160$$

$$x = \frac{160}{4} = 40$$

Pocket money of Ravi, $5x = 5 \times 40 = ₹200$

Pocket money of Sanjeev $7x = 7 \times 40 = ₹280$

Comp 2012

66. If $y = \dfrac{(p+1)^{1/3}+(p-1)^{1/3}}{(p+1)^{1/3}-(p-1)^{1/3}}$, find the value of y^3-3py^2+3y-p

Sol :

We have $y = \dfrac{(p+1)^{1/3}+(p-1)^{1/3}}{(p+1)^{1/3}-(p-1)^{1/3}}$

$$\frac{y}{1} = \frac{(p+1)^{1/3}+(p-1)^{1/3}}{(p+1)^{1/3}-(p-1)^{1/3}}$$

Applying componendo and dividendo, we get

$$\frac{y+1}{y-1} = \frac{[(p+1)^{1/3}+(p-1)^{1/3}]+[(p+1)^{1/3}+(p-1)^{1/3}]}{[(p+1)^{1/3}+(p-1)^{1/3}]-[(p+1)^{1/3}-(p-1)^{1/3}]}$$

$$\frac{y+1}{y-1} = \frac{2(p+1)^{1/3}}{2(p-1)^{1/3}}$$

$$\frac{(y+1)^3}{(y-1)^3} = \frac{p+1}{p-1}$$

$$\frac{y^3+3y^2+3y+1}{y^3-3y^2+3y-1} = \frac{p+1}{p-1}$$

Applying componendo and dividendo, we get

$$\frac{(y^3+3y^2+3y+1)+(y^3-3y^2+3y-1)}{(y^3+3y^2+3y+1)-(y^3-3y^2+3y-1)} = \frac{(p+1)+(p-1)}{(p+1)-(p-1)}$$

$$\frac{2y^3+6y}{6y^2+2} = \frac{2p}{2}$$

$$\frac{y^3+3y}{3y^2+1} = \frac{p}{1}$$

$$y^3+3y = p(3y^2+1)$$

$$y^3-3py^2+3y-1 = 0$$

Main 2011

67. Using componendo and dividendo, find the value of x.

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$$

Sol :

We have $\dfrac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$

Applying componendo and dividendo, we get

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}} = \frac{9+1}{9-1}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides, we get

$$\frac{\left(\sqrt{3x+4}\right)^2}{\left(\sqrt{3x-5}\right)^2} = \frac{5^2}{4^2}$$

$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

Again applying conponendo and dividendo, we get

$$\frac{3x+4+3x-5}{3x+4-3x+5} = \frac{25+16}{25-16}$$

$$\frac{6x-1}{9} = \frac{41}{9}$$

$$6x-1 = 41$$

$$6x = 41+1 = 42$$

$$x = \tfrac{42}{6} = 7$$

Hence, the value of x is 7.

Main 2011

68. 6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y . Find the numbers.

Sol :

As 6 is the mean proportion between x and y, we have

$$x : 6 :: 6 : y$$

$$\frac{x}{6} = \frac{6}{y}$$

$$xy = 36 \quad \ldots(i)$$

and 48 is the third proportional between x and y.

$$x : y :: y : 48$$

$$\frac{x}{y} = \frac{y}{48}$$

$$y^2 = 48x \quad \ldots(ii)$$

From Eq. (i), and Eq. (ii) we get

$$y^2 = 48 \times \frac{36}{y}$$

$$y^3 = 48 \times 36$$

$$= 8 \times 6 \times 36$$

NODIA

$= (a-d)^2$ $[a = dk^3]$

$= \text{RHS}$

$\text{LHS} = \text{RHS}$

Main 2008

76. If $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$, then prove that $\frac{a}{b} = \frac{c}{d}$.

Sol :

We have $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$

Using alternendo, we get

$$\frac{8a-5b}{8a+5b} = \frac{8c-5d}{8c+5d}$$

Using conponendo and dividendo, we get

$$\frac{(8a-5b)+(8a+5b)}{(8a-5b)-(8a+5b)} = \frac{(8c-5d)+(8c+5d)}{(8c-5d)-(8c+5d)}$$

$$\frac{16a}{-10b} = \frac{16c}{-10d}$$

$$\frac{a}{b} = \frac{c}{d}$$ Hence proved.

Main 2006

77. If $\frac{3x+5y}{3x-5y} = \frac{7}{3}$, then find $x : y$.

Sol :

We have, $\frac{3x+5y}{3x-5y} = \frac{7}{3}$

Applying componendo and dividendo, we get

$$\frac{(3x+5y)+(3x-5y)}{(3x+5y)-(3x-5y)} = \frac{7+3}{7-3}$$

$$\frac{6x}{10y} = \frac{10}{4}$$

$$\frac{x}{y} = \frac{10 \times 10}{6 \times 4}$$

$$\frac{x}{y} = \frac{25}{6}$$

Hence, $x : y = 25 : 6$

Comp 2008

78. If a, b, c, d are in continued proportion, prove that:

$$(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$$

Sol :

As a, b, c, d are in continued proportion, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$. Then

$a = bk,\ b = ck,\ c = dk$

$b = ck = dk \cdot k = dk^2$ and

$a = bk = dk^2 \cdot k = dk^3$

Substituting $a = dk^3$, $b = dk^2$ and $c = dk$ on each side of the required equality, we obtain

$$(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$$

$\text{LHS} = (a^2+b^2+c^2)(b^2+c^2+d^2)$

$= (d^2k^6 + d^2k^4 + d^2k^2)(d^2k^4 + d^2k^2 + d^2)$

$= d^2k^2(k^4+k^2+1) \cdot d^2(k^4+k^2+1)$

$= d^4k^2(k^4+k^2+1)^2$

$\text{RHS} = (ab+bc+cd)^2$

$= (dk^3 \cdot dk^2 + dk^2 \cdot dk + dk \cdot d)^2$

$= d^4k^2(k^4+k^2+1)^2$

$\text{LHS} = \text{RHS}$

Main 2007

79. If $y = \frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$, show that

$$3by^2 - 2ay + 3b = 0$$

Sol :

We have $y = \frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$

$$\frac{y}{1} = \frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$$

Applying componendo and dividendo, we get

$$\frac{y+1}{y-1} = \frac{(\sqrt{a+3b}+\sqrt{a-3b})+(\sqrt{a+3b}-\sqrt{a-3b})}{(\sqrt{a+3b}+\sqrt{a-3b})-(\sqrt{a+3b}-\sqrt{a-3b})}$$

$$\frac{y+1}{y-1} = \frac{2\sqrt{a+3b}}{2\sqrt{a-3b}}$$

$$\frac{y+1}{y-1} = \frac{\sqrt{a+3b}}{\sqrt{a-3b}}$$

$$\frac{(y+1)^2}{(y-1)^2} = \frac{a+3b}{a-3b}$$

Applying componendo and dividendo, we get

$$\frac{(y+1)^2+(y-1)^2}{(y+1)^2-(y-1)^2} = \frac{(a+3b)+(a-3b)}{(a+3b)-(a-3b)}$$

$$\frac{(y^2+2y+1)+(y^2-2y+1)}{(y^2+2y+1)-(y^2-2y+1)} = \frac{2a}{6b}$$

$$\frac{2(y^2+1)}{4y} = \frac{a}{3b}$$

$$\frac{y^2+1}{2y} = \frac{a}{3b}$$

$$3b(y^2+1) = 2ay$$

$$3by^2 - 2ay + 3b = 0$$

Comp 2007

80. If $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z}$, then show that each ratio is equal to $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$.

Sol :

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Substituting $a = dk^3$, $b = dk^2$ and $c = dk$ on each side of the required equality, we obtain

$$\begin{aligned}
\text{LHS} &= \sqrt{(a+b+c)(b+c+d)} \\
&= \sqrt{(dk^3+dk^2+dk)(dk^2-dk+d)} \\
&= \sqrt{dk(k^2+k+1)\cdot d(k^2+k+1)} \\
&= d\sqrt{k}\,(k^2+k+1) \\
\text{RHS} &= \sqrt{ab}+\sqrt{bc}+\sqrt{cd} \\
&= \sqrt{dk^3\cdot dk^2}+\sqrt{dk^2\cdot dk}+\sqrt{dk\cdot d} \\
&= \sqrt{d^2k^5}+\sqrt{d^3k^3}+\sqrt{d^2k} \\
&= d\sqrt{k}\,(k^2+k+1) \\
\text{LHS} &= \text{RHS}.
\end{aligned}$$

Main 2009

73. What least number must be added to each of the numbers 5, 11, 19 and 37, so that they are in proportion?

Sol :

Let x be number added to each of the numbers 5, 11, 19 and 37 to make them in proportion.
If x is added to each number, then they become proportional. Thus

$$\begin{aligned}
(5+x) : (11+x) &:: (19+x) : (37+x) \\
\frac{5+x}{11+x} &= \frac{19+x}{37+x} \\
(5+x)(37+x) &= (11+x)(19+x) \\
185+5x+37x+x^2 &= 209+11x+19x+x^2 \\
185+42x &= 209+30x \\
42x-30x &= 209-185 \\
12x &= 24 \\
x &= \frac{24}{12} = 2
\end{aligned}$$

Hence, the added number is 2.

Main 2009

74. Find $a : b$ using componendo and dividendo if given that

$$\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$$

Sol :

We have $\dfrac{a^3+3ab^2}{b^3+3a^2b} = \dfrac{63}{62}$

Applying componendo and dividendo, we get

$$\begin{aligned}
\frac{(a^3+3ab^2)+(b^2+3a^2b)}{(a^3+3ab^2)-(b^3+3a^2b)} &= \frac{63+62}{63-62} \\
\frac{a^3+b^3+3a^2b+3ab^2}{a^3-b^3-3a^2b+3ab^2} &= \frac{125}{1} \\
\frac{(a+b)^3}{(a-b)^3} &= \frac{(5)^3}{1^3}
\end{aligned}$$

Taking cube root both sides, we get

$$\frac{a+b}{a-b} = \frac{5}{1}$$

Again, using componendo and dividendo, we get

$$\begin{aligned}
\frac{(a+b)+(a-b)}{(a+b)-(a-b)} &= \frac{5+1}{5-1} \\
\frac{2a}{2b} &= \frac{6}{4} \\
\frac{a}{b} &= \frac{3}{2} \\
a &: b = 3 : 2
\end{aligned}$$

Here we have used following identity

$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$ and

$(a-b)^3 = a^3-b^3-3a^2b+3ab^2]$

Comp 2009

75. If a, b, c, d are in continued proportion, prove that:

$$(b-c)^2+(c-a)^2+(d-b)^2 = (a-d)^2$$

Sol :

As a, b, c, d are in continued proportion, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$. Then

$a = bk$, $b = ck$, $c = dk$

$b = ck = dk\cdot k = dk^2$ and

$a = bk = dk^2\cdot k = dk^3$

$(b-c)^2+(c-a)^2+(d-b)^2 = (a-d)^2$

Substituting $a = dk^3$, $b = dk^2$ and $c = dk$ we have

$$\begin{aligned}
\text{LHS} &= (b-c)^2+(c-a)^2+(d-b)^2 \\
&= (dk^2-dk)^2+(dk-dk^3)^2+(d-dk^2)^2 \\
&= d^2k^2(k-1)^2+d^2k^2(1-k^2)^2+d^2(1-k^2)^2 \\
&= d^2k^2(k-1)^2+d^2k^2(k^2-1)^2+d^2(k^2-1)^2 \\
&= d^2[k^2(k-1)^2+k^2(k-1)^2(k+1)^2 \\
&\qquad +(k-1)^2(k+1)^2] \\
&= d^2(k-1)^2[k^2+k^2(k+1)^2+(k+1)^2] \\
&= d^2(k-1)^2[k^2+k^2(k^2+2k+1) \\
&\qquad +k^2+2k+1] \\
&= d^2(k-1)^2[k^4+2k^3+3k^2+2k+1] \\
&= d^2(k-1)^2(k^2+k+1)^2 \\
&= d^2[(k-1)(k^2+k+1)]^2 \\
&= d^2(k^3-1)^2 \\
&= (dk^3-d)^2
\end{aligned}$$

NODIA

Comp 2000

85. If x and y be unequal and $x : y$ is the duplicate ratio of $x + z$ and $y + z$, prove that z is mean proportional between x and y.

Sol :

Since $x : y$ is the duplicate ratio of $x + z$ and $y + z$

$$x : y = (x+z)^2 : (y+z)^2$$
$$\frac{x}{y} = \frac{(x+z)^2}{(y+z)^2}$$
$$x(y+z)^2 = y(x+z)^2$$
$$x(y^2+z^2+2yz) = y(x^2+z^2+2xy)$$
$$xy^2+xz^2+2xyz = yx^2+yz^2+2xyz$$
$$xy^2+xz^2 = yx^2+yz^2$$
$$x^2y-xy^2 = xz^2-yz^2$$
$$xy(x-y) = z^2(x-y)$$
$$xy = z^2$$
$$\frac{x}{z} = \frac{z}{y}$$
$$x : z :: z : y$$

Thus z is mean proportional between x and y.

FOUR MARK QUESTIONS

Main 2017

86. If $\frac{7m+2n}{7m-2n} = \frac{5}{3}$, use properties of proportion to find

(i) $m : n$. (ii) $\frac{m^2+n^2}{m^2-n^2}$.

Sol :

(i) $m : n$.

We have $$\frac{7m+2n}{7m-2n} = \frac{5}{3}$$

Applying componendo and dividendo, we get

$$\frac{(7m+2n)+(7m-2n)}{(7m+2n)-(7m-2n)} = \frac{5+3}{5-3}$$
$$\frac{14m}{4n} = \frac{8}{2}$$
$$\frac{m}{n} = \frac{8\times 4}{2\times 14} = \frac{8}{7}$$

(ii) $\frac{m^2+n^2}{m^2-n^2}$.

Dividing numerator and denominator by n^2 we have

$$\frac{m^2+n^2}{m^2-n^2} = \frac{\frac{m^2}{n^2}+1}{\frac{m^2}{n^2}-1}$$
$$= \frac{\left(\frac{m}{n}\right)^2+1}{\left(\frac{m}{n}\right)^2-1}$$
$$= \frac{\left(\frac{8}{7}\right)^2+1}{\left(\frac{8}{7}\right)^2-1}$$
$$= \frac{\frac{64}{49}+1}{\frac{64}{49}-1}$$
$$= \frac{64+49}{64-49}$$
$$= \frac{113}{15}$$

Comp 2015

87. If $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z}$, then prove that each ratio is equal to $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$.

Sol :

We have $$\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z}$$

$$\text{Each ratio} = \frac{\text{sum of antecedents}}{\text{sum of consequents}}$$
$$= \frac{(b+c-a)+(c+a-b)+(a+b-c)}{(y+z-x)+(z+x-y)+(x+y-z)}$$
$$= \frac{a+b+c}{x+y+z}$$

Now, $$\frac{b+c-a}{y+z-x} = \frac{a+b+c}{x+y+z}$$

$$\frac{b+c-a}{a+b+c} = \frac{y+z-x}{x+y+z} \quad \text{(By alternendo)}$$

$$\frac{(b+c-a)-(a+b+c)}{a+b+c} = \frac{(y+z-x)-(x+y+z)}{x+y+z} \quad \text{(By dividendo)}$$

$$\frac{-2a}{a+b+c} = \frac{-2x}{x+y+z}$$

$$\frac{a}{a+b+c} = \frac{x}{x+y+z}$$

$$\frac{a}{x} = \frac{a+b+c}{x+y+z} \quad \text{(By alternendo)}$$

Similarly, $$\frac{c+a-b}{z+x-y} = \frac{a+b+c}{x+y+z}$$

$$\frac{b}{y} = \frac{a+b+c}{x+y+z}$$

and $$\frac{a+b-c}{x+y-z} = \frac{a+b+c}{x+y+z}$$

$$\frac{c}{z} = \frac{a+b+c}{x+y+z}$$

Hence, each given ratio $= \frac{a}{x} = \frac{b}{y} = \frac{c}{z}$.

Comp 2014

88. If $x = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$, find the value of

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}}$$

Sol :

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We have, $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z}$

$$\text{Each given ratio} = \frac{\text{Sum of antecedents}}{\text{Sum of consequents}}$$

$$= \frac{a+b+c}{x+y+z}$$

Now $\frac{b+c-a}{y+z-x} = \frac{a+b+c}{x+y+z}$

$$\frac{b+c-a}{a+b+c} = \frac{y+z-x}{x+y+z}$$

$$\frac{-2a}{a+b+c} = \frac{-2x}{x+y+z} \qquad \text{[By dividendo]}$$

$$\frac{a}{x} = \frac{a+b+c}{x+y+z}$$

Similarly, we have

$$\frac{c+a-b}{z+x-y} = \frac{a+b+c}{x+y+z}$$

$$\frac{b}{y} = \frac{a+b+c}{x+y+z}$$

and, $\frac{a+b-c}{x+y-z} = \frac{a+b+c}{x+y+z}$

$$\frac{c}{z} = \frac{a+b+c}{x+y+z}$$

$$\frac{a}{x} = \frac{b}{y} = \frac{c}{x} = \frac{a+b+c}{x+y+z}$$

$$= \text{Each given ratio.}$$

Main 2004

81. What number should be subtracted from each of the following numbers 23, 30, 57 and 78, so that the remainders are in proportion?

Sol :

Given numbers are 23, 30, 57 and 78.
Let the number x be subtracted from each number so that the remainders are in proportion
Thus

$$(23-x) : (30-x) :: (57-x) : (78-x)$$

$$\frac{23-x}{30-x} = \frac{57-x}{78-x}$$

$$(23-x)(78-x) = (57-x)(30-x)$$

$$1794 - 23x - 78x + x^2 = 1710 - 57x - 30x + x^2$$

$$1794 - 101x = 1710 - 87x$$

$$1794 - 1710 = 101x - 87x$$

$$84 = 14x \Rightarrow x = \frac{84}{14}$$

$$x = 6$$

Main 2002

82. If $a : b = 5 : 3$, then find $(5a+8b) : (6a-7b)$.

Sol :

We have $a : b = 5 : 3$

$$\frac{a}{b} = \frac{5}{3}$$

Now, $(5a+8b) : (6a-7b) = \frac{5a+8b}{6a-7b}$

Divide numerator and denominator by b we have

$$(5a+8b) : (6a-7b) = \frac{\frac{5a}{b} + \frac{8b}{b}}{\frac{6a}{b} - \frac{7b}{b}}$$

$$= \frac{5 \times \frac{5}{3} + 8}{6 \times \frac{5}{3} - 7} \qquad \left[\frac{a}{b} = \frac{5}{3}\right]$$

$$= \frac{25+24}{30-21}$$

$$= \frac{49}{9}$$

$$= 49 : 9$$

Thus $(5a+8b) : (6a-7b) = 49 : 9$

NODIA

Main 2001

83. Two numbers are in the ratio of $3 : 5$. If 8 is added to each number, then the ratio becomes $2 : 3$. Find the numbers.

Sol :

As the two numbers are in ratio $3 : 5$, let numbers be $3x$ and $5x$.
If 8 is added to each number, then the ratio becomes $2 : 3$. Thus

$$\frac{3x+8}{5x+8} = \frac{2}{3}$$

$$3(3x+8) = 2(5x+8)$$

$$9x + 24 = 10x + 16$$

$$10x - 9x = 24 - 16$$

$$x = 8$$

$$\text{First number} = 3x = 3 \times 8 = 24$$

and $\text{second number} = 5x = 5 \times 8 = 40$

Main 2000

84. Given, $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{3a-5b}{3a+5b} = \frac{3c-5d}{3c+5d}$.

Sol :

We have $\frac{a}{b} = \frac{c}{d}$

Multiplying both sides by $\frac{3}{5}$ we have

$$\frac{3a}{5b} = \frac{3c}{5d}$$

Applying componendo and dividendo, we get

$$\frac{3a+5b}{3a-5b} = \frac{3c+5d}{3c-5d}$$

Applying invertendo, we get

$$\frac{3a-5d}{3a+5d} = \frac{3c-5d}{3c+5d}$$ Hence proved.

$= \frac{1}{2}$, when $a+b+c \neq 0$

When $a+b+c=0$, then we have

$$\frac{a}{b+c} = \frac{a}{(a+b+c)-a} = \frac{a}{0-a} = -1$$

$$\frac{b}{c+a} = \frac{b}{(a+b+c)-b} = \frac{b}{0-b} = -1$$

and $$\frac{c}{a+b} = \frac{c}{(a+b+c)-c} = \frac{c}{0-c} = -1$$

In this case, each ratio $= -1$.

Hence, each of the given ratio is equal to $\frac{1}{2}$ or -1.

Main 2017

53. If b is the mean proportional between a and c, show that

$$\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}.$$

Sol :

Here b is the mean proportional between a and c.

Thus $$\frac{a}{b} = \frac{b}{c}$$

$$b^2 = ac$$

Now, $$\text{LHS} = \frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4}$$

$$= \frac{a^4+a^2(ac)+(ac)^2}{(ac)^2+(ac)c^2+c^4} \quad [\because\ b^2 = ac]$$

$$= \frac{a^4+a^3c+a^2c^2}{a^2c^2+ac^3+c^4}$$

$$= \frac{a^2[a^2+ac+c^2]}{c^2[a^2+ac+c^2]}$$

$$= \frac{a^2}{c^2}$$

$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2017

54. If $\frac{a^3+3ab^2}{3a^2b+b^3} = \frac{x^3+3xy^2}{3x^2y+y^3}$, prove that $\frac{x}{a} = \frac{y}{b}$.

Sol :

We have $$\frac{a^3+3ab^2}{3a^2b+b^3} = \frac{x^3+3xy^2}{3x^2y+y^2}$$

$$\frac{a^3+3ab^2+3a^2b+b^3}{a^3+3ab^2-3a^2b-b^3} = \frac{x^3+3xy^2+3x^2y+y^3}{x^3+3xy^2-3x^2y-y^3}$$

(By componendo and dividendo)

$$\frac{(a+b)^3}{(a-b)^3} = \frac{(x+y)^3}{(x-y)^3}$$

$$\frac{a+b}{a-b} = \frac{x+y}{x-y}$$

(Taking cube root of both sides)

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{(x+y)+(x-y)}{(x+y)-(x-y)}$$

(By componendo and dividendo)

$$\frac{2a}{2b} = \frac{2x}{2y}$$

$$\frac{a}{b} = \frac{x}{y}$$

$$\frac{a}{x} = \frac{b}{y} \qquad \text{(By alternendo)}$$

$$\frac{x}{a} = \frac{y}{b} \qquad \text{(By invertendo)}$$

Main 2016

55. If $(3a+2b) : (5a+3b) = 18 : 29$. Find $a : b$.

Sol :

We have

$$(3a+2b) : (5a+3b) = 18 : 29$$

$$\frac{3a+2b}{5a+3b} = \frac{18}{29}$$

$$\frac{\frac{3a}{b}+2}{\frac{5a}{b}+3} = \frac{18}{29}$$

$$29\left(\frac{3a}{b}+2\right) = 18\left(\frac{5a}{b}+3\right)$$

$$87\frac{a}{b}+58 = 90\frac{a}{b}+54$$

$$87\frac{a}{b}-90\frac{a}{b} = 54-58$$

$$-3\frac{a}{b} = -4$$

$$\frac{a}{b} = \frac{4}{3}$$

Thus $$a : b = 4 : 3$$

Main 2016

56. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, show that $\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}$.

Sol :

Let $$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$$

Then, $x = ak,\ y = bk,\ z = ck$

Now, $$\text{LHS} = \frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3}$$

$$= \frac{a^3k^3}{a^3} + \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3}$$

$$= k^3+k^3+k^3$$

$$= 3k^3$$

and $$\text{RHS} = \frac{3xyz}{abc}$$

$$= \frac{3(ak)(bk)(ck)}{abc}$$

$$= \frac{3abck^3}{abc}$$

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We have $x = \dfrac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$$\frac{x}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}} \text{ and}$$

$$\frac{x}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}$$

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-(\sqrt{2}+\sqrt{3})} \text{ and}$$

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{\sqrt{2}+3\sqrt{3}}{\sqrt{3}-\sqrt{2}} \text{ and}$$

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

Adding above we have

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{\sqrt{2}+3\sqrt{3}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{\sqrt{2}+3\sqrt{3}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\sqrt{2}+3\sqrt{3}-3\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = 2$$

Main 2013

89. Solve the following equation for x, using properties of proportion:

$$\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = \frac{4x-1}{2}$$

Sol :

We have, $\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = \dfrac{4x-1}{2}$

Applying componendo and dividendo we have

$$\frac{(\sqrt{x+1}+\sqrt{x-1})+(\sqrt{x+1}-\sqrt{x-1})}{(\sqrt{x+1}+\sqrt{x-1})-(\sqrt{x+1}-\sqrt{x-1})} = \frac{4x-1+2}{4x-1-2}$$

$$\frac{2\sqrt{x+1}}{2\sqrt{x-1}} = \frac{4x+1}{4x-3}$$

$$\frac{\sqrt{x+1}}{\sqrt{x-1}} = \frac{4x+1}{4x-3}$$

$$\frac{x+1}{x-1} = \frac{(4x+1)^2}{(4x-3)^2}$$

Applying componendo and dividendo we have

$$\frac{(x+1)+(x-1)}{(x+1)-(x-1)} = \frac{(16x^2+8x+1)+(16x^2-24x+9)}{(16x^2+8x+1)-(16x^2-24x+9)}$$

$$\frac{2x}{2} = \frac{32x^2-16x+10}{32x-8}$$

$$x = \frac{32x^2-16x+10}{32x-8}$$

$$x(32x-8) = 32x^2-16x+10$$

$$32x^2-8x = 32x^2-16x+10$$

$$8x = 10$$

$$x = \frac{10}{8}$$

$$x = \frac{5}{4}$$

Main 2012

90. If $x = \dfrac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, then using the properties of proportion, show that $x^2-2ax+1=0$.

Sol :

We have $\dfrac{x}{1} = \dfrac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

Applying componendo and dividendo, we get

$$\frac{x+1}{x-1} = \frac{\sqrt{a+1}+\sqrt{a-1}+(\sqrt{a+1}-\sqrt{a-1})}{\sqrt{a+1}+\sqrt{a-1}-(\sqrt{a+1}-\sqrt{a-1})}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

$$\frac{x+1}{x-1} = \frac{\sqrt{a+1}}{\sqrt{a-1}}$$

Squaring both sides, we get

$$\frac{(x+1)^2}{(x-1)^2} = \frac{(\sqrt{a+1})^2}{(\sqrt{a-1})^2}$$

$$\frac{x^2+1+2x}{x^2+1-2x} = \frac{(a+1)}{(a-1)}$$

$$\frac{x^2+1+2x}{x^2+1-2x} = \frac{a+1}{a-1}$$

Again, applying compondo and dividendo, we get

$$\frac{x^2+1+2x+(x^2+1-2x)}{x^2+1+2x-(x^2+1-2x)} = \frac{(a+1)+(a-1)}{(a+1)-(a-1)}$$

$$\frac{2(x^2+1)}{4x} = \frac{2a}{2}$$

$$\frac{x^2+1}{2x} = a$$

$$x^2+1 = 2ax$$

$$x^2-2ax+1 = 0$$

Here we have used following identities

$$(a+b)^2 = a^2+b^2+2ab$$

and $\quad (a-b)^2 = a^2+b^2-2ab]$

Main 2007

91. If $x = \dfrac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$, then prove that

$$3bx^2-2ax+3b = 0.$$

Sol :

We have $\dfrac{x}{1} = \dfrac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$

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or $p(-b)=k$

Thus substituting $x+1=0$ or $x=-1$ in given polynomial we have

$$(-1)^3-2(-1)^2+a(-1)+12=20$$
$$-1-2-a+12=20$$
$$-a=20-9$$
$$a=-11$$

Thus (d) is correct option.

Main 2022 Sem I

6. If a polynomial $2x^2-7x-1$ is divided by $x+3$, then the remainder is :

(a) -4 (b) 38
(c) -3 (d) 2

Sol :

We have $p(x)=2x^2-7x-1$

If $p(x)$ divided by $(x+b)$ leaves remainder k then from remainder theorem we have

$$p(x+b=0)=k$$

or $p(-b)=k$

Thus substituting $x+3=0$ or $x=-3$ in given polynomial we have

$$f(-3)=2(-3)^2-7(-3)-1$$
$$=18+21-1$$
$$=38$$

Thus (b) is correct option.

Main 2022 Sem I

7. If $(x-2)$ is a factor of the polynomial $x^3+2x^2-13x+k$, then k is :

(a) -10 (b) 26
(c) -26 (d) 10

Sol :

We have $p(x)=x^3+2x^2-13x+k$

If $(x+a)$ is a factor of $p(x)$ then we must have

$$p(-a)=0$$

Thus substituting $x-2=0$ or $x=2$ in given polynomial we get

$$f(2)=0$$
$$8+8-26+k=0$$
$$16-26+k=0$$
$$k=10$$

Thus (d) is correct option.

8. The remainder when $f(x)=x^2-5x+7$ is divided by $x-1$, is:

(a) 3 (b) 8
(c) 4 (d) 12

Sol :

We have $f(x)=x^2-5x+7$

Let $x-1=0$, then $x=1$, Thus $\alpha=1$

Remainder theorem says that if $f(x)$ is divided by $(x-\alpha)$, it leaves remainder b, then $f(\alpha)=b$.

Remainder

$$p(1)=(1)^2-5(1)+7$$
$$=1-5+7=3$$

Thus (a) is correct option.

9. What is the remainder obtained on dividing $f(x)=6x^3-3x^2-8x+7$ by $x-2$:

(a) – 37 (b) 27
(c) 37 (d) –27

Sol :

We have $f(x)=6x^3-3x^2-8x+7$

Let $x-2=0$, then $x=2$, Thus $\alpha=2$

Remainder theorem says that if $f(x)$ is divided by $(x-\alpha)$, it leaves remainder b, then $f(\alpha)=b$.

Remainder

$$f(2)=6(2)^3-3(2)^2+7$$
$$=48-12-16+7$$
$$=27$$

Thus (b) is correct option.

10. What is the remainder when $f(x)=x^2-4x+2$ is divided by $2x+1$?

(a) $\frac{1}{2}$ (b) $\frac{9}{4}$
(c) $\frac{3}{4}$ (d) $\frac{17}{4}$

Sol :

We have $f(x)=x^2-4x+2$

Let $2x+1=0$, then $x=-\frac{1}{2}$, Thus $\alpha=-\frac{1}{2}$

Remainder theorem says that if $f(x)$ is divided by $(x-\alpha)$, it leaves remainder b, then $f(\alpha)=b$.

Remainder

$$f\left(-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)^2-4\left(-\frac{1}{2}\right)+2$$
$$=\frac{1}{4}+2+2=\frac{17}{4}$$

Thus (d) is correct option.

NODIA

$$a = ck \times k = ck^2$$

$$(a+b):(b+c) = \frac{a+b}{b+c}$$

$$= \frac{ck^2+ck}{ck+c}$$

$$= \frac{ck(k+1)}{c(k+1)}$$

$$= k$$

and, $$a^2(b-c):b^2(a-b) = \frac{a^2(b-c)}{b^2(a-b)}$$

$$= \frac{c^2k^4(ck-c)}{c^2k^2(ck^2-ck)}$$

$$= \frac{c^3k^4(k-1)}{c^3k^3(k-1)}$$

$$= k$$

$$(a+b):(b+c)::a^2(b-c):b^2(a-b)$$

Main 2003

95. Solve the following equation for x, using properties of proportion: $\frac{1-px}{1+px}\sqrt{\frac{1+qx}{1-qx}} = 1$

Sol :

We have $$\frac{1-px}{1+px}\sqrt{\frac{1+qx}{1-qx}} = 1$$

$$\frac{1-px}{1+px} = \sqrt{\frac{1-qx}{1+qx}}$$

$$\frac{(1-px)^2}{(1+px)^2} = \frac{1-qx}{1+qx}$$

Applying componendo and dividendo, we get

$$\frac{(1-px)^2+(1+px)^2}{(1-px)^2-(1-px)^2} = \frac{(1-qx)+(1+qx)}{(1-qx)-(1+qx)}$$

$$\frac{(1-2px+p^2x^2)+(1+2px+p^2x^2)}{(1-2px+p^2x^2)-(1+2px+p^2x^2)} = \frac{2}{-2qx}$$

$$\frac{2(1+p^2x^2)}{-4px} = \frac{2}{-2qx}$$

$$\frac{1+p^2x^2}{-2px} = \frac{1}{-qx}$$

$$-qx(1+p^2x^2) = -2px$$

$$qx(1+p^2x^2) = 2px$$

$$x[q(1+p^2x^2)-2p] = 0$$

$$x[p^2qx^2+q-2p] = 0$$

$$x = 0 \text{ or } p^2qx^2+q-2p = 0$$

From $p^2qx^2+q-2p=0$ we have

$$p^2qx^2 = 2p-q$$

$$x^2 = \frac{2p-q}{p^2q}$$

$$x = \pm\frac{1}{p}\sqrt{\frac{2p-q}{q}}$$

Comp 2003

96. Find the two numbers such that their mean proportional is 24 and the third proportional is 1536.

Sol :

Let x and y be the required numbers. Then mean proportional between x and y is 24.

Now $x:24::24:y$

$$\frac{x}{24} = \frac{24}{y}$$

$$xy = (24)^2$$

$$xy = 576 \quad (1)$$

Third proportional to x and y is 1536, therefore

$$x:y::y:1536$$

$$\frac{x}{y} = \frac{y}{1536}$$

$$y^2 = 1536x \quad (2)$$

From (1) we have, $x = \frac{576}{y}$. Substituting this value of in (2), we get

$$y^2 = 1536 \times \frac{576}{y}$$

$$= 1536 \times 576$$

$$= (16 \times 96) \times (96 \times 6)$$

$$y^3 = 96 \times 96 \times 96$$

$$y = 96$$

Substituting $y = 96$ in (1), we get

$$x = \frac{576}{96} = 16$$

Hence, the required numbers are 16, 96.

Comp 2002

97. If a, b, c are in continued proportion, prove that: $(a+b+c)^2:(a^2+b^2+c^2)::(a+b+c):(a-b+c)$

Sol :

As a, b, c are in continued proportion, $\frac{a}{b} = \frac{b}{c}$

Let $\frac{a}{b} = \frac{b}{c} = k$, then, $a = bk$, $b = ck$.

$$a = ck \times k = ck^2$$

$$(a+b+c)^2:(a^2+b^2+c^2) = \frac{(a+b+c)^2}{a^2+b^2+c^2}$$

$$= \frac{(ck^2+ck+c)^2}{c^2k^4+c^2k^2+c^2}$$

$$= \frac{c^2(k^2+k+1)^2}{c^2(k^4+k^2+1)}$$

$$= \frac{(k^2+k+1)^2}{k^4+k^2+1}$$

and,

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Since $p(-1)+p(2)=1$

$$(2k-9)+(5k+24)=0$$

$$7k=-14$$

$$k=-2$$

Thus (d) is correct option.

17. If $(x-2)$ is a factor of $2x^3-x^2-px-2$, then the value of p is:

(a) 6 (b) 4

(c) 5 (d) 8

Sol :

We have $p(x)=2x^3-x^2-px-2$

Since, $x-2$ is a factor of $p(x)$,

$$p(x2)=0$$

$$2(2)^3-(2)^2-p(2)-2=0$$

$$16-4-2p-2=0$$

$$2p=10$$

$$p=5$$

Thus (c) is correct option.

18. If both $(x-2)$ and $(x-\frac{1}{2})$ are the factors of px^2+5x+r, then:

(a) $p=r$ (b) $p=2r$

(c) $2p=r$ (d) $p=r+2$

Sol :

We have $f(x)=px^2+5x+r$

Since, $x-2$ is a factor of $f(x)$

$$f(2)=0$$

$$p(2)^2+5(2)+r=0$$

$$4p+r+10=0$$

Also $(x-\frac{1}{2})$ is a factor of $f(x)$

$$f(\tfrac{1}{2})=0$$

$$p\left(\frac{1}{2}\right)^2+5\left(\frac{1}{2}\right)+r=0$$

$$\frac{p}{4}+\frac{5}{2}+r=0$$

$$p+4r+10=0 \qquad ...(2)$$

From equations (1) and (2), we have

$$4p+r+10=p+4r+10$$

$$p=r$$

Thus (a) is correct option.

19. If $(x-a)$ is a factor of the polynomial $p(x)=x^3-ax^2+2x+2$, then the value of a is:

(a) –1 (b) –3

(c) –2 (d) – 2

Sol :

Since, $(x-a)$ is a factor of $p(x)$

$$p(a)=0$$

$$a^3-a(a)^2+2(a)+2=0$$

$$a=-1$$

Thus (a) is correct option.

20. If $x-2$ is a factor of $x^3+2x^2-kx+10$, then the value of k is:

(a) 23 (b) 5

(c) 13 (d) 16

Sol :

We have $p(x)=x^3+2x^2-kx+10$

Since, $x-2$ is a factor of $p(x)$.

$$p(2)=0$$

$$(2)^3+2(2)^2-k(2)+10=0$$

$$8+8-2k+10=0$$

$$k=13$$

Thus (c) is correct option.

21. If both $(x-2)$ and $(x+3)$ are the factors of the expression $x^3+ax^2+bx-12$, then the value of $(a+b)$ is:

(a) 14 (b) – 4

(c) 6 (d) – 1

Sol :

We have $p(x)=x^3+ax^2+bx-12$

Since, $x-2$ is a factor of $p(x)$.

$$p(2)=0$$

$$(2)^3a(2)^2+b(2)-12=0$$

$$8+4a+2b-12=0$$

$$2a+b=2 \qquad ...(1)$$

Also, $(x+3)$ is a factor of $p(x)$.

$$p(-3)=0$$

$$(-3)^3+a(-3)^3+b(-3)-12=0$$

$$-27+9a-3b-12=0$$

$$3a-b=13 \qquad ...(2)$$

Solving equations (1) and (2), we get

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CHAPTER 7

FACTORISATION OF POLYNOMIALS

MULTIPLE CHOICE QUESTIONS

Main 2024

1. What must be subtracted from the polynomial x^3+x^2-2x+1, so that the result is exactly divisible by $(x-3)$?

(a) -31 (b) -30
(c) 30 (d) 31

Sol :

Let k be the number which is to be added.

$$p(x)=x^3+x^2-2x+1+k$$

Result will be exactly divisible by $(x-3)$ if

$$p(3)=0$$
$$3^3+3^2-2\times 3+1+k=0$$
$$27+9-6+1+k=0$$
$$31+k=10$$
$$k=-31$$

Thus (a) is correct option.

SQP 2024

2. If $x^2+kx+6=(x-2)(x-3)$ for all values of x, then the value of k is:

(a) -5 (b) -3
(c) -2 (d) 5

Sol :

$$x^2+kx+6==(x-2)(x-3)$$
$$=x(x-3)-2(x-3)$$
$$=x^2-3x-2x+6$$
$$x^2+kx+6==x^2-5x+6$$

Comparing both side we get $k=-5$.

Thus (a) is correct option.

Main 2023

3. If $x-2$ is a factor of $x^3-kx-12$, then the value of k is:

(a) 3 (b) 2
(c) -2 (d) -3

Sol :

We have $f(x)=x^3-kx-12$

Since $x-2$ is a factor of $x^3-kx-12$ we have

$$f(2)=0$$

Thus substituting $x=2$ in given function and equating to zero we have

$$2^3-k(2)-12=0$$
$$8-2k-12=0$$
$$-2k=4$$
$$k=-2$$

Thus (c) is correct option.

Comp 2023

4. If a polynomial x^3+3x^2+3x+1 is divided by $(x+1)$, then the remainder is :

(a) -31 (b) 0
(c) 11 (d) -11

Sol :

We have $p(x)=x^3+3x^2+3x+1$

If $p(x)$ divided by $(x+b)$ leaves remainder k then from remainder theorem we have

$$p(x+b=0)=k$$

or

$$p(-b)=k$$
$$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$$
$$=-1+3-3+1=0$$

Thus (a) is correct option.

Main 2022 Sem I

5. Polynomial $x^2-2x^2+ax+12$ when divided by $(x+1)$ leaves a remainder 20, then a is :

(a) -31 (b) 9
(c) 11 (d) -11

Sol :

We have $p(x)=x^3-2x^2+ax+12$

If $p(x)$ divided by $(x+b)$ leaves remainder k then from remainder theorem we have

$$p(x+b=0)=k$$

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$$(a+b+c):(a-b+c)=\frac{a+b+c}{a-b+c}$$
$$=\frac{ck^2+ck+c}{ck^2-ck+c}$$
$$=\frac{k^2+k+1}{k^2-k+1}$$
$$=\frac{(k^2+k+1)^2}{(k^2+k+1)(k^2-k+1)}$$
$$=\frac{(k^2+k+1)^2}{(k^2+1)^2-k^2}$$
$$=\frac{(k^2+k+1)^2}{(k^4+2k^2+1-k^2)}$$
$$=\frac{(k^2+k+1)^2}{k^4+k^2+1}$$

$$(a+b+c)^2:(a^2+b^2+c^2)::(a+b+c):(a-b+c)$$

Main 2002

98. Solve the following equation for x, using properties of proportion:

$$\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$$

Sol :

We have $\dfrac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\dfrac{5}{2}$

Applying componendo and dividendo we have

$$\frac{(\sqrt{x+4}+\sqrt{x-10})+(\sqrt{x+4}-\sqrt{x-10})}{(\sqrt{x+4}+\sqrt{x-10})-(\sqrt{x+4}-\sqrt{x-10})}=\frac{5+2}{5-2}$$
$$\frac{2\sqrt{x+4}}{2\sqrt{x-10}}=\frac{7}{3}$$
$$\frac{\sqrt{x+4}}{\sqrt{x-10}}=\frac{7}{3}$$
$$\frac{x+4}{x-10}=\frac{49}{9}$$
$$9x+36=49x-490$$
$$40x=526$$
$$x=\frac{526}{40}=\frac{263}{20}$$

Main 2001

99. Find the numbers such that the mean proportional between them is 14 and third proportional to them is 112.

Sol :

Let x and y be the required numbers. Then, mean proportional between x and y is 14.

Now $x:14::14:y$

$$\frac{x}{14}=\frac{14}{y}$$
$$xy=196 \qquad (1)$$

Third proportional to x and y is 112.

Thus $x:y::y:112$

$$\frac{x}{y}=\frac{y}{112}$$
$$y^2=112x \qquad (2)$$

Now, $xy=196$

$$x=\frac{196}{y}$$

Substituting $x=\dfrac{196}{y}$ in (2), we have

$$y^2=112\times\frac{196}{y}$$
$$=112\times196$$
$$=(4\times4\times7)\times(7\times7\times4)$$
$$y^3=4^3\times7^3$$
$$y=4\times7=28$$

Substituting the value of y in (1), we get

$$x=\frac{196}{28}=7$$

Hence, the required numbers are 7, 28.

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END

11. What is the remainder when $2x^3-7x^2+5x-9$ is divided by $2x-3$?

(a) $-\frac{21}{2}$ (b) $-\frac{21}{4}$

(c) $-\frac{129}{4}$ (d) $-\frac{129}{2}$

Sol :

We have $f(x)=2x^3-7x^2+5x-9$

Let $2x-3=0$, then $x=\frac{3}{2}$, Thus $\alpha=\frac{3}{2}$

Remainder theorem says that if $f(x)$ is divided by $(x-\alpha)$, it leaves remainder b, then $f(\alpha)=b$.

Remainder

$$f(\tfrac{3}{2})=2\left(\frac{3}{2}\right)^3-7\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)-9$$
$$=\frac{27}{4}-\frac{63}{4}+\frac{15}{2}-9$$
$$=-\frac{42}{4}=-\frac{21}{2}$$

Thus (a) is correct option.

12. What is the value of the polynomial $3x^2+5x-4$ at $x=-3$?

(a) 28 (b) 22

(c) 64 (d) 8

Sol :

We have $p(x)=3x^2+5x-4$

$$p(-3)=3(-3)^2+5(-3)-4$$
$$=27-15-4=8$$

Thus (d) is correct option.

13. What is the value of the polynomial $3x^3+7x^2+11x-1$ at $x=1$?

(a) 20 (b) 30

(c) – 30 (d) – 8

Sol :

We have $p(x)=3x^3+7x^2+11x-1$

$$p(1)=3(1)^3+7(1)^2+11(1)-1$$
$$=3+7+11-1$$
$$=20$$

Thus (a) is correct option.

14. What is the remainder on dividing $3x^2-5x+11$ by $2x+5$?

(a) $\frac{39}{4}$ (b) $\frac{169}{4}$

(c) $\frac{69}{4}$ (d) $\frac{139}{4}$

Sol :

We have $p(x)=3x^2-5x+11$

Let $2x+5=0$, then $x=-\frac{5}{2}$, Thus $\alpha=-\frac{5}{2}$

Remainder theorem says that if $f(x)$ is divided by $(x-\alpha)$, it leaves remainder b, then $f(\alpha)=b$.

Remainder

$$p(-\tfrac{5}{2})=3\left(-\frac{5}{2}\right)^2-5\left(-\frac{5}{2}\right)+11$$
$$=\frac{75}{4}+\frac{25}{2}+11=\frac{169}{4}$$

Thus (b) is correct option.

15. What is the remainder on dividing $2x^3+6x^2-17x-4$ by $2x+1$?:

(a) $\frac{39}{4}$ (b) $\frac{123}{4}$

(c) $\frac{23}{4}$ (d) $\frac{139}{4}$

Sol :

We have $p(x)=2x^3+6x^2-17x-4$

Let $2x+1=0$, then $x=-\frac{1}{2}$, Thus $\alpha=-\frac{1}{2}$

Remainder theorem says that if $f(x)$ is divided by $(x-\alpha)$, it leaves remainder b, then $f(\alpha)=b$.

Remainder

$$p(-\tfrac{1}{2})=2\left(-\frac{1}{2}\right)^3+6\left(-\frac{1}{2}\right)^2-17\left(-\frac{1}{2}\right)-4$$
$$=-\frac{1}{4}+\frac{3}{2}+\frac{17}{2}-4$$
$$=\frac{23}{4}$$

Thus (c) is correct option.

16. If the sum of remainders obtained on dividing $x^3+(kx+8)x+k$ by $x+1$ and $x-2$ is 1, then the value of k is:

(a) 2 (b) 1

(c) – 1 (d) – 2

Sol :

We have $p(x)=x^3+(kx+8)x+k$

If $p(x)$ is divide by $x+1$ then remainder

$$p(-1)=(-1)^3+[k(-1)+8](-1)+k$$
$$=-1+k-8+k$$
$$=2k-9$$

If $p(x)$ is divide by $x-2$ then remainder

$$p(2)=(2)^3+[k(2)+8](2)+k$$
$$=8+4k+16+k$$
$$=5k+24$$

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Sol :

We have $p(x) = x^3 + ax^2 + bx + 6$

Since $x-2$ is a factor of $p(x)$,

$$p(2) = 0$$
$$(2)^3 + a(2)^2 + b(2) + 6 = 0$$
$$2a + b + 7 = 0 \quad \ldots(1)$$

Also, $$p(3) = 0$$
$$(3)^3 + a(3)^2 + b(3) + 6 = 0$$
$$3a + b + 11 = 0 \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$a = -4, b = 1$$

Thus (a) is correct option.

28. If the polynomials $kx^3 - 7x^2 + 7x - 2$ and $x^3 - 2kx^2 + 8x - 8$ leave the same remainder when divided by $x-2$, then the value of k is:

(a) -1 (b) 1

(c) -2 (d) 2

Sol :

Let $p(x) = kx^3 - 7x^2 + 7x - 2$

and $f(x) = x^3 + 2kx^2 + 8x - 8$

On dividing by $x-2$, they leave the same remainder.

$$p(2) = k(2)^3 - 7(2)^2 + 7(2) - 2$$
$$= 8k - 28 + 14 - 2$$
$$= 8k - 16$$
$$f(2) = (2)^3 - 2k(2)^2 + 7(2) - 8$$
$$= 8 - 8k + 16 - 8$$
$$= -8k + 16$$

Now $p(2) = f(2)$

$$8k - 16 = -8k + 16$$
$$k - 2 = -k + 2$$
$$2k = 4$$
$$k = 2$$

Thus (d) is correct option.

29. If $(x+2)$ and $(x+3)$ are the factors of the polynomial $f(x) = x^3 + ax + b$, then the value of a is_____.

(a) -19 (b) -30

(c) 30 (d) 19

Sol :

Since $(x+2)$ is the factor of $f(x)$ we have

$$f(-2) = 0$$
$$(-2)^3 + a(-2) + b = 0$$
$$-8 - 2a + b = 0$$
$$2a - b = -8 \quad (1)$$

Since $(x+3)$ is the factor of $f(x)$ we have

$$f(-3) = 0$$
$$(-3)^3 + a(-3) + b = 0$$
$$-27 - 3a + b = 0$$
$$3a - b = -27 \quad (2)$$

Subtracting eq (1) from (1) we have

$$a = -19$$

Thus (d) is correct option.

30. If the polynomial $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$ leaves the same remainder when divided by $x+3$, then the value of a is_____.

(a) 3 (b) -3

(c) 6 (d) -6

Sol :

Let $p(x) = ax^3 + 3x^2 - 9$ and $f(x) = 2x^3 + 4x + a$

On dividing by $(x+3)$, they leave the same remainder.

$$p(-3) = f(-3)$$
$$a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$
$$-27a + 27 - 9 = -54 - 12 + a$$
$$-28a = -84 \Rightarrow a = 3$$

Thus (a) is correct option.

31. If dividing $2x^3 + 3x^2 - kx + 5$ by $x-2$, we get a remainder 7, then the value of k is:

(a) 13 (b) -13

(c) 26 (d) -26

Sol :

We have $p(x) = 2x^3 + 3x^2 - kx + 5$

$$p(2) = 7$$
$$2(2)^3 + 3(2)^2 - k(2) + 5 = 7$$
$$16 + 12 - 2k + 5 = 7$$
$$-2k = -26$$
$$k = 13$$

Thus (a) is correct option.

32. If polynomial $f(x) = x^3 - 3x^2 + 4x + 50$ is divided

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$$a = 3, b = -4$$

$$a + b = 3 + (-4) = -1$$

Thus (d) is correct option.

22. Given that $(x+2)$ and $(x+4)$ are the factors of $3x^3 + ax^2 - 6x - b$. The values of a and b respectively are:

(a) 4, 7 (b) 7, 4
(c) 13, 40 (d) 40, 13

Sol :

We have $p(x) = 3x^3 + ax^2 - 6x - b$

Since, $x-2$ is a factor of $p(x)$.

$$p(-2) = 0$$

$$3(-2)^3 + a(-2)^2 - b(-2) - b = 0$$

$$-24 + 4a + 12 - b = 0$$

$$4a - b = 12 \quad ...(1)$$

Also, $(x+4)$ is a factor of $p(x)$.

$$p(-4) = 0$$

$$3(-4)^3 + a(-4)^2 - 6(-4) - b = 0$$

$$-192 + 16a + 24 - b = 0$$

$$16a - b = 168 \quad ...(2)$$

Solving equations (1) and (2), we get

$$a = 13, b = 40$$

Thus (c) is correct option.

23. If $8x^4 - 8x^2 + 7$ is divided by $2x+1$, the remainder is

(a) $\frac{17}{2}$ (b) $\frac{11}{2}$
(c) $\frac{15}{2}$ (d) $\frac{19}{2}$

Sol :

We have $p(x) = 8x^4 - 8x^2 + 7$

When $p(x)$ is divided by $2x+1$, remainder is

$$p\left(-\frac{1}{2}\right) = 8\left(\frac{-1}{2}\right)^4 - 8\left(\frac{-1}{2}\right)^2 + 7$$

$$= \frac{11}{2}$$

Thus (b) is correct option.

24. If $2x^3 + kx^2 - (5x-3)x + 8$ is divide by $x+2$, the remainder is 30, then the value of k is:

(a) 8 (b) 9
(c) 10 (d) 11

Sol :

We have $p(x) = 2x^3 + kx^2 - (5k-3)x + 8$

Since $p(-2) = 30$

$$2(-2)^3 + k(-2)^2 - (5k-3)(-2) + 8 = 30$$

$$-16 + 4k + 10k - 6 + 8 = 30$$

$$4k = 44$$

$$k = 11$$

Thus (d) is correct option.

25. When the polynomial $x^3 + 2x^2 - kx + 4$ is divided by $x-2$, the remainder is k. The value of k is:

(a) 10 (b) $-\frac{28}{3}$
(c) $-\frac{20}{3}$ (d) 20

Sol :

We have $p(x) = x^3 + 2x^2 - kx + 4$

$$p(2) = k$$

$$(2)^3 + 2(2)^2 - b(2) + 4 = k$$

$$20 = 3k$$

Also, $k = \frac{20}{3}$

Thus (c) is correct option.

26. What number should be added to the polynomial $p(x) = 2x^3 - 3x^2 - 8x$ so that it leaves a remainder 10 when divided by $2x+1$?

(a) 3 (b) 7
(c) 10 (d) 13

Sol :

Let k be the number which is to be added'

$$p(x) = 2x^3 - 3x^2 - 8x + k$$

$$p\left(\tfrac{1}{2}\right) = 10$$

$$2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + k = 10$$

$$-\frac{1}{4} - \frac{3}{4} + 4 + k = 10$$

$$k + 3 = 10$$

$$k = 7$$

Thus (b) is correct option.

27. If $x-2$ is a factor of the expression $x^3 + ax^2 + bx + 6$. When the expression is divided by $(x-3)$, it leaves a remainder 3. Then the values of a and b respectively are:

(a) – 4, 1 (b) 1, 4
(c) – 1, 4 (d) 4, – 1

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Applying componendo and dividendo, we get

$$\frac{x+1}{x-1}=\frac{\sqrt{a+3b}+\sqrt{a-3b}+(\sqrt{a+3b}-\sqrt{a-3b})}{\sqrt{a+3b}+\sqrt{a-3b}-(\sqrt{a+3b}-\sqrt{a-3b})}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{a+3b}}{2\sqrt{a-3b}}$$

$$\frac{x+1}{x-1}=\frac{\sqrt{a+3b}}{\sqrt{a-3b}}$$

Squaring both sides, we get

$$\frac{(x+1)^2}{(x-1)^2}=\frac{a+3b}{a-3b}$$

$$\frac{x^2+1+2x}{x^2+1-2x}=\frac{(a+3b)}{(a-3b)}$$

$$\frac{x^2+1+2x}{x^2+1-2x}=\frac{(a+3b)}{(a-3b)}$$

Again, applying compondo and dividendo, we get

$$\frac{x^2+1+2x+(x^2+1-2x)}{x^2+1+2x-(x^2+1-2x)}=\frac{(a+3b)+(a-3b)}{(a+3b)-(a-3b)}$$

$$\frac{2(x^2+1)}{4x}=\frac{2a}{6b}$$

$$\frac{x^2+1}{2x}=\frac{a}{3b}$$

$$3bx^2+3b=2ax$$

$$3bx^2-2ax+3b=0 \qquad \text{Hence Proved}$$

Comp 2006

92. If $\frac{by+cz}{b^2+c^2}=\frac{cz+ax}{c^2+a^2}=\frac{ax+by}{a^2+b^2}$, then show that each ratio is equal to $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$.

Sol :

We have $\frac{by+cz}{b^2+c^2}=\frac{cz+ax}{c^2+a^2}=\frac{ax+by}{a^2+b^2}$,

$$\text{Each given ratio}=\frac{\text{Sum of antecedents}}{\text{Sum of consequents}}$$

$$=\frac{2ax+2by+2cz}{2a^2+2b^2+2c^2}$$

$$=\frac{ax+by+cz}{a^2+b^2+c^2}$$

Now, $$\frac{by+cz}{b^2+c^2}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

$$\frac{a^2+b^2+c^2}{b^2+c^2}=\frac{ax+by+cz}{by+cz}$$

Using dividendo we have

$$\frac{a^2}{b^2+c^2}=\frac{ax}{by+cz}$$

$$\frac{b^2+c^2}{a^2}=\frac{by+cz}{ax}$$

Using componendo we have

$$\frac{a^2+b^2+c^2}{a^2}=\frac{ax+by+cz}{ax}$$

$$\frac{ax}{a^2}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

$$\frac{x}{a}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

Similarly, we have

$$\frac{cz+ax}{c^2+a^2}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

$$\frac{y}{b}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

and, $$\frac{ax+by}{a^2+b^2}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

$$\frac{z}{c}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

$$\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$$

$$=\frac{ax+by+cz}{a^2+b^2+c^2}$$

= Each given ratio

Comp 2005

93. If a, b, c are in continued proportion, prove that: $(a+b+c)(a-b+c)=a^2+b^2+c^2$

Sol :

As a, b, c are in continued proportion, $\frac{a}{b}=\frac{b}{c}$

Let $\frac{a}{b}=\frac{b}{c}=k$, then, $a=bk$, $b=ck$.

$$a=ck\times k=ck^2$$

$$(a+b+c)(a-b+c)$$

$$=(ck^2+ck+c)(ck^2-ck+c)$$

$$=c(k^2+k+1)\cdot c(k^2-k+1)$$

$$c^2(k^2+k+1)(k^2-k+1)$$

$$c^2[(k^2+1)+k][(k^2+1)-k]$$

$$=c^2[(k^2+1)^2-k^2]$$

$$=c^2(k^4+k^2+1)$$

$$=(ck^2)^2+(ck)^2+c^2$$

$$=a^2+b^2+c^2 \qquad [a=ck^2, b=ck]$$

Comp 2004

94. If a, b, c are in continued proportion, prove that: $(a+b):(b+c)::a^2(b-c):b^2(a-b)$

Sol :

As a, b, c are in continued proportion, $\frac{a}{b}=\frac{b}{c}$

Let $\frac{a}{b}=\frac{b}{c}=k$, then, $a=bk$, $b=ck$.

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by $x+3$, the remainder will be

(a) -16 (b) -12

(c) -20 (d) -10

Sol :

We have $f(x) = x^3 - 3x^2 + 4x + 50$

Remainder

$$f(-3) = \left[(-3)^2 - 3 \times (-3)^2 + 4 \times (-3) + 50\right]$$
$$= (-27 - 27 - 12 + 50)$$
$$= -16$$

Thus (a) is correct option.

33. The value of a for which $(x+a)$ is a factor of the polynomial $x^3 + ax^2 - 2x + a + 6$ is

(a) 4 (b) 2

(c) -4 (d) -2

Sol :

We have $f(x) = x^3 + ax^2 - 2x + a + 6$

Since $(x+a)$ is a factor of $f(x)$,

$$f(-a) = 0$$
$$(-a)^3 + a(-a)^2 - 2(-a) + a + 6 = 0$$
$$3a = -6$$
$$a = -2$$

Thus (d) is correct option.

34. If $(x-3)$ is a factor of $x^2 + x - a$, then the value of a is_____.

(a) – 12 (b) 12

(c) 6 (d) – 6

Sol :

We have $p(x) = x^2 + x - a$

Since $(x-3)$ is a factor of $p(x)$

$$p(3) = 0$$
$$(3)^3 + 3 - a = 0$$
$$9 + 3 - a = 0$$
$$a = 12$$

Thus (b) is correct option.

35. Assertion : $(x+2)$ and $(x-1)$ are factors of the polynomial $x^4 + x^3 + 2x^2 + 4x - 8$.

Reason : For a polynomial $p(x)$ of degree≥ 1, $x-a$ is a factor of the polynomial $p(x)$ if and only if $p(a) \geq 1$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Sol :

$$p(x) = x^4 + x^3 + 2x^2 + 4x - 8$$
$$p(-2) = (-2)^4 + (-2)^3 + 2(-2)^2 + 4(-2) - 8$$
$$= 16 - 8 + 8 - 8 - 8$$
$$= 0$$

So, $(x+2)$ is a factor of $p(x)$.

$$p(1) = (1)^4 + (1)^3 + 2(1)^2 + 4(1) - 8$$
$$= 1 + 1 + 2 + 4 - 8$$
$$= 0$$

Thus $(x-1)$ is a factor of $p(x)$.

Assertion is true but reason is false.

Thus (c) is correct option.

36. Assertion : The remainder when $p(x)$ $p(x) = x^3 - 6x^2 + 2x - 4$ is divided by $(3x-1)$ is $\frac{-107}{27}$.

Reason : If a polynomial $p(x)$ is divided by $ax-b$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Sol :

$$p(x) = x^3 - 6x^2 + 2x - 4$$
$$p\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 6\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 4$$
$$= \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 4 = \frac{1}{27} - 4$$
$$= \frac{1 - 4 \times 27}{27} = \frac{-107}{27}$$

Both assertion and reason are true and reason is the correct explanation of assertion.

Thus (a) is correct option.

THREE MARKS QUESTIONS

Main 2023

37. Factorize completely using factor theorem:

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$$(-2)^3+a(-2)+b=0$$
$$-8-2a+b=0 \quad ...(1)$$

Also $(x+3)$ is factor of $f(x)$, therefore we have

$$f(-3)=0$$
$$(-3)^3+a(-3)+b=0$$
$$-27-3a+b=0 \quad ...(2)$$

Subtract Eq. (2) from (1),

$$-8-2a+27+3a=0$$
$$a+19=0$$
$$a=-19$$

Substituting the value of a in Eq. (i), we get

$$-8-2(-19)+b=0$$
$$-8+38+b=0$$
$$30+b=0$$
$$b=-30$$

Main 2018

44. Use Remainder theorem to factorize the polynomial $2x^3+3x^2-9x-10$.

Sol :

Let $f(x)=2x^3+3x^2-9x-10$

Using hit and trial method, we put $x=-1$ to get

$$f(-1)=2(-1)^3+3(-1)^2-9(-1)-10$$
$$=-2+3+9-10$$
$$=12-12=0$$

Hence, $(x+1)$ is a factor of $f(x)$. Now we divide $f(x)$ by $(x+1)$ as shown below.

$$\begin{array}{r|l} & 2x^2+x-10 \\ \hline x+1 & 2x^3+3x^2-9x-10 \\ & \underline{-2x^3-2x^2} \\ & \quad x^2-9x-10 \\ & \quad \underline{-x^2-x} \\ & \quad\quad -10x-10 \\ & \quad\quad \underline{-10x-10} \\ & \quad\quad\quad 0 \end{array}$$

$$2x^3+3x^2-9x-10=(x+1)(2x^2+x-10)$$

Now $(2x^2+x-10)=2x^2+5x-4x-10$

$$=x(2x+5)-2(2x+5)$$
$$=(x-2)(2x+5)$$

Thus $2x^3+3x^2-9x-10=(x+1)(x-2)(2x+5)$

Comp 2018

45. Let R_1 and R_2 are remainders when the polynomials $x^3+2x^2-5ax-7$ and $x^3+ax^2-12x+6$ are divided by $x+1$ and $x-2$ respectively. If $2R_1+R_2=6$, find the value of a.

Sol :

If $p(x)$ is divided by $(x+a)$, it leaves remainder b, then $p(-a)=b$.

Let, $p(x)=x^3+2x^3-5ax-7$

Substituting $x+1=0\Rightarrow x=-1$ in $p(x)$ we have

$$p(-1)=(-1)^3+2(-1)^2-5a\times(-1)-7$$
$$R_1=-1+2+5a-7$$
$$=-6+5a$$

Thus $R_1=-6+5a \quad ...(1)$

Again, $q(x)=x^3+ax^2-12+6$

Substituting $x-2=0\Rightarrow x=2$ in $q(x)$ we have

$$q(2)=2^3+a(2)^2-12(2)+6$$
$$R_2=8+4a-24+6=4a-10$$

Thus $R_2=4a-10 \quad ...(2)$

We have $2R_1+R_2=6 \quad ...(3)$

Substituting the values of R_1 and R_2, we obtain

$$2(-6+5a)+4a-10=6$$
$$-12+10a+4a-10=6$$
$$14a=28$$
$$a=\frac{28}{14}=2$$

Main 2017

46. What must be subtracted from $16x^3-8x^2+4x+7$ so that the resulting expression has $2x+1$ as a factor?

Sol :

Let $f(x)=16x^3-8x^2+4x+7$

and $g(x)=2x+1$

By remainder theorem, when $f(x)$ is divided by $g(x)=2x+1$, then remainder is equal to $f(-\frac{1}{2})$.

Now,

$$f\left(\frac{-1}{2}\right)=16\left(\frac{-1}{2}\right)^3-8\left(\frac{-1}{2}\right)^2+4\left(\frac{-1}{2}\right)+7$$
$$=16\times\left(\frac{-1}{8}\right)-8\times\frac{1}{4}-4\times\frac{1}{2}+7$$
$$=-2-2-2+7$$
$$=1$$

Thus 1 must be subtracted from $16x^3-8x^2+4x+7$

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If $(x+a)$ is a factor of $f(x)$ then we must have

$$f(-a)=0$$

If $(x-a)$ is a factor of $f(x)$ then we must have

$$f(a)=0$$

Since $(x-2)$ is a factor of x^3+x^2-4x-4 we must have

$$\begin{aligned}f(2)&=(2)^3+(2)^2-4\times 2-4\\&=8+4-8-4\\&=0\end{aligned}$$

Hence, factor theorem is satisfied.

Now, using long division, we divide $f(x)$ by $(x-2)$

$$\begin{array}{r|l} & x^2+3x+2 \\ \hline x-2 & x^3+x^2-4x-4 \\ & \underline{-x^3+2x^2} \\ & \quad 3x^2-4x-4 \\ & \quad \underline{-3x^2+6x} \\ & \quad\quad 2x-4 \\ & \quad\quad \underline{-2x+4} \\ & \quad\quad\quad 0 \end{array}$$

$$\begin{aligned}f(x)&=(x-2)(x^2+3x+2)\\&=(x-2)[x^2+2x+x+2]\\&=(x-2)[x(x+2)+1(x+2)]\\&=(x-2)[(x+2)(x+1)]\end{aligned}$$

Main 2019

41. Using the remainder theorem find the remainders obtained when $x^3+(kx+8)x+k$ is divided by $x+1$ and $x-2$. Hence find k, if the sum of the two remainders is 1.

Sol :

We have $f(x)=x^3+(kx+8)x+k$

If $f(x)$ divided by $(x+a)$ leaves remainder λ then from remainder theorem we have

$$f(x+a=0)=\lambda$$

or $$f(-a)=\lambda$$

Thus substituting $x=-1$ in $f(x)$ we have

$$\begin{aligned}f(-1)&=(-1)^3+[k(-1)+8](-1)+k\\&=-1+(-k+8)(-1)+k\\&=-1+k-8+k\\&=2k-9\end{aligned}$$

and substituting $x=+2$ in $f(x)$ we have

$$\begin{aligned}f(+2)&=(2)^3+(k\times 2+8)2+k\\&=8+(4k+16)+k\\&=5k+24\end{aligned}$$

According to the given condition,

$$\begin{aligned}f(-1)+f(2)&=1\\2k-9+5k+24&=1\\7k+15&=1\\7k&=-14\\k&=-2\end{aligned}$$

Comp 2019

42. The polynomials kx^3+3x^2-8 and $3x^3-5x+8$ are divided by $x+2$. If the remainder in each case is the same, find the value of k.

Sol :

If $p(x)$ is divided by $(x+a)$, it leaves remainder b, then $p(-a)=-b$.

Let, $p(x)=kx^3+3x^2-8$

Substituting $x+2=0\Rightarrow x=-2$ in $p(x)$ we have

$$\begin{aligned}p(-2)&=k(-2)^3+3(-2)^2-8\\&=-8k+12-8\end{aligned}$$

or $$p(-2)=-8k+4 \quad ...(1)$$

Let, $q(x)=3x^3-5x+8$

Substituting $x+2=0\Rightarrow x=-2$ in $p(x)$ we have

$$\begin{aligned}q(-2)&=3(-2)^3-5(-2)+8\\&=3\times(-8)+10+8\\&=-24+18\end{aligned}$$

or $$q(-2)=-6 \quad ...(2)$$

In both cases remainders are same. Thus

$$\begin{aligned}p(-2)&=q(-2)\\-8k+4&=-6\\-8k&=-10\\k&=\frac{10}{8}=\frac{5}{4}\end{aligned}$$

Main 2018

43. If $(x+2)$ and $(x+3)$ are factors of x^3+ax+b, find the values of a and b.

Sol :

Let $f(x)=x^2+ax+b$

Since $(x+2)$ is a factor of $f(x)$, therefore by factor theorem we have

$$f(-2)=0$$

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$$= (2x-1)(x+3)$$

Eq. (i) can be rewritten as

$$f(x) = (x-2)(2x-1)(x+3)$$

Main 2010

58. When divided by $x-3$, the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p+3)\ x - 6$ leave the same remainder. Find the value of p.

Sol :

Let $f(x) = x^3 - px^2 + x + 6$

and $g(x) = 2x^3 - x^2 - (p+3)x - 6$

If both are divided by $x-3$ and have same remainder, then

$$f(3) = g(3)$$

$$(3)^3 - p(3)^2 + 3 + 6 = 2(3)^3 - (3)^2 - (p+3)3 - 6$$

$$27 - 9p + 3 + 6 = 54 - 9 - (p+3)3 - 6$$

$$36 - 9p = 39 - (p+3)3$$

$$36 - 9p = 39 - 3p - 9$$

$$36 - 9p = 30 - 3p$$

$$36 - 30 = -3p + 9p$$

$$6p = 6$$

$$p = 1$$

Hence, the value of p is 1.

Comp 2010

59. If the $(x-2)$ and $(x+3)$ are factors of the quadratic polynomial $x^2 + (a+1)x + b$, then find the value of a and b.

Sol :

If $x - a$ is a factor of the polynomial, then

$$f(a) = 0$$

Here, $(x-2)$ and $(x+3)$ are factors of the polynomial $x^2 + (a+1)x + b$

So, $f(2) = 0$

$$(2)^2 + (a+1)(-3) + b = 0$$

$$4 + 2a + 2 + b = 0$$

$$6 + 2a + b = 0$$

$$2a + b = -6 \qquad ...(1)$$

Again, $f(-3) = 0$

$$(-3)^2 + (a+1)2 + b = 0$$

$$9 - 3(a+1) + b = 0$$

$$9 - 3a - 3 + b = 0$$

$$6 - 3a + b = 0$$

$$-3a + b = -6$$

$$3a - b = 6 \qquad ...(2)$$

Adding equations (1) and (2), we get

$$5a = 0 \ \Rightarrow \ a = 0$$

Substituting value of a in equation (1), we get

$$b = -6$$

Hence, $a = 0$ and $b = -6$.

Comp 2009

60. Factorise $x^3 - 6x^2 + 11x - 6$

Sol :

We have $p(x) = x^3 - 6x^2 + 11x - 6$

Assume $x - 1$ be a factor of $p(x)$, then substituting $x = 1$ in $p(x)$ we have

$$p(1) = 1^3 + 6(1)^2 + 11(1) - 6$$

$$= 1 - 6 + 11 - 6$$

$$= 12 - 12 = 0$$

Since $p(1) = 0$, thus $x - 1$ is a factor of $p(x)$ and our assumption is correct.

Now $p(x)\ x^3 - 6x^2 + 11x - 6$

$$= x^3 - x^2 - 5x^2 - 5x + 6x - 6$$

$$= x^2(x-1) - 5x(x-1) - 6(x-1)$$

$$= (x-1)(x^2 - 5x - 6)$$

$$= (x-1)\{x^2 - 3x - 2x - 6\}$$

$$= (x-1)\{x(x-3) - 2(x-3)\}$$

$$= (x-1)(x-3)(x-2)$$

Main 2007

61. Show that $(x-1)$ is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise above expression.

Sol :

Let $f(x) = x^3 - 7x^2 + 14x - 8$ ...(i)

Substituting $x = 1$ in Eq. (i), we get

$$f(1) = (1)^3 - 7(1)^2 + 14 \times 1 - 8$$

$$= 1 - 7 + 14 - 8$$

$$= 15 - 15 = 0$$

Since $f(1) = 0$, thus $(x-1)$ is one of the factor of $f(x)$. Now, we divide $f(x)$ by $(x-1)$ using long division method.

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for getting $(2x+1)$ as a factor.

Comp 2017

47. Find the remainders when $3x^3-4x^2+7x-5$ is divided by $x-3$ and $x+3$.

Sol :

If $p(x)$ is divided by $(x+a)$, it leaves remainder b, then $p(-a)=b$.

Let, $p(x)=3x^3-4x^2+7x-5$

Substituting $x-3=0\Rightarrow x=3$ in $p(x)$ we have

$$\begin{aligned}p(3)&=3(3)^3-4(3)^2+7\times 3-5\\&=3\times 27-4\times 9+21-5\\&=81-36+21-5\\&=61\end{aligned}$$

Again, substituting $x+3=0\Rightarrow x=-3$ in $p(x)$ we have

$$\begin{aligned}p(-3)&=3(-3)^3-4(-3)^2+7(-3)-5\\&=3\times(-27)-4\times 9-21-5\\&=-81-36-21-5\\&=-143\end{aligned}$$

Remainders are 61 and -143.

Main 2016

48. Using remainder theorem, find the value of k, if one dividing $2x^3+3x^2-kx+5$ by $x-2$, leaves a remainder 7.

Sol :

Let $f(x)=2x^3+3x^2-kx+5$ ...(i)

If $f(x)$ is divided by $x-2$, remainder is the value of $f(x)$ at $x-2=0$ i.e. at $x=2$.

Thus $f(2)=7$

$$\begin{aligned}2(2)^3+3(2)^2-k\times 2+5&=7\\16+12-2k+5&=7\\33-2k&=7\\2k&=33-7=26\\k&=13\end{aligned}$$

Comp 2016

49. The polynomial $p(x)=kx^3+9x^2+4x-8$ when divided by $(x+3)$ leaves a remainder $10(1-k)$. Find the value of k.

Sol :

If $p(x)$ is divided by $(x+a)$, it leaves remainder b, then $p(-a)=b$.

We have $p(x)=kx^3+9x^2+4x-8$

Substituting $x+3=0\Rightarrow x=-3$ in $p(x)$ we have

$$\begin{aligned}p(-3)&=k(-3)^3+9(-3)^2+4(-3)-8\\&=-27k+81-12-8\\&=-27k+61\end{aligned}$$

Also, $p(-3)=10(1-k)$

Thus $10(1-k)=-27k+61$

$$\begin{aligned}10-10k&=-27k+61\\17k&=51\ \Rightarrow\ k=3\end{aligned}$$

Main 2015

50. Find a, if the two polynomials ax^3+3x^2-9 and $2x^3+4x+a$ leave the same remainder when divided by $x+3$.

Sol :

Let $f(x)=ax^3+3x^2-9$

and $g(x)=2x^3+4x+a$

If both are divided by $x+3$ and have same remainder, then

$$\begin{aligned}f(-3)&=g(-3)\\a(-3)^3+3(-3)^2-9&=2(-3)^3+4(-3)+a\\-27a+27-9&=-54-12+a\\-28a&=-66-18\\-28a&=-84\\a&=\frac{-84}{-28}\Rightarrow a=3\end{aligned}$$

Hence, the value of a is 3.

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Main 2014

51. Using the remainder and factor theorem, factorize the following polynomial $x^3+10x^2-37x+26$.

Sol :

Let $f(x)=x^3+10x^2-37x+26$ ...(1)

Using hit and trial method, we put $x=1$ to get

$$\begin{aligned}f(1)&=1^3+10(1)^2-37(1)+26\\&=1+10-37+26\\&=37-37=0\end{aligned}$$

Since $f(1)=0$, $(x-1)$ is a factor of $f(x)$.

Now, we divide $f(x)$ by $x-1$ using long division method.

$2x^3 - x^2 - 13x - 6$

Sol :

We have $p(x) = 2x^3 - x^2 - 13x - 6$

We use hit and trial method.

Substituting $x = -2$ we get

$$\begin{aligned} p(-2) &= 2(-2)^3 - (-2)^2 - 13(-2) - 6 \\ &= -16 - 4 + 26 - 6 \\ &= -26 + 26 \\ &= 0 \end{aligned}$$

Hence $x = -2$ or $x + 2$ is a factor of $p(x)$.

Now we divide $p(x)$ by $x + 2$ as given follow.

$$\begin{array}{r} 2x^2 - 5x - 3 \\ x+2 \overline{)\, 2x^3 - x^2 - 13x - 6} \\ \underline{-\,2x^3 + 4x^2} \\ -5x^2 - 13x - 6 \\ \underline{-5x^2 - 10x} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

Thus

$$2x^3 - x^2 - 13x - 6 = (x+2)(2x^2 - 5x - 3)$$

Now $$\begin{aligned} 2x^2 - 5x - 3 &= 2x^2 - (6-1)x - 3 \\ &= 2x^2 - 6x + x - 3 \\ &= 2x(x-3) + 1(x-3) \\ &= (2x+1)(x-3) \end{aligned}$$

Hence

$$2x^3 - x^2 - 13x - 6 = (x+2)(2x+1)(x-3)$$

Main 2020

38. Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely.

Sol :

We have $f(x) = 6x^3 + 17x^2 + 4x - 12$

We use hit and trial method.

Substituting $x = -2$ we get

$$\begin{aligned} f(-2) &= 6(-2)^3 + 17(-2)^2 + 4(-2) - 12 \\ &= -48 + 68 - 8 - 12 \\ &= 0 \end{aligned}$$

So, by factor theorem, $(x + 2)$ will be a factor of $f(x)$. Now, to find other factor, we divide $f(x)$ by $(x + 2)$ using long division method.

$$\begin{array}{r} 6x^2 + 5x - 6 \\ x+2 \overline{)\, 6x^3 + 17x^2 + 4x - 12} \\ \underline{-\,6x^3 + 12x^2} \\ 5x^2 + 4x - 12 \\ \underline{-5x^2 + 10x} \\ -6x - 12 \\ \underline{-6x - 12} \\ 0 \end{array}$$

Clearly, $6x^2 + 5x - 6$ is another factor of $f(x)$

Now $f(x) = (x+2)(6x^2 + 5x - 6)$

Now $$\begin{aligned} 6x^2 + 5x - 6 &= 6x^2 + 9x - 4x - 6 \\ &= 3x(2x+3) - 2(2x+3) \\ &= (2x+3)(3x-2) \end{aligned}$$

Hence,

$$f(x) = (x+2)(2x+3)(3x-2)$$

Main 2020

39. What must be added to the polynomial $2x^3 - 3x^2 - 8x$ so that it leaves a remainder 10 when divided by $2x + 1$?

Sol :

Let $f(x) = 2x^3 - 3x^2 - 8x$

and $g(x) = 2x + 1$

By remainder theorem, when $f(x)$ is divided by $g(x) = 2x + 1$, then remainder is equal to $f(-\frac{1}{2})$.

Now,

$$\begin{aligned} f\left(\frac{-1}{2}\right) &= 2\left(\frac{-1}{2}\right)^3 - 3\left(\frac{-1}{2}\right)^2 - 8\left(\frac{-1}{2}\right) \\ &= 2 \times \left(\frac{-1}{8}\right) - 3 \times \frac{1}{4} + 8 \times \frac{1}{2} \\ &= -\frac{1}{4} - \frac{3}{4} + 4 \\ &= -1 + 4 \\ &= 3 \end{aligned}$$

So it leaves remainder 3 but we want remainder 10. So we must add 7 in $f(x)$ to get remainder 10.

Main 2019

40. Using the factor theorem, show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.
Hence factorise the polynomial completely.

Sol :

We have $f(x) = x^3 + x^2 - 4x - 4$ .....(1)

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Thus $x^2-3x-m(m+3)=[x+m][x-(m+3)]$

Main 2011

55. Find the value of k, if $(x-2)$ is a factor of $x^3+2x^2-kx+10$. Hence, determine whether $(x+5)$ is also a factor.

Sol :

Let $f(x)=x^3+2x^2-kx+10$ ...(1)

Since, $(x-2)$ is a factor of $f(x)$ be factor theorem we have

$$f(2)=0$$
$$(2)^3+2(2)^2-k(2)+10=0$$
$$8+8-2k+10=0$$
$$2k=26$$
$$k=\frac{26}{2}=13$$

Substitution $k=13$ in Eq. (1), we get

$f(x)=x^3+2x^2-13x+10$ ...(2)

Substituting $x=-5$ in Eq. (ii), we get

$$f(-5)=(-5)^3+2(-5)^2-13(-5)+10$$
$$=-125+50+65+10$$
$$=-125+125=0$$

Since $f(-5)=0$, thus $(x+5)$ is also a factor of $f(x)$.

Comp 2011

56. If both $(x-2)$ and $(2x-1)$ are factors of ax^2+5x+b, show that $a-b=0$.

Sol :

We have $p(x)=ax^2+5x+b$

If $x-2$ is a factor of $p(x)$ then by factor theorem, $p(-a)=0$.

Substituting $x=a$ in $p(x)$ and equating to zero we have

$$a(2)^2+5(2)+b=0$$
$$4a+10+b=0$$
$$4a+b=-10 \quad ...(1)$$

Now, $2x-1=0 \Rightarrow x=\frac{1}{2}$

If $2x-1$ is a factor of $p(x)$ then by factor theorem, $p(\frac{1}{2})=0$.

Substituting $x=\frac{1}{2}$ in $p(x)$ and equating to zero we have

$$a\left(\frac{1}{2}\right)^2+5\left(\frac{1}{2}\right)+b=0$$
$$\frac{a}{4}+\frac{5}{4}+b=0$$
$$\frac{a+5\times 2+4b}{4}=0$$
$$a+4b+10=0$$
$$a+4b=-10 \quad ...(2)$$

Subtracting equation (2) from (1), we get

$$3a-3b=0$$
$$a-b=0$$

Hence proved.

Main 2010

57. Use the factor theorem to factorise the following expression $2x^3+x^2-13x+6$.

Sol :

Let $f(x)=2x^3+x^2-13x+6$

Here we will use hit and trial method.

For $x=1$:

$$f(1)=2(1)^3+1^2-13(1)+6$$
$$=2+1-13+6$$
$$=-4\neq 0$$

For $x=-1$:

$$f(-1)=2(-1)^3+(-1)^2-13(-1)+6$$
$$=-2+1+13+6$$
$$=18\neq 0$$

For $x=2$:

$$f(2)=2(2)^3+(2)^2-13(2)+6$$
$$=2\times 8+4-26+6$$
$$=16-16$$
$$=0$$

Since $f(2)=0$, thus $(x-2)$ is one of the factor of $f(x)$. Now, we divide $f(x)$ by $x-2$, using long division method.

$$\begin{array}{r|l} & 2x^2+5x-3 \\ \hline x-2 & 2x^3+x^2-13x+6 \\ & 2x^3-4x^2 \\ & - \quad + \\ \hline & 5x^2-13x+6 \\ & 5x^2-10x \\ & - \quad + \\ \hline & -3x+6 \\ & -3x+6 \\ & + \quad - \\ \hline & 0 \end{array}$$

Thus $f(x)=(x-2)(2x^2+5x-3)$ ...(i)

Now, $2x^2+5x-3=2x^2+6x-x-3$

$$=2x(x+3)-1(x+3)$$

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$$= 105 - 105$$

or $p(3) = 0$

(iii) $(x - \frac{1}{2})$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence $(x-2)$, $(x-3)$ and $(x-\frac{1}{2})$ are the factors of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

Main 2005

65. $(x-2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$. When, this expression is divided by $(x-3)$, it leaves remainder 3. Find the values of a and b.

Sol :

Let $p(x) = x^3 + ax^2 + bx + 6$ (1)

Since, $(x-2)$ is a factor of $p(x)$, by factor theorem we get

$$p(2) = 0$$
$$(2)^3 + a(2)^2 + b(2) + 6 = 0$$
$$8 + 4a + 2b + 6 = 0$$
$$4a + 2b + 14 = 0$$
$$2a + b + 7 = 0 \quad (2)$$

Also, when $p(x)$ is divided by $(x-3)$, it leaves the remainder 3, thus using remainder theorem

$$p(3) = 3$$
$$(3)^3 + a(3)^2 + b(3) + 6 = 3$$
$$27 + 9a + 3b + 6 - 3 = 0$$
$$9a + 3b + 30 = 0$$
$$3a + b + 10 = 0 \quad (3)$$

Subtracting Eq. (ii) from Eq. (iii), we get

$$a + 3 = 0$$
$$a = -3$$

Substituting $a = -3$ in Eq. (3), we get

$$3(-3) + b + 10 = 0$$
$$-9 + b + 10 = 0$$
$$b + 1 = 0$$
$$b = -1$$

Hence, $a = -3$ and $b = -1$

Comp 2005

66. Use factor theorem to verify that $x + a$ is a factor of $x^n + a^n$ for any odd positive integer n.

Sol :

Let, $p(x) = x^n + a^n$

If $x + a$ is a factor of $p(x)$ then by factor theorem, $p(-a) = 0$.

Substituting $x = a$ in $p(x)$ we have

$$p(-a) = (-a)^n + a^n$$
$$= -a^n + a^n$$
(n is an odd positive integer)
$$p(-a) = 0 \qquad ((-1)^n = -1)$$

Hence by factor theorem $x + a$ is a factor of $x^n + a^n$ for any odd positive integer n.

Main 2004

67. Use the factor theorem to factorise completely $x^3 + x^2 - 4x - 4$.

Sol :

Let $p(x) = x^3 + x^2 - 4x - 4$

Using hit and trial method, substituting $x = -1$ we have

$$p(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$$
$$= -1 + 1 + 4 - 4$$
$$= 0$$

Since $p(-1) = 0$, thus $(x+1)$ is one of the factor of $p(x)$. Now, we divide $p(x)$ by $x+1$, using long division method.

$$\begin{array}{r|l} & x^2 - 4 \\ \hline x+1 & x^3 + x^2 - 4x - 4 \\ & x^3 + x^2 \\ & - \quad - \\ \hline & -4x - 4 \\ & -4x - 4 \\ & + \quad + \\ \hline & 0 \end{array}$$

Thus $p(x) = (x+1)(x^2 - 4)$ ...(i)

Now, $x^2 - 4 = x^2 - 2x + 2x - 4$

$$= x(x-2) + 2(x-2)$$
$$= (x-2)(x+2)$$

Eq. (i) can be rewritten as

$$p(x) = (x+1)(x-3)^2$$

Comp 2004

68. Find the value of c for which the polynomial $2x^3 - 7x^2 - 3x + c$ is exactly divisible by $(2x+3)$. Hence factorise the polynomial.

$$
\begin{array}{r|l}
x-1 & x^3-7x^2+14x-8 \\
 & x^3-x^2 \\
 & \hline
 & -6x^2+14x-8 \\
 & -6x^2+6x \\
 & \hline
 & +8x-8 \\
 & 8x-8 \\
 & \hline
 & 0
\end{array}
$$

Thus
$$
\begin{aligned}
f(x) &= (x-1)(x^2-6x+8) \\
&= (x-1)[x^2-4x-2x+8] \\
&= (x-1)[x(x-4)-2(x-4)] \\
&= (x-1)(x-2)(x-4)
\end{aligned}
$$

Comp 2007

62. For what value of a the polynomial $2x^3+ax^2+11x+a+3$ is exactly divisible by $2x-1$?

Sol :

Let, $p(x) = 2x^3+ax^2+11x+a+3$

Now $2x-1=0 \Rightarrow x=\frac{1}{2}$

If $p(x)$ is divisible by $2x-1$, then by factor theorem $p(\frac{1}{2})=0$.

Substituting $x=-\frac{3}{2}$ in $p(x)$ and equating to zero we have

$$
\begin{aligned}
p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^2+a\left(\frac{1}{2}\right)^2+11\left(\frac{1}{2}\right)+a+3 \\
0 &= 2\times\frac{1}{8}+a\times\frac{1}{4}+\frac{11}{2}+a+3 \\
0 &= \frac{1}{4}+\frac{a}{4}+\frac{11}{2}+a+3 \\
0 &= \frac{1+a+22+4a+12}{4} \\
0 &= 5a+35 \\
5a &= -35 \Rightarrow a=-7
\end{aligned}
$$

Main 2006

63. Show that $2x+7$ is a factor of $2x^3+5x^2-11x-14$. Hence, factorise the given expression completely, using factor theorem.

Sol :

Let $p(x) = 2x^3+5x^2-11x-14$ (1)

Substituting $2x+7=0$ i.e $x=-\frac{7}{2}$ in above we get

$$
\begin{aligned}
p\left(-\frac{7}{2}\right) &= 2\left(-\frac{7}{2}\right)^3+5\left(-\frac{7}{2}\right)^2-11\times\left(-\frac{7}{2}\right)-14 \\
&= -\frac{343}{8}\times 2+5\times\frac{49}{4}+\frac{77}{2}-14 \\
&= -\frac{686}{8}+\frac{245}{4}+\frac{77}{2}-14 \\
&= \frac{-686+490+308-112}{8} \\
&= \frac{-798+798}{8}=0
\end{aligned}
$$

Hence, $(2x+7)$ in one of the factors of $p(x)$.

Now, we divide $p(x)$ by $2x+7$, using long division method.

$$
\begin{array}{r|l}
2x+7 & 2x^3+5x^2-11x-14 \\
 & 2x^3+7x^2 \\
 & \hline
 & -2x^2-11x-14 \\
 & -2x^2-7x \\
 & \hline
 & -4x-14 \\
 & -4x-14 \\
 & \hline
 & 0
\end{array}
$$

Thus
$$
\begin{aligned}
p(x) &= (2x+7)(x^2-x-2) \\
&= (2x+7)[x^2-2x+x-2] \\
&= (2x+7)[x(x-2)+1(x-2)] \\
&= (2x+7)(x-2)(x+1)
\end{aligned}
$$

Comp 2006

64. Verify whether $(x-2)$, $(x-3)$ and $(x-\frac{1}{2})$ are the factors of the polynomial $p(x)=2x^3-11x^2+17x-6$.

Sol :

If $(x-a)$ is a factor of $p(x)$, then using factor theorem

$$p(a)=0$$

(i) $(x-2)$

$$
\begin{aligned}
p(x) &= 2x^2-11x^2+17x-6 \\
p(2) &= 2(2)^3-11(2)^2+17(2)-6 \\
&= 16-44+34-6 \\
&= 50-50
\end{aligned}
$$

or $p(2)=0$

(ii) $(x-3)$

$$
\begin{aligned}
p(3) &= 2(3)^3-11(3)^2+17(3)-6 \\
&= 54-99+51-6
\end{aligned}
$$

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$$
\begin{array}{r|l}
 & x^2+11x-26 \\
\hline
x-1 & x^3+10x^2-37x+26 \\
 & \underline{-x^3+x^2} \\
 & \quad 11x^2-37x+26 \\
 & \quad \underline{11x^2-11x} \\
 & \quad\quad -26x+26 \\
 & \quad\quad \underline{-26x+26} \\
 & \quad\quad\quad 0
\end{array}
$$

Thus $f(x)=(x-1)(x^2+11x-26)$ ...(2)

Now $x^2+11x-26=x^2+13x-2x-26$

$=x(x+13)-2(x+13)$

$=(x-2)(x+13)$

Eq. (2) becomes,

$f(x)=(x-1)(x-2)(x+13)$

Comp 2014

52. Check whether $(p+1)$ is a factor of $(p^{100}-1)$ and $(p^{101}-1)$.

Sol :

If $p+a$ is a factor of $f(p)$ then by factor theorem, $f(a)=0$

We have, $f(p)=p^{100}-1$

If $p+1$ is a factor of $f(p)$ then by factor theorem, $f(-1)=0$.

Substituting $p=-1$ in $f(p)$ we have

$f(-1)=(-1)^{100}-1$

$=1-1=0$

Since $f(-1)=0$, thus $p+1$ is a factor of $f(p)$.

Now we have $g(p)=p^{101}-1$

If $p+1$ is a factor of $g(p)$ then by factor theorem, $g(-1)=0$.

Substituting $p=-1$ in $g(p)$ we have

$g(-1)=(-1)^{101}-1$

$=-1-1$

$=-2$

Since $g(-1)=-2\neq 0$, hence $p+1$ is not a factor of $g(p)$.

Main 2012

53. If $(x-2)$ is a factor of the expression $2x^3+ax^2+bx-14$ and when the expression is divided by $(x-3)$, it leaves a remainder 52. Find the values of a and b.

Sol :

Let $f(x)=2x^3+ax^2+bx-14$ ...(i)

Since, $x-2$ is a factor of $f(x)$, by factor theorem

$f(2)=0$

$2(2)^3+a(2)^2+b(2)-14=0$

$16+4a+2b-14=0$

$4a+2b+2=0$

$4a+2b=-2$

$2a+b=-1$ ...(1)

Also, given that when $f(x)$ is divided by $(x-3)$, the remainder is 52.

By remainder theorem we have

$f(3)=52$

$2(3)^3+a(3)^2+b(3)-14=52$

$54+9a+3b-14=52$

$9a+3b=52-54+14$

$9a+3b=12$

$3a+b=4$ ...(2)

Subtracting Eq. (1) from Eq. (2), we get

$a=5$

Substituting $a=5$ in Eq. (1), we get

$2(5)+b=-1$

$b=-1-10=-11$

Hence, $a=5$ and $b=-11$

Comp 2012

54. What are the factors of the polynomial $x^2-3x-m(m+3)$?

Sol :

We have $p(x)=x^2-3x-m(m+3)$

Substituting $x=-m$ in $p(x)$ we have

$p(-m)=(-m)^2-3(-m)-m(m+3)$

$=m^2+3m-m^2-3m$

$=0$

Thus $x+m$ is a factor of given polynomial.

Now substituting $x=m+3$ in given polynomial we have

$p(x)=(m+3)^2-3(m+3)-m(m+3)$

$=(m+3)[m+3-3-m]$

$=(m+3)[0]=0$

Thus $x-(m+3)$ is also a factor of given polynomial.

NODIA

Sol :

We have $p(x) = 2x^3 - 7x^2 - 3x + c$

Here $2x + 3 = 0$

$$2x = -3 \Rightarrow x = \frac{-3}{2}$$

Since $p(x)$ is exactly by $2x+3$, then by factor theorem $p(-\frac{3}{2})$ must be zero.
Substituting $x = -\frac{3}{2}$ in $p(x)$ and equating to zero we have

$$2\left(\frac{-3}{2}\right)^3 - 7\left(\frac{-3}{2}\right)^2 - 3\left(\frac{-3}{2}\right) + c = 0$$

$$-2 \times \frac{27}{8} - \frac{7 \times 9}{4} + \frac{9}{2} + c = 0$$

$$\frac{-27}{4} - \frac{63}{4} + \frac{9}{2} + c = 0$$

$$\frac{-90}{4} + \frac{9}{2} + c = 0$$

$$c = \frac{90 - 18}{4}$$

$$= \frac{72}{4} = 18$$

Thus

$$p(x) = 2x^3 - 7x^2 - 3x + 18$$
$$= 2x^3 + 3x^2 - 10x^2 - 15x + 12x + 18$$
$$= x^2(2x+3) - 5x(2x+3) + 6(2x+3)$$
$$= (2x+3)(x^2 - 3x - 2x + 6)$$
$$= (2x+3)[x(x-3) - 2(x-3)]$$
$$= (2x+3)(x-3)(x-2)$$

Main 2003

69. Find the value of a, if $(x-a)$ is a factor of $x^3 - a^2x + x + 2$.

Sol :

Let $p(x) = x^3 - a^2x + x + 2$
Since $(x-a)$ is a factor of $p(x)$, we have

$$p(a) = 0$$
$$a^3 - a^3 + a + 2 = 0$$
$$a + 2 = 0$$
$$a = -2$$

Comp 2003

70. Show that $x-3$ is a factor of the polynomial $2x^3 - x^2 - 13x - 6$. Hence completely factorise the given polynomial.

Sol :

We have $p(x) = 2x^3 - x^2 - 13x - 6$

Substituting $x = 3$ in $p(x) = 0$ we have

$$p(x) = 2(3)^3 - (3)^2 - 13(3) - 6$$
$$= 2(27) - 9 - 39 - 6$$
$$= 54 - 54 = 0$$

So, $x-3$ is a factor of $p(x)$. Now by long division,

$$\begin{array}{r|l} & 2x^2 + 5x + 2 \\ \hline x - 3 & 2x^3 - x^2 - 13x - 6 \\ & \underline{-\ 2x^3 + 6x^2} \\ & \quad 5x^2 - 13x - 6 \\ & \quad \underline{-\ 5x^2 + 15x} \\ & \qquad 2x - 6 \\ & \qquad \underline{2x - 6} \\ & \qquad\quad 0 \end{array}$$

Factorizing the quotient, we get

$$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2$$
$$= 2x(x+2) + (x+2)$$
$$= (2x+1)(x+2)$$

Thus $2x^3 - x^2 - 13x - 6 = (x-3)(2x+1)(x+2)$

Main 2002

71. Using factor theorem, show that $(x-3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence, factorise the given expression completely.

Sol :

Let $f(x) = x^3 - 7x^2 + 15x - 9$
Substituting $x = 3$ in Eq. (i), we get

$$f(3) = (3)^3 - 7(3)^2 + 15 \times 3 - 9$$
$$= 27 - 63 + 45 - 9$$
$$= 0$$

Since $f(3) = 0$, thus $(x-3)$ is one of the factor of $f(x)$. Now, we divide $f(x)$ by $(x-3)$ using long division method.

$$\begin{array}{r|l} & x^2 - 4x + 3 \\ \hline x - 3 & x^3 - 7x^2 + 15x - 9 \\ & \underline{x^3 - 3x^2} \\ & \quad -4x^2 + 15x - 9 \\ & \quad \underline{-4x^2 + 12x} \\ & \qquad 3x - 9 \\ & \qquad \underline{3x - 9} \\ & \qquad\quad 0 \end{array}$$

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$$19a = 33$$
$$a = \frac{33}{19}$$

Comp 2015

79. If $x+a$ is a factor of the polynomial x^2+px+q and x^2+mx+n, prove that $a = \dfrac{n-q}{m-p}$.

Sol :

Let, $p(x) = x^2+px+q$ and
$q(x) = x^2+mx+n$

If $x+a$ is a factor of $p(x)$ then by factor theorem, $p(-a)=0$.
Substituting $x=a$ in $p(x)$ and equating to zero we have

$$(-a)^2+p(-a)+q=0$$
$$a^2-pa+q=0 \qquad ...(1)$$

If $x+a$ is a factor of $q(x)$ then by factor theorem, $q(-a)=0$.
Substituting $x=a$ in $q(x)$ and equating to zero we have

$$(-a)^2+m(-a)+n=0$$
$$a^2-ma+n=0 \qquad ...(2)$$

Rewriting eq (1) and (2) we have

$$a^2-pa+q=0 \qquad ...(1)$$
$$a^2-ma+n=0 \qquad ...(2)$$

Subtracting eqn. (1) from (2), we get

$$pa-ma+n-q=0$$
$$a(p-m)+n-q=0$$
$$a(p-m) = -(n-q)$$
$$-a(m-p) = -(n-q)$$
$$a = \frac{n-q}{m-p}$$ Hence proved.

Comp 2013

80. If $(x+1)$ and $(x-1)$ are factors of the polynomial ax^3+x^2+2x+b, then find the values of a and b. Also find the third factor.

Sol :

We have, $p(x) = ax^3+x^2+2x+b$
If $x+1$ is a factor of $p(x)$ then by factor theorem, $p(-1)=0$.
Substituting $x=-1$ in $p(x)$ we have

$$p(-1) = a(-1)^3+(-1)^2+2(-1)+b$$
$$0 = -a+1-2+b$$
$$0 = -a+b-1$$
$$a-b = -1 \qquad ...(1)$$

If $x-1$ is a factor of $p(x)$ then by factor theorem, $p(1)=0$.
Substituting $x=1$ in $p(x)$ we have

$$p(1) = a(1)^3+1^2+2\times 1+b$$
$$0 = a+1+2+b$$
$$0 = a+b+3$$
$$a+b = -3 \qquad ...(2)$$

Rewriting eq (1) and (2) we have

$$a-b = -1 \qquad (1)$$
$$a+b = -3 \qquad (2)$$

Adding eqn. (1) and (2) we have

$$2a = -4 \Rightarrow a = -2$$

Substituting $a=-2$ in eq (2) we have

$$-2+b = -3$$
$$b = -3+2 \Rightarrow b = -1$$

Now, $p(x) = -2x^3+x^2+2x-1$
$= -2x^3+2x^2-x^2+x+x-1$
$= -2x^2(x-1)-x(x-1)+1(x-1)$
$= (x-1)(-2x^2-x+1)$
$= (x-1)(-2x^3-2x+x+1)$
$= (x-1)[-2x(x+1)+1(x+1)]$
$= (x-1)(x+1)(1-2x)$

Third factor of $p(x)$ is $(1-2x)$.

Main 2012

81. Using the factor theorem, factorise completely the following polynomial $3x^3+2x^2-19x+6$.

Sol :

Let $f(x) = 3x^3+2x^2-19x+6$
Here we use hit and trial method.
For $x=1$:

$$f(1) = 3(1)^3+2(1)^2-19(1)+6$$
$$= 3+2-19+6$$
$$= -8 \neq 0$$

For $x=-1$:

$$f(-1) = 3(-1)^3+2(-1)^2-19(-1)+6$$
$$= -3+2+19+6$$
$$= -3+27$$
$$= 24 \neq 0$$

For $x=2$:

$$f(2) = 3(2)^3+2(2)^2-19(2)+6$$
$$= 3\times 8+2\times 4-38+6$$

NODIA

Thus value of $f(x)$ at $x=1$,

$$\begin{aligned} f(1) &= 2(1)^3 - 3(1)^2 + 7(1) - 8 \\ &= 2 - 3 + 7 - 8 \\ &= 9 - 11 \\ &= -2 \end{aligned}$$

Hence, the required remainder is -2.

FOUR MARK QUESTIONS

Main 2023

76. Find the value of a if $x-a$ is a factor of the polynomial $3x^3 + x^2 - ax - 81$

Sol :

We have $p(x) = 3x^3 + x^2 - ax - 81$

If $(x+a)$ is a factor of $p(x)$ then we must have

$$p(-a) = 0$$

If $(x-a)$ is a factor of $p(x)$ then we must have

$$p(a) = 0$$

Since $(x-a)$ is a factor of $3x^3 + x^2 - ax - 81$ we must have

$$\begin{aligned} p(a) &= 0 \\ 3a^3 + a^2 - a(a) - 81 &= 0 \\ 3a^3 + a^2 - a^2 - 81 &= 0 \\ 3a^3 - 81 &= 0 \\ 3a^3 &= 81 \\ a^3 &= 27 \\ a &= (27)^{1/3} \\ &= 3 \end{aligned}$$

Hence, the value of a is 3.

Comp 2023

77. If $p(x) = x^3 - ax^2 + bx + 3$ leaves a remainder -19 when divided by $(x+2)$ and remainder 17 when divided by $(x-2)$, prove that $a+b=6$.

Sol :

We have $p(x) = x^3 - ax^2 + bx + 3$

If $p(x)$ is divided by $(x+2)$, it leaves remainder -19, thus $p(-2) = -19$.

Thus substituting $x+2=0 \Rightarrow x=-2$ in $p(x)$ we have

$$\begin{aligned} p(-2) &= (-2)^3 - a(-2)^2 + b(-2) + 3 \\ -19 &= -8 - 4a - 2b + 3 \\ 4a + 2b &= -5 + 19 \\ 4a + 2b &= 14 \qquad \text{(Divided by 2)} \\ 2a + b &= 7 \qquad \ldots(1) \end{aligned}$$

If $p(x)$ is divided by $(x-2)$, it leaves remainder 17, thus $p(2) = 17$

Thus substituting $x-2=0 \Rightarrow x=2$ in $p(x)$ we have

$$\begin{aligned} p(2) &= 2^3 - a \times (2)^2 + b \times 2 + 3 \\ 17 &= 8 - 4a + 2b + 3 \\ 17 &= 11 - 4a + 2b \\ 6 &= -4a + 2b \\ 3 &= -2a + b \\ 2a - b &= -3 \qquad \ldots(2) \end{aligned}$$

Adding eq (1) and (2) we have

$$4a = 4 \Rightarrow a = 1$$

Substituting $a=1$ in eq (2) we have

$$\begin{aligned} 2 - b &= -3 \Rightarrow -b = -3 - 2 = 5 \\ -b &= -3 - 2 = -5 \Rightarrow b = 5 \end{aligned}$$

Thus, $a + b = 1 + 5 = 6$ Hence proved.

Comp 2023

78. $x^2 + 2x^2 - 5ax - 8$ and $x^3 + ax^2 - 12x - 6$ when divided by $(x-2)$ and $(x-3)$ leave remainder p and q respectively. If $q - p = 10$, find the value of a.

Sol :

If $p(x)$ is divided by $(x+a)$, it leaves remainder b, then $p(-a) = b$.

Let, $P(x) = x^3 + 2x^2 - 5ax - 8$

Substituting $x-2=0 \Rightarrow x=2$ in $P(x)$ we have

$$\begin{aligned} P(2) &= 2^3 + 2(2)^2 - 5a(2) - 8 \\ p &= 8 + 2 \times 4 - 10a - 8 \\ &= 8 + 8 - 10a - 8 \end{aligned}$$

or $p = -10a + 8 \qquad \ldots(1)$

Let, $Q(x) = x^3 + ax^2 - 12x - 6$

Substituting $x-3=0 \Rightarrow x=3$ in $Q(x)$ we have

$$\begin{aligned} Q(3) &= 3^3 + a(3)^2 - 12(3) - 6 \\ q &= 27 + 9a - 36 - 6 \end{aligned}$$

Thus $q = 9a - 15 \qquad \ldots(2)$

Now, $q - p = 10$

Substituting the values of q and p, we obtain

$$\begin{aligned} 9a - 15 - (-10a + 8) &= 10 \\ 9a - 15 + 10a - 8 &= 10 \\ 19a - 23 &= 10 \end{aligned}$$

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(a) $\begin{bmatrix} -3 & -5 \\ 3 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} -7 & 1 \\ 9 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & -1 \\ -9 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 5 \\ -3 & 8 \end{bmatrix}$

Sol :

Since, $A + B - C = 0$

$$C = A + B$$
$$= \begin{bmatrix} -2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 5 \\ -3 & 8 \end{bmatrix}$$

Thus (d) is correct option.

19. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 7 \end{bmatrix}$, then the values of x and y are :

(a) $x = 3,\ y = -8$

(b) $x = 4,\ y = -4$

(c) $x = 4,\ y = -8$

(d) $x = 2,\ y = -4$

Sol :

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 7 \end{bmatrix}$$
$$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 7 \end{bmatrix}$$
$$\begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 7 \end{bmatrix}$$
$$8 + y = 0 \Rightarrow y = -8$$
$$2x + 1 = 7 \Rightarrow x = 3$$

Thus (a) is correct option.

20. If $M = [1,\ 2]$ and $N = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the order of matrix MN is :

(a) 1×2 (b) 2×1

(c) 2×2 (d) 1×1

Sol :

$$MN = [1\ \ 2]_{1\times 2}\begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2\times 1}$$
$$= [4]_{1\times 1}$$

Thus (d) is correct option.

21. If $A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$, then $A^2 - 4A$ will be

(a) $\begin{bmatrix} 3 & -3 \\ 4 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -3 \\ 9 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 0 \\ 4 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Sol :

$$A^2 - 4A = A \cdot A - 4A$$
$$= \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Thus (d) is correct option.

22. If $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$, then $(A + 2I)\ (A - 3I)$ is equal to :

(a) $\begin{bmatrix} 4 & -4 \\ 8 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & -4 \\ 8 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 4 \\ -8 & -8 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 8 \\ -4 & -8 \end{bmatrix}$

Sol :

$$(A + 2I) = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$
$$(A - 3I) = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$$
$$(A + 2I)\ (A - 3I) = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 6-2 & -6+2 \\ 2+6 & -2-6 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -4 \\ 8 & -8 \end{bmatrix}$$

Thus (a) is correct option.

23. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $A^2 + xI = yA$, then the value of x is :

(a) 8 (b) 4

(c) -4 (d) -8

Sol :

$$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 7 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}$$
$$= \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

NODIA

$$= 24 + 8 - 38 + 6$$

$$= 38 - 38 = 0$$

Since $f(2) = 0$, thus by factor theorem $(x-2)$ is a factor of $f(x)$.

Now, we divide $f(x)$ by $x-2$, using long division method.

$$\begin{array}{r|l} x-2 & 3x^2+8x-3 \\ & 3x^3+2x^2-19x+6 \\ & 3x^3-6x^2 \\ & - \quad + \\ \hline & 8x^2-19x+6 \\ & 8x^2-16x \\ & - \quad + \\ \hline & -3x+6 \\ & -3x+6 \\ & - \quad - \\ \hline & 0 \end{array}$$

Thus $f(x) = (x-2)(3x^2+8x-3)$ ...(i)

Now $3x^2+8x-3 = 3x^2+9x-x-3$

$$= 3x(x+3) - 1(x+3)$$

$$= (3x-1)(x+3)$$

From Eq. (i), we get

$$f(x) = (x-2)(3x-1)(x+3)$$

Main 2009

82. Given that $x+2$ and $x+3$ are factors of $2x^3+ax^2+7x-b$. Determine the values of a and b.

Sol :

Let $p(x) = 2x^3+ax^2+7x-b$ (1)

Since, $(x+2)$ is the factor of $p(x)$, we have

$$p(-2) = 0$$

$$2(-2)^3 + a(-2)^2 + 7(-2) - b = 0$$

$$-16 + 4a - 14 - b = 0$$

$$4a - b - 30 = 0 \quad (2)$$

Since, $(x+3)$ is also a factor of $p(x)$, we have

$$p(-3) = 0$$

$$2(-3)^3 + a(-3)^2 + 7(-3) - b = 0$$

$$-54 + 9a - 21 - b = 0$$

$$9a - b - 75 = 0 \quad (3)$$

Subtracting Eqs. (2) from (3), we get

$$5a - 45 = 0$$

$$5a = 45$$

$$a = \frac{45}{5} = 9$$

Substituting $a = 9$ in Eq. (2), we get

$$4(9) - b - 30 = 0$$

$$36 - b - 30 = 0$$

$$6 - b = 0$$

$$b = 6$$

Hence, the values of a and b are 9 are and 6.

Main 2008

83. If $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$.

(i) Find the value of p.

(ii) With the value of p, factorise the above expression completely.

Sol :

Let $f(x) = 2x^3 - x^2 - px - 2$ ...(1)

Since, $(x-2)$ is a factor of $f(x)$, by factor theorem

$$f(2) = 0$$

$$2(2)^3 - (2)^2 - p(2) - 2 = 0$$

$$2 \times 8 - 4 - 2p - 2 = 0$$

$$16 - 6 - 2p = 0$$

$$10 - 2p = 0$$

$$-2p = -10$$

$$2p = 10$$

$$p = 5$$

Substituting $p = 5$ in Eq. (1), we get

$$f(x) = 2x^3 - x^2 - 5x - 2$$

Since, $(x-2)$ is factor of $f(x)$, so we divide $f(x)$ by $(x-2)$ using long division method.

$$\begin{array}{r|l} x-2 & 3x^2+8x-3 \\ & 3x^3+2x^2-19x+6 \\ & 3x^3-6x^2 \\ & - \quad + \\ \hline & 8x^2-19x+6 \\ & 8x^2-16x \\ & - \quad + \\ \hline & -3x+6 \\ & -3x+6 \\ & - \quad - \\ \hline & 0 \end{array}$$

$$f(x) = (x-2)(2x^2+3x+1)$$

$$= (x-2)\left[(2x^2+2x)+(x+1)\right]$$

$$= (x-2)[2x(x+1)+1(x+1)]$$

Hence, $f(x) = (x-2)(2x+1)(x+1)$

Comp 2008

84. Let S_1 and S_2 are remainders when polynomials

NODIA

$$5A - BC = 5\begin{bmatrix}3 & 5\\ 1 & 4\end{bmatrix} - \begin{bmatrix}2 & 4\\ 0 & 3\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$$
$$= \begin{bmatrix}15 & 25\\ 5 & 20\end{bmatrix} - \begin{bmatrix}2\times1+4\times2 & 2\times-1+4\times1\\ 0\times1+3\times2 & 0\times-1+3\times1\end{bmatrix}$$
$$= \begin{bmatrix}15 & 25\\ 5 & 20\end{bmatrix} - \begin{bmatrix}10 & 2\\ 6 & 3\end{bmatrix}$$
$$= \begin{bmatrix}5 & 23\\ -1 & 17\end{bmatrix}$$

Thus (d) is correct option.

6. If a matrix has 4 elements, then which of the following cannot be the order of the matrix?
(a) 2×2 (b) 1×4
(c) 2×3 (d) 4×1

Sol :

If a matrix has P elements, then the order $m \times n$ of the matrix must be such that $m \times n = p$.
So, out of the given options, $2 \times 3 \neq 4$.
Thus (c) is correct option.

7. Which of the following is a row matrix?
(a) $[1\ 2\ 3]$ (b) $\begin{bmatrix}2\\2\\1\end{bmatrix}$
(c) $\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$ (d) $\begin{bmatrix}1 & 4\\ 2 & 5\\ 3 & 6\end{bmatrix}$

Sol :

A row matrix is one which has only one row.
Thus (a) is correct option.

8. A matrix which has only one column is called a :
(a) row matrix (b) column matrix
(c) square matrix (d) identity matrix

Sol :

Thus (b) is correct option.

9. If $A = \begin{bmatrix}5\\3\end{bmatrix}$, $B = \begin{bmatrix}2\\0\end{bmatrix}$ and matrix $C = 2A + B$, then the matrix C is :
(a) $\begin{bmatrix}12\\6\end{bmatrix}$ (b) $\begin{bmatrix}12\\4\end{bmatrix}$
(c) $\begin{bmatrix}7\\4\end{bmatrix}$ (d) $\begin{bmatrix}8\\8\end{bmatrix}$

Sol :

$$C = 2A + B$$
$$= 2\begin{bmatrix}5\\3\end{bmatrix} + \begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}10\\6\end{bmatrix} + \begin{bmatrix}2\\0\end{bmatrix}$$
$$= \begin{bmatrix}10+2\\6+0\end{bmatrix} = \begin{bmatrix}12\\6\end{bmatrix}$$

Thus (a) is correct option.

10. If $x\begin{bmatrix}-1\\2\end{bmatrix} + 4\begin{bmatrix}3\\-y\end{bmatrix} = \begin{bmatrix}11\\-6\end{bmatrix}$, then the respective values of x and y are :
(a) $-1, 2$ (b) $1, 2$
(c) $1, -2$ (d) $-1, -2$

Sol :

$$x\begin{bmatrix}-1\\2\end{bmatrix} + 4\begin{bmatrix}3\\-y\end{bmatrix} = \begin{bmatrix}11\\-6\end{bmatrix}$$
$$\begin{bmatrix}-x\\2x\end{bmatrix} + \begin{bmatrix}12\\-4y\end{bmatrix} = \begin{bmatrix}11\\-6\end{bmatrix}$$
$$\begin{bmatrix}-x+12\\2x-4y\end{bmatrix} = \begin{bmatrix}11\\-6\end{bmatrix}$$
$$-x + 12 = 11 \Rightarrow x = 1$$

and $2x - 4y = -6$

So, $2(1) - 4y = -6 \Rightarrow y = 2$

Thus (c) is correct option.

11. If $A = [0,\ 1]$ and $B = [1\ 0]$, then the value of $3A + 2B$ is :
(a) $[2\ \ 3]$ (b) $[3\ \ 2]$
(c) $[2\ \ 2]$ (d) $[3\ \ 3]$

Sol :

$$3A + 2B = 3[0\ \ 1] + 2[1\ \ 0]$$
$$= [0\ \ 3] + [2\ \ 0]$$
$$= [0+2\ \ \ 3+0]$$
$$= [2\ \ 3]$$

Thus (a) is correct option.

12. If $A = \begin{bmatrix}2 & 3\\ 4 & 2\end{bmatrix}$ and $B = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then the matrix BA is :
(a) $\begin{bmatrix}16 & 10\\ 10 & 16\end{bmatrix}$ (b) $\begin{bmatrix}11 & 10\\ 16 & 16\end{bmatrix}$
(c) $\begin{bmatrix}7 & 22\\ 17 & 10\end{bmatrix}$ (d) $\begin{bmatrix}10 & 7\\ 22 & 17\end{bmatrix}$

Sol :

We have $BA = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 2\end{bmatrix}$
$$= \begin{bmatrix}1\times2+2\times4 & 1\times3+2\times2\\ 3\times2+4\times4 & 3\times3+4\times2\end{bmatrix}$$

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CHAPTER 8

MATRICES

MULTIPLE CHOICE QUESTIONS

Main 2024

1. If matrix $A=\begin{bmatrix}2 & 2\\ 0 & 2\end{bmatrix}$ and $A^2=\begin{bmatrix}4 & x\\ 0 & 4\end{bmatrix}$, then the value of x is:

(a) 2 (b) 4

(c) 8 (d) 10

Sol :

We have, $A=\begin{bmatrix}2 & 2\\ 0 & 2\end{bmatrix}$

$$A^2=A\cdot A$$

$$=\begin{bmatrix}2 & 2\\ 0 & 2\end{bmatrix}\begin{bmatrix}2 & 2\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4+0 & 4+4\\ 0+0 & 0+4\end{bmatrix}$$

$$=\begin{bmatrix}4 & 8\\ 0 & 4\end{bmatrix}$$

Thus $x=8$

Thus (c) is correct option.

SQP 2024

2. If $A=[-1 \quad 2]$ and $B=\begin{bmatrix}1 & -2\\ 0 & 3\end{bmatrix}$

Which of the following operation is possible?

(a) $A-B$ (b) $A+B$

(c) AB (d) BA

Sol :

Here subtraction or addition is not possible because these operation are done on same order matrices.

Now AB is possible if number of column in A is equal to number of row in B which is the case.

Main 2023

3. If $\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ -8\end{bmatrix}$, the value of x and y respectively are :

(a) 1, −2 (b) −2, 1

(c) 1, 2 (d) −2, −1

Sol :

We have, $\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ -8\end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$2x=2 \text{ and } 4y=-8$$

$$x=1;\ y=-2$$

Thus (a) is correct option.

Main 2022 Sem I

4. If $A\begin{bmatrix}2 & 0\\ -1 & 7\end{bmatrix}$ then A^2 is :

(a) $\begin{bmatrix}4 & 0\\ 1 & 49\end{bmatrix}$ (b) $\begin{bmatrix}4 & 0\\ -9 & 49\end{bmatrix}$

(c) $\begin{bmatrix}4 & 0\\ 9 & 49\end{bmatrix}$ (d) $\begin{bmatrix}4 & 9\\ -9 & 49\end{bmatrix}$

Sol :

We have, $A=\begin{bmatrix}2 & 0\\ -1 & 7\end{bmatrix}$

$$A^2=A\cdot A$$

$$=\begin{bmatrix}2 & 0\\ -1 & 7\end{bmatrix}\begin{bmatrix}2 & 0\\ -1 & 7\end{bmatrix}$$

$$=\begin{bmatrix}4+0 & 0+0\\ -2-7 & 0+49\end{bmatrix}$$

$$=\begin{bmatrix}4 & 0\\ -9 & 49\end{bmatrix}$$

Thus (b) is correct option.

Main 2022 Sem I

5. If $A=\begin{bmatrix}3 & 5\\ 1 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & 4\\ 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$ then $5A-BC$ is :

(a) $\begin{bmatrix}-5 & -23\\ 1 & 17\end{bmatrix}$ (b) $\begin{bmatrix}5 & 23\\ 1 & 17\end{bmatrix}$

(c) $\begin{bmatrix}-2 & 8\\ -3 & 3\end{bmatrix}$ (d) $\begin{bmatrix}5 & 23\\ -1 & 17\end{bmatrix}$

Sol :

We have, $A=\begin{bmatrix}3 & 5\\ 1 & 4\end{bmatrix}$

$$B=\begin{bmatrix}2 & 4\\ 0 & 3\end{bmatrix}$$

$$C=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$$

NODIA

$x^3+2x^2-5ax-7$ and $x^3+ax^2-12x+6$ are divided by $(x+1)$ and $(x-2)$ respectively. If $2S_1-S_2=10$, find the value of a.

Sol :

If $p(x)$ is divided by $(x+a)$, it leaves remainder b, then $p(-a)=-b$.

Let, $p(x)=x^3+2x^2-5ax-7$

Substituting $x+1=0\Rightarrow x=-1$ in $p(x)$ we have

$$p(-1)=(-1)^3+2(-1)^2-5a(-1)-7$$
$$S_1=-1+2+5a-7$$
$$=-8+2+5a \qquad (p(-1)=S_1)$$

Thus $S_1=5a-6$ ...(1)

Let, $q(x)=x^3+ax^2-12x+6$

Substituting $x-2=0\Rightarrow x=2$ in $q(x)$ we have

$$q(2)=(2)^3+a\times(2)^2-12\times(2)+6$$
$$S_2=8+4a-24+6 \qquad (q(2)=S_2)$$
$$S_2=4a-10 \qquad ...(2)$$

Now, $2S_1-S_2=10$

Substituting values of S_1 and S_2 we have

$$2(5a-6)-(4a-10)=10$$
$$10a-12-4a+10=10$$
$$6a=12 \Rightarrow a=2$$

FIVE MARK QUESTIONS

Main 2001

85. Find the value of the constants a and b, if $(x-2)$ and $(x+3)$ are both factors of the expression $x^3+ax^2+bx-12$.

Sol :

Let $p(x)=x^3+ax^2+bx-12$ ...(1)

Since, $(x-2)$ is the factor of $p(x)$, we have

$$p(2)=0$$
$$(2)^3+a(2)^2+b(2)-12=0$$
$$8+4a+2b-12=0$$
$$4a+2b-4=0$$
$$2a+b-2=0 \qquad ...(2)$$

Since, $(x+3)$ is the factor of $p(x)$, we have

$$p(-3)=0$$
$$(-3)^3+a(-3)^2+b(-3)-12=0$$
$$-27+9a-3b-12=0$$
$$9a-3b-39=0$$
$$3a-b-13=0 \qquad ...(3)$$

Adding Eqs. (2) and (3), we get

$$5a-15=0$$
$$5a=15$$
$$a=\frac{15}{5}=3$$

Substituting $a=3$ in Eq. (2), we get

$$2(3)+b-2=0$$
$$6+b-2=0$$
$$b+4=0$$
$$b=-4$$

Hence, the values of a and b are 3 are and -4.

END

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$$= \begin{bmatrix} 10 & 7 \\ 22 & 17 \end{bmatrix}$$

Thus (d) is correct option.

13. If $A = \begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix}$ then A^2 will be

(a) $\begin{bmatrix} 14 & 9 \\ 48 & 25 \end{bmatrix}$ (b) $\begin{bmatrix} 25 & 21 \\ 49 & 46 \end{bmatrix}$

(c) $\begin{bmatrix} 14 & 25 \\ 48 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 25 & 49 \\ 15 & 46 \end{bmatrix}$

Sol :

$$A^2 = A \cdot A$$
$$= \begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 2\times 2+3\times 7 & 2\times 3+3\times 5 \\ 7\times 2+5\times 7 & 7\times 3+5\times 5 \end{bmatrix}$$
$$= \begin{bmatrix} 25 & 21 \\ 49 & 46 \end{bmatrix}$$

Thus (b) is correct option.

14. If $\begin{bmatrix} 1 & 2 \\ 2 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$, then the values of x and y are :

(a) $x = 10,\ y = 0$ (b) $x = 5,\ y = 4$

(c) $x = 0,\ y = 10$ (d) $x = 4,\ y = 5$

Sol :

$$\begin{bmatrix} 1 & 2 \\ 2 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$
$$\begin{bmatrix} 1\times x+2\times y \\ 2\times x+9\times y \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$
$$\begin{bmatrix} x+2y \\ 2x+9y \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$x + 2y = 2 \text{ and } 2x + 9y = 9$$

Solving the two linear equations for x and y, we get

$$x = 0,\ y = 1$$

Thus (c) is correct option.

15. If $A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$ and $A^2 = I_2$, then the values of a and b are :

(a) $a = 1,\ b = 0$ (b) $a = b = 1$

(c) $a = 0,\ b = 1$ (d) $a = b = 0$

Sol :

Since, $A^2 = I$

$$A \cdot A = I$$
$$\begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}\begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1+a & 1+b \\ -a+ab & a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By equality of matrices, we have

$$-1 + b = 0 \text{ and } 1 + a = 1$$
$$b = 1 \text{ and } a = 0$$

Thus (d) is correct option.

NODIA

16. The simplified form of $\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix}\begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol :

$$\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix}\begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \sqrt{2}\times 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & 0+\frac{1}{2} \\ 1+0 & 0+0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

Thus (b) is correct option.

17. If $A = \begin{bmatrix} 3 & 9 \\ 7 & 2 \end{bmatrix}$ and I is an identity matrix of order 2, then the value of $A + 5I$ is :

(a) $\begin{bmatrix} 8 & 9 \\ 7 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 9 \\ 7 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 8 \\ 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 4 \\ 3 & 7 \end{bmatrix}$

Sol :

$$A + 5I = \begin{bmatrix} 3 & 9 \\ 7 & 2 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 9 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 9 \\ 7 & 7 \end{bmatrix}$$

Thus (a) is correct option.

18. If $A = \begin{bmatrix} -2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ -7 & 3 \end{bmatrix}$, then the matrix C, such that $A + B - C = O$ is :

Thus $f(x)=(x-3)(x^2-4x+3)$ ...(i)

Now, $x^2-4x+3=x^2-3x-x+3$

$=x(x-3)-1(x-3)$

$=(x-1)(x-3)$

Eq. (i) can be rewritten as

$f(x)=(x-3)(x-1)(x-3)$

$=(x-1)(x-3)^2$

Comp 2002

72. If $x-4$ is a factor of the cubic polynomial $x^3-3x^2-10x+24$, find its other two factor.

Sol :

If $x-4$ is a factor of the cubic polynomial $x^3-3x^2-10x+24$ then, $x-4$ must divide it completely and we can write

$p(x)=x^3-3x^2-10x+24$

$=(x-4)\,q(x)$

$q(x)=\dfrac{x^3-3x^2-10x+24}{x-4}$

Dividing $x^3-3x^2-10x+24$ by $x-4$ we have

$$\begin{array}{r|l} x-4 & x^2+x-6 \\ \hline & x^3-4x^2 \\ & \underline{} \\ & \quad x^2-10x+24 \\ & \quad \underline{-x^2-4x} \\ & \quad\quad -6x+24 \\ & \quad\quad \underline{-6x+24} \\ & \quad\quad\quad 0 \end{array}$$

Now $q(x)=x^2+x-6$

$=x^2+3x-2x-6$

$=x(x+3)-2(x+3)$

$=(x+3)(x-2)$

Thus $p(x)=(x-4)\,q(x)$

$=(x-4)(x+3)(x-2)$

Main 2001

73. A function f is defined by $f(x)=144-16x^2$, calculate $f(2)$, find the value of x, when $f(x)=0$.

Sol :

We have $f(x)=144-16x^2$

Substituting $x=2$, we have

$f(2)=144-16(2)^2$

$=144-16\times 4$

$=144-64=80$

Also, given $f(x)=0$

Thus $144-16x^2=0$

$16x^2=144$

$x^2=\dfrac{144}{16}=9$

$x=\pm 3$

Comp 2001

74. Find all factors of the polynomial $3x^3+10x^2-9x-4$ if one of its factor is $x-1$.

Sol :

Let $p(x)=3x^3+10x^2-9x-4$

Since, $(x-1)$ is the factor of $3x^3+10x^2-9x-4$, polynomial $3x^3+10x^2-9x-4$ can be written as

$p(x)=(x-1)q(x)$

$q(x)=\dfrac{p(x)}{(x-1)}$

Now we divide $3x^3+10x^2-9x-4$ by $(x-1)$,

$$\begin{array}{r|l} x-1 & 3x^2+13x+4 \\ \hline & 3x^3+10x^2-9x-4 \\ & \underset{-}{3x^3}\underset{+}{-3x^2} \\ & \underline{} \\ & \quad 13x^2-9x-4 \\ & \quad \underset{-}{13x^2}\underset{+}{-13x} \\ & \quad\quad \underline{} \\ & \quad\quad\quad 4x-4 \\ & \quad\quad\quad \underset{-}{4x}\underset{+}{-4} \\ & \quad\quad\quad \underline{} \\ & \quad\quad\quad\quad 0 \end{array}$$

Hence $q(x)=(3x^2+13x+4)$

Now polynomial $3x^3+10x^2-9x-4$ can be written as

$p(x)=(x-1)(3x^2+13x+4)$

$=(x-1)(3x^2+12x+x+4)$

$=(x-1)[3x(x+4)+1(x+4)]$

$=(x-1)(3x+1)(x+4)$

Main 2000

75. Find the remainder, when $2x^3-3x^2+7x-8$ is divided by $x-1$.

Sol :

Let $f(x)=2x^3-3x^2+7x-8$ ...(i)

If $f(x)$ is divided by $x-1$, remainder is the value of $f(x)$ at $x-1=0$ i.e. at $x=1$.

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Now, $A^2+xI=yA$

$$\begin{bmatrix}16 & 8\\56 & 32\end{bmatrix}+x\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=y\begin{bmatrix}3 & 1\\7 & 5\end{bmatrix}$$

$$\begin{bmatrix}16 & 8\\56 & 32\end{bmatrix}+\begin{bmatrix}x & 0\\0 & x\end{bmatrix}=\begin{bmatrix}3y & y\\7y & 5y\end{bmatrix}$$

$$\begin{bmatrix}16+x & 8\\56 & 32+x\end{bmatrix}=\begin{bmatrix}3y & y\\7y & 5y\end{bmatrix}$$

$$y=8 \text{ and } 16+x=3y$$

$$16+x=3\times 8=24$$

$$x=8$$

Thus (a) is correct option.

24. If $[x\ \ 1]\begin{bmatrix}1 & 0\\-2 & 0\end{bmatrix}=0$, then the value of x is :

(a) 0 (b) 2

(c) 1 (d) -1

Sol :

$$[x\ \ 1]\begin{bmatrix}1 & 0\\-2 & 0\end{bmatrix}=0$$

$$[x-2\quad 0]=0$$

$$[x-2]=0$$

$$x=2$$

Thus (b) is correct option.

25. If $A=\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}$, then which of the following is correct?

(a) $AB=BA$ (b) $AB\neq BA$

(c) $AB=\begin{bmatrix}7 & -1\\9 & 22\end{bmatrix}$ (d) $BA=\begin{bmatrix}16 & -5\\39 & 25\end{bmatrix}$

Sol :

We have $AB=\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}$

$$=\begin{bmatrix}10-3 & 5-4\\12-21 & 6-28\end{bmatrix}$$

$$=\begin{bmatrix}7 & 1\\-9 & -22\end{bmatrix}$$

$$BA=\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$$

$$=\begin{bmatrix}10+6 & -2+7\\15+24 & -3+28\end{bmatrix}$$

$$=\begin{bmatrix}16 & 5\\39 & 25\end{bmatrix}$$

So, $AB\neq BA$

Thus (b) is correct option.

26. If $A=\begin{bmatrix}7 & 3\\5 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 5\\4 & 5\end{bmatrix}$, then the matrix C such that $2A+3C=8B$, is :

(a) $\begin{bmatrix}\frac{2}{3} & \frac{22}{3}\\\frac{36}{3} & \frac{34}{3}\end{bmatrix}$ (b) $\begin{bmatrix}2 & 12\\22 & 24\end{bmatrix}$

(c) $\begin{bmatrix}7 & 17\\11 & 16\end{bmatrix}$ (d) $\begin{bmatrix}\frac{2}{3} & \frac{34}{3}\\\frac{22}{3} & \frac{36}{3}\end{bmatrix}$

Sol :

We have, $2A+3C=8B$

$$3C=8B-2A$$

$$=8\begin{bmatrix}2 & 5\\4 & 5\end{bmatrix}-2\begin{bmatrix}7 & 3\\5 & 2\end{bmatrix}$$

$$=\begin{bmatrix}16 & 40\\32 & 40\end{bmatrix}-\begin{bmatrix}14 & 6\\10 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2 & 34\\22 & 36\end{bmatrix}$$

$$C=\frac{1}{3}\begin{bmatrix}2 & 34\\22 & 36\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{3} & \frac{34}{3}\\\frac{22}{3} & \frac{36}{3}\end{bmatrix}$$

Thus (d) is correct option.

27. Given $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}M=7I$. If I is matrix of order 2×2, then the order of matrix M is ..... .

(a) 2×2 (b) 2×1

(c) 2×3 (d) 3×3

Sol :

$$\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}_{2\times 2}[M]_{p\times q}=[71]_{2\times 2}$$

$$2\times q=2\times 2 \text{ and } 2=p$$

$$q=2 \text{ and } p=2$$

Thus (a) is correct option.

28. Simplified form of $\begin{bmatrix}4\sin 30^\circ & 2\cos 60^\circ\\\sin 90^\circ & 2\cos 0^\circ\end{bmatrix}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$ is

(a) $\begin{bmatrix}13 & 13\\14 & 14\end{bmatrix}$ (b) $\begin{bmatrix}14 & 14\\13 & 13\end{bmatrix}$

(c) $\begin{bmatrix}13 & 14\\14 & 13\end{bmatrix}$ (d) $\begin{bmatrix}14 & 13\\13 & 14\end{bmatrix}$

Sol :

$$\begin{bmatrix}4\sin 30^\circ & 2\cos 60^\circ\\\sin 90^\circ & 2\cos 0^\circ\end{bmatrix}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}=\begin{bmatrix}4\times\frac{1}{2} & 2\times\frac{1}{2}\\1 & 2\times 1\end{bmatrix}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$$

$$=\begin{bmatrix}13 & 14\\14 & 13\end{bmatrix}$$

NODIA

$$C = \frac{1}{5}A^2 - B^2$$

Now, $A^2 = AA$

$$= \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix}$$
$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$
$$\frac{1}{5}A^2 = \frac{1}{5}\begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$
$$= \begin{bmatrix} 10 \times \frac{1}{5} & 15 \times \frac{1}{5} \\ 15 \times \frac{1}{5} & 25 \times \frac{1}{5} \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

and $B^2 = BB$

$$= \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$C = \frac{1}{5}A^2 - B^2$$
$$= \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2-1 & 3-0 \\ 3-0 & 5-1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

Main 2017, Main 2003

46. Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$. Hence solve for a and b given

$$X = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

Sol :

We have $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

Here, $B^2 = BB$

$$= \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$$

and $4B = 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4 \times 1 & 4 \times 1 \\ 4 \times 8 & 4 \times 3 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

Now $X = B^2 - 4B$

$$= \begin{bmatrix} 0 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$
$$= \begin{bmatrix} 9-4 & 4-4 \\ 32-32 & 17-12 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

We have $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$
$$\begin{bmatrix} 5a+0 \\ 0+5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$5a = 5 \Rightarrow a = 1 \text{ and } 5b = 50 \Rightarrow b = 10$$

Comp 2017

47. If $A = \begin{bmatrix} 2 & -3 \\ p & q \end{bmatrix}$, find p and q so that $A^2 = I$.

Sol :

We have $A^2 = I$

$$\begin{bmatrix} 2 & -3 \\ p & q \end{bmatrix}\begin{bmatrix} 2 & -3 \\ p & q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2 \cdot 2 + (-3) \cdot p & 2 \cdot (-3) + (-3) \cdot q \\ p \cdot 2 + q \cdot p & p \cdot (-3) + q \cdot q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 4-3p & -6-3q \\ 2p+qp & -3p+q^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$4 - 3p = 1 \quad \text{.....(1)}$$
$$-6 - 3q = 0 \quad \text{.....(2)}$$
$$2p + pq = 0 \quad \text{.....(3)}$$
$$-3p + q^2 = 1 \quad \text{.....(4)}$$

From (1) and (2), we get $p = 1$ and $q = -2$. These values of p and q also satisfy (3) and (4). Hence, $p = 1, q = -2$.

Main 2016

48. Given, matrix $A = \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$ and

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Reason : The product AB of two matrices A and B is possible if number of columns in A is equal to the number of rows in B. Also, the order of the product matrix AB is number of rows in AX number of columns in B.

(a) Both assertion and reason are correct and reason is the correct explanation of assertion.
(b) Bot assertion and reason are correct but reason is not the correct explanation of assertion.
(c) Assertion is correct but reason is incorrect.
(d) Assertion is incorrect but reason is correct.

Sol :

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}_{2\times 2}[X]_{p\times q}=\begin{bmatrix}2\\ 1\end{bmatrix}_{2\times 1}$$

$$2\times q=2\times 1$$

and $$2=p$$

$$q=1,\ p=2$$

So, the order of matrix X is 2×1.

Assertion is incorrect and reason is correct.

Thus (d) is correct option.

35. Assertion : If A and B are square matrices of order 2, then $AB=BA$ is not always true.

Reason : Matrix multiplication is associative.

(a) Both assertion and reason are correct and reason is the correct explanation of assertion.
(b) Bot assertion and reason are correct but reason is not the correct explanation of assertion.
(c) Assertion is correct but reason is incorrect.
(d) Assertion is incorrect but reason is correct.

Sol :

Both assertion and reason are correct but reason is not the correct explanation of assertion.

Thus (b) is correct option.

THREE MARKS QUESTIONS

Main 2023

36. If $A=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$, $C=\begin{bmatrix}4 & 1\\ 1 & 5\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ Find $A(B+C)-14I$.

Sol :

We have $$A=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix},\ B=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix},\ C=\begin{bmatrix}4 & 1\\ 1 & 5\end{bmatrix}$$

and $$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Now $$B+C=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}+\begin{bmatrix}4 & 1\\ 1 & 5\end{bmatrix}$$

$$=\begin{bmatrix}1+4 & 2+1\\ 2+1 & 4+5\end{bmatrix}$$

$$=\begin{bmatrix}5 & 3\\ 3 & 9\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}\begin{bmatrix}5 & 3\\ 3 & 9\end{bmatrix}$$

$$=\begin{bmatrix}1\times 5+3\times 3 & 1\times 3+3\times 9\\ 2\times 5+4\times 3 & 2\times 3+4\times 9\end{bmatrix}$$

$$=\begin{bmatrix}14 & 30\\ 22 & 42\end{bmatrix}$$

Now $$A(B+C)-14I=\begin{bmatrix}14 & 30\\ 22 & 42\end{bmatrix}-14\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}14 & 30\\ 22 & 42\end{bmatrix}-\begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix}$$

$$=\begin{bmatrix}0 & 30\\ 22 & 28\end{bmatrix}$$

Comp 2023

37. Find the values of x, y, a and b if $\begin{bmatrix}x+y & a+b\\ a-b & x-3y\end{bmatrix}=\begin{bmatrix}5 & -1\\ 3 & -7\end{bmatrix}$

Sol :

We have $$\begin{bmatrix}x+y & a+b\\ a-b & x-3y\end{bmatrix}=\begin{bmatrix}5 & -1\\ 3 & -7\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$x+y=5 \quad (1)$$

$$a+b=-1 \quad (2)$$

$$a-b=3 \quad (3)$$

$$x-3y=-7 \quad (4)$$

Adding (2) and (3), we get

$$2a=2 \Rightarrow a=1$$

Substituting $a=1$ in (2) we get

$$1+b=-1 \Rightarrow b=-2$$

Now to find x and y, subtracting (4) from (1), we get

$$4y=12 \Rightarrow y=3 \quad (5)$$

Substituting $y=3$ in (1), we get

$$x+3=5 \Rightarrow x=5-3=2$$

Hence, $x=2, y=3, a=1$ and $b=-2$

Main 2020

38. If $A=\begin{bmatrix}3 & 0\\ 5 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 2\\ 1 & 0\end{bmatrix}$ find $A^2-2AB+B^2$

Sol :

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We have $A=\begin{bmatrix}3 & 0\\ 5 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 2\\ 1 & 0\end{bmatrix}$

Now $A^2=\begin{bmatrix}3 & 0\\ 5 & 1\end{bmatrix}\begin{bmatrix}3 & 0\\ 5 & 1\end{bmatrix}$

$=\begin{bmatrix}3\times 3+0\times 5 & 3\times 0+0\times 1\\ 5\times 3+1\times 5 & 5\times 0+1\times 1\end{bmatrix}$

$=\begin{bmatrix}9+0 & 0+0\\ 15+5 & 0+1\end{bmatrix}$

$=\begin{bmatrix}9 & 0\\ 20 & 1\end{bmatrix}$

$AB=\begin{bmatrix}3 & 0\\ 5 & 1\end{bmatrix}\begin{bmatrix}-4 & 2\\ 1 & 0\end{bmatrix}$

$=\begin{bmatrix}-3\times 4+0\times 1 & 3\times 2+0\times 0\\ -5\times 4+1\times 1 & 5\times 2+1\times 0\end{bmatrix}$

$=\begin{bmatrix}-12+0 & 6+0\\ -20+1 & 10+0\end{bmatrix}$

$=\begin{bmatrix}-12 & 6\\ -19 & 10\end{bmatrix}$

$2AB=2\begin{bmatrix}-12 & 6\\ -19 & 10\end{bmatrix}$

$=\begin{bmatrix}-24 & 12\\ -38 & 20\end{bmatrix}$

$B^2=\begin{bmatrix}-4 & 2\\ 1 & 0\end{bmatrix}\begin{bmatrix}-4 & 2\\ 1 & 0\end{bmatrix}$

$=\begin{bmatrix}4\times 4+2\times 1 & -4\times 2+2\times 0\\ -1\times 4+0\times 1 & 1\times 2+0\times 0\end{bmatrix}$

$=\begin{bmatrix}16+2 & -8+0\\ -4+0 & 2+0\end{bmatrix}$

$=\begin{bmatrix}18 & -8\\ -4 & 2\end{bmatrix}$

Now

$A^2-2AB+B^2=\begin{bmatrix}9 & 0\\ 20 & 1\end{bmatrix}-\begin{bmatrix}-24 & 12\\ -38 & 20\end{bmatrix}+\begin{bmatrix}18 & -8\\ -4 & 2\end{bmatrix}$

$=\begin{bmatrix}9+24+18 & 0-12-8\\ 20+38-4 & 1-20+2\end{bmatrix}$

$=\begin{bmatrix}51 & -20\\ 54 & -17\end{bmatrix}$

Comp 2020

39. Given $A=\begin{bmatrix}x & 3\\ y & 3\end{bmatrix}$. If $A^2=3I$, where I is the identity matrix of order 2, find x and y.

Sol :

We have $=\begin{bmatrix}x & 3\\ y & 3\end{bmatrix}$

Now, $A^2=A\cdot A$

$=\begin{bmatrix}x & 3\\ y & 3\end{bmatrix}\begin{bmatrix}x & 3\\ y & 3\end{bmatrix}$

$=\begin{bmatrix}x^2+3y & 3x+9\\ xy+3y & 3y+9\end{bmatrix}$

and $3I=3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

Since, $A^2=3I$

$\begin{bmatrix}x^2+3y & 3x+9\\ xy+3y & 3y+9\end{bmatrix}=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$3x+9=0$ and $3y+9=3$

$x+3=0$ and $y+3=1$

$x=-3$ and $y=-2$

Thus $x=-3$, $y=-2$ which satisfied $x^2+3y=3$ and $xy+3y=0$

Main 2019

40. Simplify

$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}+\cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$

Sol :

$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}+\cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$

$=\begin{bmatrix}\sin^2 A & -\sin A\cos A\\ \sin A\cos A & \sin^2 A\end{bmatrix}+\begin{bmatrix}\cos^2 A & \cos A\sin A\\ -\sin A\cos A & \cos^{20} A\end{bmatrix}$

$=\begin{bmatrix}\sin^2 A+\cos^2 A & -\sin A\cos A+\cos A\sin A\\ \sin A\cos A-\sin A\cos A & \sin^2 A+\cos^2 A\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $[\cos^2\theta+\sin^2\theta=1]$

Comp 2019

41. If $A=\begin{bmatrix}1 & 3\\ -2 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & 4\\ 5 & 1\end{bmatrix}$ and $C=\begin{bmatrix}-3 & 4\\ 6 & 0\end{bmatrix}$, verify that $(AB)C=A(BC)$.

Sol :

$AB=\begin{bmatrix}1 & 3\\ -2 & 0\end{bmatrix}\begin{bmatrix}2 & 4\\ 5 & 1\end{bmatrix}$

$=\begin{bmatrix}1\cdot 2+3\cdot 5 & 1\cdot 4+3\cdot 1\\ (-2)\cdot 2+0\cdot 5 & (-2)\cdot 4+0\cdot 1\end{bmatrix}$

$=\begin{bmatrix}17 & 7\\ -4 & -8\end{bmatrix}$

$(AB)C=\begin{bmatrix}17 & 7\\ -4 & -8\end{bmatrix}\begin{bmatrix}-3 & 4\\ 6 & 0\end{bmatrix}$

$=\begin{bmatrix}17\cdot(-3)+7\cdot 6 & 17\cdot 4+7\cdot 0\\ (-4)\cdot(-3)+(-8)\cdot 6 & (-4)\cdot 4+(-8)\cdot 0\end{bmatrix}$

$=\begin{bmatrix}-9 & 68\\ -36 & -16\end{bmatrix}$ (1)

NODIA

$$BC=\begin{bmatrix}2 & 4\\ 5 & 1\end{bmatrix}\begin{bmatrix}-3 & 4\\ 6 & 0\end{bmatrix}$$
$$=\begin{bmatrix}2\cdot(-3)+4\cdot 6 & 2\cdot 4+4\cdot 0\\ 5\cdot(-3)+1\cdot 6 & 5\cdot 4+1\cdot 0\end{bmatrix}$$
$$=\begin{bmatrix}18 & 8\\ -9 & 20\end{bmatrix}$$
$$A(BC)=\begin{bmatrix}1 & 3\\ -2 & 0\end{bmatrix}\begin{bmatrix}18 & 8\\ -9 & 20\end{bmatrix}$$
$$=\begin{bmatrix}1\cdot 18+3\cdot(-9) & 1\cdot 8+3\cdot 20\\ (-2)\cdot 18+0\cdot(-9) & (-2)\cdot 8+0\cdot 20\end{bmatrix}$$
$$=\begin{bmatrix}-9 & 68\\ -36 & -16\end{bmatrix} \quad (2)$$

From (1) and (2), we find that $(AB)C=A(BC)$

Main 2018

42. Find the values of x and y, if
$$2\begin{bmatrix}x & 7\\ 9 & y-5\end{bmatrix}+\begin{bmatrix}6 & -7\\ 4 & 5\end{bmatrix}=\begin{bmatrix}10 & 7\\ 22 & 15\end{bmatrix}.$$

Sol :

We have
$$2\begin{bmatrix}x & 7\\ 9 & y-5\end{bmatrix}+\begin{bmatrix}6 & -7\\ 4 & 5\end{bmatrix}=\begin{bmatrix}10 & 7\\ 22 & 15\end{bmatrix}$$
$$\begin{bmatrix}2x & 14\\ 18 & 2y-10\end{bmatrix}+\begin{bmatrix}6 & -7\\ 4 & 5\end{bmatrix}=\begin{bmatrix}10 & 7\\ 22 & 15\end{bmatrix}$$
$$\begin{bmatrix}2x+6 & 7\\ 22 & 2y-5\end{bmatrix}=\begin{bmatrix}10 & 7\\ 22 & 15\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$2x+6=10 \text{ and } 2y-5=15$$
$$2x=10+6 \text{ and } 2y=15+5$$
$$2x=16 \text{ and } 2y=20$$

Hence, $x=2$ and $y=10$.

Main 2018

43. If $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$, $B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$, find $AC+B^2-10C$.

Sol :

We have
$$A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix},$$
$$B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$$
and
$$C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$
$$AC=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$
$$=\begin{bmatrix}2-3 & 0+12\\ 5-7 & 0+28\end{bmatrix}=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}$$
and
$$B^2=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$$
$$=\begin{bmatrix}0-4 & 0+28\\ 0-7 & -4+49\end{bmatrix}=\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}$$

Now, $AC+B^2-10C$
$$=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-10\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$
$$=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-\begin{bmatrix}10 & 0\\ -10 & 40\end{bmatrix}$$
$$=\begin{bmatrix}-1-4-10 & 12+28-0\\ -2-7+10 & 28+45-40\end{bmatrix}$$
$$=\begin{bmatrix}-15 & 40\\ 1 & 33\end{bmatrix}$$

Comp 2018

44. If $A=\begin{bmatrix}2 & -2\\ 5 & -5\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4\\ 3 & 4\end{bmatrix}$, compute AB and BA. What conclusions can you draw?

Sol :

$$AB=\begin{bmatrix}2 & -2\\ 5 & -5\end{bmatrix}\begin{bmatrix}3 & 4\\ 3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}2\cdot 3+(-2)\cdot 3 & 2\cdot 4+(-2)\cdot 4\\ 5\cdot 3+(-5)\cdot 3 & 5\cdot 4+(-5)\cdot 4\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=\text{O and}$$
$$BA=\begin{bmatrix}3 & 4\\ 3 & 4\end{bmatrix}\begin{bmatrix}2 & -2\\ 5 & -5\end{bmatrix}$$
$$=\begin{bmatrix}3\cdot 2+4\cdot 5 & 3\cdot(-2)+4\cdot(-5)\\ 3\cdot 2+4\cdot 5 & 3\cdot(-2)+4\cdot(-5)\end{bmatrix}$$
$$=\begin{bmatrix}26 & -26\\ 26 & -26\end{bmatrix}$$

From the above products, it follows that

(i) AB = O, even though A ≠ O and B ≠ O.

(ii) AB ≠ BA i.e. multiplication of matrices is not commutative

Main 2017

45. If $A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 1\\ -3 & 2\end{bmatrix}$ and $A^2-5B^2=5C$. Find matrix C is a by 2 matrix.

Sol :

We have
$$A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$$
and
$$B=\begin{bmatrix}-2 & 1\\ -3 & 2\end{bmatrix}$$

Also, we have, $A^2-4B^2=5C$

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Main 2015

51. If $A=\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}$ and $C=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$, find $AB-5C$.

Sol :

We have $A=\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}$,

$B=\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}$

and $C=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

Here, $AB=\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}$

$=\begin{bmatrix}0+35 & 6+12\\ 0+20 & 4+12\end{bmatrix}$

$=\begin{bmatrix}35 & 27\\ 20 & 16\end{bmatrix}$

and $5C=5\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$=\begin{bmatrix}5 & -25\\ -20 & 30\end{bmatrix}$

$AB-5C=\begin{bmatrix}35 & 27\\ 20 & 16\end{bmatrix}-\begin{bmatrix}5 & -25\\ -20 & 30\end{bmatrix}$

$=\begin{bmatrix}30 & 27\\ 40 & 16\end{bmatrix}$

and $5C=5\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$=\begin{bmatrix}5 & -25\\ -20 & 30\end{bmatrix}$

$AB-5C=\begin{bmatrix}35 & 27\\ 20 & 16\end{bmatrix}-\begin{bmatrix}5 & -25\\ -20 & 30\end{bmatrix}$

$=\begin{bmatrix}30 & 52\\ 40 & -14\end{bmatrix}$

Comp 2015

52. If $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}3 & 3\\ 1 & -11\end{bmatrix}$, find matrix B such that $BA=C$.

Sol :

BA is a 2×2 matrix, but A is a 2×2 matrix. Thus B is a 2×2 matrix.

Let $B=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $BA=C$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 3\\ 1 & -1\end{bmatrix}$$

$$\begin{bmatrix}a+2b & -a+3b\\ c+2d & -c+3d\end{bmatrix}=\begin{bmatrix}2 & 3\\ 1 & -11\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$a+2b=2 \quad .....(1)$$

$$-a+3b=3 \quad .....(2)$$

$$c+2d=1 \quad .....(3)$$

$$-c+3d=-11 \quad .....(4)$$

Solving (1) and (2), we get

$$a=0,\ b=1.$$

Solving (3) and (4), we get

$$c=5,\ d=-2$$

Thus $B=\begin{bmatrix}0 & 1\\ 5 & -2\end{bmatrix}$

Main 2014

53. Find x and y, if $\begin{bmatrix}-2 & 0\\ 3 & 1\end{bmatrix}\begin{bmatrix}-1\\ 2x\end{bmatrix}+3\begin{bmatrix}-2\\ 1\end{bmatrix}=2\begin{bmatrix}y\\ 3\end{bmatrix}$.

Sol :

We have $\begin{bmatrix}-2 & 0\\ 3 & 1\end{bmatrix}\begin{bmatrix}-1\\ 2x\end{bmatrix}+3\begin{bmatrix}-2\\ 1\end{bmatrix}=2\begin{bmatrix}y\\ 3\end{bmatrix}$

$$\begin{bmatrix}-2\times -1+0\times 2x\\ 2\times -1+1\times 2x\end{bmatrix}+\begin{bmatrix}-2\times 3\\ 1\times 3\end{bmatrix}=\begin{bmatrix}2\times y\\ 2\times 3\end{bmatrix}$$

$$\begin{bmatrix}2\\ -3+2x\end{bmatrix}+\begin{bmatrix}-6\\ 3\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

$$\begin{bmatrix}2-6\\ -3+2x+3\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

$$\begin{bmatrix}-4\\ 2x+0\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

$$\begin{bmatrix}-4\\ 2x\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$-4=2y \text{ and } 2x=6$$

$$y=\frac{-4}{2}=-2 \text{ and } x=\frac{6}{2}=3$$

Thus $y=-2$ and $x=3$

Comp 2014

54. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then find k so that $A^2=8A+kI$.

Sol :

We have, $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$

$A^2=AA$

$=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$

NODIA

$B=\begin{bmatrix}4\\5\end{bmatrix}$. If $AX=B$, then

(i) write the order of matrix X.

(ii) find the matrix 'X'.

Sol :

We have $A=\begin{bmatrix}4\sin 30^\circ & \cos 0^\circ\\ \cos 0^\circ & 4\sin 30^\circ\end{bmatrix}_{2\times 2}$

and $B=\begin{bmatrix}4\\5\end{bmatrix}_{2\times 1}$

Since $AX=B$, here product of matrices A and X is possible. Thus number of columns in A is equal to the number of rows in X i.e 2. Number of columns in X is equal to number of columns in product result matrix B i.e 1. Hence, X has 2 row and 1 column. Consequently, X is of order 2×1.

Now $A=\begin{bmatrix}4\sin 30^\circ & \cos 0^\circ\\ \cos 0^\circ & 4\sin 30^\circ\end{bmatrix}$

$=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ $\left[\begin{matrix}\sin 30^\circ=\frac{1}{2}\\ \text{and } \cos 0^\circ=1\end{matrix}\right]$

Let $X=\begin{bmatrix}x_1\\x_2\end{bmatrix}_{2\times 1}$

Now, $AX=B$

$$\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}4\\5\end{bmatrix}$$
$$\begin{bmatrix}2x_1+x_2\\x_1+2x_2\end{bmatrix}=\begin{bmatrix}4\\5\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$2x_1+x_2=4 \quad ...(i)$$

and $$x_1+2x_2=5 \quad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$x_1=1 \text{ nad } x_2=2$$

Thus $X=\begin{bmatrix}1\\2\end{bmatrix}$

(i) Order is 2×1

(ii) $X=\begin{bmatrix}1\\2\end{bmatrix}$

Comp 2016

49. If $A=\begin{bmatrix}2 & 3\\0 & -2\end{bmatrix}$ and $B=\begin{bmatrix}-8\\8\end{bmatrix}$, find matrix X such that $2AX=B$.

Sol :

Since $2AX=B$ and B is a 2×1 matrix.

$2AX$ is a 2×1 matrix and AX is also a 2×1 matrix, but A is 2×2 matrix.

Thus X is a 2×1 matrix.

Let $X=\begin{bmatrix}x\\y\end{bmatrix}$, then $2AX=B$

$$2\begin{bmatrix}2 & 3\\0 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-8\\8\end{bmatrix}$$
$$\begin{bmatrix}4 & 6\\0 & -4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-8\\8\end{bmatrix}$$
$$\begin{bmatrix}4x+6y\\0\cdot x+(-4)y\end{bmatrix}=\begin{bmatrix}-8\\8\end{bmatrix}$$
$$4x+6y=-8 \quad .....(1)$$

and $$-4y=8 \Rightarrow y=-2$$

Substituting $y=-2$ in (1), we get

$$4x+6(-2)=-8$$
$$4x=12-8=4$$
$$x=1$$

Thus $X=\begin{bmatrix}1\\-2\end{bmatrix}$.

Main 2015

50. If $A=\begin{bmatrix}3 & x\\0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}9 & 16\\0 & -y\end{bmatrix}$, find x and y when $A^2=B$.

Sol :

We have $A=\begin{bmatrix}3 & x\\0 & 1\end{bmatrix}$

and $B=\begin{bmatrix}9 & 16\\0 & -y\end{bmatrix}$

Also, it is given $A^2=B$. ...(ii)

Now $A^2=A\cdot A$

$$=\begin{bmatrix}3 & x\\0 & 1\end{bmatrix}\begin{bmatrix}3 & x\\0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}9+0 & 3x+x\\0+0 & 0+1\end{bmatrix}$$
$$=\begin{bmatrix}9 & 4x\\0 & 1\end{bmatrix}$$

From given $A^2=B$ we have

$$\begin{bmatrix}9 & 4x\\0 & 1\end{bmatrix}=\begin{bmatrix}9 & 16\\0 & -y\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$4x=16 \text{ and } 1=-y$$
$$x=\tfrac{16}{4}=4 \text{ and } y=-1$$

Thus $x=4$ and $y=-1$.

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and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

we have to find $A^2-5A+7I$.

Here, $A^2=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$=\begin{bmatrix}3\times 3+1\times(-1) & 3\times 1+1\times 2\\ (-1)\times 3+2\times(-1) & (-1)\times 1+2\times 2\end{bmatrix}$

$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}$

$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

$A^2-5A+7I$

$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}+\begin{bmatrix}(-5)\times 3 & (-5)\times 1\\ (-5)\times(-1) & (-5)\times 2\end{bmatrix}+\begin{bmatrix}7\times 1 & 7\times 0\\ 7\times 0 & 7\times 1\end{bmatrix}$

$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}+\begin{bmatrix}-15 & -5\\ 5 & -10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$=\begin{bmatrix}8-15+7 & 5-5+0\\ -5+5+0 & 3-10+7\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Main 2012

59. Given, $\begin{bmatrix}2 & 1\\ -3 & 4\end{bmatrix}X=\begin{bmatrix}7\\ 6\end{bmatrix}$, write

(i) the order of the matrix X.

(ii) the matrix X.

Sol :

We have $\begin{bmatrix}2 & 1\\ -3 & 4\end{bmatrix}X=\begin{bmatrix}7\\ 6\end{bmatrix}$ ...(i)

The right side of above matrix is of order 2×1 . Therefore, the matrix of left side should be of order 2×1. As left side if the product of two matrices. One is of order 2×2, therefore the other matrix X should be of order 2×1.

Hence, order of the matrix X is 2×1.

(ii) Let $X=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

From Eq. (i), we get

$\begin{bmatrix}2 & 1\\ -3 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}7\\ 6\end{bmatrix}$

$\begin{bmatrix}2\times x_1+1\times x_2\\ -3\times x_1+4\times x_2\end{bmatrix}=\begin{bmatrix}7\\ 6\end{bmatrix}$

$\begin{bmatrix}2x_1+x_2\\ -3x_1+4x_2\end{bmatrix}=\begin{bmatrix}7\\ 6\end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$2x_1+x_2=7$ ...(ii)

and $-3x_1+4x_2=6$ ...(iii)

Solving above equation we get $x_1=2$ and $x_2=3$.

Thus $X=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$

Comp 2012

60. Find a 2×2 matrix X which satisfies the equation.

$\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

Sol :

We have,

$\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$\begin{bmatrix}0+35 & 6+21\\ 0+20 & 4+12\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$\begin{bmatrix}35 & 27\\ 20 & 16\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$2X=\begin{bmatrix}1-35 & -5-27\\ -4-20 & 6-16\end{bmatrix}$

$=\begin{bmatrix}-34 & -32\\ -24 & -10\end{bmatrix}$

$X=\frac{1}{2}\begin{bmatrix}-34 & -32\\ -24 & -10\end{bmatrix}$

$=\begin{bmatrix}-17 & -16\\ -12 & -5\end{bmatrix}$

Main 2011

61. If $A=\begin{bmatrix}3 & 5\\ 4 & -2\end{bmatrix}$ and $B=\begin{bmatrix}2\\ 4\end{bmatrix}$. Is the product AB possible? Give a reason. If yes, then find.

Sol :

We have $A=\begin{bmatrix}3 & 5\\ 4 & -2\end{bmatrix}$ and $B=\begin{bmatrix}2\\ 4\end{bmatrix}$

Since, matrix A has two rows and two columns, therefore order of A is 2×2 and matrix B has two rows and one column, therefore order of B is 2×1.

Now $A_{2\times 2}\cdot B_{2\times 1}$ is possible because number of columns in A is equal to the number of rows in B.

Hence, product of A and B is possible.

Now, $AB=\begin{bmatrix}3 & 5\\ 4 & -2\end{bmatrix}\begin{bmatrix}2\\ 4\end{bmatrix}$

$=\begin{bmatrix}3\times 2+5\times 4\\ 4\times 2-2\times 4\end{bmatrix}$

$=\begin{bmatrix}6+20\\ 8-8\end{bmatrix}$

NODIA

$$= \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

and, $8A + kI = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$= \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

Now $A^2 = 8A + kI$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

From $1 = 8 + k$ we get $k = -7$ which also satisfy $56 + k = 49$.

Main 2013

55. Given, $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

Find the matrix X such that $A + 2X = 2B + C$.

Sol :

We have $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$

$$B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$$

and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Now $A + 2X = 2B + C$

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -3 \times 2 & 2 \times 2 \\ 4 \times 2 & 0 \times 2 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6+4-2 & 4+0+6 \\ 8+0-2 & 0+2-0 \end{bmatrix}$$

$$2X = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \times \frac{1}{2} & 10 \times \frac{1}{2} \\ 6 \times \frac{1}{2} & 2 \times \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

Main 2013, Main 2003

56. Find x and y, if $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$.

Sol :

We have $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$$\begin{bmatrix} x \times 2 + 3x \times 1 \\ y \times 2 + 4y \times 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal.

Thus $5x = 5$ and $6y = 12$

$x = 1$ and $y = 2$

Comp 2013

57. Find a matrix A satisfying the equation

$$A\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}.$$

Sol :

Let $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ Then, the given equation is

$$AB = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

Number of columns in A is equal to the number of rows in B i.e 2. Number of rows in A is equal to number of rows in AB i.e 2. Hence, A has 2 row and 2 columns. Consequently, it is of order 2×2.

Let $A = \begin{bmatrix} a & b \\ x & y \end{bmatrix}$. Then,

$$\begin{bmatrix} a & b \\ x & y \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2a+4b & 3a+5b \\ 2x+4y & 3x+5y \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

$2a + 4b = 0$ .....(1)

$3a + 5b = -4$ .....(2)

$2x + 4y = 10$ .....(3)

$3x + 5y = 3$ .....(4)

Solving, (1) and (2), we get $a = -8$, $b = 4$. Solving (3) and (4), we get $x = -19$, $y = 12$.
Hence, $a = -8$, $b = 4$, $x = -19$ and $y = 12$.

Main 2012

58. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find $A^2 - 5A + 7I$.

Sol :

We have $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

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Thus (c) is correct option.

29. Given $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times X = \begin{bmatrix} p \\ q \end{bmatrix}$. The order of matrix X is :

(a) 2×2 (b) 1×2

(c) 2×1 (d) 1×1

Sol :

$$AX = B$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{(2\times 2)}$$

$$B = \begin{bmatrix} p \\ q \end{bmatrix}_{(2\times 1)}$$

Let the order of X be $(m \times n)$

The order of matrix X is (2×1).

Thus (c) is correct option.

30. If $\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$ the value of x is

(a) 5 (b) 1

(c) 4 (d) 3

Sol :

We have $\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 12 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2+6 & x+3 \\ 0+12 & 1+0 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 8 & x+3 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$$

$$x + 3 = 8$$

$$x = 8 - 3 = 5$$

Thus (a) is correct option.

31. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 + 5A$ will be

(a) $\begin{bmatrix} 23 & 10 \\ 10 & 13 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 23 \\ 13 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 23 & 10 \\ 10 & 23 \end{bmatrix}$ (d) $\begin{bmatrix} 23 & 10 \\ -10 & 13 \end{bmatrix}$

Sol :

$$A^2 + 5A = A \cdot A + 5A$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 10 \\ -10 & 13 \end{bmatrix}$$

Thus (d) is correct option.

32. If $A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$ and $A^2 = B$, then the value of x is

(a) 36 (b) 32

(c) 42 (d) 48

Sol :

$$A^2 = B$$

$$A \cdot A = B$$

$$\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$x = 36$$

Thus (a) is correct option.

33. Assertion : Let $A = \begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} m-n & 6 \\ 14 & m+n \end{bmatrix}$, of $2A = B$ then $m = 7$ and $n = 3$.

Reason : Two equal matrices have the same order and their corresponding elements are also equal.

(a) Both assertion and reason are correct and reason is the correct explanation of assertion.

(b) Bot assertion and reason are correct but reason is not the correct explanation of assertion.

(c) Assertion is correct but reason is incorrect.

(d) Assertion is incorrect but reason is correct.

Sol :

Both assertion and reason are correct and reason is the correct explanation of assertion.

$$2A = B$$

$$2\begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} m-n & 6 \\ 14 & m+n \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 \\ 14 & 10 \end{bmatrix} = \begin{bmatrix} m-n & 6 \\ 14 & m+n \end{bmatrix}$$

$$m - n = 4 \text{ and } m + n = 10$$

Solving the two equations for m and n, we get

$$m = 7 \text{ and } n = 3$$

Thus (a) is correct option.

34. Assertion : If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the order of matrix X is 1×2.

NODIA

$$= \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

Main 2011

62. If $A = \begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$ find a matrix D such that $:3A - 2B + 2D = 0$.

Sol :

We have, $A = \begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$

Now $3A - 2B + 2D = 0$

$$2D = 2B - 3A$$

$$= 2\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} - 3\begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 6 & -4 \end{bmatrix} - \begin{bmatrix} 12 & 12 \\ -6 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 4-12 & 2-12 \\ 6+6 & -4-18 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -10 \\ 12 & -22 \end{bmatrix}$$

$$2D = 2\begin{bmatrix} -4 & -5 \\ 6 & -11 \end{bmatrix}$$

$$D = \begin{bmatrix} -4 & -5 \\ 6 & -11 \end{bmatrix}$$

Comp 2011

63. Find x, y, if $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

Sol :

We have, $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 2 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

$$\begin{bmatrix} 6x & -2 \\ -2x & +4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-2-8 \\ -2x+4+10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$6x - 10 = 8 \quad (1)$$

and $-2x + 14 = 4y \quad (2)$

From eq (1) we have

$$6x = 8 + 10 = 18$$

$$x = \tfrac{18}{6} = 3$$

Substituting $x = 3$ in eq (2) we have

$$6 + 4y = 14$$

$$4y = 14 - 6 = 8$$

$$y = \tfrac{8}{4} = 2$$

Thus $x = 3$ and $y = 2$.

Main 2010

64. Given, $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find $AB + 2C - 4D$.

Sol :

NODIA

We have $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$,

$$C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}, \; D = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

Here, $AB = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 3 \times 6 - 2 \times 1 \\ -1 \times 6 + 4 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18 - 2 \\ -6 + 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$$

Now $AB + 2C - 4D = \begin{bmatrix} 16 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$$= \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 8 - 8 \\ -2 + 10 - 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Main 2010

65. Evaluate $\begin{bmatrix} 3\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$.

Sol :

We know that $\sin 30^\circ = \frac{1}{2}$

$$\cos 60^\circ = \tfrac{1}{2}$$

$$\sin 90^\circ = 1$$

and $\cos 0^\circ = 1$

Thus $\begin{bmatrix} 3\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 4 + 1 \times 5 & 2 \times 5 + 1 \times 4 \\ 1 \times 4 + 2 \times 5 & 1 \times 5 + 2 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 + 5 & 10 + 4 \\ 4 + 10 & 5 + 8 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Comp 2010

66. If $A = \begin{bmatrix} 2 & -3 \\ x & y \end{bmatrix}$, find x and y so that $A^2 = I_2$.

M should be the order of 2×2.

Let $$M=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

From Eq. (1), we have

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=6\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4a+2c & 4b+2d \\ -a+c & -b+d \end{bmatrix}=\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$4a+2c=6 \quad .....(1)$$

$$4b+2d=0 \quad .....(2)$$

$$-a+c=0 \quad .....(3)$$

and $$-b+d=6 \quad .....(4)$$

Solving Eqs. (1) and (3), we get

$$a=1 \text{ and } c=1$$

Solving Eqs. (2) and (4), we get

$$b=-2 \text{ and } d=4$$

Matrix $$M=\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$$

Comp 2018

88. If $\mathrm{A}=\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$, $\mathrm{B}=\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix}$ and $\mathrm{C}=\begin{bmatrix} -3 & 4 \\ 6 & 0 \end{bmatrix}$, verify that $A(B+C)=AB+AC$.

Sol :

$$B+C=\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix}+\begin{bmatrix} -3 & 4 \\ 6 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 2+(-3) & 4+4 \\ 5+6 & 1+0 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 8 \\ 11 & 1 \end{bmatrix}$$

$$A(B+C)=\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} -1 & 8 \\ 11 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1\cdot(-1)+3\cdot 11 & 1\cdot 8+3\cdot 1 \\ (-2)\cdot(-1)+0\cdot 11 & (-2)\cdot 8+0\cdot 1 \end{bmatrix}$$

$$=\begin{bmatrix} 32 & 11 \\ 2 & -16 \end{bmatrix} \quad (1)$$

$$\mathrm{AB}=\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1\cdot 2+3\cdot 5 & 1\cdot 4+3\cdot 1 \\ (-2)\cdot 2+0\cdot 5 & (-2)\cdot 4+0\cdot 1 \end{bmatrix}$$

$$=\begin{bmatrix} 17 & 7 \\ -4 & -8 \end{bmatrix}$$

and

$$AC=\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 6 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 1\cdot(-3)+3\cdot 6 & 1\cdot 4+3\cdot 0 \\ (-2)\cdot(-3)+0\cdot 6 & (-2)\cdot 4+0\cdot 0 \end{bmatrix}$$

$$=\begin{bmatrix} 15 & 4 \\ 6 & -8 \end{bmatrix}$$

$$AB+AC=\begin{bmatrix} 17 & 7 \\ -4 & -8 \end{bmatrix}+\begin{bmatrix} 15 & 4 \\ 6 & -8 \end{bmatrix}$$

$$=\begin{bmatrix} 17+5 & 7+4 \\ -4+6 & -8+(-8) \end{bmatrix}$$

$$=\begin{bmatrix} 32 & 11 \\ 2 & -16 \end{bmatrix} \quad (2)$$

From (1) and (2), we find that

$$\mathrm{A(B + C)= AB + AC.}$$

Main 2016

89. Given, $A=\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2=9A+mI$. Find m.

Sol :

We have $$A=\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} \text{ and } I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also, it is given that, $A^2=9A+mI$

Here, $A^2=A\cdot A$

$$=\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 2\times 2+0\times(-1) & 2\times 0+0\times 7 \\ -1\times 2+7\times(-1) & -1\times 0+7\times 7 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

Now $$A^2=9A+mI$$

$$\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}=9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}+m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}=\begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix}+\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}=\begin{bmatrix} 18+m & 0 \\ -9 & 63+m \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$18+m=4 \text{ and } 63+m=49$$

$$m=4-18=-14$$

Here $m=-14$ also satisfy $63+m=49$.

Thus $m=-14$

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Here, $BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0\times p+(-q)\times 0 & 0\times 0-q\times 2 \\ 1\times p+0\times 0 & 1\times 0+0\times 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$

and $C^2 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2\times 2+(-2)\times 2 & 2\times(-2)+(-2)\times 2 \\ 2\times 2+2\times 2 & 2\times(-2)+2\times 2 \end{bmatrix}$

$= \begin{bmatrix} 4-4 & -4-4 \\ 4+4 & -4+4 \end{bmatrix}$

$= \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$

Now $BA = C^2$

$\begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$-2q = -8$ and $p = 8$

$q = \frac{-8}{-2} = 4$

Hence, the values of p and q are 8 and 4, respectively.

Comp 2008

71. Find x, y, u and v if $\begin{bmatrix} x+5 & 3 \\ y-2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 2u-1 \\ 7 & v-2 \end{bmatrix}$.

Sol :

We have $\begin{bmatrix} x+5 & 3 \\ y-2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 2u-1 \\ 7 & v-2 \end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$x+5=8,\ 2u-1=3,\ y-2=7$ and $v-2=5$

$x = 8-5 = 3$

$2u = 3+1 = 4 \Rightarrow u = \frac{4}{2} = 2,$

$y = 7+2 = 9$

and $v = 2+5 = 7$

Hence, $x=3, y=9, u=2$ and $v=7$

Main 2007

72. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then find the values of x and y.

Sol :

We have $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 2\times 3 & 2\times 4 \\ 2\times 5 & 2\times x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$8+y=0$ and $2x+1=5$

From $8+y=8$ we get $y=-8$.

From $2x+1=5$ we get $x=2$.

Main 2007

73. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, then find $A^2 + AB + B^2$.

Sol :

We have $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

Now $A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times 1+0\times 2 & 1\times 0+0\times 1 \\ 2\times 1+1\times 2 & 2\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1\times 2+0\times -1 & 1\times 3+0\times 0 \\ 2\times 2+1\times -1 & 2\times 3+1\times 0 \end{bmatrix}$

$= \begin{bmatrix} 2-0 & 3+0 \\ 4-1 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$

$B^2 = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2\times 2+3\times -1 & 2\times 3+3\times 0 \\ -1\times 2+0\times -1 & -1\times 3+0\times 0 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & 6+0 \\ -2-0 & -3+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$

Now

$A^2 + AB + B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1+2+1 & 0+3+6 \\ 4+3-2 & 1+6-3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$

NODIA

Main 2006

74. Let A be a matrix satisfying $A\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix} = [4 \quad -7]$
(i) State the order of A
(ii) Find A

Sol :

(i) Order of A

Let $\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix} = B$ then we have

$$AB = [4 \quad -7]$$

Number of columns in A is equal to the number of rows in B i.e 2. Number of rows in A is equal to number of rows in AB i.e 1. Hence, A has 1 row and 2 columns. Consequently, it is of order 1×2.

(ii) A

Let $A = [x \quad y]$. Then, we have

$$A\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix} = [4 \quad -7]$$

$$[x \quad y]\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix} = [4 \quad -7]$$

$$[2x+0y \quad x+3y] = [4 \quad -7]$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$2x+0y = 4 \Rightarrow x = 2$$

and

$$x+3y = -7$$

$$2+3y = -7$$

$$3y = -7-2 = -9$$

$$y = -3$$

Hence, $A = [2 \quad -3]$

Comp 2006

75. Find a matrix X satisfying $A^2 = A + 2X$ if

$$A = \begin{bmatrix}3 & 4\\ 2 & -1\end{bmatrix}$$

Sol :

We have $A = \begin{bmatrix}3 & 4\\ 2 & -1\end{bmatrix}$

Now $A^2 = A + 2X$

$$A^2 - A = 2X$$

$$X = \frac{1}{2}(A^2 - A)$$

Now $A^2 = AA$

$$A^2 = \begin{bmatrix}3 & 4\\ 2 & -1\end{bmatrix}\begin{bmatrix}3 & 4\\ 2 & -1\end{bmatrix}$$

$$= \begin{bmatrix}9+8 & 12-4\\ 6-2 & 8+1\end{bmatrix}$$

$$= \begin{bmatrix}17 & 8\\ 4 & 9\end{bmatrix}$$

$$A^2 - A = \begin{bmatrix}17 & 8\\ 4 & 9\end{bmatrix} - \begin{bmatrix}3 & 4\\ 2 & -1\end{bmatrix}$$

$$= \begin{bmatrix}17-3 & 8-4\\ 4-2 & 9+1\end{bmatrix}$$

$$= \begin{bmatrix}14 & 4\\ 2 & 10\end{bmatrix}$$

$$\frac{1}{2}(A^2 - A) = \frac{1}{2}\begin{bmatrix}14 & 4\\ 2 & 10\end{bmatrix} = \begin{bmatrix}7 & 2\\ 1 & 5\end{bmatrix}$$

$$X = \frac{1}{2}(A^2 - A)$$

$$X = \begin{bmatrix}7 & 2\\ 1 & 5\end{bmatrix}$$

Main 2005

76. Find the value of x, given that $A^2 = B$,

$$A = \begin{bmatrix}2 & 12\\ 0 & 1\end{bmatrix} \text{ and } B = \begin{bmatrix}4 & x\\ 0 & 1\end{bmatrix}.$$

Sol :

We have $A^2 = B$, $A = \begin{bmatrix}2 & 12\\ 0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}4 & x\\ 0 & 1\end{bmatrix}$,

Here, $A^2 = \begin{bmatrix}2 & 12\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & 12\\ 0 & 1\end{bmatrix}$

$$= \begin{bmatrix}4+0 & 24+12\\ 0+0 & 0+1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 36\\ 0 & 1\end{bmatrix}$$

As given, $A^2 = B$

$$\begin{bmatrix}4 & 36\\ 0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\ 0 & 1\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$x = 36$$

Comp 2005

77. If $A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I_2 = O$

Sol :

We have $A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$A^2 = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}$$

$$= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

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$$-5A = \begin{bmatrix} (-5)\times 3 & (-5)\times 1 \\ (-5)\times(-1) & (-5)\times 2 \end{bmatrix}$$
$$= \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$$

and $$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Hence,

$$A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Main 2004

78. Given, $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, find the matrix X such that $A + X = 2B + C$.

We have $$A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}, \text{ and}$$
$$B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$$

and $$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Now $$A + X = 2B + C$$
$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$
$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -6+1 & 4+0 \\ 8+0 & 0+2 \end{bmatrix}$$
$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix}$$
$$X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -5-2 & 4+1 \\ 8-2 & 2-0 \end{bmatrix}$$
$$= \begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$$

Thus $X = \begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$

Comp 2004

79. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, find a and b.

Sol :

We have $$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \text{ and}$$

Now $$(A+B)^2 = A^2 + B^2$$
$$(A+B)(A+B) = A^2 + B^2$$
$$(A+B)A + (A+B)B = A^2 + B^2 \quad \text{[Distributive]}$$
$$A^2 + BA + AB + B^2 = A^2 + B^2 \quad \text{[Distributive]}$$
$$BA + AB = O$$
$$\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} + \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 2a-b+2 & -a+1 \\ 2a-2 & -b+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$2a - b + 2 = 0 \quad (1)$$
$$-a + 1 = 0 \quad (2)$$
$$2a - 2 = 0 \quad (3)$$
and $$-b + 4 = 0 \quad (4)$$

From eq (2) and eq (4) we get $a = 1$, $b = 4$ which also satisfy eq (1) and eq (3).

Comp 2003

80. Prove that $(A+B)^2 \neq A^2 + 2AB + B^2$ if $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

Sol :

We have $$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

Now $$A^2 = AA$$
$$= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$
$$AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 7 & 2 \end{bmatrix}$$
$$2AB = \begin{bmatrix} 2 & 2 \\ 14 & 4 \end{bmatrix}$$
$$B^2 = BB$$
$$= \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

NODIA

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

and,

$$BA=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1-1 & 1-1\\ -1+1 & -1+1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$AB=BA=O$$

Main 2006

93. If $A=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}$, $B=\begin{bmatrix}0 & 2\\ 1 & -1\end{bmatrix}$, $C=\begin{bmatrix}-2 & 3\\ 1 & -1\end{bmatrix}$, then find A^2-A+BC.

Sol :

We have $A=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}$,

$B=\begin{bmatrix}0 & 2\\ 1 & -1\end{bmatrix}$

and $C=\begin{bmatrix}-2 & 3\\ 1 & -1\end{bmatrix}$,

Now $A^2=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}$

$$=\begin{bmatrix}4\times 4+(-2)\times 6 & 4\times(-2)+(-2)\times(-3)\\ 6\times 4+(-3)\times 6 & 6\times(-2)+(-3)\times(-3)\end{bmatrix}$$

$$=\begin{bmatrix}16-12 & -8+6\\ 24-18 & -12+9\end{bmatrix}$$

$$=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}$$

and $BC=\begin{bmatrix}0 & 2\\ 1 & -1\end{bmatrix}\begin{bmatrix}-2 & 3\\ 1 & -1\end{bmatrix}$

$$=\begin{bmatrix}0\times(-2)+2\times 1 & 0\times 3+2\times(-1)\\ 1\times(-2)+(-1)\times 1 & 1\times 3+(-1)\times(-1)\end{bmatrix}$$

$$=\begin{bmatrix}0+2 & 0-2\\ -2-1 & 3+1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

$$A^2-A+BC=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}-\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}+\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

Main 2003

94. Find x and y, if $\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}\begin{bmatrix}2x\\ 1\end{bmatrix}+2\begin{bmatrix}-4\\ 5\end{bmatrix}=4\begin{bmatrix}2\\ y\end{bmatrix}$.

Sol :

We have $\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}\begin{bmatrix}2x\\ 1\end{bmatrix}+2\begin{bmatrix}-4\\ 5\end{bmatrix}=4\begin{bmatrix}2\\ y\end{bmatrix}$

$$\begin{bmatrix}3\times 2x-2\times 1\\ -1\times 2x+4\times 1\end{bmatrix}+\begin{bmatrix}-4\times 2\\ 5\times 2\end{bmatrix}=\begin{bmatrix}4\times 2\\ 4\times y\end{bmatrix}$$

$$\begin{bmatrix}6x-2\\ -2x+4\end{bmatrix}+\begin{bmatrix}-8\\ 10\end{bmatrix}=\begin{bmatrix}8\\ 4y\end{bmatrix}$$

$$\begin{bmatrix}6x-2-8\\ -2x+4+10\end{bmatrix}=\begin{bmatrix}8\\ 4y\end{bmatrix}$$

$$\begin{bmatrix}6x-10\\ -2x+14\end{bmatrix}=\begin{bmatrix}8\\ 4y\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$6x-10=8$$

and $$-2x+14=4y$$

Solving $6x-10=8$ we get $x=3$.

Substituting $x=3$ in $-2x+14=4y$ we have

$$-2\times 3+14=4y$$

$$-6+14=4y$$

$$4y=8$$

$$y=\frac{8}{4}=2$$

Hence, $x=3$ and $y=2$.

Comp 2000

95. Find the value of x, y, u and v, when: $\begin{bmatrix}x+2y & u-2v\\ x-2y & 2u+2v\end{bmatrix}=\begin{bmatrix}11 & 8\\ 3 & 6\end{bmatrix}$

Sol :

We have $\begin{bmatrix}x+2y & u-2v\\ x-2y & 2u+v\end{bmatrix}=\begin{bmatrix}11 & 8\\ 3 & 6\end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$x+2y=11,\ x-2y=3$$

$$u+2v=8 \text{ and } 2u+v=6$$

Thus, we have two systems of simultaneous linear equations as given below:

$$x+2y=11,\ x-2y=3 \text{ and,}$$

$$u-2v=8,\ 2u+v=6$$

Consider the first system of equations:

$$x+2y=11 \qquad .....(1)$$

$$x-2y=3 \qquad .....(2)$$

Adding eq (1) and eq (2) we have

$$x-2y+x+2y=3+11$$

$$2x=14 \Rightarrow x=\tfrac{14}{2}=7$$

Subtracting eq (2) from eq (1), we have

$$(x+2y)-(x-2y)=11-3$$

$$x+2y-x+2y=8$$

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We have $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(i) $A^2 \neq A$

Now $A^2 = A \cdot A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + 0 & 1 \times 0 + 0 \times (-1) \\ 0 + (-1) \times 0 & 0 + (-1) \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Clearly, $A^2 \neq A$.

(ii) $A^3 \neq A$

$$A^3 = A^2 A$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times (-1) \\ 0 \times 1 + 1 \times 0 & 0 \times 0 + 1 \times -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Clearly, $A^3 = A$

Main 2000

85. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

Sol :

We have, $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

Now $2A - 3B + 5C = O$

$$2A = 3B - 5C$$

$$= 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} -10 & 0 & 10 \\ -35 & -5 & -30 \end{bmatrix}$$

$$= \begin{bmatrix} -6-10 & 6+0 & 0+10 \\ 9-35 & 3-5 & 12-30 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$

FOUR MARK QUESTIONS

Comp 2020

86. If $\begin{bmatrix} 3x+2 & y^2+2 \\ 7 & y^2-5y \end{bmatrix} = \begin{bmatrix} 2x+4 & 3y \\ 7 & -6 \end{bmatrix}$, find the values of x and y.

Sol :

We have $\begin{bmatrix} 3x+2 & y^2+2 \\ 5 & y^2-5y \end{bmatrix} = \begin{bmatrix} 2+4 & 3y \\ 5 & -6 \end{bmatrix}$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$3x + 2 = 2x + 4 \quad (1)$$

$$y^2 + 2 = 3y \quad (2)$$

$$y^2 - 5y = -6 \quad (3)$$

From (1), we get

$$3x - 2x = 4 - 2 \Rightarrow x = 2$$

From (2), we get

$$y^2 + 2 = 3y$$

$$y^2 - 3y + 2 = 0$$

$$(y-1)(y-2) = 0$$

$$y - 1 = 0 \text{ or } y - 2 = 0$$

Thus $y = 1, 2$

From (3), we get

$$y^2 - 5y = -6$$

$$y^2 - 5y + 6 = 0$$

$$(y-2)(y-3) = 0$$

$$y - 2 = 0 \text{ or } y - 3 = 0$$

Thus $y = 2, 3$. Since (2) and (3) must be true simultaneously, we take the common value of y.

Therefore, $y = 2$

Hence, $x = 2$ and $y - 2$.

Main 2019

87. Given $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I$, where M is a matrix and I is unit matrix of order 2×2.

(i) State the order of matrix M.

(ii) Find the matrix M.

Sol :

We have, $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I$ .....(1)

Since, $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ is a order of 2×2 and right side of the equation I is also a 2×2 order. So, the matrix

NODIA

Sol :

We have $A=\begin{bmatrix}2 & -3\\ x & y\end{bmatrix}$

Now $A^2 = I_2$

$$AA = I_2$$

$$\begin{bmatrix}2 & -3\\ x & y\end{bmatrix}\begin{bmatrix}2 & -3\\ x & y\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}4-3x & -6-3y\\ 2x+xy & -3x+y^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$4-3x=1 \quad (1)$$

$$-6-3y=0 \quad (2)$$

$$2x+xy=0 \quad (3)$$

and, $$-3x+y^2=1 \quad (4)$$

Solving (1) and (2), we get $x=1$, $y=-2$. Clearly, these values satisfy equations (3) and (4). Hence, $x=1$, $y=-2$.

Comp 2009

67. Find x and y, if $\begin{bmatrix}2x & x\\ y & 3y\end{bmatrix}=\begin{bmatrix}16\\ 9\end{bmatrix}$.

Sol :

We have $$\begin{bmatrix}2x & x\\ y & 3y\end{bmatrix}\begin{bmatrix}3\\ 2\end{bmatrix}=\begin{bmatrix}16\\ 9\end{bmatrix}$$

$$\begin{bmatrix}6x+2x\\ 3y+6y\end{bmatrix}=\begin{bmatrix}16\\ 9\end{bmatrix}$$

$$\begin{bmatrix}8x\\ 9y\end{bmatrix}=\begin{bmatrix}16\\ 9\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$8x=16 \Rightarrow x=\frac{16}{8}=2$$

and $$9y=9 \Rightarrow y=\frac{9}{9}=1$$

Thus $x=2$ and $y=1$

Comp 2009

68. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$, find AB and BA. Show that $AB \neq BA$.

Sol :

We have $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$,

$$AB=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$$

$$AB=\begin{bmatrix}1\times5+2\times7 & 1\times6+2\times8\\ 3\times5+4\times7 & 3\times6+4\times8\end{bmatrix}$$

$$=\begin{bmatrix}5+14 & 6+16\\ 15+28 & 18+32\end{bmatrix}$$

$$=\begin{bmatrix}19 & 22\\ 43 & 50\end{bmatrix}$$

and, $$BA=\begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$$

$$=\begin{bmatrix}5\times1+6\times3 & 5\times2+6\times4\\ 7\times1+8\times3 & 7\times2+8\times8\end{bmatrix}$$

$$=\begin{bmatrix}23 & 34\\ 31 & 46\end{bmatrix}$$

Clearly, $AB \neq BA$.

Main 2008

69. If $\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}+2M=3\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}$, then find the matrix M.

Sol :

We have $$\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}+2M=3\begin{bmatrix}3 & 2\\ 0 & -3\end{bmatrix}$$

$$\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}+2M=\begin{bmatrix}3\times3 & 3\times2\\ 3\times0 & 3\times-3\end{bmatrix}$$

$$\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}+2M=\begin{bmatrix}9 & 6\\ 0 & -9\end{bmatrix}$$

$$2M=\begin{bmatrix}9 & 6\\ 0 & -9\end{bmatrix}-\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}$$

$$2M$$

$$M=\frac{1}{2}\begin{bmatrix}8 & 2\\ 2 & -12\end{bmatrix}$$ [dividing both sides by 2]

$$=\begin{bmatrix}\frac{1}{2}\times8 & \frac{1}{2}\times2\\ \frac{1}{2}\times2 & \frac{1}{2}\times-12\end{bmatrix}$$

[multiplying each element of a matrix by $\frac{1}{2}$]

$$M=\begin{bmatrix}4 & 1\\ 1 & -6\end{bmatrix}$$

Main 2008

70. Given, $A=\begin{bmatrix}p & 0\\ 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}0 & -q\\ 1 & 0\end{bmatrix}$, $C=\begin{bmatrix}2 & -2\\ 2 & 2\end{bmatrix}$ and $BA=C^2$, then find the values of p and q.

Sol :

We have $A=\begin{bmatrix}p & 0\\ 0 & 2\end{bmatrix}$,

$B=\begin{bmatrix}0 & -q\\ 1 & 0\end{bmatrix}$,

and $C=\begin{bmatrix}2 & -2\\ 2 & 2\end{bmatrix}$

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Comp 2015

90. If $\mathrm{A}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, show that $(aI+bA)^3 = aI+3a^2bA$ where I is the unit matrix of order 2.

Sol :

We have $\mathrm{X}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$

$$a\mathrm{I}+b\mathrm{A} = a\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+b\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}a & 0\\ 0 & a\end{bmatrix}+\begin{bmatrix}0 & b\\ 0 & 0\end{bmatrix}=\begin{bmatrix}a & b\\ 0 & a\end{bmatrix}$$
$$(a\mathrm{I}+b\mathrm{A})^2 = \begin{bmatrix}a & b\\ 0 & a\end{bmatrix}\begin{bmatrix}a & b\\ 0 & a\end{bmatrix}$$
$$=\begin{bmatrix}a\cdot a+b\cdot 0 & a\cdot b+b\cdot a\\ 0\cdot a+a\cdot 0 & 0\cdot b+a\cdot a\end{bmatrix}$$
$$=\begin{bmatrix}a^2 & 2ab\\ 0 & a^2\end{bmatrix}$$
$$(a\mathrm{I}+b\mathrm{A})^3 = (a\mathrm{I}+b\mathrm{A})^2(a\mathrm{I}+b\mathrm{A})$$
$$=\begin{bmatrix}a^2 & 2ab\\ 0 & a^2\end{bmatrix}\begin{bmatrix}a & b\\ 0 & a\end{bmatrix}$$
$$=\begin{bmatrix}a^2\cdot a+2ab\cdot 0 & a^2\cdot b+2ab\cdot a\\ 0\cdot a+a^2\cdot 0 & 0\cdot b+a^2\cdot a\end{bmatrix}$$
$$=\begin{bmatrix}a^3 & 3a^2b\\ 0 & a^3\end{bmatrix} \quad \text{.....(1)}$$
$$a^3\mathrm{I}+3a^2b\mathrm{A} = a^3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+3a^2b\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}a^3 & 0\\ 0 & a^3\end{bmatrix}+\begin{bmatrix}0 & 3a^2b\\ 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}a^3 & 3a^2b\\ 0 & a^3\end{bmatrix} \quad \text{.....(2)}$$

From (1) and (2), we get

$$(a\mathrm{I}+b\mathrm{A})^3 = a^3\mathrm{I}+3a^2b\mathrm{A}$$

Main 2014

91. If $a=\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}$, $B=\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$ and $C=\begin{bmatrix}-3 & 2\\ -1 & 4\end{bmatrix}$, then find $A^2+AC-5B$.

Sol :

We have $A=\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}$,

$B=\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$

and $C=\begin{bmatrix}-3 & 2\\ -1 & 4\end{bmatrix}$,

Here, $A^2=\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}$

$$=\begin{bmatrix}2\times 2+1\times 0 & 2\times 1+1\times(-2)\\ 0\times 2+(-2)\times 0 & 0\times 1+(-2)\times(-2)\end{bmatrix}$$
$$=\begin{bmatrix}4+0 & 2-2\\ 0-0 & 0+4\end{bmatrix}$$
$$=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$$
$$AC=\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}-3 & 2\\ -1 & 4\end{bmatrix}$$
$$=\begin{bmatrix}2\times(-3)+1\times(-1) & 2\times 2+1\times 4\\ 0\times(-3)+(-2)\times(-1) & 0\times 2+(-2)\times 4\end{bmatrix}$$
$$=\begin{bmatrix}-6-1 & 4+4\\ 0+2 & 0-8\end{bmatrix}$$
$$=\begin{bmatrix}-7 & 8\\ 2 & -8\end{bmatrix}$$

and $5B=5\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$

$$=\begin{bmatrix}5\times 4 & 5\times 1\\ 5\times(-3) & 5\times(-2)\end{bmatrix}$$
$$=\begin{bmatrix}20 & 5\\ -15 & -10\end{bmatrix} \quad \text{...(iii)}$$

Now,

$$A^2+AC-5B=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}+\begin{bmatrix}-7 & 8\\ 2 & -8\end{bmatrix}-\begin{bmatrix}20 & 5\\ -15 & -10\end{bmatrix}$$
$$=\begin{bmatrix}4-7-20 & 0+8-5\\ 0+2+15 & 4-8+10\end{bmatrix}$$
$$=\begin{bmatrix}-23 & 3\\ 17 & 6\end{bmatrix}$$

Comp 2013

92. Give an example of two matrices A and B such that

(i) $A\neq O, B\neq O, AB=O$ and $BA\neq O$

(ii) $A\neq O, B\neq O, AB=BA=O$

Sol :

(i) $A\neq O, B\neq O, AB=O$ and $BA\neq O$

Let $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 0\\ 3 & 0\end{bmatrix}$. Then,

$$AB=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\ 3 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O \text{ and}$$
$$BA=\begin{bmatrix}0 & 0\\ 3 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 3 & 0\end{bmatrix}\neq O$$

(ii) $A\neq O, B\neq O, AB=BA=O$

Let $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$. Then,

$$AB=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}=\begin{bmatrix}1-1 & -1+1\\ 1-1 & -1+1\end{bmatrix}$$

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$$A+B=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 0\\ 3 & 3\end{bmatrix}$$

$$(A+B)^2=\begin{bmatrix}3 & 0\\ 3 & 3\end{bmatrix}\begin{bmatrix}3 & 0\\ 3 & 3\end{bmatrix}$$

$$=\begin{bmatrix}9 & 0\\ 18 & 9\end{bmatrix} \quad .....(1)$$

Also,

$$A^2+2AB+B^2=\begin{bmatrix}-1 & -4\\ 8 & 7\end{bmatrix}+\begin{bmatrix}2 & 2\\ 14 & 4\end{bmatrix}+\begin{bmatrix}5 & 2\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}6 & 0\\ 24 & 12\end{bmatrix} \quad .....(2)$$

From (1) and (2), we obtain that

$$(A+B)^2 \neq A^2+2AB+B^2.$$

Main 2002

81. If $A=\begin{bmatrix}2 & 2\\ a & b\end{bmatrix}$, find a and b so that $A^2=O_{2\times 2}$

Sol :

We have $$A=\begin{bmatrix}2 & 2\\ a & b\end{bmatrix}$$

Now $$A^2=O_{2\times 2}$$

$$AA=O_{2\times 2}$$

$$\begin{bmatrix}2 & 2\\ a & b\end{bmatrix}\begin{bmatrix}2 & 2\\ a & b\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}4+2a & 4+2b\\ 2a+ab & 2a+b^2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$4+2a=0 \quad .....(1)$$

$$4+2b=0 \quad .....(2)$$

$$2a+ab=0 \quad .....(3)$$

and, $$2a+b^2=0 \quad .....(4)$$

Solving (1) and (2), we get

$$a=-2 \text{ and } b=-2.$$

Clearly, these values satisfy equations (3) and (4).

Hence, $a=-2$ and $b=-2$.

Comp 2002

82. Let X be a matrix such that $\begin{bmatrix}6 & -2\\ 2 & 3\end{bmatrix}X=\begin{bmatrix}10\\ -4\end{bmatrix}$

(i) State the order of X

(ii) Find X

Sol :

(i) Order of X

Let $A=\begin{bmatrix}6 & -2\\ 2 & 3\end{bmatrix}$. Then,

$$\begin{bmatrix}6 & -2\\ 2 & 3\end{bmatrix}X=\begin{bmatrix}10\\ -4\end{bmatrix}$$

$$AX=\begin{bmatrix}10\\ -4\end{bmatrix}$$

Clearly, AX exists and is of order 2×1.

Number of columns in A is equal to the number of rows in X i.e 2. Number of columns in AX is equal to number of columns in X i.e 1.

Hence, matrix X is of order 2×1.

(ii) X

Let $$X=\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{bmatrix}6 & -2\\ 2 & 3\end{bmatrix}X=\begin{bmatrix}10\\ -4\end{bmatrix}$$

$$\begin{bmatrix}6 & -2\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}10\\ -4\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$6x-2y=10 \quad .....(1)$$

and, $$2x+3y=-4 \quad .....(2)$$

Solving above we get $x=1$ and $y=-2$.

Main 2001

83. Find x and y, if $\begin{bmatrix}-3 & 2\\ 0 & -5\end{bmatrix}\begin{bmatrix}x\\ 2\end{bmatrix}=\begin{bmatrix}-5\\ y\end{bmatrix}$.

Sol :

We have $$\begin{bmatrix}-3 & 2\\ 0 & -5\end{bmatrix}\begin{bmatrix}x\\ 2\end{bmatrix}=\begin{bmatrix}-5\\ y\end{bmatrix}$$

$$\begin{bmatrix}-3\times x+2\times 2\\ 0\times x-5\times 2\end{bmatrix}=\begin{bmatrix}-5\\ y\end{bmatrix}$$

$$\begin{bmatrix}-3x+4\\ 0-10\end{bmatrix}=\begin{bmatrix}-5\\ y\end{bmatrix}$$

$$\begin{bmatrix}-3x+4\\ -10\end{bmatrix}=\begin{bmatrix}-5\\ y\end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$-3x+4=-5 \text{ and } -10=y$$

$$-3x=-9 \text{ and } y=-10$$

$$x=3 \text{ and } y=-10$$

Comp 2001

84. If $A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$, show that (i) $A^2 \neq A$ (ii) $A^3 \neq A$

Sol :

$$4y = 8 \Rightarrow y = 2$$

Thus $x = 8$ and $y = 2$

Now, consider the second system of equations:

$$u - 2v = 8 \quad \text{.....(3)}$$

$$2u + v = 6 \quad \text{.....(4)}$$

Multiplying eq (4) by 2 and adding it to eq (3), we get

$$(u - 2v) + 2(2u + v) = 8 + 2 \times 6$$

$$u - 2v + 4u + 2v = 8 + 12$$

$$5u = 20 \Rightarrow u = 4$$

Multiplying eq (3) by 2 and subtracting from eq (4), we get

$$(2u + v) - 2(u - 2v) = 6 - 2 \times 8$$

$$2u + v - 2u + 4v = 6 - 16$$

$$5v = -10 \Rightarrow u = -2$$

Hence, $x = 7, y = 2, u = 4$ and $v = -2$

END

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(c) 18 (d) -18

Sol :

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$(x+10)-2x=(3x+2)-(x+10)$$
$$-x+10=2x-8$$
$$-x-2x=-8-10$$
$$-3x=-18 \Rightarrow x=6$$

Thus (a) is correct option.

13. In an AP, if $d=-4$, $n=7$ and $a_n=4$, then a is equal to

(a) 6 (b) 7
(c) 20 (d) 28

Sol :

In an AP, $a_n=a+(n-1)d$

$$4=a+(7-1)(-4)$$
$$4=a+6(-4)$$
$$4+24=a \Rightarrow a=28$$

Thus (d) is correct option.

14. In an AP, if $a=3.5$, $d=0$ and $n=101$, then a_n will be

(a) 0 (b) 3.5
(c) 103.5 (d) 104.5

Sol :

As, $d=0$ all the terms are same whatever the value of n. So, $a_n=3.5$.

Alternate Method :

In an AP, $a_n=a+(n-1)d$

$$a_n=3.5+(101-1)\times 0$$
$$=3.5$$

Thus (b) is correct option.

15. The 11th term of an AP -5, $\frac{-5}{2}$, 0, $\frac{5}{2}$, ..., is

(a) -20 (b) 20
(c) -30 (d) 30

Sol :

Here, $a=-5$, $d=\frac{-5}{2}-(-5)=\frac{5}{2}$

nth term, $a_n=a+(n-1)d$

$$a_{11}=-5+(11-1)\times\left(\frac{5}{2}\right)$$
$$a_{11}=-5+25=20$$

Thus (b) is correct option.

16. In an AP, if $a=3.5$, $d=0$ and $n=101$, then a_n will be

(a) 0 (b) 3.5
(c) 103.5 (d) 104.5

Sol :

For an AP, $a_n=a+(n-1)d$

$$=3.5+(101-1)\times 0$$
$$=3.5$$

Thus (b) is correct option.

17. Which term of an AP, 21, 42, 63, 84, ... is 210?

(a) 9th (b) 10th
(c) 11th (d) 12th

Sol :

Let nth term of given AP be 210,

First term, $a=21$

Common difference, $d=42-21=21$

and $a_n=210$

In an AP, $a_n=a+(n-1)d$

$$210=21+(n-1)21$$
$$210=21+21n-21$$
$$210=21n \Rightarrow n=10$$

Hence, the 10th term of the given AP is 210.
Thus (b) is correct option.

18. If the common difference of an AP is 5, then what is $a_{18}-a_{13}$?

(a) 5 (b) 20
(c) 25 (d) 30

Sol :

Given, the common difference of AP i.e, $d=5$

Using, $a_n=a+(n-1)d$

We have, $a_{18}=a+(18-1)d$

and $a_{13}=a+(13-1)d$

Now, $a_{18}-a_{13}=a+(18-1)d-[a+(13-1)d]$

$$=a+17\times 5-a-12\times 5$$
$$=85-60=25$$

Thus (c) is correct option.

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and $n = 10$

$$a_n = a + (n-1)\,d$$
$$a_{10} = 70 + (10 - 1)5$$
$$= 70 + 45$$
$$= 115$$

Thus (c) is correct option.

Main 2022 Sem I

7. The first four terms of an Arithmetic Progression (A.P.) whose first term is 4 and common difference is -6, are :
(a) 4, -10, -16, -22
(b) 4, 10, 16, 22
(c) 4, -2, -8, -14
(d) 4, 2, 8, 14

Sol :

We have $a = 4,\ d = -6$

First term $a_1 = a = 4$

Second term $a_2 = a + d = 4 + (-6) = -2$

Third term $a_3 = a + 2d = 4 + 2(-6) = -8$

Fourth term $a_4 = a + 3d = 4 + 3(-6) = -14$

Hence, the first four terms are 4, -2, -8, -14.
Thus (c) is correct option.

Main 2022 Sem I

8. The first three terms of an Arithmetic Progression (A.P.) are 1, 9, 17 then the next two terms are :
(a) 25 and 35 (b) 27 and 37
(c) 25 and 33 (d) none of these

Sol :

The given A.P. is 1, 9, 17 .....

Here, first term $a = 1$

Common difference, $d = 9 - 1 = 8$

Fourth term, $a_4 = a + 3d$
$= 1 + 3 \times 8 = 25$

Fifth term, $a_5 = a + 4d$
$= 1 + 4 \times 8 = 33$

Thus (c) is correct option.

Main 2022 Sem I

9. If 73 is the n^{th} term of the arithmetic progression 3, 8, 13, 18 ..... then n is :
(a) 13 (b) 14
(c) 15 (d) 16

Sol :

Given A.P. is 3, 8, 13, 18, ...

Here, $8 - 3 = 5$
$13 - 8 = 5$
$18 - 13 = 5$

So, given is an AP in which

$$a = 3,\ d = 5,\ a_n = 73$$

Now, $a_n = a + (n-1)\,d$

$$73 = 3 + (n-1)\,5$$
$$= 3 + 5n - 5$$
$$73 - 3 + 5 = 5n$$
$$75 = 5n$$
$$n = 15$$

Thus (c) is correct option.

10. The n^{th} term of the AP $a,\ 3a,\ 5a,\ ...$ is
(a) na (b) $(2n-1)\,a$
(c) $(2n+1)\,a$ (d) $2na$

Sol :

Given AP is $a,\ 3a,\ 5a,\ ...$

First term is a and $d = 3a - a = 2a$

n^{th} term $a_n = a + (n-1)\,d$
$= a + (n-1)\,2a$
$= a + 2na - 2a$
$= 2na - a = (2n-1)\,a$

Thus (b) is correct option.

11. The common difference of the AP $\frac{1}{p},\ \frac{1-p}{p},\ \frac{1-2p}{p},\ ...$ is
(a) 1 (b) $\frac{1}{p}$
(c) -1 (d) $-\frac{1}{p}$

Sol :

Given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}\ ...$

Common difference

$$d = \frac{1-p}{p} - \frac{1}{p}$$
$$= \frac{1-p-1}{p} = \frac{-p}{p} = -1$$

Thus (c) is correct option.

12. The value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP, is
(a) 6 (b) -6

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For an AP, $a_n = a+(n-1)d$

So, $99 = 12+(n-1)\times 3$

$99-12 = 3n-3$

$99-12+3 = 3n$

$90 = 3n \Rightarrow n = 30$

Thus (c) is correct option.

38. The list of numbers $-10, -6, -2, 2, ...$ is
(a) an AP with $d=-16$ (b) an AP with $d=4$
(c) an AP with $d=-4$ (d) not an AP

Sol :

The given numbers are $-10, -6, -2, 2, ...$

Here, $a_1 = 10$, $a_2 = -6$, $a_3 = -2$ and $a_4 = 2$, ...

Since, $d_1 = a_2 - a_1$

$= -6-(-10) = -6+10 = 4$

$d_2 = a_3 - a_2$

$= -2-(-6) = -2+6 = 4$

$d_3 = a_4 - a_3$

$= 2-(-2) = 2+2 = 4$

Since, $d_1 = d_2 = d_3 = ... = 4$

i.e., each successive term of given list has same difference. So, the given list forms an AP with common difference, $d=4$.

Thus (b) is correct option.

39. If the nth term of an AP is $4n+1$, then the common difference is
(a) 3 (b) 4
(c) 5 (d) 6

Sol :

Given that the n^{th} term of an AP is $4n+1$.

$a_n = 4n+1$

Substituting $n = 1, 2, 3, ...$ we have

$a_1 = 4(1)+1 = 5$

$a_2 = 4(2)+1 = 9$

Common difference,

$d = a_2 - a_1 = 9-5 = 4$

Thus (b) is correct option.

40. If a, b, c, d, e, f are in AP, then $e-c$ is equal to
(a) $2(c-a)$ (b) $2(d-c)$
(c) $2(f-d)$ (d) $(d-c)$

Sol :

Let x be the common difference of the AP a, b, c, d, e, f.

For an AP, $a_n = a+(n-1)d$

$e = a+(5-1)x$

$e = a+4x$ ...(1)

and $c = a+(3-1)x$

$c = a+2x$ ...(2)

Using equation (1) and (2), we get

$e-c = a+4x-a-2x$

$= 2x = 2(d-c)$

Thus (b) is correct option.

41. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its term will be
(a) 7 (b) 11
(c) 18 (d) 0

Sol :

In an AP, $a_n = a+(n-1)d$

Now, according to the question,

$7a_7 = 11a_{11}$

$7[a+(7-1)d] = 11[a+(11-1)d]$

$7(a+6d) = 11(a+10d)$

$7a+42d = 11a+110d$

$4a+68d = 0$

$4(a+17d) = 0$

$a+17d = 0$ ...(1)

18th term of an AP,

$a_{18} = a+(18-1)d = a+17d$

But from equation (1) this is zero.

42. What is the common difference of four terms in AP such that the ratio of the product of the first and fourth term to that of the second and third term is 2:3 and the sum of all four terms is 20?
(a) 3 (b) 1
(c) 4 (d) 2

Sol :

We take the four terms as $a-3x$, $a-x$, $a+x$ and $a+3x$.

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19. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8

(c) -4 (d) 4

Sol :

We have $a_{18} - a_{14} = 32$

In an AP, $a_n = a + (n-1)\,d$

$$a + (18-1)\,d - [a + (14-1)\,d] = 32$$

$$a + 17d - a - 13d = 32$$

$$4d = 32 \Rightarrow d = 8$$

Hence, the required common difference of the given AP is 8.

Thus (a) is correct option.

20. The 4th term from the end of an AP -11, -8, -5, ..., 49 is

(a) 37 (b) 40

(c) 43 (d) 58

Sol :

Common difference,

$$d = -8 - (-11)$$

$$= -8 + 11 = 3$$

Last term, $l = 49$

nth term of an AP from the end is

$$a_n = l - (n-1)\,d$$

,
$$a_4 = 49 - (4-1) \times 3$$

$$= 49 - 9 = 40$$

21. The famous mathematician associated with finding the sum of the first 100 natural numbers is

(a) Pythagoras (b) Newton

(c) Gauss (d) Euclid

Sol :

Gauss is the famous mathematician associated with finding the sum of the first 100 natural numbers, i.e., 1, 2, 3, ..., 100 = 5050.

Thus (c) is correct option.

22. If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is

(a) 0 (b) 5

(c) 6 (d) 15

Sol :

We have $a = -5$ and $d = 2$

$$S_n = \frac{n}{2}\{2a + (n-1)\,d\}$$

$$S_6 = \frac{6}{2}[2a + (6-1)\,d]$$

$$= 3[2(-5) + 5(2)]$$

$$= 3(-10 + 10) = 0$$

Thus (a) is correct option.

23. The sum of first 16 terms of the AP 10, 6, 2, ... is

(a) -320 (b) 320

(c) -352 (d) -400

Sol :

Given, AP, is 10, 6, 2 .....

We have $a = 10$ and $d = (6-10) = -4$

$$S_n = \frac{n}{2}\{2a + (n-1)\,d\}$$

$$S_{16} = \frac{16}{2}[2a + (16-1)\,d]$$

$$= 8[2 \times 10 + 15(-4)]$$

$$= 8(20 - 60)$$

$$= 8(-40) = -320$$

Thus (a) is correct option.

24. In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(a) 19 (b) 21

(c) 38 (d) 42

Sol :

We have $a = 1$, $a_n = 20$ and $S_n = 399$

Now, $S_n = \frac{n}{2}(a + a_n)$

$$399 = \frac{n}{2}(1 + 20)$$

$$n = \frac{399 \times 2}{21} = 38.$$

25. The sum of first five multiples of 3 is

(a) 45 (b) 55

(c) 65 (d) 75

Sol :

The first five multiples of 3 are 3, 6, 9, 12 and 15.

Here, first term, $a = 3$, $d = 6 - 3 = 3$ and $n = 5$

$$S_n = \frac{n}{2}\{2a + (n-1)\}\,d$$

$$S_5 = \frac{5}{2}[2a + (5-1)\,d]$$

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$$= \frac{5}{2}[2 \times 3 + 4 \times 3]$$

$$= \frac{5}{2}(6+12) = \frac{5}{2} \times 18 = 45$$

Thus (a) is correct option.

26. If the sum of the series $2+5+8+11$ ..... is 60100, then the number of terms are

(a) 100 (b) 200
(c) 150 (d) 250

Sol :

We have $a = 2$. $d = 5-2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a+(n-1)d] = S_n$$

$$\frac{n}{2}[4+(n-1)3] = 60100$$

$$n(3n+1) = 120200$$

$$3n^2 + n - 120200 = 0$$

$$(n-200)(3n+601) = 0 \Rightarrow n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Thus (b) is correct option.

27. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20
(c) 25 (d) 30

Sol :

Given, the common difference of AP i.e., $d = 5$

Now $a_n = a+(n-1)d$

Now, $a_{18} - a_{13} = a+(18-1)d - [a+(13-1)d]$

$$= a + 17 \times 5 - a - 12 \times 5$$

$$= 85 - 60 = 25$$

Thus (c) is correct option.

28. There are 60 terms is an AP of which the first term is 8 and the last term is 185. The 31st term is

(a) 56 (b) 94
(c) 85 (d) 98

Sol :

Let d be the common difference;

Now $a_n = a+(n-1)d$

Then 60th term, $a_{60} = 8+(60-1)d$

$$185 = 8 + 59d$$

$$59d = 177 \Rightarrow d = 3$$

31th term $a_{31} = 8 + 30 \times 3 = 98$

Thus (d) is correct option.

29. The first and last term of an AP are a and ℓ respectively. If S is the sum of all the terms of the AP and the common difference is $\frac{\ell^2 - a^2}{k-(\ell+a)}$, then k is equal to

(a) S (b) $2S$
(c) $3S$ (d) None of these

Sol :

We have, $S = \frac{n}{2}(a+\ell)$

$$\frac{2S}{a+\ell} = n \qquad (1)$$

Also, $\ell = a+(n-1)d$

$$d = \frac{\ell - a}{n-1} = \frac{\ell - a}{\frac{2S}{a+\ell} - 1}$$

$$= \frac{\ell^2 - a^2}{2S-(\ell+a)}$$

Thus $k = 2S$

Thus (b) is correct option.

30. If the nth term of an AP is given by $a_n = 5n-3$, then the sum of first 10 terms if

(a) 225 (b) 245
(c) 255 (d) 270

Sol :

We have $a_n = 5n-3$

Substituting $n = 1$ and 10 we have

$$a = 2$$

$$a_{10} = 47$$

Thus $S_n = \frac{n}{2}(a+a_n)$

$$S_{10} = \frac{10}{2}(2+47)$$

$$= 5 \times 49 = 245$$

Thus (b) is correct option.

31. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

(a) -1 (b) -8
(c) 7 (d) -9

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Sol :

4th term of first AP,

$$a_4 = -1 + (4-1)\,d$$
$$= -1 + 3d$$

and 4th term of second AP,

$$a'_4 = -8 + (4-1)\,d$$
$$= -8 + 3d$$

Now, the difference between their 4th terms,

$$a'_4 - a_4 = (-1 + 3d) - (-8 + 3d)$$
$$= -1 + 3d + 8 - 3d = 7$$

Hence, the required difference is 7.
Thus (c) is correct option.

32. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is

(a) 2 (b) 3
(c) 5 (d) 6

Sol :

We have $S_{11} = 33$

$$\frac{11}{2}[2a + 10d] = 33$$

$$a + 5d = 3$$

i.e. $a_6 = 3 \Rightarrow a_4 = 2$

Since, alternate terms are integers and the given sum is possible, $a_4 = 2$.
Thus (a) is correct option.

33. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61, ..., then n is equal to

(a) 10 (b) 12
(c) 11 (d) 13

Sol :

$$\frac{2n}{2}\{2 \times 2 + (2n-1)3\} = \frac{n}{2}\{2 \times 57 + (n-1)2\}$$

$$2(6n+1) = 112 + 2n$$

$$10n = 110 \Rightarrow n = 11$$

Thus (c) is correct option.

34. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6 (b) 7
(c) 20 (d) 28

Sol :

In an AP, $a_n = a + (n-1)d$

$$4 = a + (7-1)(-4)$$
$$4 = a + 6(-4)$$
$$4 + 24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

35. The first four terms of an AP whose first term is -2 and the common difference is -2 are

(a) $-2, 0, 2, 4$ (b) $-2, 4, -8, 16$
(c) $-2, -4, -6, -8$ (d) $-2, -4, -8, -16$

Sol :

Let the first four terms of an AP are $a, a+d, a+2d$ and $a+3d$.
Given, that first term, $a = -2$ and common difference, $d = -2$, then we have an AP as follows

$$-2, \quad -2-2, \quad -2+2(-2), \quad -2+3(-2)$$
$$= -2, -4, -6, -8$$

Thus (c) is correct option.

36. The 21^{th} term of an AP whose first two terms are -3 and 4, is

(a) 17 (b) 137
(c) 143 (d) -143

Sol :

Given, first two terms of an AP are

$$a = -3$$

and $a + d = 4$

$$-3 + d = 4 \Rightarrow d = 7$$

For an AP, $a_n = a + (n-1)d$

Thus $a_{21} = a + (21-1)d$

$$= -3 + (20)7$$
$$= -3 + 140 = 137$$

Thus (b) is correct option.

37. The number of two digit numbers which are divisible by 3 is

(a) 33 (b) 31
(c) 30 (d) 29

Sol :

Two digit numbers which are divisible by 3 are 12, 15, 18, ..., 99;

Here $a = 12$, $d = 3$ and $a_n = 99$

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Thus (a) is correct option.

47. **Assertion :** Common difference of the AP $-5, -1, 3, 7, \ldots\ldots$ is 4.
Reason : Common difference of the AP $a, a+d, a+2d, \ldots\ldots$ is given by $d = a_2 - a_1$
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Common difference, $d = -1-(-5) = 4$
So, both A and R are correct and R explains A.
Thus (c) is correct option.

48. **Assertion :** If sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term is $a_n = 6n - 7$.
Reason : n^{th} term of an AP, whose sum to n terms is S_n, is given by $a_n = S_n - S_{n-1}$
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

nth term of an AP,

$$\begin{aligned} a_n &= S_n - S_{n-1} \\ &= 3n^2 - 4n - 3(n-1)^2 + 4(n-1) \\ &= 6n - 7 \end{aligned}$$

So, both A and R are correct and R explains A.
Thus (a) is correct option.

49. **Assertion :** Sum of first 10 terms of the arithmetic progression $-0.5, -1.0, -1.5, \ldots\ldots$ is 31.
Reason : Sum of n terms of an AP is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a is first term and d common difference.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Assertion, $S_{10} = \frac{10}{2}[2(-0.5) + (10-1)(-0.5)]$

$$= 5[-1 - 4.5]$$

$$= 5(-5.5) = 27.5$$

Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

50. **Assertion :** $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.
Reason : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Common difference of an AP $d = a_n - a_{n-1}$ is independent of n or constant.
So, A is correct but R is incorrect.
Thus (d) is correct option.

THREE MARKS QUESTIONS

Main 2023

51. The 5^{th} term and the 9th term of an Arithmetic Progression are 4 and -12 respectively. Find:
(a) the first term
(b) common difference
(c) sum of 16 terms of the AP.

Sol :

We have $a_5 = 4$ and $a_9 = -12$

Now $a_n = a + (n-1)d$

$$a_5 = a + 4d = 4 \quad \ldots(1)$$

and $$a_9 = a + 8d = -12 \quad \ldots(2)$$

Salving equations (1) and (2), we get

$$d = -4 \text{ and } a = 20$$

(a) First term of the A.P. is 20.

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Sum $S_4 = 4a = 20$

Thus $a = 5$

Also, $\frac{(a-3x)(a+3x)}{(a-x)(a+x)} = \frac{2}{3}$

$$3(a^2-(3x)^2) = 2(a^2-x^2)$$
$$3a^2-27x^2 = 2a^2-2x^2$$
$$a^2 = 25x^2$$
$$a = 5x \Rightarrow x = \frac{a}{5} = \frac{5}{5} = 1$$

Thus $x = 1$. Common difference, $d = 2x = 2$

Thus (d) is correct option.

43. The sum of 11 terms of an AP whose middle term is 30, is

(a) 320 (b) 330
(c) 340 (d) 350

Sol :

Middle term is $\frac{11+1}{2} = 6$ th term.

Now $a_n = a+(n-1)d$

$a_6 = a+5d$

$30 = a+5d$

$a = 30-5d$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{11} = \frac{11}{2}(2a+10d)$

Substituting value of a we have

$$S_{11} = \frac{11}{2}[2(30-5d)+10d]$$
$$= \frac{11}{2}[60-10d+10d]$$
$$= 11 \times 30$$
$$S_{11} = 330$$

Thus (b) is correct option.

44. Five distinct positive integers are in a arithmetic progression with a positive common difference. If their sum is 10020, then the smallest possible value of the last term is

(a) 2002 (b) 2004
(c) 2006 (d) 2007

Sol :

Let the five integers be $a-2d$, $a-d$, a, $a+d$, $a+2d$.

Then, we have,

$(a-2d)+(a-d)+a+(a+d)+(a+2d) = 10020$

$5a = 10020 \Rightarrow a = 2004$

Now, as smallest possible value of d is 1.

Hence, the smallest possible value of $a+2d$ is 2004+2 = 2006

Thus (c) is correct option.

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45. If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33
(c) 37 (d) 38

Sol :

We have $a_2 = 13$, and $a_5 = 25$

In an AP, $a_n = a+(n-1)d$

$a_2 = a+(2-1)d = 13$

$a+d = 13$ ...(1)

and $a_5 = a+(5-1)d = 25$

$a+4d = 25$ ...(2)

Subtracting equation (1) from equation (2), we get

$3d = 25-13 = 12 \Rightarrow d = 4$

From equation (1), $a = 13-4 = 9$

Now, 7th term, $a_7 = a+(7-1)d$

$= 9+6 \times 4 = 33$

Thus (b) is correct option.

46. **Assertion :** If n^{th} term of an AP is $7-4n$, then its common differences is -4.

Reason : Common difference of an AP is given by $d = a_{n+1}-a_n$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Assertion, $a_n = 7-4n$

$d = a_{n+1}-a_n$

$= 7-4(n+1)-(7-4n)$

$= 7-4n-4-7+4n$

$= -4$

Both are correct. Reason is the correct explanation.

CHAPTER 9

ARITHMETIC PROGRESSION

MULTIPLE CHOICE QUESTIONS

Main 2024

1. The 7^{th} term of the given Arithmetic Progression (A.P.) : $\frac{1}{a}$, $(\frac{1}{a}+1)$, $(\frac{1}{a}+2)$.... is :
(a) $(\frac{1}{a}+6)$ (b) $(\frac{1}{a}+7)$
(c) $(\frac{1}{a}+8)$ (d) $(\frac{1}{a}+7^7)$

Sol :

We can see easily that $A=\frac{1}{a}$ and $d=1$.

Now $a_n = a+(n-1)\,d$

$a_7 = a+6d$

$=\frac{1}{a}+6$

Thus (a) is correct option.

SQP 2024

2. Which of the following is/are an Arithmetic Progression (A.P.)?
1. 1, 4, 9, 16,.....
2. $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}$,.....
3. 8, 6, 4, 5
(a) only 1
(b) only 2
(c) only 2 and 3
(d) all 1, 2 and 3

Sol :

We can see easily that only 2 is a AP because there is common difference $\sqrt{3}$.
In 1 and 3, there is no common difference.

Thus (b) is correct option.

Main 2023

3. The n^{th} term of an Arithmetic Progression (A.P.) is $2n+5$. The 10^{th} term is:
(a) 7 (b) 15
(c) 25 (d) 45

Sol :

n^{th} term, $a_n = 2n+5$

10^{th} term, $a_{10} = 2(10)+5$

$= 20+5$

$= 25$

Thus (c) is correct option.

Comp 2023

4. The first term of AP is p and the common difference is q, then its 10th term is
(a) $q+9p$ (b) $p-9q$
(c) $p+9q$ (d) $2p+9q$

Sol :

We have $a=p$ and $d=q$

$a_{10} = a+(10-1)\,d$

$= p+9q$

Thus (c) is correct option.

Main 2022 Sem I

5. In an Arithmetic Progression (A.P.), if first term is 5, common difference is -3 and the n^{th} term is -7, then n is :
(a) 5 (b) 17
(c) -13 (d) 7

Sol :

We have $a=5,\ d=-3,\ a_n=-7$

Now $a_n = a+(n-1)\,d$

$-7 = 5+(n-1)(-3)$

$= 5-3n+3$

$3n = 8+7 = 15$

$n = \frac{15}{3} = 5$

Thus (a) is correct option.

Main 2022 Sem I

6. 70, 75, 80, 85 are the first four terms of an Arithmetic Progression, then the 10^{th} term is :
(a) 35 (b) 25
(c) 115 (d) 105

Sol :

The given A.P. is 70, 75, 80, 85 .....

Here, $75-70=5,\ 80-75=5$

$85-80=5$

So, given sequence, is an AP in which

$a=70,\ d=5$

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(b) Common difference of the A.P. is -4

(c) We know that,

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{16} = \frac{16}{2}[2 \times 20 + (16-5) \times (-4)]$$

$$= 8[40 - 44]$$

$$= 8 \times (-4)$$

$$= -32$$

(a) First term of the A.P. is 20.

(b) Common difference of the A.P. is -4

(c) The sum of 16 terms of the A.P. is -32.

Main 2023

52. Which term of the Arithmetic Progression (A.P.) 15, 30, 45, 60... is 300? Hence find the sum of all the terms of the Arithmetic Progression (A.P.)

Sol :

Given, A.P. is 15, 30, 45, 60, ...

Here, $a = 15$,

$d = 30 - 15 = 15$

Let 300 be n^{th} term of this A.P.

Then, $a_n = 300$

$$a + (n-1)d = 300$$

$$15 + (n-1)15 = 300$$

$$15n = 300$$

$$n = 20$$

Hence, the required number of term in the A.P. is 20.

Now, $S_n = \frac{n}{2}(a + a_n)$

$$S_{20} = \frac{20}{2}(15 + 300)$$

$$= 10 \times 315$$

$$= 3150$$

Hence, sum of all the terms of A.P. is 3150.

Comp 2023

53. Find the 20^{th} term of an AP whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Sol :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$a_7 = 3a_3 + 2$

$$a + 6d = 3 \times 7 + 2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$

$$a + 8 = 7 \Rightarrow a = -1$$

$$a_{20} = a + 19d = -1 + 19 \times 4 = 75$$

$$a_n = a + (n-1)d$$

$$= -1 + 4n - 4 = 4n - 5.$$

Hence n^{th} term is $4n - 5$.

NODIA

Comp 2023

54. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Sol :

We have $S_7 = 63$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$63 = \frac{7}{2}[2a + 6d]$$

$$9 = a + 3d \quad \ldots(1)$$

Now, sum of next 7 terms,

$$S_{8-14} = 161$$

$$S_{8-14} = \frac{7}{2}(a_8 + a_{14})$$

$$161 = \frac{7}{2}(a + 7d + a + 13d)$$

$$161 = \frac{7}{2}(2a + 20d)$$

$$23 = a + 10d \quad \ldots(2)$$

Subtracting equation (1) from (2) we have

$$14 = 7d \Rightarrow d = 2$$

Substituting the value of d in (1), we get

$$a = 3$$

Hence, the AP is 3, 5, 7, 9, ...

Main 2020

55. The 4th, 6th and the last term of a geometric progression are 10, 40 and 640, respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series.

Sol :

Let a and r the first term and common ratio of GP respectively and n be total number of terms. According to the question,

4th term $a_4 = 10$

$$ar^{4-1} = 10$$

$$ar^3 = 10 \quad \ldots..(1)$$

Here, $a = 24,\ d = 21 - 24 = -3$

Sum of n term, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$78 = \frac{n}{2}[2 \times 24 + (n-1)(-3)]$$

$$156 = n(48 - 3n + 3)$$

$$156 = n(51 - 3n)$$

$$3n^2 - 51n + 156 = 0$$

$$n^2 - 17n + 52 = 0$$

$$n^2 - 13n - 4n + 52 = 0$$

$$(n-4)(n-13) = 0 \Rightarrow n = 4, 13$$

When $n = 4$, $S_4 = \frac{4}{2}[2 \times 24 + (4-1)(-3)]$

$$= 2(48 - 9)$$

$$= 2 \times 39 = 78$$

When $n = 13$,

$$S_{13} = \frac{13}{2}[2 \times 24 + (13-1)(-3)]$$

$$= \frac{13}{2}[48 + (-36)] = 78$$

Hence, the number of terms $n = 4$ or $n = 13$.

87. Find the sum of the following series.
$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$

Sol :

The given series can be written as sum of two series

$(5 + 9 + 13 + ... + 81) +$
$+(-41) + (-39) + (-37) + (-35)$
$...(-5) + (-3)$

For the series $(5 + 9 + 13 .....81)$

$a = 5,\ d = 4$ and $a_n = 81$

Now $a_n = a + (n-1)d$

$$81 = 5 + (n-1)4$$

$$81 = 5 + (n-1)4$$

$$(n-1)4 = 76 \Rightarrow n = 20$$

$$S_n = \frac{20}{2}(5 + 81) = 860$$

For series

$(-41) + (-39) + (-37) + ... + (-5) + (-3)$

$a_n = -3,\ a = -41$ and $d = 2$

$a_n = -41 + (n-1)(2)$

$-3 = -41 + 2n - 2 \Rightarrow n = 20$

Now $S_n = \frac{20}{2}[-41 + -3] = -440$

Sum of the series $= 860 - 440 = 420$

88. The ratio of the sums of first m and first n terms of an AP is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}$$

$$\frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m^2}{n^2} \times \frac{n}{m} = \frac{m}{n}$$

$$m[2a + (n-1)d] = n[2a + (m-1)d]$$

$$2ma + mnd - md = 2na + nmd - nd$$

$$2ma - 2na = md - nd$$

$$d = 2a$$

Now, $\frac{a_m}{a_n} = \frac{a + (m-1)d}{a + (n-1)d}$

$$= \frac{a + (m-1) \times 2a}{a + (n-1) \times 2a}$$

$$= \frac{a + 2ma - 2a}{a + 2na - 2a}$$

$$= \frac{2ma - a}{2na - a} = \frac{a(2m-1)}{a(2n-1)}$$

$$= 2m - 1 : 2n - 1$$

89. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X.

Sol :

The houses are numbered consecutively from 1 to 49 i.e. 1, 2, 3 ..... $x-1$, x, $x+1$, ... 49.
Sum of numbers of houses proceeding X number house,

$=$ sum of no. following X

$$\frac{x-1}{2}[1 + (x-1)] = [(1 + ...49) - (1 + 2 + ...x)]$$

$$\frac{x-1}{2}[1 + x - 1] = \left[\frac{49}{2} \times (1 + 49) - \left[\frac{x}{2}(1 + x)\right]\right]$$

$$\frac{(x-1)x}{2} = \frac{49 \times 50}{2} - \frac{x}{2}(x+1)$$

$$\frac{x(x-1)}{2} + (x+1)\frac{x}{2} = 49 \times 25$$

$$\frac{x^2 - x + x^2 + x}{2} = 49 \times 25$$

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$$6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$$
$$(6-5) \times 196 = 49d^2$$
$$d^2 = \frac{196}{49} = 4$$
$$d = \pm 2$$

Thus numbers are $a - 3d = 14 - 3 \times 2 = 8$

$$a - d = 14 - 2 = 12$$
$$a + d = 14 + 2 = 16$$
$$a + 3d = 14 + 3 \times 2 = 20$$

Thus required AP is $8, 12, 16, 20$.

Main 2019

59. In an Arithmetic Progression (AP) the fourth and sixth terms are 8 and 14 respectively. Find the
(i) first term.
(ii) common difference.
(iii) sum of the first 20 terms.

Sol :

Let a and d be the first term and common difference of an AP.

Then, $a_4 = 8$ and $a_6 = 14$

Now $a_n = a + (n-1)d$

Thus $8 = a + (4-1)d$ (1)

and $14 = a + (6-1)d$ (2)

Simplifying above equation we have

Thus $8 = a + 3d$ (3)

and $14 = a + 5d$ (4)

Solving Eqs. (3) and (4), we get

$$a = -1 \text{ and } d = 3$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{20} = \frac{20}{2}[2 \times (-1) + (20-1)3]$$
$$= 10[-2 + 57]$$
$$= 10 \times 55$$
$$= 550$$

(i) First term is -1.
(ii) Common difference is 3
(iii) Sum of first 20 terms is 550.

Main 2019

60. The first and last term of a Geometric Progression (GP) are 3 and 96 respectively. If the common ratio is 2, then find
(i) n in the number of terms of the GP.
(ii) Sum of the n terms.

Sol :

We have $a = 3$, $l = a_n = 96$ and $r = 2$

nth term of a GP,

$$a_n = ar^{n-1}$$
$$96 = 3(2)^{n-1}$$
$$32 = 2^{n-1}$$
$$2^5 = 2^{n-1}$$

Equating the power 2, we get

$$5 = n - 1$$
$$n = 6$$

Hence, the number of terms in a GP is 6.
Now, sum of n terms of a GP is

$$S_n = \frac{a(r^n - 1)}{(r-1)},\ r > 1$$
$$S_6 = \frac{3(2^6 - 1)}{2 - 1}$$
$$= \frac{3 \times (64 - 1)}{1}$$
$$= 3 \times 63$$
$$= 189$$

Main 2019

61. The sum of the first three terms of an Arithmetic Progression (AP) is 42 and the product of the first and third term is 52. Find the first term and the common difference.

Sol :

Let a and d be the first term and common difference of an AP and first three terms of an AP are a, $a + d$ and $a + 2d$.
Since sum of the first three terms of an Arithmetic Progression (AP) is 42, we have

$$a + (a + d) + (a + 2d) = 42$$
$$3a + 3d = 42$$
$$a + d = 14$$

Since product of the first and third term is 52, we have

$$a \times (a + 2d) = 52$$
$$a \times [a + 2(14 - a)] = 52$$
$$a \times (28 - a) = 52$$
$$28a - a^2 = 52$$
$$a^2 - 28a + 52 = 0$$
$$a^2 - 26a - 2a + 52 = 0$$
$$a(a - 26) - 2(a - 26) = 0$$
$$(a - 2)(a - 26) = 0$$

NODIA

79. The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages of all the participants is 168 years, find the age of eldest participant in the painting competition.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a=8, d=4$ month $=\frac{1}{3}$ years, $S_n=168$

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$
$$168=\frac{n}{2}\left[2(8)+(n-1)\frac{1}{3}\right]$$
$$n^2+47n-1008=0$$
$$n^2+63n-16n-1008=0$$
$$(n-16)(n+63)=0$$

Thus $n=16$ or $n=-63$

As n cannot be negative, we take $n=16$

Age of the eldest participant $=a+15d=13$ years

80. A thief runs with a uniform speed of 100 m/ minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100/minute in the first minute and increased his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

Sol :

Let total time to catch the thief be n minutes
Total distance covered by thief will be $100n$
Total distance covered by policeman,

$$=100+110+120+....+(n-1) \text{ terms}$$
$$100n=\frac{n-1}{2}[200+(n-2)10]$$
$$n^2-3n-18=0$$
$$(n-6)(n+3)=0$$
$$n=6$$

Policeman takes 5 minutes to catch the thief.

81. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle terms separately.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
The three digit numbers which leaves 3 as remainder when divided by 4 are: $103, 107, 111, ... 999$.
Now, the first number $a=103$, last number $a_n=999$ and common difference $d=4$
Let the number of terms in this sequence be n.

$$a_n=a+(n-1)d$$
$$999=103+(n-1)4$$
$$896=(n-1)4$$
$$(n-1)=\frac{896}{4}=224$$
$$n=224+1=225$$
$$\text{Middle term}=\frac{225+1}{2}$$
$$=113^{th} \text{ term}$$
$$a_{113}=103+112\times 4=551$$

and $a_{112}=551-4=547$

Sum of first 112 terms,

$$S_{112}=\frac{112}{2}(a+a_{112})$$
$$=56(103+547)$$
$$=56\times 650=36400$$

and $a_{114}=551+4=555$

The sum of last 112 terms,

$$S_{114-225}=\frac{112}{2}(a_{114}+a_{225})$$
$$=56(555+999)$$
$$=56\times 1554=87024$$

82. Find the middle term of the sequence formed all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

Sol :

The sequence is 10, 13, ... 94
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
Here $a=10,\ d=3,\ S_n=94$

$$94=10+(n-1)3$$
$$84=(n-1)3$$
$$n=\frac{84}{3}+1=29$$

Therefore $\frac{29+1}{2}=15^{th}$ term is the middle term.

Middle term $a_{15}=a+(15-1)d$

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$$140 = 2x - 60$$
$$x = \frac{140+60}{2} = 100$$

Thus prizes are Rs. 100, Rs. 80, Rs. 60, Rs. 40.

67. In a garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row. How many rows are there of rose plants> also find the total number of roes plants in the garden.

Sol :

The number of rose plants in the $1^{st}, 2^{nd}, \ldots..$ are $23, 21, 19, \ldots\ldots, 5$.
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here $a = 23, d = -2, a_n = 5$

$$a_n = a + (n-1)d$$
$$5 = 23 + (n-1)(-2)$$
$$n = 10$$

Total number of roes plants in the flower bed,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = 5(46 - 18) = 140$$

68. A sum of Rs. 1890 is to be used to given seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 50 less than its preceding prize, find the value of each of the prizes.

Sol :

Let first prize be Rs. x, then series of prize is $x,\ x-50,\ x-100,\ x-150, \ldots..$
Here series is AP and $a = x,\ d = -50,\ n = 7$ and $S_n = 1890$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$1890 = \frac{7}{2}[2x + (-50) \times 6]$$
$$270 = x + (-50) \times 3 = x - 150$$
$$x = 270 + 150 = 420$$

The prizes are Rs. 420, Rs. 370, Rs. 320, Rs. 270, Rs. 220, Rs. 170, Rs. 120.

69. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here $a = 20, d = 15$ and $S_n = 3250$
Now

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$3250 = \frac{n}{2}[2a + (n-1) \times 15]$$
$$3250 \times 2 = n[40 + 15n - 15]$$
$$6500 = n[25 + 15n]$$
$$1300 = n[5 + 3n]$$
$$3n^2 + 65n - 60n - 1300 = 0$$
$$n(3n + 65) - 20(3n + 65) = 0$$
$$(n - 20)(3n + 65) = 0$$

Since $n = -65/3$, is not possible, $n = 20$

Man will repay loan in 20 months.

FOUR MARK QUESTIONS

Main 2018

70. The 4th term of an AP, is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of series to 8 terms.

Sol :

Let a and d be the first term and common difference of an AP.
Then, $a_4 = 22$ and $a_{15} = 66$
Now

$$a_n = a + (n-1)\,d$$
$$22 = a + (4-1)\,d \quad (1)$$

and

$$66 = a + (15-1)\,d \quad (2)$$

Simplifying above equation we have

$$22 = a + 3d \quad (3)$$

and

$$66 = a + 14d \quad (4)$$

Subtracting eq (3) from eq (4), we get

$$44 = 11d$$
$$d = \frac{44}{11} = 4$$

Substituting $d = 4$ in eq (3) we have

$$22 = a + 3 \times 4$$
$$22 = a + 12$$
$$a = 22 - 12 = 10$$

Now

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$
$$S_8 = \frac{8}{2}[2 \times 10 + (8-1) \times 4]$$

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$$a = 2,\ a = 26$$

When $a = 2$, then $d = 14 - a$

$$= 14 - 2 = 12$$

When $a = 26$ then $d = 14 - 26$

$$= -12$$

Comp 2019

62. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28^{th} term of AP.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Now, $S_7 = 63$

$$\frac{7}{2}\left[2a + 6d\right] = 63$$

$$2a + 6d = 18 \qquad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}\left[2a + 13d\right] = 224$$

$$2a + 13d = 32 \qquad (2)$$

Subtracting equation (1) form (2) we get

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now $a_n = a + (n-1)d$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus 28^{th} term is 57.

Comp 2019

63. Determine an AP whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Sol :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = 9$

$$a + 2d = 9 \qquad ...(1)$$

and $a_8 - a_5 = 6$

$$(a + 7d) - (a + 4d) = 6$$

$$3d = 6 \Rightarrow d = 2$$

Substituting this value of d in (1), we get

$$a + 2(2) = 9 \Rightarrow a = 5$$

So, AP is 5, 7, 9, 11, ...

Main 2018

64. If $(k-3)$, $(2k+1)$ and $(4k+3)$ are three consecutive terms of an AP, find the value of k.

Sol :

Let $a_1 = k-3$, $a_2 = 2k+1$ and $a_3 = 4k+3$ are consecutive terms of an AP, then

$$a_2 - a_1 = a_3 - a_2$$

$$(2k+1) - (k-3) = (4k+3) - (2k+1)$$

$$2k + 1 - k + 3 = 4k + 3 - 2k - 1$$

$$k + 1 = 2k + 2 \Rightarrow k = 2$$

Comp 2018

65. If the sum of first m terms of an AP is same as the sum of its first n terms $(m \neq n)$, show that the sum of its first $(m+n)$ terms is zero.

Sol :

Let the first term be a, common difference be d, nth term be a_n, and sum of n term be S_n

Now $S_m = S_n$

$$\frac{m}{2}\left[2a + (m-1)d\right] = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$2ma + m(m-1)d = 2na + n(n-1)d$$

$$2a(m-n) + \left[(m^2 - n^2) - m + n\right]d = 0$$

$$2a(m-n) + \left[(m-n)(m+n) - (m-n)\right]d = 0$$

$$(m-n)\left[2a + (m+n-1)d\right] = 0$$

$$2a + (m+n-1)d = 0 \qquad [m - n \neq 0]$$

$$S_{m+n} = \frac{m+n}{2}\left[2a + (m+n-1)d\right]$$

$$= \frac{m+n}{2} \times 0 = 0$$

NODIA

66. A sum of Rs. 280 is to be used towards four prizes. If each prize after the first is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Sol :

Let first prize be Rs. x, then series of prize is $x,\ x-20,\ x-40,\ x-60, ...$

Here series is AP and $a = x,\ d = -20,\ n = 4$ and $S_n = 280$.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$280 = \frac{4}{2}\left[2x + 3(-20)\right]$$

$$280 = 2\left[2x - 60\right]$$

$$63+(n-1)2=3+(n-1)7$$
$$63+2n-2=3+7n-7$$
$$61+2n=7n-4$$
$$65=5n \Rightarrow n=13$$

When n is 13, the n^{th} terms are equal i.e., $a_{13}=A_{13}$

75. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1+S_3=2S_2$.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1+2+3+.....n$

$S_2 = 1+3+5+.....$ up to n terms

$S_3 = 1+4+7+.....$ upto n terms

Now $S_n=\frac{n(n+1)}{2}$

$$S_2=\frac{n}{2}[2+(n-1)2]=\frac{n}{2}[2n]=n^2$$

and $$S_3=\frac{n}{2}[2+(n-1)3]=\frac{n(3n-1)}{2}$$

Now, $$S_1+S_3=\frac{n(n+1)}{2}+\frac{n(3n-1)}{2}$$
$$=\frac{n[n+1+3n-1]}{2}=\frac{n[4n]}{2}$$
$$=2n^2=2s_2 \quad \text{Hence Proved}$$

76. If S_n denotes, the sum of the first n terms of an AP prove that $S_{12}=3(S_8-S_4)$.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n=\frac{n}{2}[2a+(n-1)d]$$
$$S_{12}=6[2a+11d]$$
$$=12a+66d$$
$$S_8=4[2a+7d]=8a+28d$$
$$S_4=2[2a+3d]=4a+6d$$
$$3(S_8-S_4)=3[(8a+28d)-(4a+6d)]$$
$$=3[4a+22d]=12a+66d$$
$$=6[2a+11d]=S_{12} \text{Hence Proved}$$

77. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_{14}=2a_8$ and $a_6=-8$

Now $$a+13d=2(a+7d)$$
$$a+13d=2a+14d$$
$$a=-d \quad ...(1)$$

and $$a_6=-8$$
$$a+5d=-8 \quad ...(2)$$

Solving (1) and (2), we get

$$a=2, d=-2$$

Now $$S_{20}=\frac{20}{2}[2\times 2+(20-1)(-2)]$$
$$=10[4+19\times(-2)]$$
$$=10(4-38)$$
$$=10\times(-34)=-340$$

78. The digit of a positive number of three digits are in AP and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number.

Sol :

Let these digit of 3 digit number be $a-d, a, a+d$

Since their sum is 15,

$$a-d+a+a+d=15$$
$$3a=15 \Rightarrow a=5$$

Required 3 digit number $=100(a-d)+10a+a+d$
$$=100a-100d+10a+a+d$$
$$=111a-99d$$

Number obtained by reversing digit,
$$=100(a+d)+10a+a-d$$
$$=100a+100d+10a+a-d$$
$$=111a+99d$$

According the question, we have
$$111a+99d=111a-99d-594$$
$$2\times 99d=-594 \Rightarrow d=-3$$

Thus number is
$$111a-99d=111\times 5-99\times -3$$
$$=555+297=852$$

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6th term $a_6 = 40$

$$ar^{6-1} = 40$$

$$ar^5 = 40 \quad \text{.....(2)}$$

Last term $a_n = 640$

$$ar^{n-1} = 640 \quad \text{.....(3)}$$

Dividing Eq. (2) by Eq. (1), we get

$$\frac{ar^5}{ar^3} = \frac{40}{10}$$

$$r^2 = 4$$

$$r = 2 \quad \text{[common ratio is positive]}$$

Now, from Eq. (1),

$$a(2)^3 = 10$$

$$a = \frac{10}{8} = \frac{5}{4}$$

Submitting value of a and r in Eq (3), we get

$$\left(\frac{5}{4}\right)(2)^{n-1} = 640$$

$$2^{n-1} = 640 \times \frac{4}{5}$$

$$= 512$$

$$2^{n-1} = 2^9$$

$$n - 1 = 9$$

$$n = 10$$

Thus first terms is $\frac{5}{4}$, common ration is 2 and number of term of the series is 10.

Main 2020

56. If the 6th term of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference.

Sol :

Let a be the first term and d be the common difference of the A.P.

Since 6th term of the A.P. is equal to four times its first term, thus

$$a_6 = 4a_1$$

$$a + (6-1)\,d = 4a$$

$$3a = 5d \quad \text{.....(1)}$$

Since sum of first six terms is 75,

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$75 = \frac{6}{2}[2a + (6-1)\,d]$$

$$75 = 3\,[2a + 5d]$$

$$75 = 6a + 15d \quad (2)$$

Substitution $3a = 5d$ in above equation we have

$$75 = 2 \times 5d + 15d$$

$$= 10d + 15d$$

$$= 25d$$

$$d = \frac{75}{25} = 3$$

Thus first term is 5 and common difference is 3.

Comp 2020

57. In an AP of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_{10} = 210$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$210 = \frac{10}{2}(2a + 9d)$$

$$42 = 2a + 9d \quad (1)$$

Now $a_{36} = a + 35d$

$$a_{50} = a + 49d$$

Sum of last 15 terms,

$$S_{36-50} = \frac{n}{2}(a_{36} + a_{50})$$

$$2565 = \frac{15}{2}(a + 35d + a + 49d)$$

$$171 = \frac{1}{2}(2a + 84d)$$

$$171 = a + 42d \quad (2)$$

Solving (1) and (2) we get $a = 3$ and $d = 4$

Hence, AP is 3, 7, 11, ...

Comp 2020

58. Divide 56 in four parts in AP such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Sol :

Let the four numbers be $a-3d, a-d, a+d, a+3d$

Now $a - 3d + a - d + a + d + a + 3d = 56$

$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14-3d, 14-d, 14+d, 14+3d$

Now, according to question, we have

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$

$$\frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$$6(196 - 9d^2) = 5(196 - d^2)$$

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$$= 10 + 14 \times 3 = 52$$

$$a_{14} = 52 - 3 = 49$$

$$a_{16} = 52 + 3 = 55$$

Sum of first 14 terms,

$$s_{14} = \frac{14}{2}(a + a_{14})$$

$$= \frac{14}{2}[10 + 49] = 413$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Sum of the last 14 terms,

$$s_{16-29} = \frac{14}{2}(a_{16} + a_{29})$$

$$= \frac{14}{2}[55 + 94]$$

$$= 7 \times 149 = 1043$$

83. Show that the sum of all terms of an AP whose first term is a, the second term is b and last term is c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Sol :

Given, first term, $A = a$

and second term $A_2 = b$

Common difference, $D = b - a$

Last term, $A_n = c$

$$A + (n-1)d = c$$

$$a + (n-1)(b-a) = c$$

$$(b-a)(n-1) = c - a$$

$$n - 1 = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1$$

$$= \frac{c - a + b - a}{b-a}$$

$$n = \frac{b + c - 2a}{b-a}$$

Now sum of all terms

$$S_n = \frac{n}{2}[A + A_n]$$

$$= \frac{(b+c-2a)}{2(b-a)}[a+c]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$ Hence Proved

84. If in an AP, the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m+n)$ terms is $-(m+n)$.

Sol :

Let 1st term of series be a and common difference be d, then we have

$$S_m = n$$

and $$S_n = m$$

$$\frac{m}{2}[2a + (m-1)d] = n \quad ...(1)$$

$$\frac{n}{2}[2a + (n-1)d] = m \quad ...(2)$$

Subtracting we have

$$a(m-n) + \frac{d}{2}[m(m-1) - n(n-1)] = n - m$$

$$2a(m-n) + d[m^2 - n^2 - (m-n)] = 2(n-m)$$

$$2a(m-n) + d(m-n)[(m+n) - 1] = 2(n-m)$$

$$2a + d[(m+n) - 1] = -2$$

Now, $$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2}(-2)$$

$$= -(m+n)$$

85. The 17th term of an AP is 5 more than twice its 8th term. If 11th term of AP is 43, then find its n^{th} term.

Sol :

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a + (n-1)d$$

Since 17th term of an AP is 5 more than twice of its 8th term, thus

$$a + (17-1)d = 5 + 2[a + (8-1)d]$$

$$a + 16d = 5 + 2(a + 7d)$$

$$a + 16d = 5 + 2a + 14d$$

$$2d - a = 5 \quad ...(1)$$

Since 11th term of AP is 43,

$$a + (11-1)d = 43$$

$$a + 10d = 43 \quad ...(2)$$

Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

Hence, n^{th} term would be

$$a_n = 3 + (n-1)4 = 4n - 1$$

86. How many terms of the AP 24, 21, 18, ... must be taken so that their sum is 78?

Sol :

Given : 24, 21, 18, ... are in AP.

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$$= 4[20 + 28]$$
$$= 4 \times 48$$
$$= 192$$

Comp 2018

71. The sum of the 3^{rd} and 7^{th} terms of an AP is 6 and their product is 8. Find the sum of first 20 terms of the AP.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a_3 + a_7 = 6$

$$a + 2d + a + 6d = 6$$
$$a + 4d = 3 \quad (1)$$

and $a_3 \times a_7 = 8$

$$(a + 2d)(a + 6d) = 8 \quad (2)$$

Substituting the value $a = (3 - 4d)$ in (2) we get

$$(3 - 4d + 2d)(3 - 4d + 6d) = 8$$
$$(3 + 2d)(3 - 2d) = 8$$
$$9 - 4d^2 = 8$$
$$4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d = \frac{1}{2}$ in equation (1), $a = 1$.

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{20}{2}\left[2 + \frac{19}{2}\right] = 115$$

Thus $d = \frac{1}{2}, a = 1$ and $S_{20} = 115$

CASE 2 : Substituting $d = -\frac{1}{2}$ in equation (1) $a = 5$

$$S_{20} = \frac{20}{2}\left[2 \times 5 + 19 \times \left(-\frac{1}{2}\right)\right]$$
$$= 10\left[10 - \frac{19}{2}\right] = 15$$

Thus $d = -\frac{1}{2}, a = 5$ and $S_{20} = 15$

72. The ninth term of an AP is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

Sol :

Let the first term be a, common difference be d and nth term be a_n.

Now $a_9 = 7a_2$

$$a + 8d = 7(a + d)$$
$$a + 8d = 7a + 7d$$
$$-6a + d = 0 \quad (1)$$

and $a_{12} = 5a_3 + 2$

$$a + 11d = 5(a + 2d) + 2$$
$$a + 11d = 5a + 10d + 2$$
$$-4a + d = 2 \quad ...(2)$$

Subtracting (2) from (1), we get

$$-2a = -2 \Rightarrow a = 1$$

Substituting this value of a in equation (1) we get

$$-6 + d = 0 \Rightarrow d = 6$$

Hence first term is 1 and common difference is 6.

73. The sum of n terms of an AP is $3n^2 + 5n$. Find the AP. Hence find its 15^{th} term.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $S_n = 3n^2 + 5n$

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$
$$= 3(n^2 + 1 - 2n) + 5n - 5$$
$$= 3n^2 + 3 - 6n + 5n - 5$$
$$= 3n^2 - n - 2$$
$$a_n = S_n - S_{n-1}$$
$$= 3n^2 + 5n - (3n^2 - n - 2)$$
$$= 6n + 2$$

Thus AP is 8, 14, 20, ...

Now $a_{15} = a + 14d = 8 + 14(6) = 92$

74. For what value of n, are the n^{th} terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Sol :

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.

n is same

For 1st AP, $a = 63,\ d = 2$

For 2nd AP, $A = 3, D = 7$

Since nth term is same,

$$a_n = A_n$$
$$a + (n-1)d = A + (n-1)D$$

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$$\frac{2x^2}{2} = 49 \times 25$$

$$x^2\ x^2 = 7 \times 7 \times 5 \times 5$$

$$x = 7 \times 5 = 35$$

90. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Sol :

Given AP is 45, 39, 33, ...

Here, $a = 45$, $d = 39 - 45 = -6$ and $S_n = 180$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$

$$360 = n(90 - 6n + 6)$$

$$360 = n(96 - 6n)$$

$$60 = n(16 - n)$$

$$n^2 - 16n + 60 = 0$$

$$n^2 - 6n - 10n + 60 = 0$$

$$n(n-6) - 10(n-6) = 0$$

$$(n-10)(n-6) = 0$$

$$n = 10 \text{ or } n = 6$$

Hence, 10 terms or 6 terms can be taken to get the sum of AP as 180.

Now, sum of 6 terms,

$$S_6 = \frac{6}{2}[2 \times 45 + (6-1)(-6)]$$

$$= 3(90 - 30)$$

$$= 3 \times 60 = 180$$ Hence, verified.

and sum of 10 terms,

$$S_{10} = \frac{10}{2}[2 \times 45 + (10-1)(-6)]$$

$$= 5(90 - 54)$$

$$= 5 \times 36 = 180$$ Hence, verified.

Here we have two values of n because d is negative. There will be negative terms after some positive terms. Thus first 6 term will give sum 180 and after 10 term it will be again 180 because negative term cancel positive term.

Series will be : 45, 39, 33, 27, 21, 15, 9, 3, -3, -9 ...

Here it may be easily seen that sum of initial 6 terms is 180. Sum of next 4 terms is zero. Thus sum of 10 terms is also 180.

END

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CHAPTER 10

GEOMETRIC PROGRESSION

NODIA

MULTIPLE CHOICE QUESTIONS

1. What is the next term of the series $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{3}$ (d) $\frac{5}{3}$

Sol :

Given series is $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$ which seems to be GP.

Here, $a = \frac{1}{6}$,

$$r = \frac{a_2}{a} = \frac{1}{3} \div \frac{1}{6} = 2$$

$$r = \frac{a_3}{a_2} = \frac{2}{3} \div \frac{1}{3} = 2$$

Thus above series is GP and $r = 2$.

Now next term $a_4 = a_3 r$

$$= \frac{2}{3} \times 2 = \frac{4}{3}$$

Thus (a) is correct option.

2. What is the next term of the GP $\frac{3}{16}, -\frac{3}{8}, \frac{3}{4}, -\frac{3}{2} \ldots$

(a) 1 (b) 2
(c) 3 (d) 4

Sol :

Given series is $\frac{3}{16}, -\frac{3}{8}, \frac{3}{4}, -\frac{3}{2}, \ldots$

Here, $a = \frac{3}{16}$,

$$r = \frac{-3}{8} \div \frac{3}{16}$$

$$= \frac{-3}{8} \times \frac{16}{3} = -2$$

Now next term $a_5 = a_4 r$

$$= \frac{-3}{2} \times (-2) = 3$$

Thus (c) is correct option.

3. What is the 15^{th} term of the GP $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \ldots$

(a) $\frac{1}{3^{13}}$ (b) $\frac{\sqrt{3}}{3^{14}}$
(c) $\frac{1}{3^{15}}$ (d) $\frac{\sqrt{3}}{3^{13}}$

Sol :

Given series is $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \ldots$ ...

Here, $a = \sqrt{3}$

$$r = \frac{1}{\sqrt{3}} \div \sqrt{3}$$

$$= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$

Now $a_n = ar^{n-1}$

$$a_{15} = ar^{n-1}$$

$$= \sqrt{3}\left(\frac{1}{3}\right)^{15-1}$$

$$= \sqrt{3} \times \left(\frac{1}{3}\right)^{14}$$

$$= \sqrt{3} \times \frac{1}{3^{14}}$$

$$= \frac{\sqrt{3}}{3^{14}}$$

Thus (b) is correct option.

4. What is the 10^{th} and n^{th} terms of the GP 5, 25, 125, ...

(a) 5^{11} (b) 5^{10}
(c) 5^{8} (d) 5^{9}

Sol :

Given series is 5, 25, 125, ...

Here, $a = 5$,

$$r = 25 \div 5 = 5$$

Now $a_n = ar^{n-1}$

$$= 5 \times 5^{n-1}$$

$$= 5^{n-1+1}$$

$$= 5^{n}$$

$$a_{10} = 5^n$$
$$= 5^{10}$$

Thus (b) is correct option.

5. For what value of x, the numbers $-\frac{5}{7}$, x, $-\frac{7}{5}$ are in GP.?

(a) ± 3 (b) ± 2
(c) ± 4 (d) ± 1

Sol :

Since $-\frac{5}{7}$, x, $-\frac{7}{5}$ are in GP, from $b^2 = ac$ we have

$$x^2 = -\frac{5}{7} \times -\frac{7}{5}$$
$$x^2 = 1$$
$$x = \pm 1$$

Thus (d) is correct option.

6. What is the 6^{th} term of the GP $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$ ...

(a) $\frac{3}{64}$ (b) $\frac{3}{32}$
(c) $\frac{3}{128}$ (d) $\frac{3}{256}$

Sol :

Given series is $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, ...

Here, $a = \frac{3}{2}$,

$$r = \frac{3}{4} \div \frac{3}{2}$$
$$= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$
$$a_n = ar^{n-1}$$
$$a_6 = ar^{6-1}$$
$$= \frac{3}{2} \times \left(\frac{1}{2}\right)^5$$
$$= \frac{3}{2} \times \frac{1}{32}$$
$$= \frac{3}{64}$$

Thus (a) is correct option.

7. What is the sum of n terms of GP $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ...

(a) $\frac{1}{2^n}$ (b) $\frac{1}{2^{n-1}}$
(c) $1 - \frac{1}{2^n}$ (d) $1 - \frac{1}{2^{n-1}}$

Sol :

Let a be the first term and r be the common ratio.

Here, $a = \frac{1}{2}$, $r = \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$

Now $S_n = \frac{a(1-r^n)}{1-r}$ $(r < 1)$

$$= \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$$
$$= \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}}$$
$$= 1 - \left(\frac{1}{2}\right)^n$$
$$= 1 - \frac{1}{2^n}$$

Thus (c) is correct option.

8. What is the sum n terms of GP 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, ...

(a) $\frac{9}{10}\left(1 - \frac{1}{10^{n-1}}\right)$ (b) $\frac{9}{10}\left(1 - \frac{1}{10^n}\right)$
(c) $\frac{10}{9}\left(1 - \frac{1}{10^{n-1}}\right)$ (d) $\frac{10}{9}\left(1 - \frac{1}{10^n}\right)$

Sol :

Let a be the first term and r be the common ratio.

Here, $a = 1$, $r = \frac{1}{10}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r < 1)$$
$$= \frac{1(1-\frac{1}{10})^n}{1-\frac{1}{10}}$$
$$= \frac{1\left[1-\left(\frac{1}{10}\right)^n\right]}{\frac{9}{10}}$$
$$= \frac{10}{9}\left(1 - \frac{1}{10^n}\right)$$

Thus (d) is correct option.

9. What is the sum n terms of GP 5, $\frac{20}{7}$, $\frac{80}{49}$, ...

(a) $\frac{3}{35}\left[1-\left(\frac{4}{7}\right)^n\right]$ (b) $\frac{3}{35}\left[1-\left(\frac{4}{7}\right)^{n-1}\right]$
(c) $\frac{35}{3}\left[1-\left(\frac{4}{7}\right)^n\right]$ (d) $\frac{35}{3}\left[1-\left(\frac{4}{7}\right)^{n-1}\right]$

Sol :

Let a be the first term and r be the common ratio.

Here $a = 5$, $r = \frac{20}{7} \div 5 = \frac{4}{7}$

Now $S_n = \frac{a(1-r^n)}{1-r}$ $(r < 1)$

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$$= \frac{5\left[1-\left(\frac{4}{7}\right)^n\right]}{1-\frac{4}{7}}$$
$$= \frac{5\left[1-\left(\frac{4}{7}\right)^n\right]}{\frac{3}{7}}$$
$$= \frac{5\times 7}{3}\left[1-\left(\frac{4}{7}\right)^n\right]$$
$$= \frac{35}{3}\left[1-\left(\frac{4}{7}\right)^n\right]$$

Thus (c) is correct option.

10. What is the sum n terms of GP 3, 12, 48, 192, ...

(a) 4^n-3 (b) 2^n-1
(c) 2^n-3 (d) 4^n-1

Let a be the first term and r be the common ratio.

Here, $a=3,\ r=\frac{12}{3}=4$

$$S_n = \frac{a(r^n-1)}{r-1} \qquad (r>1)$$
$$= \frac{3(4^n-1)}{4-1}$$
$$= \frac{3(4^n-1)}{3}$$
$$= 4^n-1$$

Thus (d) is correct option.

11. What is the sum n terms of GP 2, -6, 18, -54, ...

(a) $\frac{1}{2}[1-(-3)^n]$ (b) $[1-(3)^n]$
(c) $[1-(-3)^n]$ (d) $\frac{1}{3}[1-(3)^n]$

Sol :

Let a be the first term and r be the common ratio.

Here, $a=2,\ r=\frac{-6}{2}=-3$

$$S_n = \frac{a[1-r^n]}{1-r}$$
$$= \frac{2[1-(-3)^n]}{1+3}$$
$$= \frac{2[1+(-3)^n]}{4}$$
$$= \frac{1}{2}[1-(-3)^n]$$

Thus (a) is correct option.

12. What is the sum of the GP 1, 2, 4, 8, ... up to 10 terms.

(a) 3078 (b) 1023
(c) 2078 (d) 2020

Sol :

Let a be the first term and r be the common ratio.

Here, $a=1,\ r=2$

$$S_{10} = \frac{a(r^n-1)}{r-1}$$
$$= \frac{1[(2)^n-1]}{2-1}$$
$$= \frac{1[(2)^n-1](2+1)}{(2-1)(2+1)}$$
$$= \frac{3[(2)^n-1]}{3}$$
$$= (2)^{10}-1$$
$$= 1024-1 = 1023$$

Thus (b) is correct option.

13. What is the sum of the GP 2, 6, 18, ... up to 8 terms ?

(a) 3078 (b) 6560
(c) 2078 (d) 2020

Sol :

Let a be the first term and r be the common ratio.

Here, $a=2,\ r=\frac{6}{2}=3$

$$S_8 = \frac{a(r^n-1)}{r-1}$$
$$= \frac{2[(3)^n-1]}{3-1}$$
$$= \frac{2[(3)^n-1]}{2}$$
$$= (3)^8-1$$
$$= 6561-1 = 6560$$

Thus (b) is correct option.

14. What is the sum n terms of GP 6, -3, $\frac{3}{2}$, $-\frac{3}{4}$, $\frac{3}{8}$, ...

(a) $2\left[1-\left(\frac{1}{2}\right)^n\right]$ (b) $2\left[1-\left(\frac{-1}{2}\right)^n\right]$
(c) $4\left[1-\left(\frac{1}{2}\right)^n\right]$ (d) $4\left[1-\left(\frac{-1}{2}\right)^n\right]$

Sol :

Let a be the first term and r be the common ratio.

Here, $a=6$

$$r = \frac{-3}{6} = \frac{-1}{2}$$
$$S_n = \frac{a(r^n-1)}{r-1} = \frac{6\left[\left(\frac{-1}{2}\right)^n-1\right]}{\frac{-1}{2}-1}$$
$$= \frac{6\left[\frac{(-1)^n}{2^n}-1\right]}{\frac{-3}{2}} = \frac{6\frac{[(-1)^n-(2)^n]}{2^n}}{\frac{-3}{2}}$$

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$$-2\left[\frac{(-1)^n-(2)^n}{(2)^n}\right]\times\left(\frac{-2}{1}\right)$$
$$=-4\left[\frac{(-1)^n-(2)^n}{(2)^n}\right]$$
$$=-4\left[\frac{(2)^n-(-1)^n}{(2)^n}\right]$$
$$=4\left[1-\left(\frac{-1}{2}\right)^n\right]$$

Thus (d) is correct option.

15. What is the 12^{th} term of a GP whose 8^{th} term is 192 and common ratio is 2.

(a) 3078 (b) 3072
(c) 2078 (d) 2072

Sol :

Let a be the first term and r be the common ratio then

We have $a_8 = 192$ and $r = 2$

Since $a_8 = 192$, from $a_n = ar^{n-1}$ we get

$$a_8 = a(2)^{8-1}$$
$$192 = a2^7$$
$$a = \frac{192}{2^7} = \frac{192}{128} = \frac{3}{2}$$

Now $a_n = a(r)^{n-1}$

$$a_{12} = \frac{3}{2}(2)^{12-1} = \frac{3}{2}\times 2^{11}$$
$$= \frac{3}{2}\times 2048 = 3072$$

Thus (b) is correct option.

16. What is the value of x such that $-\frac{2}{7}$, x, $-\frac{7}{2}$ are three consecutive terms of a GP

(a) 1 (b) 2
(c) 3 (d) 4

Sol :

If a, b, c are in GP, the $b^2 = ac$. Since $-\frac{2}{7}$, x, $-\frac{7}{2}$ are three consecutive terms of a GP, we obtain

$$x^2 = \frac{-2}{7}\times\frac{-7}{2}$$
$$= 1$$
$$= (\pm 1)^2$$
$$x = \pm 1$$

Hence, $x = 1$ or -1

Thus (a) is correct option.

17. What is the value of x such that $x+9$, $x-6$ and 4 are three consecutive terms of a GP

(a) 14 (b) 15
(c) 16 (d) 17

Sol :

If a, b, c are in GP, the $b^2 = ac$. Since $x+9$, $x-6$ and 4 are three consecutive terms of a GP, then

$$(x-6)^2 = (x+9)\times 4$$
$$x^2-12x+36 = 4x+36$$
$$x^2-12x-4x+36-36 = 0$$
$$x^2-16x = 0$$
$$x(x-16) = 0$$

Thus $x = 0, 16$

Thus (c) is correct option.

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18. What is the value of x such that x, $x+3$, $x+9$ are first three terms of a GP

(a) 3 (b) 5
(c) 7 (d) 9

Sol :

If a, b, c are in GP, the $b^2 = ac$. Since x, $x+3$, $x+9$ are first three terms of a GP, we have

$$(x+3)^2 = x(x+9)$$
$$x^2+6x+9 = x^2+9x$$
$$9 = 9x-6x = 3x$$
$$x = \frac{9}{3} = 3$$

Thus (a) is correct option.

19. How many terms of the GP 1, 2, 4, 8, 16, ... will make a sum of 4096?

(a) 10 (b) 11
(c) 12 (d) 13

Sol :

Given GP is 1, 2, 4, 8, 16, ...

Here $a = 1$, $r = 2$

Let n be the terms in GP which gives sum 4095.

Now $S_n = \frac{a(r^n-1)}{r-1}$ when $r > 1$

$$4095 = \frac{1(2^n-1)}{2-1}$$
$$4095 = 2^n-1$$
$$4096 = 2^n$$

If x, y and z are in GP, then

$$y^2 = xz$$

Now, $ar^6 \times ar^6 = ar^3 \times ar^9$

$$a^2 r^{6+6} = a^2 r^{3+9}$$

$$a^2 r^{12} = a^2 r^{12}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Thus x, y and z are in GP

27. Which term of the GP 2, $2\sqrt{2}$, 4, ... is 128?

Sol :

Given, series is 2, $2\sqrt{2}$, 4, ... is 128

Here, $a = 2$

$$r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$l = 128$$

Let 128 be the n^{th} term, then

$$a_n = ar^{n-1}$$

$$128 = ar^{n-1}$$

$$128 = 2(\sqrt{2})^{n-1}$$

$$2(\sqrt{2})^{n-1} = 2^7$$

$$(\sqrt{2})^{n-1} = 2^{7-1} = 2^6$$

$$(\sqrt{2})^{n-1} = (\sqrt{2})^{12}$$

Comparing above equation, we get

$$n - 1 = 12$$

$$n = 12 + 1 = 13$$

Thus 128 is the 13^{th} term.

28. Which term of the GP 1, $\frac{1}{3}$, $\frac{1}{9}$, ... is $\frac{1}{243}$?

Sol :

Given series is 1, $\frac{1}{3}$, $\frac{1}{9}$, ... is $\frac{1}{243}$

Here, $a = 1$,

$$r = \frac{1}{3} \div 1 = \frac{1}{3},$$

$$l = \frac{1}{243}$$

Let $\frac{1}{243}$ is the n^{th} term, then

$$a_n = ar^{n-1}$$

$$\frac{1}{243} = 1 \times \left(\frac{1}{3}\right)^{n-1}$$

$$\left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^{n-1}$$

Comparing above equation, we get

$$n - 1 = 5$$

$$n = 5 + 1 = 6$$

Thus $\frac{1}{243}$ is the 6^{th} term of given GP

29. Which term of the GP $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, ... is $\frac{1}{19683}$?

Sol :

Given series is $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, ... is $\frac{1}{19683}$

Here, $a = \frac{1}{3}$

$$r = \frac{1}{9} \div \frac{1}{3}$$

$$= \frac{1}{9} \times \frac{3}{1} = \frac{1}{3}$$

$$l = \frac{1}{19683}$$

Let $\frac{1}{19683}$ is the n^{th} term, then

$$a_n = ar^{n-1}$$

$$\frac{1}{19683} = \frac{1}{3} \times \left(\frac{1}{3}\right)^{n-1}$$

$$\left(\frac{1}{3}\right)^9 = \left(\frac{1}{3}\right)^n$$

Comparing above equation, we get

$$n = 9$$

Hence, $\frac{1}{19683}$ is the 9^{th} term of given GP

30. Which term of the GP. 3, $-3\sqrt{3}$, 9, $-9\sqrt{3}$, ... is 729?

Sol :

Given GP is 3, $-3\sqrt{3}$, 9, $-9\sqrt{3}$, ... is 729

Here, $a = 3$

$$d = \frac{-3\sqrt{3}}{3} = -\sqrt{3}$$

$$l = 729$$

Let 729 is the n^{th} term, then

$$a_n = ar^{n-1}$$

$$729 = 3(-\sqrt{3})^{n-1}$$

$$\frac{729}{3} = (-\sqrt{3})^{n-1}$$

$$243 = (-\sqrt{3})^{n-1}$$

$$(-\sqrt{3})^{10} = (-\sqrt{3})^{n-1}$$

Comparing above equation, we get

$$n - 1 = 10$$

$$n = 10 + 1 = 11$$

Thus 729 is the 11^{th} term of given GP.

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Dividing Eq. (2) by Eq. (1), we get

$$4 = r^2$$

Since common ration is positive,

$$r = 2$$

Now substituting $r = 2$ in eq (1) we get

$$a = \frac{10}{8} = \frac{5}{4}$$

Substituting value of a and r in eq (3), we get

$$640 = \tfrac{5}{4}2^{n-1}$$

$$640 \times \tfrac{4}{5} = 2^{n-1}$$

$$512 = 2^{n-1}$$

$$2^9 = 2^{n-1}$$

$$n - 1 = 9$$

$$n = 10$$

First terms is $\frac{5}{4}$, common ration is 2 and number of term of the series is 10.

Comp 2020

24. The sum of the first 3 terms of a GP is 1 and the sum of the next 3 terms is 27. Determine the first term, the common ratio and the sum of the GP to n terms.

Sol :

Let a be the first term and r be the common ratio. Then we can write first three term as a, ar and ar^2 and next three term ar^3, ar^4 and ar^5.
Since sum of first 3 terms of GP is 1, thus

$$a + ar + ar^2 = 1$$

$$a(1 + r + r^2) = 1 \quad \ldots(1)$$

Since sum of next 3 terms of GP is 27, thus

$$ar^3 + ar^4 + ar^5 = 27$$

$$ar^3(1 + r + r^2) = 27 \quad \ldots(2)$$

Dividing eq (2) by eq (1) we have

$$\frac{ar^3(1 + r + r^2)}{a(1 + r + r^2)} = 27$$

$$r^3 = 27$$

$$r = 3$$

Now substituting $r = 3$ in eq (1) we have

$$a(1 + 3 + 3^2) = 1$$

$$a(1 + 3 + 9) = 1$$

$$a(13) = 1$$

$$a = \frac{1}{13}$$

Now $$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1}{13}\left[\frac{3^n - 1}{3 - 1}\right]$$

$$= \frac{1}{26}(3^n - 1)$$

Main 2019

25. The first and last term of a Geometric Progression (GP) are 3 and 96 respectively. If the common ratio is 2, then find
(i) n in the number of terms of the GP.
(ii) Sum of the n terms.

Sol :

We have $a = 3$, $a_n = 96$ and $r = 2$
(i) n in the number of terms of the GP.
nth term of a GP is $a_n = ar^{n-1}$

$$96 = 3(2)^{n-1}$$

$$32 = 2^{n-1}$$

$$2^5 = 2^{n-1}$$

Equating the power 2, we get

$$5 = n - 1$$

$$n = 6$$

Hence, the number of terms in a GP is 6.
(ii) Sum of the n terms.
Sum of n terms of a GP is

$$S_n = \frac{a(r^n - 1)}{(r - 1)},\ r > 1$$

$$= \frac{3(2^6 - 1)}{2 - 1}$$

$$= \frac{3 \times (64 - 1)}{1}$$

$$= 3 \times 63$$

$$= 189$$

Comp 2019

26. If the fourth, seventh and tenth terms of a GP. are x, y, z respectively, prove that x, y, z are in GP

Sol :

Let a be the first term and r be the common ratio of given GP.
In given GP, we have

$$a_4 = x,\ a_7 = y,\ a_{10} = z$$

Let a be the first term and r be the common ratio.

From $a_n = ar^{n-1}$ we can write

$$a_3 = ar^3 = x$$

$$a_7 = ar^6 = y$$

and $$a_{10} = ar^9 = z$$

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$$10^{2bx+20} = 10^{ax+10+cx+10}$$

$$10^{2bx+20} = 10^{ax+cx+20}$$

Comparing, above we get

$$2bx+20 = ax-cx+20$$

$$2bx = ax+cx$$

$$2b = a+c \qquad \text{which is true}$$

This 10^{ax+10}, 10^{bx+10}, 10^{cx+10}, $x \neq 0$ are in GP.

37. If a a^2+2 and a^3+10 are in GP, then find the values(s) of a.

Sol :

Since a a^2+2 and a^3+10 are in GP, we have

$$(a^2+2)^2 = a(a^3+10)$$

$$a^4+4a^2+4 = a^4+10a$$

$$4a^2-10a+4 = 0$$

$$2a^2-5a+2 = 0$$

$$2a^2-a-4a+2 = 0$$

$$a(2a-1)-2(2a-1) = 0$$

$$(2a-1)(a-2) = 0$$

Thus $a = 2$ or $\frac{1}{2}$

38. The first and the second terms of a GP are x^{-4} and x^m. If its 8^{th} term is x^{52}, then find the value of m.

Sol :

Let a be the first term and r be the common ratio of given GP.

First term, $a = x^{-4}$ ...(1)

Second term, $a_2 = x^m$

Common ratio, $r = \frac{a_2}{a_1} = \frac{x^m}{x^{-4}} = x^{m+4}$ ...(2)

Eighth term $a_8 = x^{52}$

Now, $a_8 = ar^7$

$$x^{52} = x^{-4} \times r^7$$

$$= x^{-4} \times x^{7(m+4)}$$

$$= x^{-4+7m+28}$$

$$x^{52} = x^{7m+24}$$

On comparing, we have

$$52 = 7m+24$$

$$7m = 52-24 = 28$$

$$m = \frac{28}{7} = 4$$

Hence $m = 4$

39. Find the geometric progression whose 4^{th} term is 54 and the 7^{th} term is 1458.

Sol :

Let a be the first term and r be the common ratio.

Now we have $a_4 = 54$ and $a_7 = 1458$

Now $a_n = ar^{n-1}$

From $a_4 = 54$ we have

$$54 = ar^3 \qquad (1)$$

From $a_7 = 1458$ we have

$$1458 = ar^6 \qquad (2)$$

Dividing eq (2) we eq (1), we have

$$\frac{1458}{54} = \frac{ar^6}{ar^3}$$

$$27 = r^3$$

$$3^3 = r^3$$

$$r = 3$$

Substituting $r = 3$ in eq (1) we have

$$54 = a27$$

$$a = \frac{54}{27} = 2$$

Hence required GP is a, ar, ar^2, ...or 2, 6, 18, 54, ...

40. If a, b, c are in GP show that $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are also in GP

Sol :

Since a, b, c are in GP, we can write

$$b^2 = ac$$

$$\frac{1}{b^2} = \frac{1}{ac}$$

$$\left(\frac{1}{b}\right)^2 = \frac{1}{a} \times \frac{1}{c}$$

From above eqaution we get that $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are also in GP. Hence proved.

41. The terms of a GP with first term a and common ratio r are squared. Prove that resulting numbers form a GP. Find its first term, common ratio and the n^{th} term.

Sol :

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31. The first term of a GP is 1 and the sixth term is 1024. Find the common ratio and insert 4 terms between them.

Sol :

Let a be the first term and r be the common ratio.

Thus $a = 1,\ a_6 = 1024$

Since 6th term is 1024, we have

$$ar^{6-1} = 1024$$
$$(1)\,r^5 = 1024$$
$$r^5 = 4^5$$

Comparing both sides, we get $r = 4$.

Now
$$a_2 = ar^{2-1} = ar = 1 \times 4 = 4$$
$$a_3 = ar^{3-1} = ar^2 = 1 \times (4)^2 = 16$$
$$a_4 = (1)\,(4)^{4-1} = (4)^3 = 64$$
$$a_5 = (4)^4 = 256$$

The GP is 4, 16, 64, 256 ...

32. Find the number of terms of a GP whose first term is $\frac{3}{4}$, common ratio is 2 and the last term is 384.

Sol :

Let a be the first term and r be the common ratio of given GP.

Let number of terms in given GP be n.

First term of a GP, $a = \frac{3}{4}$

and common ratio $r = 2$

Last term $l = 384$

Now
$$a_n = ar^{n-1}$$
$$384 = \frac{3}{4}(2)^{n-1}$$
$$2^{n-1} = \frac{384 \times 4}{3}$$
$$512 = 2^9$$
$$n - 1 = 9$$
$$n = 9 + 1 = 10$$

Number of terms in given GP is 10.

33. The 5^{th}, 8^{th} and 11^{th} terms of a GP are p, q and s respectively. Show that $q^2 = ps$.

Sol :

Let a be the first term and r be the common ratio of given GP.

In given GP, we have

$$a^5 = p,\ a_8 = q \text{ and } a_{11} = s$$

Let a be the first term and r be the common ratio.

From $a_n = ar^{n-1}$ we can write

$$a_5 = ar^6 = p$$
$$a_8 = ar^7 = q$$

and $a_{11} = ar^{10} = s$

Now, $q^2 = ps$

$$ar^8 \times ar^8 = ar^5 \times ar^{11}$$
$$a^2 r^{16} = a^2 r^{16}$$

L.H.S. = R.H.S. Hence Proved

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34. If a, b, c are in GP., then show that a^2, b^2, c^2 are also in GP

Sol :

Since a, b, c are in GP, then

$$b^2 = ac \quad \ldots(1)$$

Term a^2, b^2, c^2 will be in GP if

$$(b^2)^2 = a^2 \times c^2$$
$$(ac)^2 = a^2 c^2 \quad \text{[From (1)]}$$
$$a^2 c^2 = a^2 c^2$$

Thus a^2, b^2, c^2 are in GP.

35. If a, b, c are in A.P., then show that 3^a, 3^b, 3^c are in GP

Sol :

Since a, b and c are in AP we have

$$2b = a + c$$

Now 3^a, 3^b, 3^c will be in GP if

$$(3b)^2 = 3^a \cdot 3^c$$
$$3^{2b} = 3^{a+c}$$

Comparing, above we get $2b = a + c$, which is true.

Thus 3^a, 3^b, 3^c are in GP.

36. If a, b, c are in AP, then show that 10^{ax+10}, 10^{bx+10}, 10^{cx+10}, $x \neq 0$, are in GP

Sol :

Since a, b and c are in AP we have

$$2b = a + c$$

Now 10^{ax+10}, 10^{bx+10}, 10^{cx+10}, $x \neq 0$ are in GP if

$$(10^{bx+10})^2 = (10^{ax+10}) \times (10^{cx+10})$$

$$2^{12} = 2^n$$

Comparing, we get $n = 12$. Thus 12 terms will make sum 4095.
Thus (c) is correct option.

20. What is the 6th term from the end of the GP 3, -6, 12, -24, ..., 12288.
(a) 284 (b) 384
(c) -384 (d) -284

Sol :

Given series is 3, -6, 12, -24, ..., 12288

Here, $a = 3$,

$$r = -6 \div 3 = -2,$$

$$= 12288$$

Now, 6th term from the end

$$= l \times \left(\frac{1}{r}\right)^{n-1}$$

$$= 12288 \times \left(\frac{1}{-2}\right)^{6-1}$$

$$= 12288 \times \frac{1}{(-2)^5}$$

$$= \frac{12288}{-32}$$

$$= -384$$

Thus (c) is correct option.

21. The fourth term of a GP is square of its second term and the first term is -3. Which of the following is the sixth term ?
(a) 829 (b) 529
(c) 629 (d) 729

Sol :

Let a be the first term and r be the common ratio.

Here $a = -3$ and $a_4 = (T_2)^2$

Since fourth term of a GP is square of its second term,

$$a_4 = (a_2)^2$$

$$ar^3 = (ar)^2$$

$$ar^3 = a^2r^2$$

$$r = a = -3$$

Now $a_6 = ar^{6-1}$

$$= ar^5$$

$$= -3 \times (-3)^5$$

$$= 729$$

Thus (d) is correct option.

22. Assertion (A) : The sum of first 10 terms of the GP 1, 2, 4, 8, 16, ... will make a sum of 1024?
Reason (R) : Sum of first n term of GP is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Sol :

Given GP is 1, 2, 4, 8, 16, ...

Here $a = 1,\ r = 2$

Now $S_n = \frac{a(r^n - 1)}{r - 1}$ when $r > 1$

$$= \frac{1(2^{10} - 1)}{2 - 1}$$

$$= 2^{10} - 1$$

$$= 1024 - 1$$

$$= 1023$$

Thus assertion is true reason are true and reason is correct explanation of assertion.
Thus (a) is correct option

THREE MARKS QUESTIONS

Main 2020

23. The 4th, 6th the last term of a geometric progression are 10, 40 and 640, respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series.

Sol :

Let the first term and common ratio of GP be a and r, respectively and total number of terms be n.
According to the question, we have

4th term $a_4 = ar^{4-1}$

$$10 = ar^3 \quad (1)$$

6th term $a_6 = ar^{6-1}$

$$40 = ar^5 \quad (2)$$

and last term $a_n = ar^{n-1}$

$$640 = ar^{n-1} \quad (3)$$

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If first term is a and common ratio is r then initial term of GP are a, ar, ar^2.
Squaring, we get a^2, a^2r^2, a^2r^4

Here $(a^2r^2)^2 = a^2 \times a^2r^4$

$$a^4r^4 = a^4r^4$$

Thus $B^2 = AC$

Thus a^2, a^2r^2, a^2r^4 are in GP and the first term is a^2. Common ratio is r^2
The nth term will be

$$a_n = ar^{n-1}$$
$$= a^2(r^{n-1})^2$$
$$= a^2r^{2n-2}$$

42. How many terms of the GP : 3, 6, 2, ..., will give a sum 3069?

Sol :

Given GP is 3, 6, 2, ...,

Here $a = 3,\ r\ \ r = \frac{6}{3} = 2$

Let n be the terms in GP which gives sum 3096.
Let number of terms $= n$

Now $S_n = \dfrac{a(r^n - 1)}{r - 1}$ when $r > 1$

$$3069 = \frac{3(2^n - 1)}{2 - 1}$$

$$3069 = 3(2^n - 1)$$

$$\frac{3069}{3} = 2^n - 1$$

$$1023 = 2^n - 1$$

$$1024 = 2^n$$

$$2^{10} = 2^n$$

Comparing, we get $n = 10$. Thus 10 terms will make sum 3069.

43. How many terms of the GP 3, 3^2, 3^3, ... are needed to get the sum 1092?

Sol :

Given GP is 3, 3^2, 3^3, ...

Here, $a = 3,\ r = 3$

Let n be the terms in GP which gives sum 1092.

Now $S_n = \dfrac{a(r^n - 1)}{r - 1}$ when $r > 1$

$$1092 = \frac{3(3^n - 1)}{3 - 1}$$

$$1092 = \frac{3(3^n - 1)}{2}$$

$$\frac{1092 \times 2}{3} = 3^n - 1$$

$$728 = 3^n - 1$$

$$729 = 3^n$$

$$3^6 = 3^n$$

Comparing, we get $n = 6$. Thus 6 terms will make sum 1092.

44. How many terms of the GP $\sqrt{5}$, 5, $5\sqrt{5}$, 25, ... will give a sum of $780 + 156\sqrt{5}$?

Sol :

Given GP is $\sqrt{5}$, 5, $5\sqrt{5}$, 25, ...

Here $a = \sqrt{5}.\ d = \sqrt{5}$

Let n be the terms in GP which gives sum $780 + 156\sqrt{5}$.

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{when } r > 1$$

$$780 + 156\sqrt{5} = \frac{\sqrt{5}(\sqrt{5^n} - 1)}{\sqrt{5} - 1}$$

$$(\sqrt{5} - 1)(780 + 156\sqrt{5}) = \sqrt{5}(\sqrt{5^n} - 1)$$

$$780\sqrt{5} - 780 + 156 \times 5 - 156\sqrt{5} = \sqrt{5}(\sqrt{5^n} - 1)$$

$$624\sqrt{5} - 780 + 780 = \sqrt{5}(\sqrt{5^n} - 1)$$

$$624\sqrt{5} = \sqrt{5}(\sqrt{5^n} - 1)$$

$$624 = \sqrt{5^n} - 1$$

$$625 = \sqrt{5^n}$$

$$5^4 = 5^{n/2}$$

Comparing, we get $\frac{n}{2} = 4 \Rightarrow n = 8$. Thus 8 terms will make sum $780 + 156\sqrt{5}$.

45. The sum of n terms of a GP is 2295. Its first term is 9 and the common ratio is 2. Find the number of terms and its fifth term.

Sol :

Let a be the first term and r be the common ratio.
We have $S_n = 2295,\ a = 9,\ r = 2$

Now $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$$2295 = \frac{9(2^n - 1)}{2 - 1}$$

$$2295 = 9(2^n - 1)$$

$$255 = 2^n - 1$$

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$$256 = 2^n$$
$$2^8 = 2^n$$

Comparing both sides, we get $n = 8$.

Now $a_5 = ar^{5-1}$
$= ar^4$
$= 9 \times (2)^4$
$= 9 \times 16 = 144$

46. The sum of first 3 terms of a GP. is $\frac{61}{20}$ and their product is 1. Find the first 3 terms.

Sol :

Let the 3 terms of GP be $\frac{a}{r}$, a, ar.

Since their product is 1, thus

$$\frac{a}{r} \times a \times ar = 1$$
$$a^3 = 1$$
$$a = 1$$

Since sum of first 3 terms $\frac{61}{20}$, we have

$$\frac{1}{r} + 1 + r = \frac{61}{20}$$
$$\frac{r^2 + r + 1}{r} = \frac{61}{20}$$
$$20r^2 + 20r + 20 = 61r$$
$$20r^2 + 20 = 61r$$
$$20r^2 - 41r + 20 = 0$$
$$20r^2 - 25r - 16r + 20 = 0$$
$$5r(4r-5) - 4(4r-5) = 0$$
$$(5r - r)(4r - 5) = 0$$

Thus $r = \frac{4}{5}$ or $\frac{5}{4}$

The 3 numbers are $\frac{4}{5}$, 1, $\frac{5}{4}$, or $\frac{5}{4}$, 1, $\frac{4}{5}$

47. Find S_n and the sum of the first 6 terms of the GP 256, 128, 64, ...

Sol :

Let a be the first term and r be the common ratio.

Here GP is 256, 128, 64, ...

Now $a = 256$, $r = \frac{128}{256} = \frac{1}{2}$

$$S_n = a\frac{(1 - r^n)}{1 - r} \qquad (r < 1)$$
$$= 256\frac{\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}}$$
$$= \frac{256}{\frac{1}{2}}\left[1 - \frac{1}{2^n}\right]$$
$$= \frac{256}{1} \times \frac{2}{1}\left[1 - \frac{1}{2^n}\right]$$
$$= 512\left(1 - \frac{1}{2^n}\right)$$

$$S_6 = 256\frac{\left[1 - \left(\frac{1}{2}\right)^6\right]}{1 - \frac{1}{2}}$$
$$= \frac{256\left(1 - \frac{1}{2^6}\right)}{\frac{1}{2}}$$
$$= 256 \times \frac{2}{1}\left(1 - \frac{1}{64}\right)$$
$$= 512 \times \frac{64 - 1}{64}$$
$$= 512 \times \frac{63}{64}$$
$$= 504$$

48. In a GP, the first term is 729, 7^{th} term is 64, calculate S_7.

Sol :

Let a be the first term and r be the common ratio.

Now $a = 729$ and $a_7 = 64$

Now $a_7 = ar^{n-1}$
$64 = 729\, r^6$
$r^6 = \frac{64}{729} = \left(\frac{2}{3}\right)^6$
$r = \frac{2}{3}$

$$S_7 = \frac{a(1 - r^n)}{1 - r} \qquad (r < 1)$$
$$= \frac{729\left[1 - \left(\frac{2}{3}\right)^7\right]}{1 - \frac{2}{3}}$$
$$= \frac{729\left[1 - \frac{128}{2187}\right]}{\frac{1}{3}}$$
$$= \frac{729 \times 3 \times (2187 - 128)^{\frac{1}{3}}}{2187}$$
$$= 2059$$

49. Find the sum of the GP 0.15, 0.015, 0.0015, ... up to 30 terms.

Sol :

Let a be the first term and r be the common ratio.

$$a = 0.15,\ r = \frac{0.15}{0.015} = \frac{1}{10}$$

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Now $S_{30} = \frac{a(1-r^{30})}{1-r}$ $(r<1)$

$= \frac{0.15\left[1-\left(\frac{1}{10}\right)^{30}\right]}{1-\frac{1}{10}}$

$= \frac{0.15\times 10\left[1-\frac{1}{10^{30}}\right]}{9}$

$= \frac{1.5}{9}\left[1-\frac{1}{10^{30}}\right]$

$= \frac{1}{6}\left[1-\frac{1}{10^{30}}\right]$

50. Find the number of terms and find their sums in GP $1+3+9+...+2187$

Sol :

Let a be the first term, r be the common ratio and n be the number of term.

Here, $a=1,\ r=\frac{3}{1}=3,\ a_n=2187$

Now $a_n = ar^{n-1}$

$2187 = 1(3)^{n-1}$

$3^7 = (3)^{n-1}$

$3^7 = (3)^{n-1}$

Comparing both side, we get

$n-1=7$

$n=8$

$S_n = \frac{a(r^n-1)}{r-n}$

$= \frac{1(3^8-1)}{3-1}$

$= \frac{3^8-1}{2}$

$= \frac{6561-1}{2} = 3280$

51. Find the number of terms and find their sums in GP $3+6+12+...+1536$

Sol :

Let a be the first term, r be the common ratio and n be the number of term.

Here, $a=3,\ r=\frac{6}{3}=2$ and $a_n=1536$

Now $a_n = ar^{n-1}$

$1536 = (3)(2)^{n-1}$

$3\times 512 = 3(2)^{n-1}$

$512 = (2)^{n-1}$

$(2)^9 = (2)^{n-1}$

Comparing both sides, we get

$n-1=9$

$n=10$

Now $S_{10} = \frac{a(r^n-1)}{r-1} = \frac{3(2^{10}-1)}{2-1}$

$= \frac{3(1024-1)}{1}$

$= 3(1023) = 3069$

52. Find the number of terms and find their sums in GP $1+(-2)+4+(-8)+...+(-2048)$

Sol :

Let a be the first term, r be the common ratio and n be the number of term.

Here, $a=1,\ r=\frac{a_2}{a_1}=\frac{-2}{1}=-2$

$a_n = ar^{n-1}$

$-2048 = (1)(-2)^{n-1}$

$= (-2)^{n-1}$

$(-2)^{11} = (-2)^{n-1}$

Comparing both sides, we get

$11 = n-1$

$n=12$

$S_{12} = \frac{a(r^n-1)}{r-1}$

$= \frac{1[(-2)^{12}-1]}{-2-1}$

$= \frac{4096-1}{-3}$

$= \frac{4095}{-3} = -1365$

53. First term of a GP is 576 and seventh term is 9. Find the common ratio and write the GP.

Sol :

Let a be the first term and r be the common ratio.

Here $a=576$ and $a_7=9$

Now $a_7 = ar^6$

$9 = 576r^6$

$1 = 64r^6$

$r^6 = \frac{1}{64} = \left(\frac{1}{2}\right)^6$

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Thus $r = \frac{1}{2}$

Now $a_2 = 576 \times \frac{1}{2} = 288$

$a_3 = 288 \times \frac{1}{2} = 144$

$a_4 = 144 \times \frac{1}{2} = 72$

$a_5 = 72 \times \frac{1}{2} = 36$

$a_6 = 36 \times \frac{1}{2} = 18$

GP is 576, 288, 144, 72, 36, 18, 9

54. In a GP the third term is 24 and the sixth term is 192. Find the tenth term.

Sol :

Let a be the first term and r be the common ratio.

Here $a_3 = 24$ and $a_6 = 192$

Now $a_3 = ar^{3-1}$

$24 = ar^2$ (1)

$a_6 = ar^{6-1}$

$192 = ar^5$ (2)

Dividing eq (2) by eq (1) we have

$8 = r^3$

$2^3 = r^3$

Comparing both sides, we get $r = 2$.
Substituting $r = 2$ in eq (1) we have

$24 = a2^2$

$24 = 4a$

$6 = a$

NNow $a_{10} = ar^9$

$= 6 \times 2^9$

$= 6 \times 512$

$= 3072$

55. The second term of a GP is $\frac{9}{4}$ and the eighth term is $\frac{16}{81}$. Find the GP.

Sol :

Let a be the first term and r be the common ratio.

Here $a_2 = \frac{9}{4}$ and $a_8 = \frac{16}{81}$

Now $a_2 = ar^{n-1}$

$\frac{9}{4} = ar$ (1)

and $a_8 = ar^{8-1}$

$\frac{16}{81} = ar^7$

Dividing eq (2) by eq (1) we have

$\frac{\frac{16}{81}}{\frac{9}{4}} = r^6$

$\frac{16 \times 4}{81 \times 9} = r^6$

$\frac{4 \times 4 \times 4}{9 \times 9 \times 9} = r^6$

$\frac{4^3}{9^3} = r^6$

$\frac{2^6}{3^6} = r^6$

Comparing both sides, we get $r = \frac{2}{3}$.
Substituting $r = \frac{2}{3}$ in eq (1) we have

$\frac{9}{4} = a \times \frac{2}{3}$

$a = \frac{9}{4} \times \frac{3}{2} = \frac{27}{8}$

Thus GP is $\frac{27}{8}, \frac{9}{4}, \frac{3}{2}, 1, \frac{2}{3}, \ldots$

56. Find the number of terms of a GP whose first term is $\frac{3}{4}$ and the common ratio is 2 and the last term is 384.

Sol :

Let a be the first term, r be the common ratio and n be the number of term in GP.

Here $a = \frac{3}{4}$ and $r = 2$ and $a_n = 384$

Now last term $a_n = ar^{n-1}$

$384 = \frac{3}{4} \times 2^{n-1}$

$128 = \frac{1}{4} \times 2^{n-1}$

$128 \times 4 = 2^{n-1}$

$2^7 \times 2^2 = 2^{n-1}$

$2^9 = 2^{n-1}$

Comparing both sides, we get,

$n - 1 = 9$

$n = 10$

57. If the third term of a GP is $\frac{5}{2}$ and the seventh term is $\frac{8}{125}$, find the GP.

Sol :

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Let a be the first term and r be the common ratio.

Here $a_3 = \frac{5}{2}$ and $a_7 = \frac{8}{125}$

Now $a_3 = ar^{3-1}$

$$\frac{5}{2} = ar^2 \quad ...(1)$$

and $a_7 = ar^{7-1}$

$$\frac{8}{125} = ar^6 \quad ...(2)$$

Dividing eq (2) by eq (1) we have

$$\frac{8}{125} \div \frac{5}{2} = \frac{ar^6}{ar^2}$$

$$\frac{16}{625} = r^4$$

$$\left(\frac{2}{5}\right)^4 = r^4$$

Comparing both sides, we get $r = \frac{2}{5}$.
Substituting $r = \frac{2}{5}$ in eq (1) we have

$$\frac{5}{2} = a\left(\frac{2}{5}\right)^2$$

$$\frac{5}{2} \times \frac{5^2}{2^2} = a$$

$$a = \frac{5}{2} \times \frac{25}{4} = \frac{125}{8}$$

Thus GP is $\frac{125}{8}$, $\frac{25}{4}$, $\frac{5}{2}$, 1, $\frac{2}{5}$, ...

58. In a GP the fifth term is 162 and the eighth term is 4374. Find the GP.

Sol :

Let a be the first term and r be the common ratio.

Here $a_5 = 162$ and $a_8 = 4374$

Now $a_5 = ar^{5-1}$

$$162 = ar^4 \quad (1)$$

and $a_8 = ar^{8-1}$

$$4374 = ar^7$$

Dividing eq (2) by eq (1) we have

$$\frac{4374}{162} = \frac{ar^7}{ar^4}$$

$$27 = a^3$$

$$3^3 = a^3$$

Comparing both sides, we get $r = 3$.
Substituting $r = 3$ in eq (1) we have

$$162 = a(3)^4$$

$$162 = a \times 81$$

$$a = \frac{162}{81} = 2$$

Thus GP is 2, 6, 8, 54 ....

59. The fifth term of a GP is 48 and the eighth term is 384. Find its 12^{th} term.

Sol :

Let a be the first term and r be the common ratio.

Here $a_5 = 48$ and $a_8 = 384$

Now $a_5 = ar^{5-1}$

$$48 = ar^4 \quad (1)$$

and $a_8 = ar^{8-1}$

$$384 = ar^7$$

Dividing eq (2) by eq (1) we have

$$\frac{384}{48} = \frac{ar^7}{ar^4}$$

$$8 = a^3$$

$$2^3 = a^3$$

Comparing both sides, we get $r = 2$.
Substituting $r = 2$ in eq (1) we have

$$48 = a(2)^4$$

$$48 = a \times 16$$

$$a = \frac{48}{16} = 3$$

Now $a_{12} = ar^{12-1}$

$$= ar^{11}$$

$$= 3(2)^{11}$$

$$= 3 \times 2048$$

$$= 6144$$

FOUR MARK QUESTIONS

60. The sum of first three terms of a GP is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Sol :

Let a be the first term and r be the common ratio then first three term are a, ar and ar^2.
Since product uf first three terms of GP is 1, we have

$$\frac{a}{r} \times a \times ar = 1$$

$$a^3 = 1$$

$$a = 1 \quad (1)$$

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$$a = \frac{15}{3} = 5$$

Thus we get $5-d$, 5 and $5+d$. By adding 1, 4, 19 in them we get $6-d$, 9 and $24+d$
Now $6-d$, 9 and $24+d$ are in GP.
Thus

$$9^2 = (6-d)(24+d)$$
$$81 = 144 - 18d + d^2$$
$$d^2 - 18d + 63 = 0$$
$$d^2 + 21d - 3d - 63 = 0$$
$$d(d+21) - 3(d+21) = 0$$
$$(d+21)(d-3) = 0$$

Thus $d = -21$ and 3.
If $d = 3$ and $a = 5$, then numbers becomes,
$5-3,\ 5,\ 5+3$ i.e. 2, 5, 8
If $d = -21$, then numbers becomes ;
$5+21,\ 5,\ 5-21$ i.e 26, 5, -16

66. If a, b, c are in A.P. as well in GP., prove that $a = b = c$.

Sol :

Given that a, b, c are in A.P. as well as in GP
Since a, b, c are in AP, we can write

$$2b = a + c$$
$$b = \frac{a+c}{2} \qquad \ldots(1)$$

Since a, b, c are in GP, we can write

$$b^2 = ac \qquad \ldots(2)$$

From (1) and (2) we have

$$\left(\frac{a+c}{2}\right)^2 = ac$$
$$(a+c)^2 = 4ac$$
$$(a+c)^2 - 4ac = 0$$
$$a^2 + 2bc + b^2 - 4ac = 0$$
$$a^2 - 2bc + b^2 = 0$$
$$(a-c)^2 = 0$$
$$a - c = 0$$

Thus $a = c$ ...(3)
Substituting $a = c$ in eq (1) we have

$$2b = a + c$$
$$= a + a$$
$$2b = 2a$$

Thus $b = a$ ...(4)
From (3) and (4), we have

$$a = b = c$$ Hence Proved

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$$2b = a + c$$
$$= a + a$$
$$2b = 2a$$

Thus $b = a$ ...(4)

From (3) and (4), we have

$$a = b = c$$

Now $b - c = 0$,

$c - a = 0$

and $a - b = 0$

Thus $a^{b-c} + b^{c-a} + c^{a-b} = a^0 + b^0 + c^0$

$= 1 + 1 + 1$

$= 3$

63. The 4^{th}, 7^{th} and the last terms of a GP are 54, 1458, 13122 respectively. Find the common ratio, first term, the number of terms and their sum.

Sol :

Let a be the first term and r be the common ratio.

We have $a_4 = 54$, $a_7 = 1458$, $a_n = 13122$

$ar^3 = 54$ ...(1)

$ar^6 = 1458$ ...(2)

Dividing eq (2) from eq (1), we get

$$\frac{ar^6}{ar^3} = \frac{1458}{54}$$
$$r^3 = 27$$
$$r^3 = 3^3$$
$$r = 3 \quad ...(3)$$

Substituting the value of r in (1) we have

$$a(3)^3 = 54$$
$$a = \frac{54}{27} = 2$$

Now last term i.e n term is

$$ar^{n-1} = 13122$$
$$2 \times 3^{n-1} = 13122$$
$$3^{n-1} = 6561$$
$$3^{n-1} = 3^8$$

Comparing above we have

$$n - 1 = 8$$
$$n = 9$$

Thus given GP has 9 terms.

Now $S_n = \frac{a(r^n - 1)}{r - 1}$

$$S_9 = \frac{a(r^8 - 1)}{r - 1}$$
$$= \frac{2(3^9 - 1)}{3 - 1}$$
$$= \frac{2(3^9 - 1)}{2}$$
$$= 3^9 - 1$$
$$= 19683 - 1 = 19682$$

64. Four numbers are in GP in which the third term is greater than the first by 9 and the second term is greater than the fourth term by 18. Find the numbers.

Sol :

Let a be the first term and r be the common ratio. Then we can write four term as a, ar, ar^2 and ar^3 Since the third term is greater than the first by 9, we have

$$ar^2 - a = 9$$
$$a(r^2 - 1) = 9 \quad ...(1)$$

Since the second term is greater than the fourth term by 18, we have

$$ar - ar^3 = 18$$
$$ar(1 - r^2) = 18 \quad ...(2)$$

Dividing eq (2) by eq (1) we have

$$\frac{ar(1 - r^2)}{a(r^2 - 1)} = \frac{18}{9}$$
$$\frac{-r(r^2 - 1)}{r^2 - 1} = 2$$
$$r = -2$$

Substitutng $r = -2$ in eq (1) we get

$$a(4 - 1) = 9$$
$$a = 3$$

Thus numbers are 3, -6, 12, -24

65. Three numbers are in A.P. and their sum is 15. If 1, 4 and 19 are added to these numbers respectively, the resulting numbers are in GP Find the numbers.

Sol :

Let $a - d$, a, $a + d$ be the three numbers in A.P.

Since their sum is 15, we have

$$a - d + a + a + d = 15$$
$$3a = 15$$

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Since sum of first three terms of GP is $\frac{39}{10}$ we have

$$\frac{a}{r}+a+ar=\frac{39}{10}$$

$$a\left(\frac{1}{r}+1+r\right)=\frac{39}{10} \quad ...(2)$$

Substitute the value of a in eqn. (2), we get

$$1\left(\frac{1}{r}+1+r\right)=\frac{39}{10}$$

$$\frac{1+r+r^2}{r}=\frac{39}{10}$$

$$10+10r+10r^2=39r$$

$$10r^2+10r-39r+10=0$$

$$10r^2-29r+10=0$$

$$10r^2-25r-4r+10=0$$

$$5r(2r-5)-2(2r-5)=0$$

$$(2r-5)(5r-2)=0$$

Hence $r=\frac{5}{2}$ or $\frac{2}{5}$

If we take $r=\frac{5}{2}$, then terms of GP be 1, $\frac{5}{2}$, $\frac{25}{4}$, $\frac{125}{8}$, ...

If we take $r=\frac{2}{5}$, then terms of GP will be 1, $\frac{2}{5}$, $\frac{4}{25}$, $\frac{8}{125}$, ...

61. Three numbers whose sum is 70 are in GP If each of the extremes is multiplied by 4 and the mean by 5, the numbers will be in A.P. Find the numbers.

Sol :

Let $\frac{a}{r}$, a, ar be three numbers in GP.

Since their sum is 70, we have

$$\frac{a}{r}+a+ar=70$$

$$a\left(\frac{1}{r}+1+r\right)=70 \quad ...(1)$$

By multiplying the extremes by 4 and mean by 5, then numbers becomes $\frac{a}{r}\times 4$, $a\times 5$ and $ar\times 4$ i.e $\frac{4a}{r}$, $5a$, $4ar$

But these are in AP. Thus

$$2(5a)=\frac{4a}{r}+4ar$$

$$10a=4a\left(\frac{1}{r}+r\right)$$

$$5=2\left(\frac{1}{r}+r\right)$$

$$5r=2+2r^2$$

$$2r^2-5r+2=0$$

$$2r^2-r-4r+2=0$$

$$r(2r-1)-2(2r-1)=0$$

$$(2r-1)(r-2)=0$$

Thus $r=\frac{1}{2}$ or $r=2$.

Substituting $r=2$ in (1), we have

$$a\left(\frac{1}{2}+1+2\right)=70$$

$$\frac{7}{2}a=70$$

$$a=70\times\frac{2}{7}=20$$

Substituting $r=\frac{1}{2}$ in (1), we have

$$a\left(2+1+\frac{1}{2}\right)=70$$

$$\frac{7}{2}a=70$$

$$a=70\times\frac{2}{7}=20$$

Thus in both case we have $a=20$.

If $r=2$, numbers are

$\frac{20}{2}$, 20, 20×2 i.e., 10, 20, 40

If $r=\frac{1}{2}$, then numbers are

$\frac{20}{\frac{1}{2}}$, 20, $20\times\frac{1}{2}$ i e $\frac{20\times 2}{1}$, 20, $20\times\frac{1}{2}$

i.e. 40, 20, 10.

62. If a, b, c are in A.P. as well as in GP, then find the value of $a^{b-c}+b^{c-a}+c^{a-b}$.

Sol :

Given that a, b, c are in A.P. as well as in GP

Since a, b, c are in AP, we can write

$$2b=a+c$$

$$b=\frac{a+c}{2} \quad ...(1)$$

Since a, b, c are in GP, we can write

$$b^2=ac \quad ...(2)$$

From (1) and (2) we have

$$\left(\frac{a+c}{2}\right)^2=ac$$

$$(a+c)^2=4ac$$

$$(a+c)^2-4ac=0$$

$$a^2+2bc+b^2-4ac=0$$

$$a^2-2bc+b^2=0$$

$$(a-c)^2=0$$

$$a-c=0$$

Thus $a=c$ ...(3)

Substituting $a=c$ in eq (1) we have

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CHAPTER 11

REFLECTION

MULTIPLE CHOICE QUESTIONS

1. The reflection of the point $P(-2, 5)$ in the x-axis is:

(a) $(2, 5)$ (b) $(2, -5)$
(c) $(-2, -5)$ (d) $(-2, 0)$

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$.
The reflection of the point $(-2, 5)$ in the x-axis is $(-2, -5)$.
Thus (c) is correct option.

2. The reflection of the point $P(-2, 5)$ in the y-axis is:

(a) $(2, 5)$ (b) $(2, -5)$
(c) $(-2, -5)$ (d) $(-2, 0)$

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$.
Hence, the reflection of the point $(-2, 5)$ in the y-axis is $(2, 5)$.
Thus (a) is correct option.

3. If the image of the point P under reflection in the x-axis is $(-3, 2)$, then the co-ordinates of the point P are:

(a) $(3, 2)$ (b) $(-3, -2)$
(c) $(3, -2)$ (d) $(-3, 0)$

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Since $P'(x, -y)$ is $(-3, 2)$, $P(x, y)$ will be $(-3, -2)$.
Thus (b) is correct option.

4. The reflection of the point $P(-3, 5)$ in the origin is:

(a) $(-3, 5)$ (b) $(3, 5)$
(c) $(-3, -5)$ (d) $(3, -5)$

Sol :

Reflection of a point $P(x, y)$ in the origin $O(0, 0)$ is $P(-x, -y)$.
Thus reflection of the point $P(-3, 5)$ in the origin is the point $(3, -5)$.
Thus (d) is correct option.

5. The point $P(3, -2)$ is reflected in the line $y = -1$, the image has co-ordinates:

(a) $(-3, -2)$ (b) $(3, -4)$
(c) $(3, 4)$ (d) $(3, 0)$

Sol :

Reflection of the point $P(x, y)$ in the line $y = a$ is the point $P'(x, -y + 2a)$.
Here, $x = 3$, $y = -2$ and $a = -1$
Thus $P'[3, -(-2) + 2(-1)]$ or $P'(3, 0)$
Thus (d) is correct option.

6. The co-ordinates of the point $(3, -2)$ after reflection in the x-axis are respectively:

(a) $(3, 2)$ (b) $(-3, 2)$
(c) $(3, -2)$ (d) $(-3, -2)$

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $(x, -y)$.
The reflection of the point $(3, -2)$ in the x-axis is $(3, 2)$.
Thus (a) is correct option.

7. The co-ordinates of the point $(-4, 7)$ after reflection in the origin are:

(a) $(4, 7)$ (b) $(4, -7)$
(c) $(-4, 7)$ (d) $(-4, -7)$

Sol :

Reflection of a point $P(x, y)$ in the origin $O(0, 0)$ is $P(-x, -y)$.
Thus reflection of the point $P(-4, 7)$ in the origin is the point $(4, -7)$.
Thus (b) is correct option.

8. The co-ordinates of the point $(7, -6)$ after

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reflection with y-axis are:
(a) $(-7, 6)$ (b) $(7, -6)$
(c) $(-7, -6)$ (d) $(7, 6)$

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$.
Hence, the reflection of the point $(7, -6)$ in the y-axis is $(-7, 6)$.
Thus (c) is correct option.

9. The image of the point (x, y) in the line $x = 0$ is $(-2, 7)$. The point (x, y) is:
(a) $(-2, -7)$ (b) $(2, -7)$
(c) $(2, 7)$ (d) $(-2, 7)$

Sol :

Reflection of a point $P(x, y)$ in the line $x = a$ is $P'(-x + 2a, y)$.
Here, $a = 0$

$$P'(-x + 2a, y) = (-2, 7)$$

Thus (c) is correct option.

10. The co-ordinate of the point $(-4, 5)$ after reflection in the line $y = 0$ are:
(a) $(4, 5)$ (b) $(4, -5)$
(c) $(-4, -5)$ (d) none of these

Sol :

The line $y = 0$ means reflection with respect to x-axis.
Reflection of the point $(-4, 5)$ in the x-axis is $(-4, -5)$
Thus (c) is correct option.

11. Point P is reflected in the x-axis to $P'(7, -3)$. The point P is:
(a) $(-3, 5)$ (b) $(-7, 3)$
(c) $(-7, -3)$ (d) $(7, 3)$

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Since $P'(x, -y)$ is $(7, -3)$, $P(x, y)$ will be $(7, 3)$.
Thus (d) is correct option.

12. Co-ordinates of the point $(-2, 3)$ under the reflection in the line $x = 0$ are:
(a) $(2, 3)$ (b) $(-2, -3)$
(c) $(3, 2)$ (d) $(3, -2)$

Sol :

The line $x = 0$ means reflection with respect to y-axis. Reflection of the point $(-2, 3)$ in the x-axis is $(-2, -3)$
Thus (b) is correct option.

13. Point P is reflected in the x-axis. Co-ordinates of its image are $(-4, 9)$. The co-ordinates of are:
(a) $(4, -9)$ (b) $(-4, 9)$
(c) $(-4, -9)$ (d) $(4, 9)$

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Here $P'(x, -y)$ is $(-4, 9)$. So $P(x, y)$ will be $(-4, -9)$.
Thus (c) is correct option.

14. The co-ordinates of point Q obtained from reflection of $P(5, 1)$ in line $y = 3$ are respectively:
(a) $(-5, 1)$ (b) $(5, -1$
(c) $(5, 5)$ (d) $(5, 4)$

Sol :

Reflection of a point $P(x, y)$ in the line $y = a$ is $P'(x, -y + 2a)$
The reflection of the point $P(5, 1)$ in the line $y = 3$ is the point $P'(5, -1 + 2\times 3)$ i.e. $(5, 5)$.
Thus (c) is correct option.

15. Point obtained after reflection of point $(3, 5)$ in the line $y = 1$ is:
(a) $(3, -3)$ (b) $(-3, 4)$
(c) $(-3, 5)$ (d) $(3, 5)$

Sol :

Reflection of a point $P(x, y)$ in the line $y = a$ is $P'(x, -y + 2a)$.
Thus $P'(x, -y + 2a) = P'(3, -5 + 2 \times 1)$

$$= (3, -3)$$

Thus (a) is correct option.

16. A point $P(7, -5)$ is first reflected in x-axis to point Q and then Q is reflected in y-axis to point R. The co-ordinates of point Q and R are respectively :
(a) $(-7, 5)$ and $(7, -5)$
(b) $(-7, 5)$ and $(7, 5)$
(c) $(7, -5)$ and $(-7, 5)$
(d) $(7, 5)$ and $(-7, 5)$

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. The reflection of the point $(7, -5)$ in the x-axis is $(7, 5)$. Thus point Q is $(7, 5)$

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Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$. The reflection of the point Q (7, 5) in the y-axis is (–7, 5). Thus point R is (–7, 5)
Thus (d) is correct option.

17. A point $P(3, -7)$ is first reflected in y-axis to point Q and then Q is reflected in x-axis to point R. The co-ordinates of point Q and R are respectively :
(a) (–3, 7) and (3, –7)
(b) (–3, 7) and (3, 7)
(c) (–3, –7) and (–3, 7)
(d) (3, 7) and (–3, 7)

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$. The reflection of the point $P(3, -7)$ in the y-axis is (–3, –7). Thus point Q is $Q(-3, -7)$
Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. The reflection of the point $Q(-3, -7)$ in the x-axis is (–3, 7). Thus point Q is (–3, 7)
Thus (c) is correct option.

18. A point $P(1, 5)$ is reflected in line $y = 2$ to point Q. Now point Q is reflected in y-axis to point R. The co-ordinates of point R are:
(a) (–1, –1) (b) (–1, 1)
(c) (1, 5) (d) (1, –5)

Sol :

Reflection of a point $P(x, y)$ in the line $y = a$ is $P'(x, -y + 2a)$. Thus point Q is $(1, -5 + 2 \times 2)$ i.e $Q(1, -1)$
When a point is reflected in y-axis its x co-ordinate changes its sign. The reflection of the point (1, –1) in the y-axis is (–1, –1). Thus point R is (–1, –1)
Thus (a) is correct option.

19. The point $P(a, b)$ is first reflected in the origin to Q and then reflected in the y-axis to R. If R has co-ordinates (3, –2) then the value of a and b are
(a) 3 and 2 (b) –3 and 2
(c) 3 and –2 (d) –3 and –2

Sol :

$P(a, b)$ when reflected in origin, gives image $Q(-a, -b)$. Now Q when reflected in y-axis gives image $R(a, -b)$

But $R(a, -b) = R(3, -2)$

$a = 3$ and $b = 2$

Thus (a) is correct option.

20. The point $P(a, b)$ is first reflected in the origin to Q and then reflected in the x-axis to R. If R has co-ordinates (3, –2) then the value of a and b are
(a) 3 and 2 (b) –3 and 2
(c) 3 and –2 (d) –3 and –2

Sol :

$P(a, b)$ when reflected in origin, gives image $Q(-a, -b)$. Now Q when reflected in x-axis gives image $R(-a, b)$

But $R(-a, b) = R(3, -2)$

$a = -3$ and $b = -2$

Thus (d) is correct option.

21. The point $P(5, 7)$ on reflection in x-axis is mapped on P'. Then P' reflection in the origin is mapped as P''. Which of the following is point P''?
(a) (–5, 7) (b) (–5, –7)
(c) (5, 7) (d) (5, –7)

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$.
Thus reflection of $P(5, 7)$ in x-axis is $P'(5, -7)$.
Reflection of the point $P'(x, y)$ in the origin is $P''(-x, -y)$.
Thus reflection of $P'(5, -7)$ in the origin is P'' (–5, 7).
Thus (a) is correct option.

22. The point $P(3, 2)$ on reflection in x-axis is mapped on P'. Then P' reflection in the origin is mapped as P''. Which of the following is point P''?
(a) (–3, 2) (b) (–3, –2)
(c) (3, 2) (d) (3, –2)

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$.
Thus reflection of $P(3, 2)$ in y-axis is $P'(-3, 2)$.
Reflection of the point $P'(x, y)$ in the origin is $P''(-x, -y)$.
Thus reflection of $P'(-3, 2)$ in the origin is P'' (3, –2).
Thus (d) is correct option.

23. If the reflection of the point $P(3, 9)$ in the line $x = a$ is the point (7, 9) the value of a will be
(a) 2 (b) 3
(c) 4 (d) 5

Sol :

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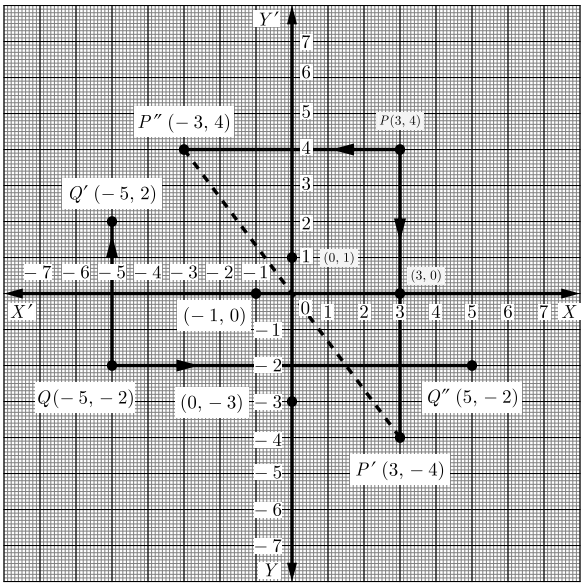


Main 2010

49. Use graph paper for this question. The points $A(2, 1)$, $B(0, 3)$ and $C(-2, -2)$ are the vertices of a triangle.

(i) Plot the points on the graph paper.

(ii) Draw the triangle formed by reflecting these points in the x-axis.

(iii) Are the two triangles congruent?

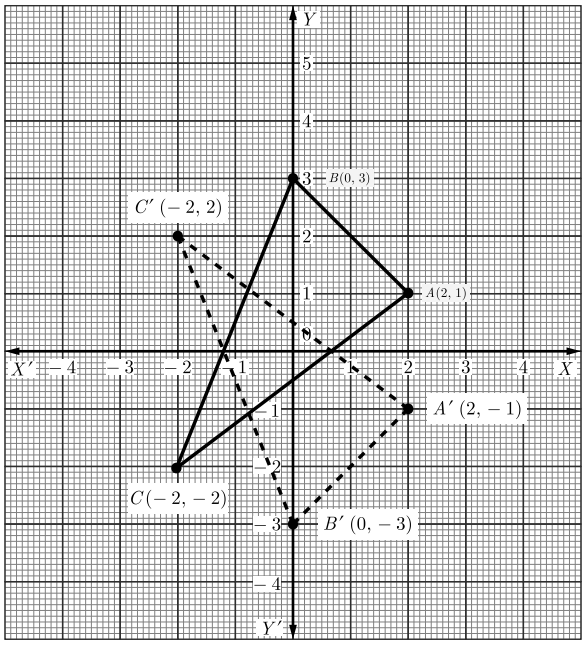
Sol :

Take 1 cm = 1 unit on both axes.

(i) Plot the points $A(2,1)$, $B(0,3)$ and $C(-2,-2)$ as shown in the figure given below.

(ii) The points $A'(2,-1)$, $B'(0,-3)$ and $C'(-2,2)$ are the reflections of the points A, B and C respectively in the x-axis. The triangle formed by the points A', B' and C' has been shown in the figure by dotted lines.

(iii) The two triangles ABC and $A'B'C'$ are congruent (measure the distances and check it).



Main 2009

50. Use graph paper to answer this question.

(i) Plot the points $A(4,6)$ and $B(1,2)$.

(ii) A' is the image of A, when reflected in X -axis.

(iii) B' is the image of B, when B is reflected in the line AA'.

(iv) Give the geometrical name for the figure $ABA'B'$.

Sol :

Take 1 cm = 2 unit on both axes.

(i) The points $A(4,6)$ and $B(1,2)$ on the graph paper are shown in figure below.

(ii) The image of A reflected in x-axis is $A'(4,-6)$ as shown in figure.

(iii) The image of B reflected in the line AA' is $B'(-1+2\times 4,\ 2)$ i.e. $(7,2)$ as shown in figure.

(iv) The geometrical name for the figure $ABA'B'$ is quadrilateral.

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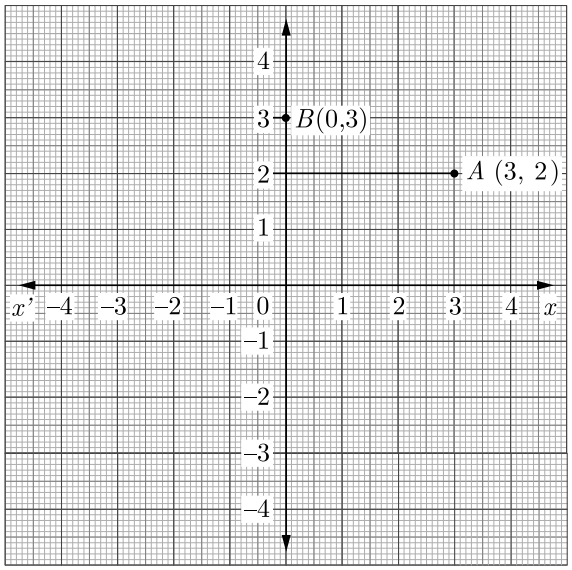

Point A' is the image of A under the reflection in the x-axis. Point B' is image of B under the reflection in the line AA'. Which of the following is point B'?

(a) (–3, –2) (b) (6, 3)
(c) (–6, 3) (d) (3, –2)

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$.
The reflection of the point $A(3, 2)$ in the x-axis is $A'(3, -2)$.
Reflection of a point $P(x, y)$ in the line $x = a$ is $P'(-x+2a, y)$.
Equation of line AA' is $x = 3$,
Thus reflection of point $B(0, 3)$ in line $x = 3$, is

$$B'(0+2\times 3, 3) \text{ i.e } B'(6, 3)$$

Thus (b) is correct option.

28. Assertion (A): The reflection of the point $P(-2, 7)$ in the y-axis is $(2, -7)$.
Reason (R): When a point is reflected in y-axis, only the sign of x-coordinate changes,

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$. Thus when a point is reflected in y-axis, only the sign of x-coordinate changes,
Hence, reflection of the point $P(-2, 7)$ in y-axis $(2, 7)$.
Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
Thus (a) is correct option.

29. Assertion (A): If the image of the Point P under reflection in the x-axis is $(-3, 4)$, then, the co-ordinates of the Point P are $(3, -4)$.
Reason (R): The reflection of the point $P(x, y)$ in the x-axis is the point $P'(x, -y)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. The image of the point P under reflection in the x-axis is $(-3, 4)$, then the co-ordinates of the point P will be $(-3, -4)$.
Here Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

30. Assertion (A): The reflection of the point $(4, 5)$ in the line $y = 2$ is $(4, -1)$.
Reason (R): The reflection of the point $P(x, y)$ in the line $y = a$ is the point $P'(x, -y+2a)$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Reflection of the point $P(x, y)$ in the line $y = a$ is the point $P'(x, -y+2a)$.
Therefore, the reflection of the point $(4, 5)$ in the line $y = 2$ is given by

$$(4, -5+2\times 2) = (4, -1)$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion

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(A).
Thus (a) is correct option.

31. Assertion (A): The reflection of point (2, 3) in the origin is (3, 2)
Reason (R): The reflection of the point $P(a, b)$ in the origin is the point $(-a, -b)$
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Reflection of the point $P(x, y)$ in the origin is the point $P'(-x, -y)$. Therefore, the reflection of the point (2, 3) in the origin is (–2, –3).
Here Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

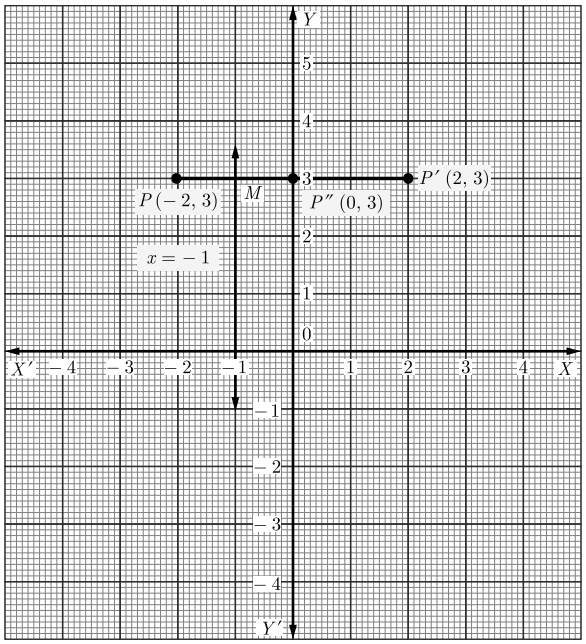
THREE MARKS QUESTIONS

Main 2016

32. Use graph paper for this question. A point P under reflection in y-axis is mapped onto $P'(4, 3)$.
(i) Find the coordinates of P.
(ii) Find the coordinates of the image of P under reflection in the line $x = -1$.

Sol :

(i) Reflection of the point (x, y) in the y-axis is $(-x, y)$. Since $P'(4, 3)$ is the image of P under reflection in the y-axis, the coordinates of P are $(-4, 3)$.
(ii) Reflection of the point (x, y) in the line $x = a$ is the point $(2a - x, y)$. Hence the image of the point $P(-4, 3)$ under reflection in the line $x = -1$ is the point $P''(2 \times (-1) + 4, 3)$ i.e. the point $P''(2, 3)$.



Main 2011

33. Write down the coordinates of the image of the point $(3, -2)$ when :
(i) reflected in the x-axis
(ii) reflected in the y-axis
(iii) reflected in the origin
(iv) reflected in the x-axis followed by reflection in the y-axis.

Sol :

(i) Image of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Thus image of $P(3, -2)$ in x-axis is $P'(3, 2)$.
(ii) Image of the point $P(x, y)$ in the y-axis is $P'(-x, y)$. Thus image of $P(3, -2)$ in y-axis is $P'(-3, -2)$.
(iii) Image of the point $P(x, y)$ in the origin is $P'(-x, -y)$. So, the image of the point $P(3, -2)$ in the origin is $P'(-3, 2)$.
(iv) Image of $P(3, -2)$ in x-axis is $P'(3, 2)$. Now, the image of $P'(3, 2)$ in y-axis is $P''(-3, 2)$.

Comp 2009

34. Find the reflection of the point $P(-2, 8)$ in the point $M(4, -2)$.

Sol :

Let $P'(a, b)$ be the reflection of the point $P(-2, 8)$ in the point $M(4, -2)$. Then, M is the mid-point of line segment PP'.

$P(-2, 8)$ $\quad$ $M(4, -2)$ $\quad$ $P'(a, b)$

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$$\left(\frac{a-2}{2}, \frac{b+8}{2}\right)(4, -2)$$

Thus $\frac{a-2}{2} = 4$ and $\frac{b+8}{2} = -2$

$$a - 2 = 8 \text{ and } b + 8 = -4$$

$$a = 10 \text{ and } b = -12$$

So, the reflection of the point $P(-2, 8)$ in the point $M(4, -2)$ is the point $P'(10, -12)$.

Main 2005

35. Use a graph paper for this question (take 10 small divisions = 1 unit on both the axes). P and Q have coordinates $(0,5)$ and $(-2,4)$.

(i) P is invariant, when reflected in an axis. Name the axis.

(ii) Find the image of Q on reflection in the axis found in (i).

(iii) $(0,k)$ on reflection in the origin in invariant. What the value of k.

(iv) Write the coordinates of the image of Q, obtained by reflection it in origin followed in X-axis.

Sol :

Take 1 cm = 1 unit on both axes.

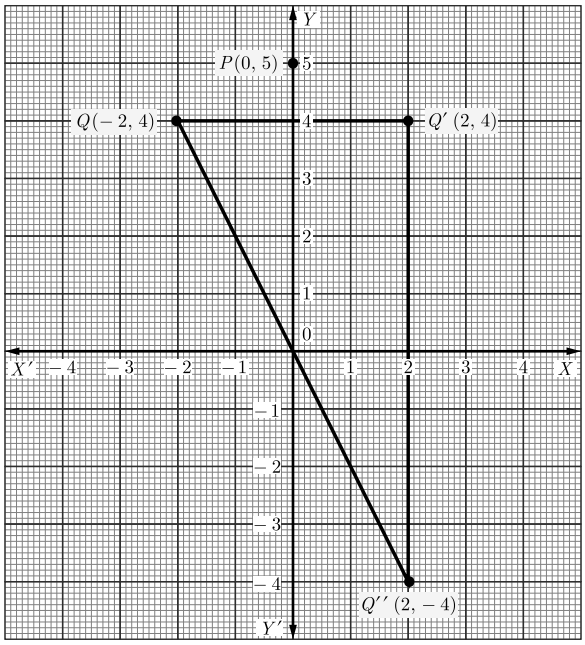
We plot the points $P(0,5)$ and $Q(-2,4)$ on the graph paper.

(i) Point P lies on y-axis, therefore P is invariant when reflected in y-axis.

(ii) Image of $Q(x,y)$ in the in y-axis is $Q'(-x, y)$ The image of $Q(-2,4)$ on the reflection in y-axis is $Q'(2,4)$.

(iii) Reflection of origin in the origin is invariant. Since point $(0,k)$ on reflection in the origin is invariant, it must be origin. Thus $k = 0$

(iv) The coordinates of the image of $Q(-2,4)$ obtained by reflecting it in the origin as $Q''(2,-4)$ and its reflection in x-axis is $Q''(2,4)$.



Main 2001

1. (i) Point $P(a, b)$ is reflected in the x-axis to $P'(5, -2)$. Write down the values of a and b.

(ii) P'' is the image of P when reflected in y-axis. Write down the coordinates of P''.

(iii) Name a single transformation that maps P' to P''.

Sol :

(i) The reflection of $P(a, b)$ in x-axis is $P'(a, -b)$. So, coordinates of P' are $(a, -b)$. But, the coordinates of P' are given as $(5, -2)$.

$$a = 5, \text{ and } -b = -2$$

$$a = 5 \text{ and } b = 2$$

So, the coordinates of P are $(5, 2)$.

(ii) Image of the point $P(x, y)$ in the y-axis is $P'(-x, y)$. Thus image of $P(5,2)$ in y-axis is $P'(-5, 2)$.

(iii) The coordinates of P' and P'' are $(5, -2)$ and $(-5, 2)$ respectively. We observe that the signs of coordinates of P' are opposite to the signs of coordinates of P''. So, reflection in the origin is the single transformation that maps P' to P''.

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Main 2000

36. The point $A(4, -1)$ is reflected as A' in y-axis. Point B on reflection in x-axis is mapped as $B'(-2, 5)$.

(i) Write the coordinates of A' and B.

(ii) Write the coordinates of the middle point of the line segment $A'B$.

Sol :

(i) The reflection of $A(4, -1)$ in y-axis is $A'(-4, -1)$.

Point B on reflection in x-axis is mapped as $B'(-2, 5)$. Reflecting $B'(-2, 5)$ in x-axis we get $B(-2, -5)$

(ii) The coordinates of A' and B are $(-4, -1)$ and $(-2, -5)$ respectively. So, the coordinates of the middle point of the line segment $A'B$ are $\left(\frac{(-4)+(-2)}{2}, \frac{(-1)+(-5)}{2}\right) \equiv (-3, -3)$

37. Find the coordinates of the image of $(-3, 4)$ under reflection in x-axis followed by reflection in the line $x = 2$.

Sol :

The image of $P(-3,4)$ in x-axis is $P'(-3,-4)$. The image (x, y) in the line $x = a$ is $(2a - x, y)$. So, image $(-3, -4)$ in the line $x = 2$ is $(2 \times 2 - (-3) - 4)$ i.e. $(7, -4)$.

38. If the reflection of the point $P(3, 5)$ in the line $x = a$ is the point $(5, 5)$ find the value of a.

Sol :

Reflection of the point (x, y) in the line $x = a$ is the point $(2a - x, y)$.
Thus image P' of the point $P(3, 5)$ in the line $x = a$ is $(2a - 3, 5)$. But, the coordinates of reflection are $(5, 5)$.

$$2a - 3 = 5$$
$$2a = 8$$
$$a = 4$$

39. The point $P(h, k)$ is first reflected in x-axis and then reflected in the origin to get the point $Q(-2, 5)$, Find the values of h and k.

Sol :

The reflection of $P(h, k)$ in x-axis is $P'(h, -k)$.
The reflection of $P'(h,-k)$ in origin is $P''(-h, k)$.
But, the P'' is given as point $Q(-2, 5)$.

$$-h = -2 \text{ and } k = 5$$
$$h = 2 \text{ and } k = 5$$

40. Point $A(5, -1)$ on reflection in x-axis is mapped as A'. Also, A on reflection in y-axis is mapped as A''. Write the coordinates A' and A''. Also, calculate the distance $A'A''$.

Sol :

The reflection of $A(5, -1)$ in x-axis is $A'(5, 1)$.
The reflection of $A(5, -1)$ in y-axis is $A''(-5,-1)$.

$$A'A'' = \sqrt{\{5-(-5)\}^2 + (-1-1)^2}$$
$$= \sqrt{10^2 + 2^2}$$
$$= \sqrt{104} \text{ units}$$

41. Point $A(1, -5)$ is mapped as A' on reflection in the x-axis. Point $B(3, 2)$ is mapped as B' on reflection in the origin. Write the coordinates of A' and B'. Calculate AB'.

Sol :

The reflection of $A(1, -5)$ in x-axis is $A'(1, 5)$.
The reflection of $B(3, 2)$ in origin is $B'(-3, -2)$.

Now
$$AB' = \sqrt{(-3-1)^2 + (-2-5)^2}$$
$$= \sqrt{16 + 49}$$
$$= \sqrt{65} \text{ units}$$

FOUR MARK QUESTIONS

Main 2020

42. Use graph paper for this question. Take 1 cm = 1 unit on both x and y-axes.

(i) Plot the following points on your graph sheets : $A(-4, 0)$, $B(-3, 2)$, $C(0, 4)$, $D(4, 1)$ and $E(7, 3)$

(ii) Reflect the points B, C, D and E on the X-axis and name them as B' , C' , D' and E', respectively.

(iii) Join the points A, B, C, D, E, E', D', C', B' and A' in order.

(iv) Name the closed figure formed.

Sol :

(i) Points $A(-4, 0)$, $B(-3, 2)$, $C(0, 4)$, $D(4, 1)$ and $E(7, 3)$ on graph papers are shown below in figure.

(ii) The reflection of the points B, C, D and E on the x-axis are the points $B'(-3,-2)$, $C'(0,-4)$, $D'(4,-1)$ and $E'(7,-3)$

(iii) We have joined the points A, B, C, D, E, E', D', C', B' and A' in order.

(iv) We get the figure of irregular decagonas shown in figure.

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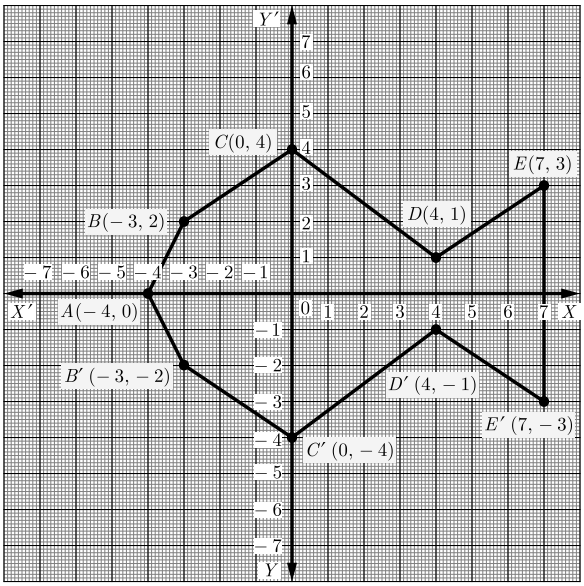


Main 2019

43. Use a graph sheet for this question. Take 1 cm = 1 unit along both x and y-axis.

(i) Plot the following points. $A(0, 5)$, $B(3, 0)$, $C(1, 0)$ and $D(1, -5)$

(ii) Reflect the points B, C and D on the Y-axis and name them as B', C' and D' respectively.

(iii) Write down the coordinates of B', C' and D'.

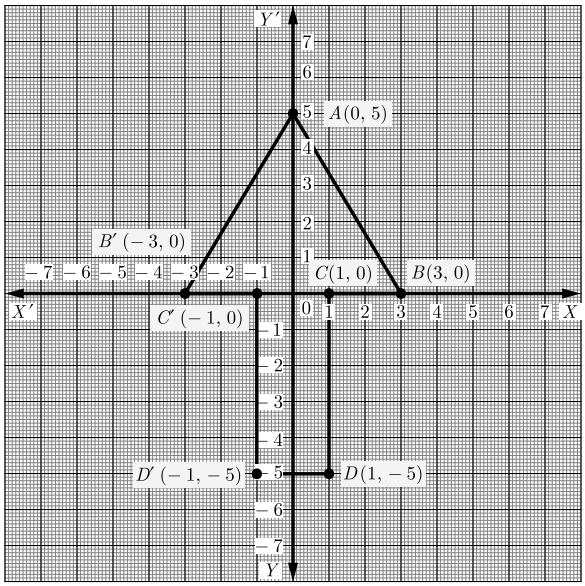
(iv) Join the points A, B, C, D, D', C', B', A' in order and give a name to the closed figure $ABCDD'C'B'$.

Sol :

Take 1 cm = 1 unit on both axes.

(i) Points $A(0, 5)$, $B(3, 0)$, $C(1, 0)$ and $D(1, -5)$ on graph papers are shown below in figure.

(ii) The reflection of the points B, C and D on the y-axis are B', C' and D' which are shown in figure.



(iii) Image of point $B(3, 0)$ on y-axis is $B'(-3, 0)$
Image of point $C(1, 0)$ on y-axis is $C'(-1, 0)$
.
Image of point D (1, –5) on y-axis is $D'(-1, -5)$

(iv) When we join the points A, B, C, D, D', C', B', A in order we get the figure of an arrow as shown in figure.

Main 2018

44. Use graph paper for this question. (Take 2 cm = 1 unit along both X and Y-axis) $ABCD$ is a quadrilateral whose vertices are $A(2,2)$, $B(2, -2)$, $C(0, -1)$ and $D(0,1)$.

(i) Reflect quadrilateral $ABCD$ on the y-axis and name it as $A'B'CD$.

(ii) Write down the coordinates of A' and B'.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon $A'B'CD$.

Sol :

Take 2 cm = 1 unit on both axes. We plot the points $A(2,2)$, $B(2, -2)$, $C(0, -1)$ and $D(0,1)$ on graph as shown in figure.

(i) Image of points $A(2,2)$, $B(2, -2)$, $C(0, -1)$ and $D(0,1)$ are shown on graph. It must be noted that point $C(0, -1)$ and $D(0,1)$ are invarint and reflection of quadrilateral $ABCD$ is $A'B'CD$

(ii) From the graph, we get $A'(-2, 2)$ and $B'(-2, -2)$

(iii) Point $C(0, -1)$ and $D(0,1)$ are invarint are invariant under the y-axis reflection.

(iv) $A'B'CD$ is a trapezium.

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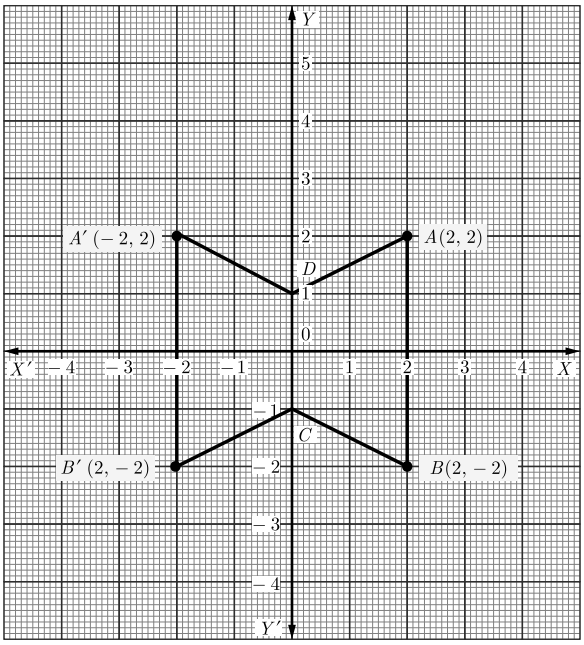


Main 2014

45. Use graph paper for this question. A point P is reflected to P' in the x-axis. The coordinates of its image are $(4, -3)$. Find :

(i) Coordinates of P.

(ii) coordinates of the image P'' of P under reflection in the y-axis.

(iii) coordinates of the image Q' of the point $Q(1, 2)$ in the line PP'.

(iv) coordinates of the image Q'' of the point $Q(1, 2)$ in the line PP''.

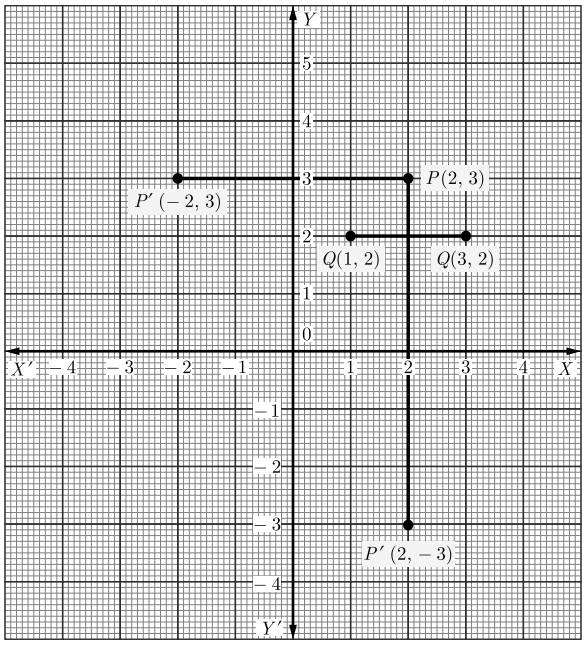
Sol :

(i) Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Since $P'(4, -3)$ is the image of P in the x-axis, the coordinates of P are $(4, 3)$.

(ii) Reflection of the point (x, y) in the y-axis is $(-x, y)$. Thus reflection of the point $P(4, 3)$ in the y-axis is $P''(-4, 3)$.

(iii) In both point $P(4, 3)$ and $P'(4, -3)$ we have $x = 4$. Thus line PP' is given by $x = 4$. Reflection of the point (x, y) in the line $x = a$ is the point $(2a - x, y)$. Thus image Q' of the point $Q(1, 2)$ in the line $PP'(x = 4)$ is $(2 \times 4 - 1, 2)$ i.e $(7, 2)$.

(iv) In both point $P(4, 3)$ and $P''(-4, 3)$ we have $y = 3$. Thus line PP'' is given by $y = 3$. Reflection of the point (x, y) in the line $y = a$ is the point $(x, 2a - y)$. Thus image Q'' of the point $Q(1, 2)$ in the line $PP''(y = 3)$ is $(1, 2 \times 3 - 2)$ i.e $(1, 4)$.



Main 2013

46. Using graph paper, plot the point $A(6,4)$ and $B(0,4)$.

(i) Reflect A and B in the origin to get the images A' and B'.

(ii) Write the coordinates of A' and B'.

(iii) State the geometrical name for the figure $ABA'B'$.

(iv) Find its perimeter.

Sol :

Take 1 cm = 2 unit on both axes.

We plot the point $A(6,4)$ and $B(0,4)$ on the graph paper.

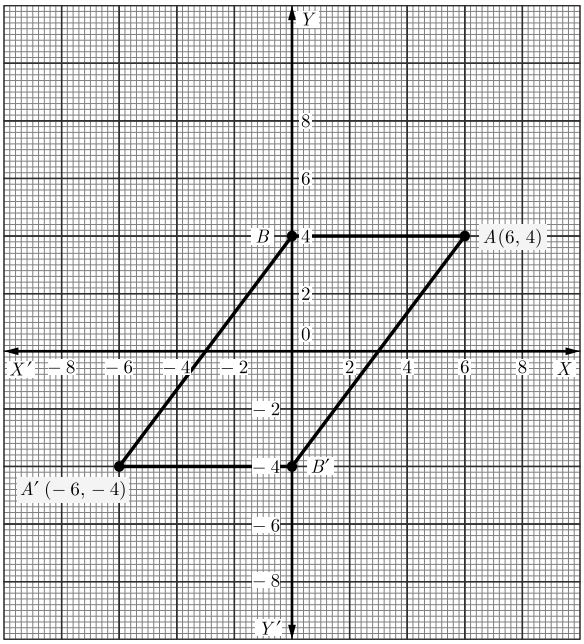
(i) Since, point $A(6,4)$ is in Ist quadrant, so the reflection of A in the origin is in IIIrd quadrant which is point A'.

Point $B(0,4)$ is in positive direction of Y y-axis, thus the reflection of B' in the origin is on the negative direction y-axis which is point B'.

(ii) Image of $P(x, y)$ in the origin is $P'(-x, -y)$. Thus coordinates of A' are $(-6, -4)$ and B' are $(0, -4)$.

(iii) We joined the points as shown in graph. It may be easily seen that opposite sides are parallel and equal and adjacent sides are not equal. Hence, it is parallelogram.

NODIA



(iv) In right $\Delta ABB'$,

$$AB' = \sqrt{AB^2 + BB'^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= 10$$

Perimeter of a parallelogram $ABA'B'$

$$= 2(AB + AB')$$
$$= 2(6 + 10)$$
$$= 2 \times 16 = 32 \text{ units}$$

Main 2012

47. Use graph paper for this question.

(i) Plot the points $A(-4, 4)$, $B(2, 2)$.

(ii) Reflect A and B in the origin to get the images A' and B' respectively. Write the coordinates of A' and B'.

(iii) Give geometrical name of the figure $ABA'B'$.

(iv) Draw and name its lines of symmetry.

Sol :

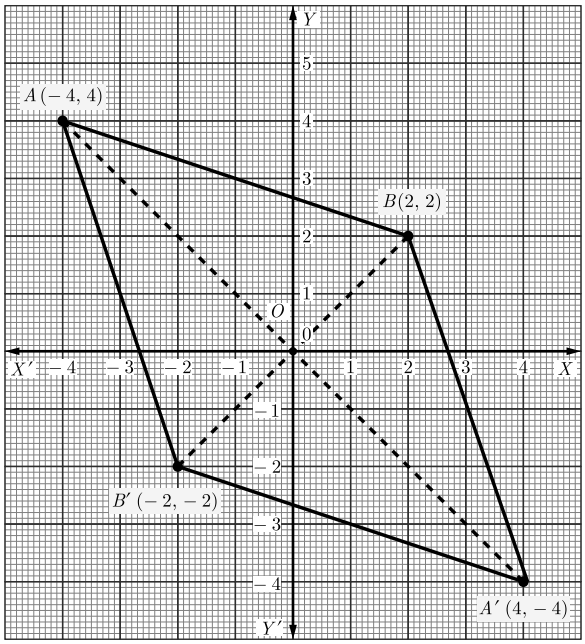
Take 1 cm = 1 unit on both axes.

(i) Point $A(-4, 4)$ and $B(2, 2)$ on the graph paper are shown in figure below.

(ii) The reflections of points $A(-4, 4)$ and $B(2, 2)$ in the origin are points $A'(4, -4)$ and $B'(-2, -2)$ as shown in figure.

(iii) Figure $ABA'B'$ is a rhombus.

(iv) Lines AA' and BB' are lines of symmetry.



Main 2011

48. Use graph paper for this question. Points (3, 0) and (−1, 0) are invariant points under reflection in the line L_1; points $(0, -3)$ and (0, 1) are invariant points on reflection in line L_2.

(i) Find the lines L_1 and L_2.

(ii) Write down the images of points $P(3, 4)$ and $Q(-5, -2)$ on reflection in L_1. Name the images as P' and Q' respectively.

(iii) Write down the images of P and Q on reflection in L_2. Name the images as P'' and Q'' respectively.

(iv) State or describe a single transformation that maps P' onto P''.

Sol :

(i) Points (3, 0) and (−1, 0) are invariant points under reflection in the line L_1. Here we observe that both points are on x-axis. A point lying on the x-axis is invariant under reflection in x-axis. Thus line L_1 is the x-axis.

Similarly, the line L_2 is the y-axis.

(ii) Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Thus images of points $P(3, 4)$ and $Q(-5, -2)$ on reflection in L_1 (x-axis) are points $P'(3, -4)$ and $Q'(-5, 2)$.

(iii) Reflection of the point (x, y) in the y-axis is $(-x, y)$. Thus images of $P(3, 4)$ and $Q(-5, -2)$ on reflection in L_2 (y-axis) are point $P''(-3, 4)$ and $Q''(5, -2)$.

(iv) The single transformation that maps $P'(3, -4)$ onto $P''(-3, 4)$ is the reflection in the origin.

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Reflection of the point (x, y) in the line $x = a$ is the point $(2a - x, y)$.
Thus image P' of the point $P(3, 9)$ in the line $x = a$ is $(2a - 3, 9)$. But, the coordinates of reflection are $(7, 9)$.

$$2a - 3 = 7$$
$$2a = 10$$
$$a = 5$$

Thus (d) is correct option.

24. If the reflection of the point $P(5, 3)$ in the line $y = a$ is the point $(5, 5)$ the value of a will be
(a) 2 (b) 3
(c) 4 (d) 5

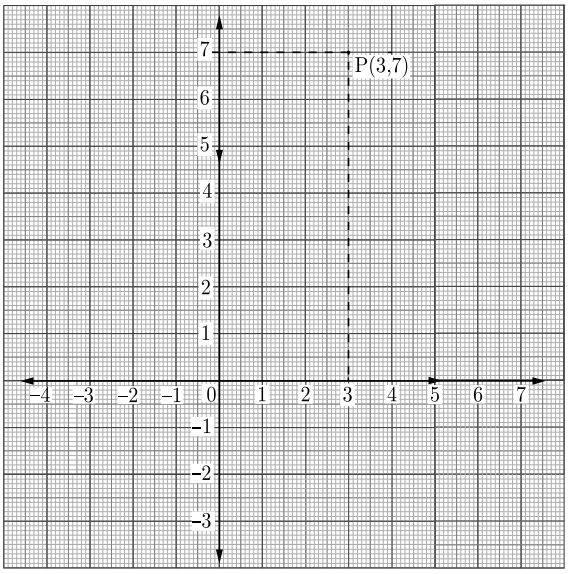
Sol :

Reflection of the point (x, y) in the line $y = a$ is the point $(x, 2a - y)$.
Thus image P' of the point $P(5, 3)$ in the line $y = a$ is $(5, 2a - 3)$. But, the coordinates of reflection are $(5, 5)$.

$$2a - 3 = 5$$
$$2a = 8$$
$$a = 4$$

Thus (c) is correct option.

25. Consider the point P plotted in cartesian plane:

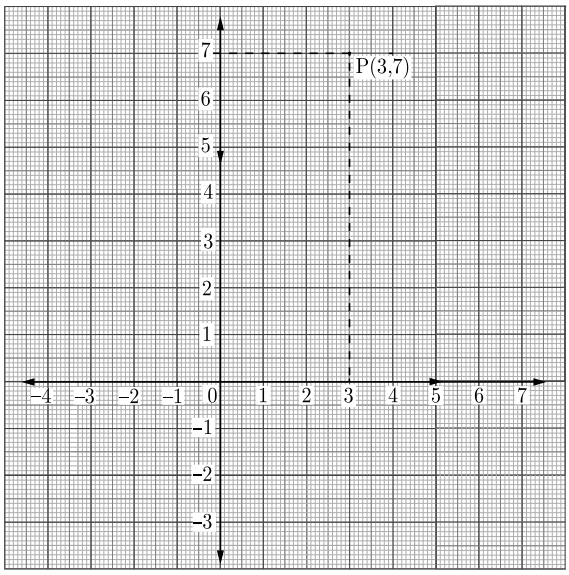


The co-ordinate of image of point P when reflected in x-axis is:
(a) $(-3, 7)$ (b) $(3, -7)$
(c) $(-3, -7)$ (d) none of these

Sol :

Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$.
The reflection of the point $(3, 7)$ in the x-axis is $(3, -7)$.
Thus (b) is correct option.

26. Consider the point P plotted in cartesian plane:

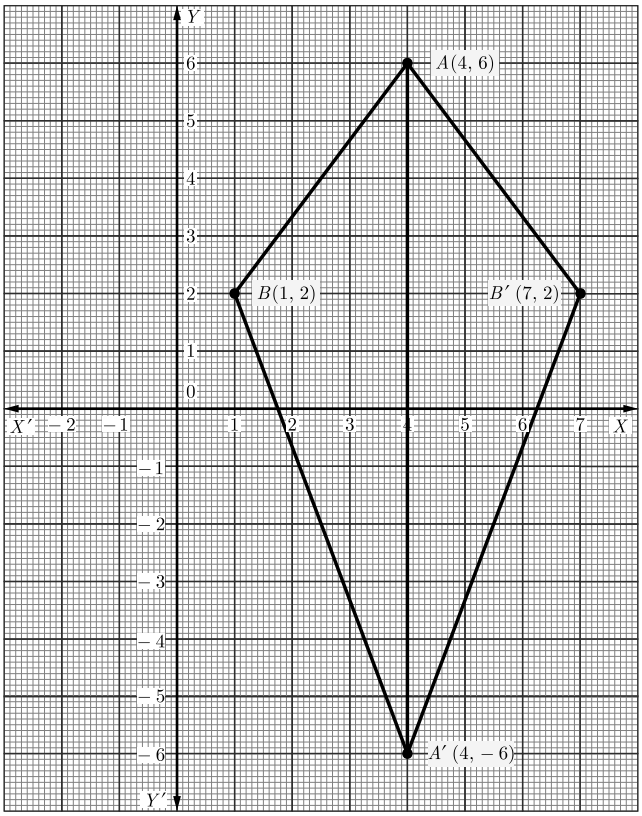


The co-ordinate of image of point P when reflected in y-axis is:
(a) $(-3, 7)$ (b) $(3, -7)$
(c) $(-3, -7)$ (d) none of these

Sol :

Reflection of the point $P(x, y)$ in the y-axis is $P'(-x, y)$.
The reflection of the point $(3, 7)$ in the y-axis is $(-3, 7)$.
Thus (a) is correct option.

27. Point A and B are plotted on a graph as shown below.



Main 2008

51. Use graph paper for this question. $A(0,\ 3)$ $B(3, -2)$ and $O(0,\ 0)$ are the vertices of triangle ABO.

(i) Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.

(ii) Plot D the reflection of B in the y-axis, and write it coordinates.

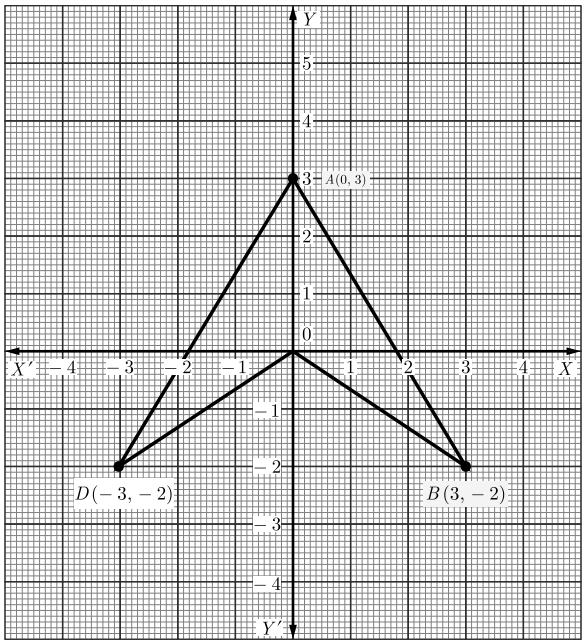
(iii) Give the geometrical name of the figure $ABOD$.

(iv) Write the equation of the line of symmetry of the figure $ABOD$.

Sol :

Take 2 cm = 1 unit on both the axes

(i) Plot the point $A(0,\ 3)$, $B(3,\ -2)$ and $O(0,\ 0)$ on the graph paper as shown in figure below.



(ii) Plot the point D which is the reflection of the point B in the y-axis. The coordinates of D are $(-3,\ -2)$.

(iii) Figure $ABOD$ is a kite (arrow).

(iv) The axis of symmetry of the kite $ABOD$ is the y-axis, its equation is $x = 0$.

Comp 2008

52. Use graph paper for this question. Plot the points $O(0,\ 0)$, $A(-4,\ 4)$, $B(-3,\ 0)$ and $C(0,\ -3)$

(i) Reflect points A and B in the y-axis and name them A' and B' respectively. Write down their coordinates.

(ii) Name the figure $OABCB'A'$.

(iii) State the line of symmetry of this figure.

Sol :

Take 2 cm = 1 unit on both the axes.

Plot the points $O(0,\ 0)$, $A(-4,\ 4)$, $B(-3,\ 0)$ and $C(0,\ -3)$ on the graph paper as shown below.

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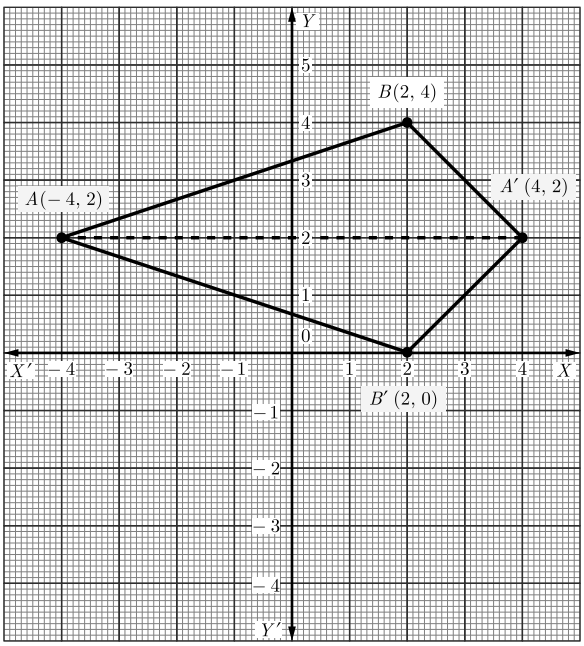
(i) Points $A(-4, 2)$ and $B(2, 4)$ on the graph paper are shown in given figure.
(ii) The image of the point $A(-4, 2)$ when reflected in the y-axis is the point $A'(4, 2)$.
(iii) The image of the point $B(2, 4)$ when reflected in the line AA' is the point $B'(2, 0)$.
(iv) The figure $ABA'B'$ is a kite.
(v) The line of symmetry of the figure $ABA'B'$ is line AA' i.e. $y = 2$.



Comp 2000

61. The point $P(3, -5)$ is reflected to P' in the x-axis. Also, point P on reflection in the y-axis is mapped as P''.
(i) Find the coordinates of P' and P''.
(ii) Compute the distance $P'P''$.
(iii) Find the middle point of the line segment $P'P''$.
(iv) On which coordinate axis does the middle point of the line segment PP'' lie?

Sol :

(i) The reflection of $P(3, -5)$ in the x-axis is $P'(3, 5)$ and the reflection of $P(3, -5)$ in y-axis is $P''(-3, -5)$.

(ii) $$P'P'' = \sqrt{(-3-3)^2 + (-5-5)^2}$$
$$= \sqrt{36 + 100}$$
$$= \sqrt{136} \text{ units}$$

(iii) The coordinates of the middle point of the line segment $P'P''$ are
$$\left(\frac{3+(-3)}{2}, \frac{5+(-5)}{2}\right) = (0, 0)$$
So, the mid-point of $P'P''$ is the origin.

(iv) The coordinates of the middle point of the line segment PP'' are
$$\left(\frac{3+(-3)}{2}, \frac{-5+(-5)}{2}\right) = (0 - 5)$$
Clearly, x-coordinate of this point is zero. So, it lies on y-axis.

FIVE MARK QUESTIONS

Main 2023

62. Use graph sheet to answer this question. Take 2 cm = 1 unit along both the axes. [5]
(i) Plot A, B, C where $A(0, 4)$, $B(1, 1)$ and $C(4, 0)$
(ii) Reflect A and B on the x-axis and name them as E and D respectively.
(iii) Reflect B through the origin and name it F. Write down the coordinates of F.
(iv) Reflect B and C on the y-axis and name them as H and G respectively.
(v) Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed.

Sol :

Take 2 cm = 1 unit on both axes.
(i) Points $A(0, 4)$, $B(1, 1)$ and $C(4, 0)$ are shown in figure given below.
(ii) Reflection of point $A(0,4)$ and $B(1,1)$ on x-axis are $E(0, -4)$ and $D(1, -1)$ as shown in figure.
(iii) Reflection of point $B(1,1)$ thorugh origin is $F(-1, -1)$ as shown in figure.
(iv) Reflection of point $B(1,1)$ and $C(4,0)$ on y-axis are $H(-1, 1)$ and $G(-4, 0)$ as shown in figure.
(v) We have joined points A, B, C, D, E, F, G, H and A in order. The closed figure formed is a star.

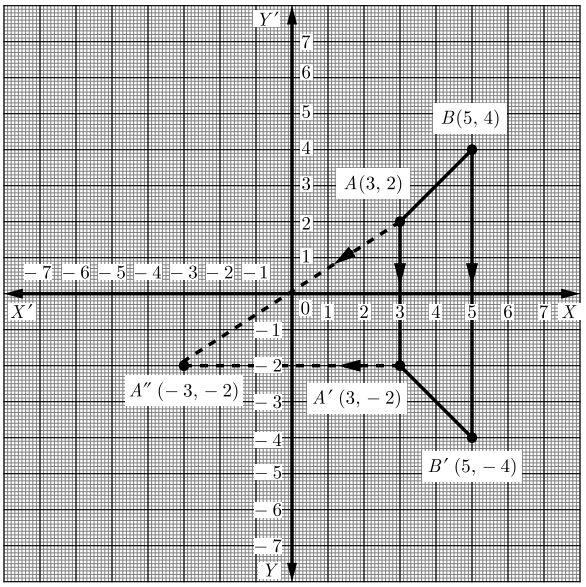
trapezium.

(b) The axis of symmetry of $ABB'A'$ is the x-axis.

(c) $\angle ABB' = 45^\circ$

(d) The coordinates of A'' are $(-3, -2)$.

(e) The single transformation that maps A' to A'' is the reflection in the y-axis.



Main 2006

55. Use graph paper for this question. The point $A(2,3)$, $B(4,5)$ and $C(7,2)$ are the vertices of ΔABC.

(i) Write down the coordinates of A', B', C', if $A'B'C'$ are the images of ΔABC, when reflected in the origin.

(ii) Write down the coordinates of A'', B'', C'', if $A''B''C''$ are the images of ΔABC, when reflected in the X-axis.

(iii) Mention the special name of the quadrilateral $BCC''B''$ and find its area.

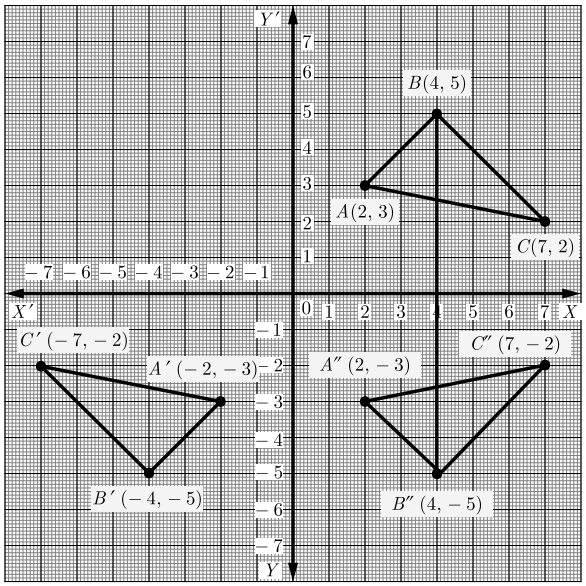
Sol :

Take 1 cm = 1 unit on both axes.

We plot point $A(2,3)$, $B(4,5)$ and $C(7,2)$ on graph as shown in figure.

(i) Reflection of $P(x,y)$ in the origin is $P'(-x,-y)$. The reflection of A, B and C in the origin are $A'(-2,-3)$, $B'(-4,-5)$ and $C'(-7,-2)$

(ii) Reflection of $P(x,y)$ in the x-axis is $(x,-y)$. Thus reflection of A, B and C in the x-axis are $A''(2,-3)$, $B''(4,-5)$ and $C''(7,-2)$.



(iii) The special name of the quadrilateral $BCC''B''$ is isosceles trapezium.

Area of trapezium

$= \frac{1}{2}$(Sum of parallel sides) $\times$ Perpendicular distance

$= \frac{1}{2}(4+10) \times 3$ sq

$= \frac{1}{2} \times 14 \times 3 = 21$ sq units

Comp 2006

56. Use graph paper for this question. The point $P(-1,3)$ is reflected in the line parallel to y-axis at a distance of 2 units to the right side of y-axis onto the point P'. The point Q is reflected in the origin onto the point $Q'(-3,-2)$. Find:

(i) the coordinates of P' and Q.

(ii) the equation of the line $P'Q$.

Sol :

Take 1 cm = 1 unit on both axes.

(i) The equation of a line parallel to y-axis at a distance of 2 units to the right side of y-axis is $x = 2$. The image of $P(-1, 3)$ in the line $x = 2$ is $P'(2\times 2-(-1),3)$ i.e. $P'(5,3)$.
Since Q and $Q'(-3,-2)$ are images of each other in the origin O. So, coordinates of Q are (3, 2).

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CHAPTER 12

SECTION FORMULA AND MID POINT

NODIA

MULTIPLE CHOICE QUESTIONS

Main 2023

1. The coordinates of the vertices of ΔABC are respectively $(-4,\ -2)$, $(6,\ 2)$ and $(4,\ 6)$. The centroid G of ΔABC is :

(a) $(2,\ 2)$ (b) $(2,\ 3)$
(c) $(3,\ 3)$ (d) $(0,\ -1)$

Sol :

Centroid,

$$G(x,\ y) = \left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$$

$$= \left(\frac{-4+6+4}{2}, \frac{-2+2+6}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{6}{2}\right)$$

$$= (3,\ 3)$$

Thus (c) is correct option.

Comp 2023

2. The centroid of the triangle whose vertices are $(3,-7),(-8,6)$ and $(5,\ 10)$ is

(a) $(0,\ 9)$ (b) $(0,\ 3)$
(c) $(1,\ 3)$ (d) $(3,\ 5)$

Sol :

Centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

i.e. $\left(\frac{3+(-8)+5}{3}, \frac{-7+6+10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right)$

$$= (0,3)$$

Thus (b) is correct option.

3. The ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and $(6,\ 3)$, hence the value of y is

(a) $2:3, y=3$ (b) $3:2, y=4$
(c) $3:2, y=3$ (d) $3:2, y=2$

Sol :

Let the required ratio be $k:1$

Then, $$2 = \frac{6k-4(1)}{k+1}$$

or $$k = \frac{3}{2}$$

The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $$y = \frac{3(3)+2(3)}{3+2} = 3$$

Thus (c) is correct option.

4. The point P on x-axis equidistant from the points $A(-1,0)$ and $B(5,0)$ is

(a) $(2,\ 0)$ (b) $(0,\ 2)$
(c) $(3,\ 0)$ (d) $(-3,5)$

Sol :

You may easily observe that both point $A(-1,0)$ and $B(5,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1,0)$ and $B(5,0)$.

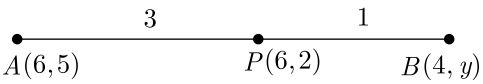
$$x = \frac{-1+5}{2} = 2$$

5. If the point $P\ (6,\ 2)$ divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$ then the value of y is

(a) 4 (b) 3
(c) 2 (d) 1

Sol :

As per given information in question we have drawn the figure below,



Here, $$x_1 = 6,\ y_1 = 5$$

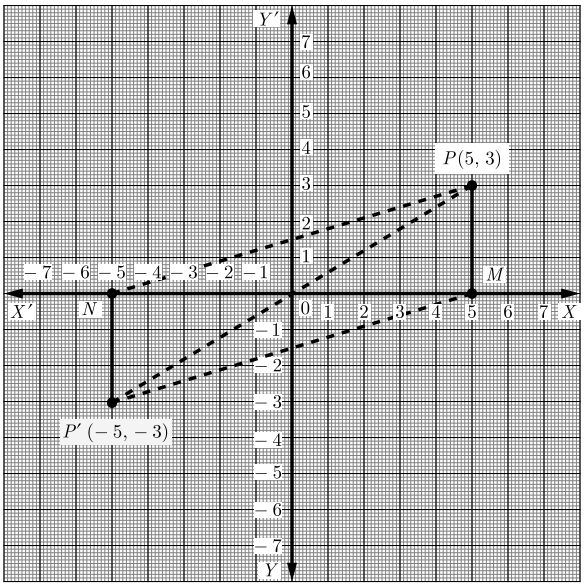
and $$x_2 = 4\ \ y_2 = y$$

Now $$y = \frac{my_2+ny_1}{m+n}$$

$$2 = \frac{3\times y+1\times 5}{3+1}$$

$$2 = \frac{3y+5}{4}$$

$$3y+5 = 8$$



Main 2002

59. The point $P(3,4)$ is reflected to P' in the x-axis and O' is the image of O (the origin), when reflected in the line PP'. Using graph paper, give
(i) the coordinates of P' and O'.
(ii) the lengths of the segments PP' and OO'.
(iii) the perimeter of the quadrilateral $POP'O'$.
(iv) the geometrical name of the figure $POP'O'$.

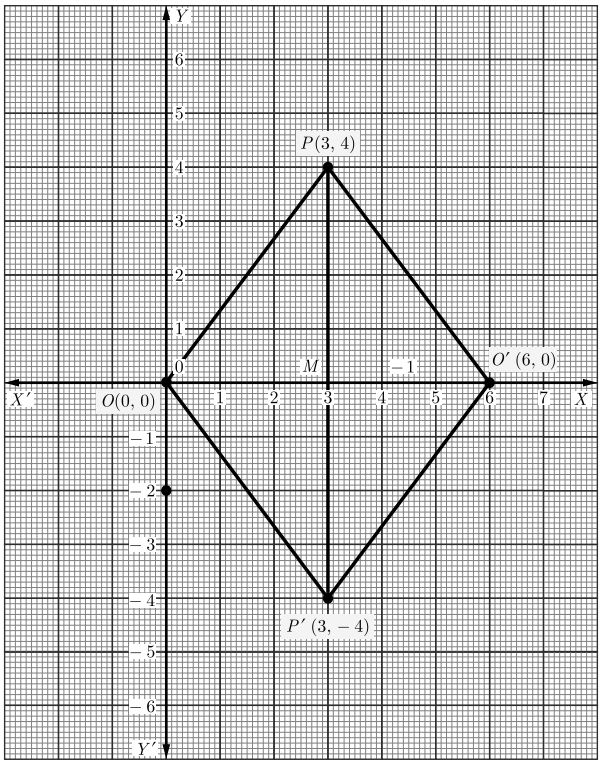
Sol :

Take 1 cm = 1 unit on both axes.

(i) Since, P' is the image of P in the x-axis. So, the coordinates of P' are $(3, -4)$.

(ii) From the graph, it is clear that length of segment PP' is 8 units, length of segment OO' is 6 units.

(iii) Draw perpendicular from P on x-axis, which intersect X-axis at M. Then, $OM = 3$ units and $MP = 4$ units



In right ΔOMP using Pythagoras theorem,

$$OP = \sqrt{OM^2 + MP^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25} = 5 \text{ units}$$

Similarly, $OP' = O'P' = O'P = 5$ units

Perimeter of the quadrilateral $POP'O'$,

$$4(OP) = 4 \times 5 = 20 \text{ units}$$

(iv) Here $OO' = 6$ and $PP' = 8$. Since, all four sides are equal and diagonals $PP' \neq OO'$ Hence, figure $POP'O'$ is a rhombus.

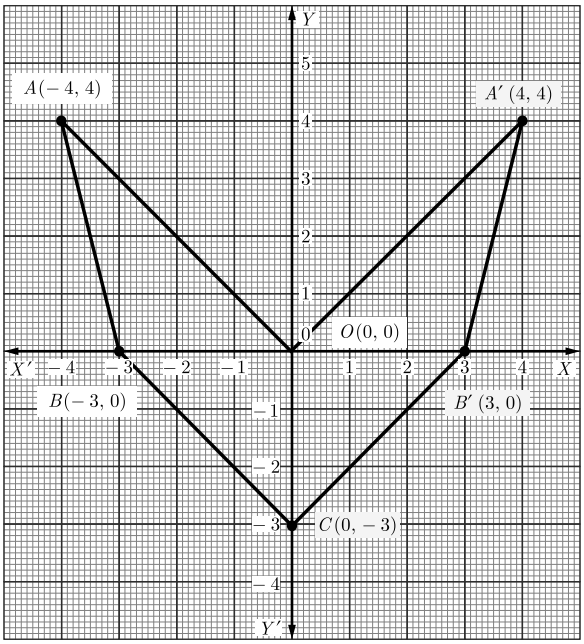
Comp 2002

60. Use graph paper to answer the following questions. (Take 2 cm = 1 unit on both axes)
(i) Plot the points $A(-4, 2)$ and $B(2, 4)$.
(ii) A' is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A'.
(iii) B' is the image of B when reflected in the line AA'. Write the coordinates of B'.
(iv) Write the geometric name of the figure $ABA'B'$.
(v) Name a line of symmetry of the figure formed.

Sol :

Take 2 cm = 1 unit on both axes.

NODIA



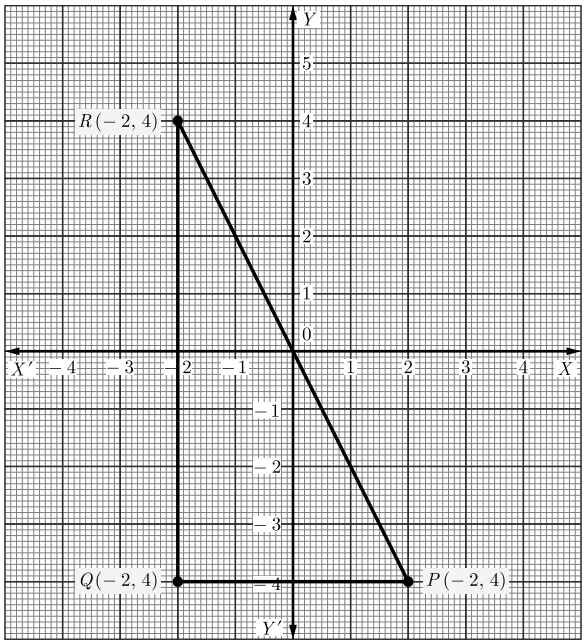
(i) The reflection of the point $A(-4, 4)$ in the y-axis is the point $A'(4, 4)$ and the reflection of the point $B(-3, 0)$ in the y-axis is the point $B'(3, 0)$.
(ii) The figure $OABCB'A'$ is a (concave) hexagon.
(iii) The line of symmetry of the figure $OABCB'A'$ is y-axis.

Main 2007

53. Use a graph paper for this question.
(i) The point $P(2, -4)$ is reflected about the line $x = 0$ to get the image Q. Find the coordinates of Q.
(ii) Point Q is reflected about the line $y = 0$ to get the image R. Find the coordinates of R.
(iii) Name the figure PQR.
(iv) Find the area of figure PQR.

Sol :

Take 1 cm = 1 unit on both axes.
(i) The reflection of $P(2, -4)$ about the line $x = 0$ (i.e. y-axis) is $Q(-2, -4)$.
(ii) The reflection of Q about the line $y = 0$ (i.e. x-axis) is $R(-2, 4)$.
(iii) On joining all adjacents points, we get a right ΔPQR.



(iv) Area of right angled triangle,

$$\Delta PQR = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times PQ \times QR$$

$$= \frac{1}{2} \times 4 \times 8$$

$$= 16 \text{ sq units}$$

Comp 2007

54. Use graph paper for this question..
(i) Plot $A(3, 2)$ and $B(5, 4)$ on the graph paper.
(ii) Reflect A and B in the x-axis to A' and B'. Plot these on the same graph paper.
(iii) Write down
(a) the geometrical name of the figure $ABB'A'$.
(b) the axis of symmetry of $ABB'A'$.
(c) the measure of the angle ABB'.
(d) the image A' of A, when A is reflected in the origin.
(e) the single transformation that maps A' to A'.

Sol :

Take 1 cm = 1 unit on both axes.
(i) Point $A(3, 2)$ and $B(5, 4)$ on the graph paper are shown in figure below.
(ii) Refletion of points $A(3, 2)$ and $B(5, 4)$ in the x-axis are $A'(3, -2)$ and $B'(5, -4)$ respectively
(iii) (a) The figure $ABB'A'$ is an isosceles

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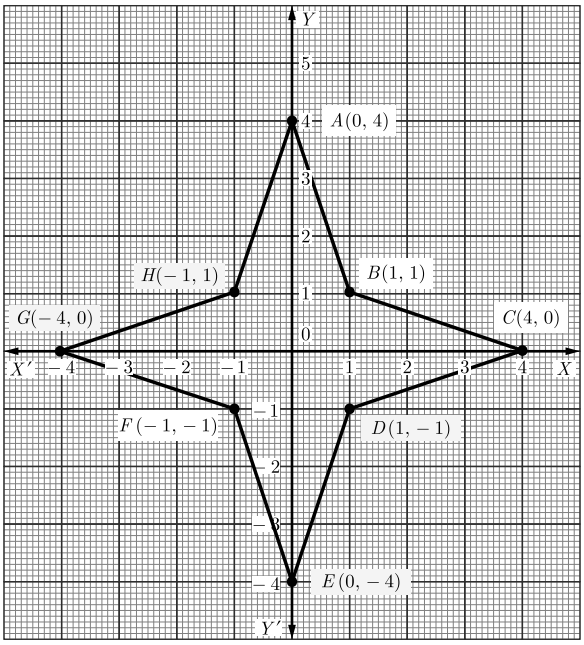


Main 2015

63. Use a graph paper for this question taking 1 cm = 1 unit along both the x and y-axis:

(i) Plot the points $A(0,5)$, $B(2,5)$, $C(5,2)$, $D(5,-2)$, $E(2,-5)$ and $F(0,-5)$.

(ii) Reflect the point B, C, D and E on the y-axis name them respectively as B', C', D' and E'.

(iii) Write the coordinates of B', C', D' and E'.

(iv) Name the figure formed by $BCDEE'D'C'B'$.

(v) Name a line of symmetry for the figure formed.

Sol :

Take 1 cm = 1 unit on both axes.

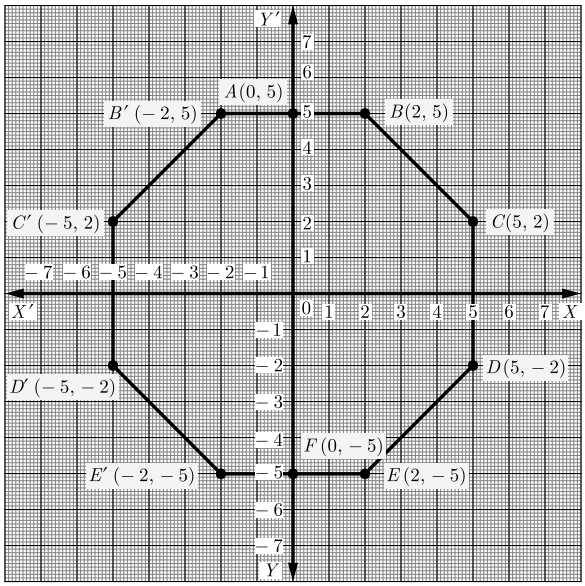
(i) Points $A(0,5)$, $B(2,5)$, $C(5,2)$, $D(5,-2)$, $E(2,-5)$ and $F(0,-5)$ are shown in the graph paper given below.

(ii) Reflection of a point (x,y) in the y-axis is $(-x,y)$. Thus reflection of all point is shown in graph below.

(iii) Image points are $B'(-2,5)$, $C'(-5,2)$, $D'(-5,-2)$ and $E'(-2,-5)$.

(iv) The figure formed by $BCDEE'D'C'B'$ is octagon.

(v) Both axes are line of symmetry for this figure.



Main 2004

64. Use the graph paper for this question. Points $A(1,1)$, $B(5,1)$, $C(4,2)$ and $D(2,2)$ are the vertices of a quadrilateral. Name the quadrilateral $ABCD$. A, B, C and D are reflected in the origin onto A', B', C' and D' respectively. Locate A', B', C' and D' on the graph sheet and write their coordinates. Are D, A, A' and D' collinear?

Sol :

Take 1 cm = 1 unit on both axes.

(i) We plot the point $A(1,1)$, $B(5,1)$, $C(4,2)$ and $D(2,2)$ on the graph paper as shown below.

(ii) The quadrilateral $ABCD$ is an isosceles trapezium.

(iii) Reflection of points A, B, C and D in the origin onto the point A', B', C' and D' respectively are shown in the figure. The coordinates of these points are $A'(-1,-1)$, $B'(-5,-1)$, $C'(-4,-2)$ and $D'(-2,-2)$.

(iv) When, we join the points D and D', we find that the points A and A' lie on it. Hence, the points D, A, A' and D' are collinear.

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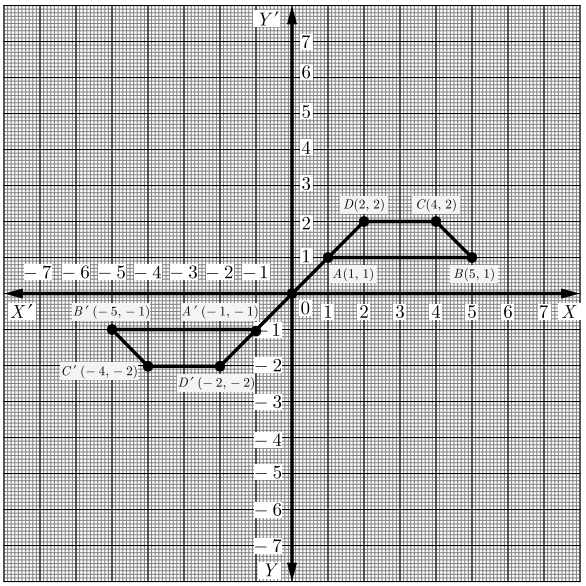


Main 2003

65. Use a graph paper for this question. (take 10 small division = 1 unit on both axes). Plot the point $P(3,2)$ and $Q(-3,-2)$. Form P and Q, draw perpendicular PM and QN on the x-axis.

(i) Name the image of P on reflection in the origin.

(ii) Assign the special name to the geometrical figure $PMQN$ and find its area.

(iii) Write the coordinates of the point to which M is mapped on reflection in

(a) X-axis

(b) Y-axis

(c) Origin.

Sol :

Take 1 cm = 1 unit on both axes.

We plot point $P(3,2)$ and $Q(-3,-2)$ on graph paper as shown in figure. We have drawn perpendicular PM and QN on the x-axis.

(i) The image of the point $P(3,2)$ on reflection in the origin is the point $Q(-3,-2)$.

(ii) $PMQN$ is a parallelogram.

Area of parallelogram $PMQN$,

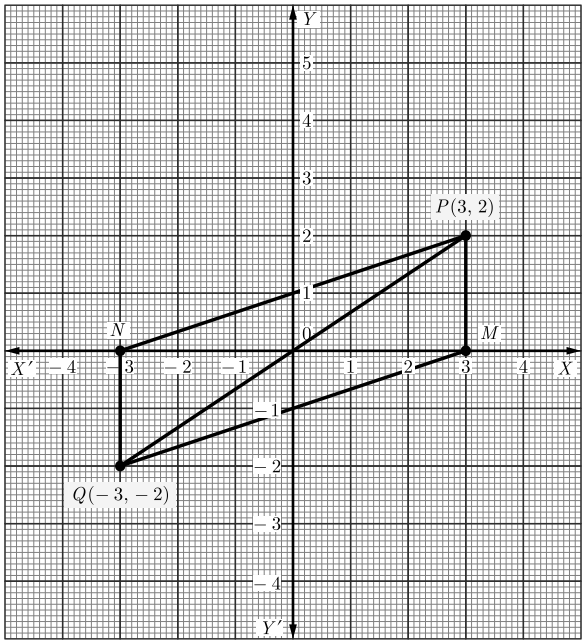
$$\begin{aligned} \square &= 2 \times \text{Area of } \Delta NMP \\ &= 2 \times \frac{1}{2} \times NM \times MP \\ &= NM \times MP \\ &= 6 \times 2 = 12 \text{ sq units} \end{aligned}$$

(iii) The coordinates of the points to which, M is mapped on reflection in

(a) X-axis is $(3,0)$

(b) Y-axis is

(c) origin is $(-3,0)$.



Comp 2001

66. Use graph paper for this question. The point $P(3,4)$ on reflection in x-axis is mapped on P'. Then P' reflection in the origin is mapped as P''.

(i) Find the coordinates of P' and P''.

(ii) Write down a single transformation that maps P onto P''.

Sol :

(i) Reflection of the point $P(x, y)$ in the x-axis is $P'(x, -y)$. Thus reflection of $P(3,4)$ in x-axis is $P'(3,-4)$.

Reflection of the point $P(x, y)$ in the origin is $P''(-x, -y)$. Thus reflection of $P'(3,-4)$ in the origin is $P'(-3, 4)$.

(ii) The reflection of the point $P(3, 4)$ in y-axis maps it onto $P''(-3, 4)$. This is single transformation.

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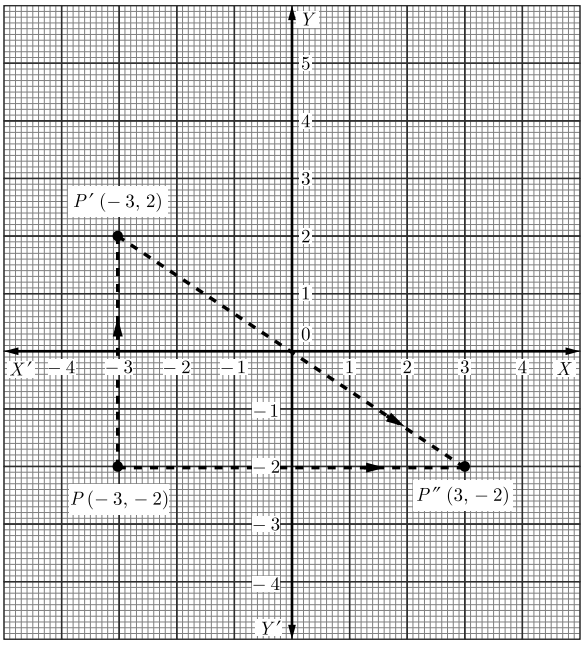

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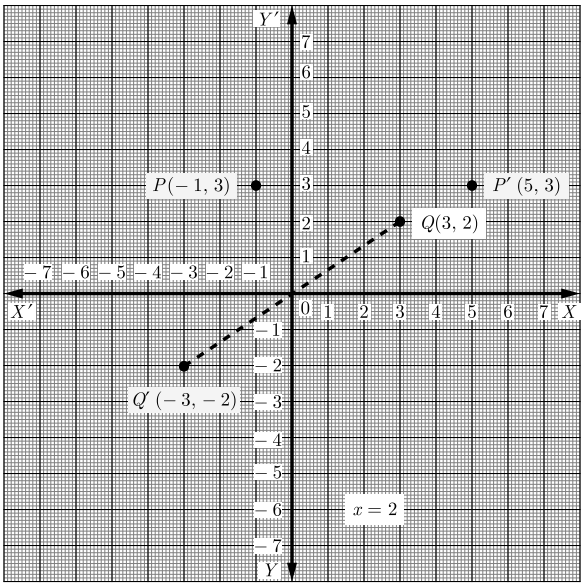


(ii) The coordinates of P' and Q are (5, 3) and (3, 2) respectively. So, using $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ equation of the line $P'Q$ is

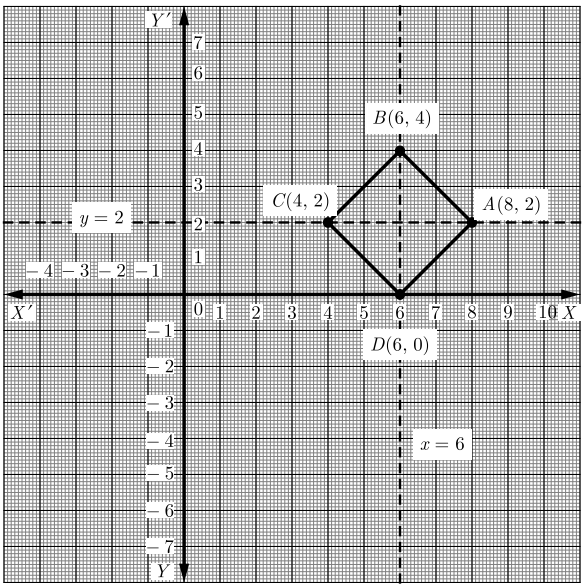
$$y - 3 = \frac{2-3}{3-5}(x-5)$$

$$y - 3 = \frac{1}{2}(x-5)$$

$$x - 2y + 1 = 0$$

Comp 2005

57. Use the graph paper for this question. Plot the points $A(8, 2)$ and $B(6, 4)$. These two points are vertices of a figure which is symmetrical about $x = 6$ and $y = 2$. Complete the figure on the graph. Write down the geometrical name of the figure



Sol :

Take 1 cm = 2 unit on both axes.

Points $A(8, 2)$ and $B(6, 4)$ are plotted on the graph paper as shown in figure given below. Clearly, $x = 6$ is a line parallel to y-axis at a distance of 6 units from it. Similarly, $y = 2$ is a line parallel to x-axis at a distance of 2 units from it. Since the figure is symmetrical about these two lines, its one vertex C(say) is the reflection (image) of vertex $A(8, 2)$ in $x = 6$ and other vertex D(say) is the image of $B(6, 4)$ in the line $y = 2$.

So, the coordinates of C and D are

$$(2 \times 6 - 8, 2) = (4, 2)$$

and $$(6, 2 \times 2 - 4) = (6, 0).$$

Complete the figure $ABCD$ as shown in given figure. Clearly, $ABCD$ is a square.

Comp 2003

58. Use graph paper for this question. The point $P(5, 3)$ was reflected in the origin to get the image P'.

(i) Write down the coordinates of P'.

(ii) If M is the foot of the perpendicular from P to the x-axis, find the coordinates of M.

(iii) If N is the foot of the perpendicular from P' to the x-axis, find the coordinates of N.

(iv) Name the figure $PMP'N$.

(v) Find the area of the figure $PMP'N$.

Sol :

Take 1 cm = 1 unit on both axes.

(i) The reflection of $P(5, 3)$ in origin is $P'(-5, -3)$.

(ii) From the plot we get that coordinates of M are $(5, 0)$.

(iii) From the plot we get that coordinates of N are $(-5, 0)$.

(iv) Clearly, figure $PMP'N$ is a parallelogram.

(v) Area of parallelogram $PMP'N$

$$= 2 \text{ Area of } \Delta NMP$$

$$= 2\left(\frac{1}{2} \times NM \times MP\right)$$

$$= NM \times MP$$

$$= 10 \times 3$$

$$= 30 \text{ sq. units}$$

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$$3y = 8 - 5 = 3 \Rightarrow y = 1$$

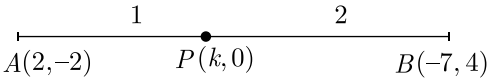
Thus (d) is correct option.

6. If the point $P(k, 0)$ divides the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ in the ratio $1 : 2$, then the value of k is

(a) 1 (b) 2
(c) -2 (d) -1

Sol :

As per question statement figure is shown below.



$$k = \frac{1(-7) + 2(2)}{1+2} \quad \left(x = \frac{mx_2 + nx_1}{m+n}\right)$$

$$= \frac{-7+4}{3} = \frac{-3}{3} = -1$$

Thus $k = -1$

Thus (d) is correct option.

7. If $A(\frac{m}{3}, 5)$ is the mid-point of the line segment joining the points $Q(-6, 7)$ and $R(-2, 3)$, then the value of m is

(a) -12 (b) -4
(c) 12 (d) -6

Sol :

Given points are $Q(-6, 7)$ and $R(-2, 3)$

Mid point $A(\frac{m}{3}, 5) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$

$$= (-4, 5)$$

Equating, $\frac{m}{3} = -4 \Rightarrow m = -12$

Thus (a) is correct option.

8. The mid-point of the line-segment AB is $P(0, 4)$, if the coordinates of B are $(-2, 3)$ then the co-ordinates of A are

(a) (2, 5) (b) $(-2, -5)$
(c) (2, 9) (d) $(-2, 11)$

Sol :

Let point A be (x, y).

Now using mid-point formula,

$$(0, 4) = \left(\frac{x-2}{2}, \frac{y+3}{2}\right)$$

Thus $0 = \frac{x-2}{2} \Rightarrow x = 2$

and $4 = \frac{y+3}{2} \Rightarrow y = 5$

Hence point A is (2, 5).

Thus (a) is correct option.

9. x-axis divides the line segment joining $A(2, -3)$ and $B(5, 6)$ in the ratio

(a) $2 : 3$ (b) $3 : 5$
(c) $1 : 2$ (d) $2 : 1$

Sol :

Let point $P(x, 0)$ on x-axis divide the segment joining points $A(2, -3)$ and $B(5, 6)$ in ratio $k : 1$, then

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{6k - 3}{k+1}$$

$$6k = 3 \Rightarrow k = \frac{1}{2}$$

Therefore ratio is $1 : 2$.

Thus (c) is correct option.

10. The point which divides the line segment joining the points $(8, -9)$ and $(2, 3)$ in the ratio $1 : 2$ internally lies in the

(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

Sol :

We have $x_1 = 8$, $y_1 = -9$, $x_2 = 2$ and $y_2 = 3$.

and $m_1 : m_2 = 1 : 2$

Let the required point be $P(x, y)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{1 \times 2 + 2 \times 8}{1+2} = 6$$

and $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$$= \frac{1 \times 3 + 2(-9)}{1+2} = -5$$

Thus $(x, y) = (6, -5)$ and this point lies in IV quadrant.

Thus (d) is correct option.

11. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is

(a) 3 (b) -3
(c) 7 (d) 4

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Sol :

Since, centre is the mid-point of end points of the diameter.

$$(3,\ 5) = \left(\frac{4+2}{2},\ \frac{7+y}{2}\right)$$

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7 + y = 10 \ \Rightarrow \ y = 3$$

Thus (a) is correct option.

12. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Sol :

Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.

We have, $\frac{4-2}{2} = y \ \Rightarrow \ y = 1$

and $\frac{4+x}{2} = -1 \ \Rightarrow \ x = -6$

Thus (a) is correct option.

13. If $x - 2y + k = 0$ is a median of the triangle whose vertices are at points $A(-1, 3)$, $B(0, 4)$ and $C(-5, 2)$,then the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

Sol :

Coordinate of the centroid G of ΔABC

$$= \left(\frac{-1+0-5}{2}, \frac{3+4+2}{3}\right)$$

$$= (-2, 3)$$

Since, G lies on the median, $x - 2y + k = 0$, it must satisfy the equation,

$$-2 - 6 + k = 0 \ \Rightarrow \ k = 8$$

Thus (d) is correct option.

14. The point which lies on the perpendicular bisector of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$ is

(a) (0, 0) (b) (0, 2)

(c) (2, 0) (d) $(-2, 0)$

Sol :

We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

Mid-point of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Thus (a) is correct option.

15. If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$

(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

Sol :

Let, $AP : AB = m : n$

Using section formula, we have,

$$4 = \frac{8m + 2n}{m + n}$$

and $$2 = \frac{4m + n}{m + n}$$

Solving these as linear equation, we get,

$$m = 1 \text{ and } n = 2$$

$$\frac{AP}{AB} = \frac{1}{2}$$

$$AP = \frac{1}{2}AB$$

Thus (d) is correct option.

16. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then the value of a is

(a) -4 (b) -12

(c) 12 (d) -6

Sol :

Since $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the points $Q(-6, 5)$ and $R(-2, 3)$,

$$\left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

Now $$\frac{a}{3} = -4 \ \Rightarrow \ a = -12$$

Thus (b) is correct option.

17. If the point $C(k, 4)$ divides the line segment

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joining two points $A(2, 6)$ and $B(5,1)$ in ratio 2 : 3, the value of k is ..... .

(a) $\frac{5}{16}$ (b) $\frac{16}{5}$

(c) $\frac{9}{5}$ (d) $\frac{5}{9}$

Sol :

We have $m:n = 2:3$

By section formula,

$$\frac{mx_2 + nx_1}{m+n} = x$$

Now, $\frac{2\times 5 + 3\times 2}{2+3} = k$

$$k = \frac{16}{5}$$

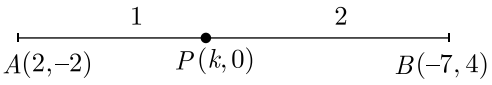
18. If the point $P(k, 0)$ divides the line segment joining the points $A(2,-2)$ and $B(-7,4)$ in the ratio 1 : 2, then the value of k is

(a) 1 (b) 2

(c) -2 (d) -1

Sol :

As per question statement figure is shown below.



$$k = \frac{1(-7)+2(2)}{1+2} \left(x = \frac{mx_2+nx_1}{m+n}\right)$$

$$= \frac{-7+4}{3} = \frac{-3}{3} = -1$$

Thus $k = -1$

Thus (d) is correct option.

19. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6,5)$ and $R(-2,3)$, then the value of a is

(a) -4 (b) -12

(c) 12 (d) -6

Sol :

Since $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the points $Q(-6,5)$ and $R(-2,3)$,

$$\left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

Now $\frac{a}{3} = -4 \Rightarrow a = -12$

Thus (b) is correct option.

20. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, the value of y will be

(a) 3 (b) 4

(c) 5 (d) 6

Sol :

Diagonals of a parallelogram bisect each other.
Mid-points of AC and BD are same.

Thus $\left(3, \frac{-1+y}{2}\right) = (3,2)$

$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

Thus (c) is correct option.

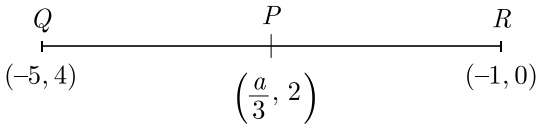
21. Select the value of a, for which point $P(\frac{a}{3},2)$ is the midpoint of the line segment joining the Points $Q(-5,4)$ and $R(-1,0)$.

(a) –9 (b) 9

(c) 8 (d) –8

Sol :

As per question, line diagram is shown below.



Since P is mid-point of QR, we have

$$\frac{a}{3} = \frac{-5+(-1)}{2} = \frac{-6}{2} = -3$$

Thus $a = -9$

Thus (a) is correct option.

22. If $A(\frac{m}{3}, 5)$ is the mid-point of the line segment joining the points $Q(-6,7)$ and $R(-2,3)$, then the value of m is

(a) -12 (b) -4

(c) 12 (d) -6

Sol :

Given points are $Q(-6,7)$ and $R(-2,3)$

Mid point $A(\frac{m}{3}, 5) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$

$$= (-4, 5)$$

Equating, $\frac{m}{3} = -4 \Rightarrow m = -12$

Thus (a) is correct option.

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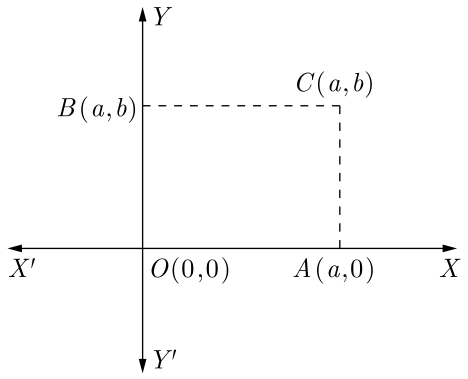


Then, the coordinates of A and B are $(a, 0)$ and $(0, b)$ respectively.
Since, $OACB$ is a rectangle. Therefore,

$$AC = OB \Rightarrow AC = b$$

Thus, we have

$$OA = a \text{ and } AC = b$$

So, the coordinates of C are (a, b).
The coordinates of the mid-point of OC are

$$\left(\frac{a+b}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

Also, the coordinates of the mid-points of AB are

$$\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

Clearly, coordinates of the mid-point of OC and AB are same.
Hence, OC and AB bisect each other.
Also,

$$OC = \sqrt{a^2 + b^2}$$

and $$AB = \sqrt{(a-0)^2 + (0-b)^2}$$

$$OC = AB$$

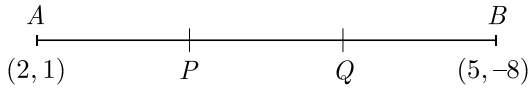
Thus diagonal of a rectangle bisect each other and are equal.

Main 2009

46. If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.

Sol :

As per question, line diagram is shown below.



Let $P(x,y)$ divides AB in the ratio 1:2
Using section formula we get

$$x = \frac{1 \times 5 + 2 \times 2}{1+2} = 3$$

$$y = \frac{1 \times -8 + 2 \times 1}{1+2} = -2$$

Hence coordinates of P are $(3,-2)$.

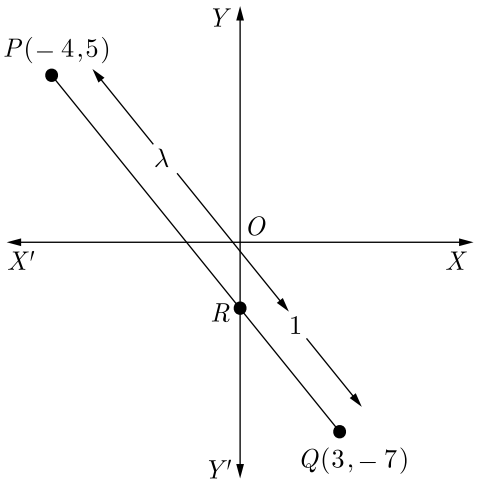
Comp 2009

47. In what ratio does the y-axis divide the line segment joining the point P (–4, 5) and $Q(3, -7)$? Also, find the coordinates of the point of intersection.

Sol :

Suppose y-axis divides PQ in the ratio λ :1. Then, the coordinates of the point of division are

$$R(x, y) = \left(\frac{3\lambda - 4}{\lambda + 1}, \frac{-7\lambda + 5}{\lambda + 1}\right)$$



Since R lies on y-axis and x-coordinate of every point on y-axis is zero.

Thus $$\frac{3\lambda - 4}{\lambda + 1} = 0$$

$$3\lambda - 4 = 0$$

$$\lambda = \frac{4}{3}$$

Hence, the required ratio is $\frac{4}{3}$:1 i.e., 4 : 3.
Putting $\lambda = \frac{4}{3}$ in the coordinates of R, we find that its coordinates are $(0, \frac{-13}{7})$

Remark:
If A (x_1, y_1) and $B(x_2, y_2)$ are two points, then the line segment joining A and B is divided by x-axis in the ratio $-\frac{x_1}{x_2}$.

Comp 2008

48. Find the coordinates of the points which divide the line segment joining the points A (-2, 2) and B (2, 8) into four equal parts.

Sol :

Let P,Q and R be the points which divide the segment AB into four equal parts, then Q is mid-point of AB, P is mid-point of AQ and R is mid-pont of QB.

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$$11+11k=15+9k$$
$$k=2$$

Thus ratio is 2 : 1.

Thus (c) is correct option.

29. If the mid-point of the line segment joining the points $A(3,4)$ and $B(k,6)$ is $P(x,y)$ and $x+y-10=0$, the value of k will be

(a) 4 (b) 5

(c) 6 (d) 7

Sol :

If $P(x,y)$ is mid point of $A(3,4)$ and $B(k,6)$, then we have

$$\frac{3+k}{2}=x \text{ and } y=\frac{4+6}{2}=\frac{10}{2}=5$$

Substituting above value in $x+y-10=0$ we have

$$\frac{3+k}{2}+5-10=0$$
$$\frac{3+k}{2}=5$$
$$3+k=10 \Rightarrow k=10-3=7$$

Thus (d) is correct option.

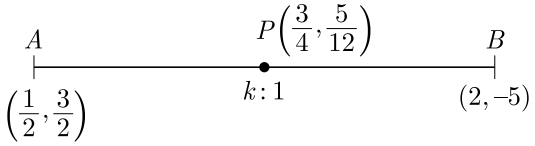
30. What is the ratio in which the point $P(\frac{3}{4},\frac{5}{12})$ divides the line segment joining the point $A\ (\frac{1}{2},\frac{3}{2})$ and $(2,-5)$.

(a) 4 : 7 (b) 3 : 7

(c) 1 : 5 (d) 2 : 5

Sol :

Let P divides AB in the ratio k:1. Line diagram is shown below.



Now $$\frac{k(2)+1(\frac{1}{2})}{k+1}=\frac{3}{4}$$
$$8k+2=3k+3$$
$$k=\frac{1}{5}$$

Thus required ratio is $\frac{1}{5}$:1 or 1:5.

Thus (c) is correct option.

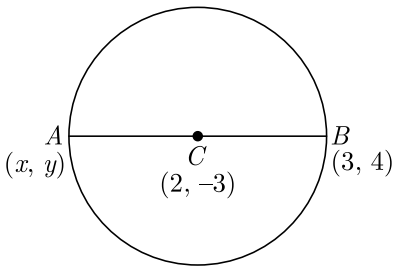
31. AB is diameter of the circle whose centre is $(2,-3)$ and B is the point $(3,4)$. The coordinates of a point A will be

(a) $(10,-1)$ (b) $(-1,10)$

(c) $(1,-10)$ (d) $(-10,1)$

Sol :

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.



Let the co-ordinates of point A be (x,y).

x-coordinate of C,

$$\frac{x+3}{2}=2$$
$$x+3=4 \Rightarrow x=1$$

and y-coordinate of C,

$$\frac{y+4}{2}=-3$$
$$y+4=-6 \Rightarrow y=-10$$

Hence, coordinates of point A is $(1,-10)$.

Thus (c) is correct option.

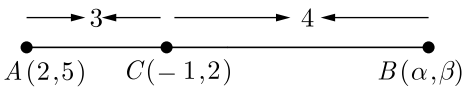
THREE MARKS QUESTIONS

Comp 2023

32. If the point C (–1, 2) divides internally the line segment joining A (2, 5) and B in ratio 3 : 4, find the coordinates of B.

Sol :

Let the point B be $B(\alpha,\ \beta)$.



Since $AC : BC = 3 : 4$.

So, the coordinates of C are

$$C(x,y)\left(\frac{3\alpha+4\times 2}{3+4},\frac{3\beta+4\times 5}{3+4}\right)$$
$$=\left(\frac{3\alpha+8}{7},\frac{3\beta+20}{7}\right)$$

But, the coordinates of C are (–1, 2).

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Using the section formula we have

$$x = \frac{m \times x_2 + n \times x_1}{m+n}$$
$$= \frac{2 \times 2 + 3 \times (-3)}{2+3}$$
$$= \frac{4-9}{5} = -\frac{5}{5} = -1 \text{ and}$$
$$y = \frac{m \times y_2 + n \times y_1}{m+n}$$
$$= \frac{2 \times (-7) + 3 \times 3}{2+3}$$
$$= \frac{-14+9}{5} = -1.$$

The coordinates of P are $(-1, -1)$.

Main 2015

39. If (a, b) is the mid-point of the line segment joining the points A $(10, -6)$ and B$(k, 4)$ and $a - 2b = 18$, find the value of k and the distance AB.

Sol :

The mid-point of the line segment joining the points A$(10, -6)$ and B$(k, 4)$ is $(\frac{10+k}{12}, \frac{-6+4}{2})$ i.e $(\frac{10+k}{2}, -1)$.

But the mid-point of the segment AB is (a, b)

Thus $a = \frac{10+k}{2}$ and $b = -1$

Also $a - 2b = 18$ (given)

Substituting the values of a and b in above,

$$\frac{10+k}{2} - 2 \times (-1) = 18$$
$$\frac{10+k}{2} = 18 + 2(-1) = 16$$
$$10 + k = 16 \times 2 = 32$$
$$k = 32 - 10 = 22$$

So, the point B is $(22, 4)$

Distance,

$$AB = \sqrt{(22-10)^2 + (4+6)^2}$$
$$= \sqrt{144+100}$$
$$= \sqrt{224} = 2\sqrt{61} \text{ units.}$$

Comp 2014

40. Prove that the points $(-2, -1)$, $(1, 0)$, $(4, 3)$ and $(1, 2)$ are the vertices of a parallelogram. Is it a rectangle?

Sol :

A quadrilateral is a parallelogram if and only if its diagonals bisect each other.

Let the given point be A, B, C and D respectively.

Then,

Here, mid-point of diagonal AC is

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = (1,1)$$

and mid-point of diagonal BD is

$$\left(\frac{1+1}{2}, \frac{0+2}{2}\right) = (1,1)$$

Thus, the diagonals AC and BD have same mid-point, so the diagonals AC and BD bisect each other. Therefore, ABCD is a parallelogram.

Now, we shall see whether $ABCD$ is a rectangle or not.

We have,

$$AC = \sqrt{(4-(-2))^2 + (3-(-1))^2}$$
$$= 2\sqrt{13}$$

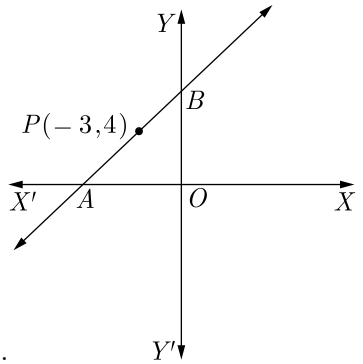
and, $BD = \sqrt{(1-1)^2 + (0-2)^2}$

$$= 2$$

Clearly, $AC \neq BD$ So, $ABCD$ is not a rectangle.

Main 2013

41. In the figure given below, the line segment AB meets X-axis and Y-axis at B. The point $P(-3,4)$ on AB divides it in the ratio $2 : 3$. Find the coordinates of A and B



Sol :

Given points A and B lie on X and Y-axis, respectively. So, let the coordinates of A and B be $(x,0)$ and $(0,y)$ respectively.

Since, the point $P(-3,4)$ on AB divides it in the ratio $2 : 3$.

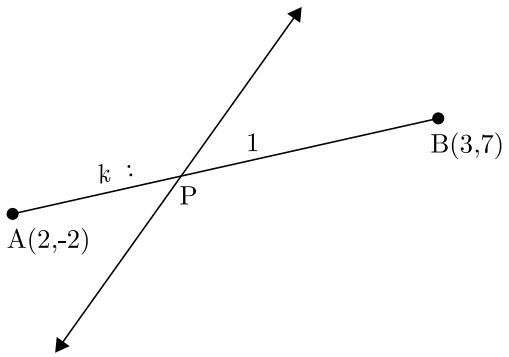
$$AP : BP = 2 : 3$$

Using section formula we get

$$P(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$
$$= \left(\frac{2 \times 0 + 3 \times x}{2+3}, \frac{2 \times y + 3 \times 0}{2+3}\right)$$

But coordinate of P is given $(-3,4)$.

NODIA



Coordinates of P are

$$P(x,y)=\left(\frac{k\times x_2+x_1}{k+1},\ \frac{k\times y_2+y_1}{k+1}\right)$$
$$=\left(\frac{k\times 3+2}{k+1},\ \frac{k\times 7+(-2)}{k+1}\right)$$
$$=\left(\frac{3k+2}{k+1},\ \frac{7k-2}{k+1}\right)$$

As the point P is on line the line (i), substituting this point on line equation we have

$$2\times\frac{3k+2}{k+1}+\frac{7k-2}{k+1}-4=0$$
$$2(3k+2)+(7k-2)-4(k+1)=0$$
$$6k+4+7k-2-4k-4=0$$
$$9k=2\Rightarrow k=\frac{2}{9}$$

The required ratio is $\frac{2}{9}:1$ i.e. 2 : 9

Main 2017

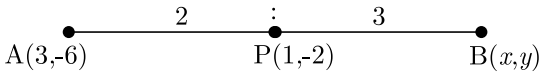
36. $P(1,-2)$ is a point on the line segment joining $A(3,-6)$ and $B(x,y)$ such that $AP:PB$ is equal to $2:3$. Find the coordinates of B.

Sol :

Here point P divides a line segment joining $A(3,-6)$ and $B(x,y)$ such that

$$AP:PB=2:3$$

i.e. P divides AB in the ratio $2:3$ internally.



Using section formula we get

$$P(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$
$$=\left(\frac{2x+3\times 3}{2+3},\frac{2y+3\times(-6)}{2+3}\right)$$
$$=\left(\frac{2x+9}{5},\frac{2y-18}{5}\right)$$

But given coordinates of P are $(1,-2)$.

Thus $\frac{2x+9}{5}=1$ and $\frac{2y-18}{5}=-2$

$$2x=5-9\Rightarrow x=-2$$

and $2y-18=-10\Rightarrow y=4$

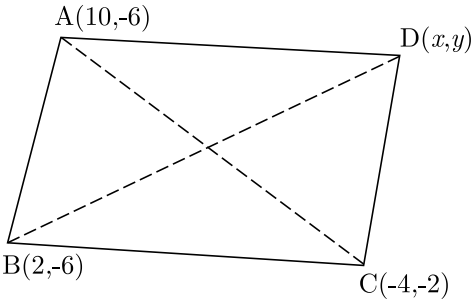
Thus, coordinate of B are $(-2,4)$

Comp 2016

37. Three consecutive vertices's of a parallelogram $ABCD$ are $A(10,-6)$, $B(2,-6)$ and $C(-4,-2)$, find the fourth vertex D.

Sol :

In the parallelogram $ABCD$, the three consecutive vertices's are $A(10,-6)$, $B(2,-6)$ and $C(-4,-2)$. Let the fourth vertex D be (x, y).



The mid-point of AC is

$$\left(\frac{10+(-4)}{2},\frac{-6+(-2)}{2}\right)\text{ i.e. }(3,-4)\quad\ldots(1)$$

The mid-point of BD is

$$\left(\frac{2+x}{2},\frac{-6+y}{2}\right)\quad\ldots(2)$$

Since the diagonals of a parallelogram bisect each other, the mid-points of AC and BD are same.

From (1) and (2), we get

$$\frac{2+x}{2}=3\text{ and }\frac{-6+y}{2}=-4$$
$$2+x=6\text{ and }-6+y=-8$$
$$x=4\text{ and }y=-2$$

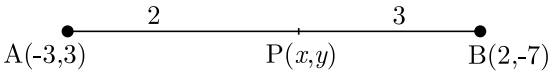
The fourth vertex of the parallelogram $ABCD$ is $D(4,-2)$.

Comp 2015

38. Calculate the coordinates of the point P which divides the line segment joining $A(-3,3)$ and $B(2,-7)$ internally in the ratio $2:3$.

Sol :

Let $P(x, y)$ be point the which divides the line segment joining A (-3, 3) and (2, -7) internally in the ratio 2 : 3. Then



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23. The co-ordinate of the point dividing the line segment joining the points $A(1,3)$ and $B(4,6)$ in the ratio $2:1$ is ..... .

(a) (5, 3) (b) (3, 5)
(c) (4, 6) (d) (6, 4)

Sol :

Let point $P(x,\ y)$ divides the line segment joining the points $A(1,3)$ and $B(4,6)$ in the ratio $2:1$.
Using section formula we have

$$(x,\ \ y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\ \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$(x,\ \ y)=\left(\frac{2\times4+1\times1}{2+1},\ \frac{2\times6+1\times3}{2+1}\right)$$

$$=\left(\frac{8+1}{3},\ \frac{12+3}{3}\right)$$

$$=\left(\frac{9}{3},\ \frac{15}{3}\right)=(3,5)$$

Thus (b) is correct option.

24. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is

(a) 3 (b) -3
(c) 7 (d) 4

Sol :

Since, centre is the mid-point of end points of the diameter.

$$(3,\ 5)=\left(\frac{4+2}{2},\ \frac{7+y}{2}\right)$$

Comparing both the sides, we get

$$5=\frac{7+y}{2}$$

$$7+y=10\ \Rightarrow\ y=3$$

Thus (a) is correct option.

25. The mid-point of the line-segment AB is $P(0,4)$, if the coordinates of B are $(-2,\ 3)$ then the co-ordinates of A are

(a) (2, 5) (b) $(-2,-5)$
(c) (2, 9) (d) $(-2,11)$

Sol :

Let point A be $(x,\ y)$.
Now using mid-point formula,

$$(0,4)=\left(\frac{x-2}{2},\ \ \frac{y+3}{2}\right)$$

Thus $$0=\frac{x-2}{2}\ \Rightarrow\ x=2$$

and $$4=\frac{y+3}{2}\ \Rightarrow\ y=5$$

Hence point A is (2, 5).
Thus (a) is correct option.

26. x-axis divides the line segment joining $A(2,-3)$ and $B(5,6)$ in the ratio

(a) $2:3$ (b) $3:5$
(c) $1:2$ (d) $2:1$

Sol :

Let point $P(x,\ 0)$ on x-axis divide the segment joining points $A(2,\ -3)$ and $B(5,\ 6)$ in ratio $k\ :\ 1$, then

$$y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$0=\frac{6k-3}{k+1}$$

$$6k=3\ \Rightarrow\ k=\frac{1}{2}$$

Therefore ratio is $1:2$.
Thus (c) is correct option.

27. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$?

(a) $7:8$ (b) $7:2$
(c) $2:8$ (d) $2:7$

Sol :

Let $$AP:PB=k:1$$

Now $$\frac{3k-6}{k+1}=-4$$

$$3k-6=-4k-4$$

$$7k=2\ \Rightarrow\ k=\frac{2}{7}$$

Hence, $$AP:PB=2:7$$

Thus (d) is correct option.

NODIA

28. What is the ratio in which $(11,15)$ divides the line segment joining the points $(15,5)$ and $(9,20)$.

(a) $1:3$ (b) $2:3$
(c) $2:1$ (d) $1:2$

Sol :

Let the two points $(15,5)$ and $(9,20)$ are divided in the ratio $k:1$ by point $P(11,15)$.
Using Section formula, we get

$$x=\frac{m_2x_1+m_1x_2}{m_2+m_1}$$

$$11=\frac{1(15)+k(9)}{1+k}$$

$$(-3,4)=\left(\frac{0+3x}{5},\frac{2y+0}{5}\right)$$

Comparing the coordinates from both sides, we get

$$-3=\frac{3x}{5} \text{ and } 4=\frac{2y}{5}$$

$$-3\times 5=3x \text{ and } 4\times 5=2y$$

$$3x=-15 \text{ and } 20=2y$$

$$x=\frac{-15}{3} \text{ and } y=\frac{20}{2}$$

$$x=-5 \text{ and } y=10$$

Hence, the coordinates of A and B are $(-5,0)$ and $(0,10)$.

Comp 2013

42. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Sol :

Let y -axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB=k:1$

Now, the coordinates of point of division P,

$$(x,y)=\left(\frac{k(-1)+1(5)}{k+1},\ \frac{k(-4)+1(-6)}{k+1}\right)$$

$$=\left(\frac{-k+5}{k+1},\ \frac{-4k-6}{k+1}\right)$$

Since P lies on y axis, therefore $x=0$, which gives

$$\frac{5-k}{k+1}=0 \Rightarrow k=5$$

Hence required ratio is $5:1$,

Now $$y=\frac{-4(5)-6}{6}=\frac{-13}{3}$$

Hence point on y-axis is $\left(0,-\frac{13}{3}\right)$.

Comp 2012

43. Determine the ratio in which the straight line $x-y-2=0$ divides the line segment joining $(3,-1)$ and $(8,9)$.

Sol :

Let co-ordinates of P be (x_1,y_1) and it divides line AB in the ratio $k:1$.

Now $$x_1=\frac{8k+3}{k+1}$$

$$y_1=\frac{9k-1}{k+1}$$

Since point $P(x_1,y_1)$ lies on line $x-y-2=0$, so co-ordinates of P must satisfy the equation of line.

Thus $$\frac{8k+3}{k+1}-\frac{9k-1}{k+1}-2=0$$

$$8k+3-9k+1-2k-2=0$$

$$-3k+2=0 \Rightarrow k=\frac{2}{3}$$

So, line $x-y-2=0$ divides AB in the ratio $2:3$

Main 2010

44. Find the ratio in which the point $(-3,\ p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,\ 3)$. Hence, find the value of p.

Sol :

Suppose the point P $(-3,\ p)$ divides the line segment joining points A $(-5,\ -4)$ and B $(-2,\ 3)$ in the ratio $k:1$.

Then, the coordinates of P are,

$$P(-3,\ p)=\left(\frac{-2k-5}{k+1},\frac{3k-4}{k+1}\right)$$

Now $$\frac{-2k-5}{k+1}=-3 \qquad (1)$$

and $$\frac{3k-4}{k+1}=p \qquad (2)$$

From eq (1) we have

$$-2k-5=-3k-3$$

$$-2k+3k=-3+5$$

$$k=2$$

Substituting $k=2$ in eq (2) we have

$$\frac{3k-4}{k+1}=p$$

$$\frac{3\times 2-4}{2+1}=p$$

$$6-4=3p$$

$$p=\tfrac{2}{3}$$

Hence, the ratio is $2:1$ and $p=\frac{2}{3}$.

Comp 2010

45. Prove that the diagonal of a rectangle bisect each other and are equal.

Sol :

Let $OACB$ be a rectangle such that OA is a along x–axis and OB is along y–axis Let $OA=a$ and $OB=b$.

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Thus $\frac{3\alpha+8}{7} = -1$ and $\frac{3\beta+20}{7} = 2$

$3\alpha+8 = -7$ and $3\beta+20 = 14$

$3\alpha = -15$ and $3\beta = -6$

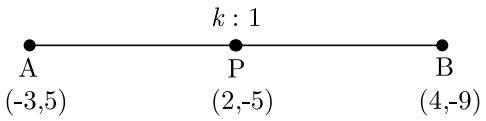
$\alpha = -5$ and $\beta = -2$

Thus, the coordinates of B are (–5, –2).

Comp 2022

33. In what ratio does the point $P(2,-5)$ divide the line segment joining the points A(-3,5) and $B(4,-9)$)?

Sol :



Let P (2,-5) divide the line segment joining the points A(-3, 5) and B (4,-9) in the ratio k : 1 *i.e.* AP : PB = k : 1.

Coordinates of P are

$$P(x,y) = \left(\frac{k\times x_2+x_1}{k+1}, \frac{k\times y_2+y_1}{k+1}\right)$$
$$= \left(\frac{k\times 4+(-3)}{k+1}, \frac{k\times(-9)+5}{k+1}\right)$$
$$= \left(\frac{4k-3}{k+1}, \frac{-9k+5}{k+1}\right)$$

But P is (2,-5), therefore we have

$$\frac{4k-3}{k+1} = 2 \text{ and } \frac{-9k+5}{k+1} = -5.$$

From $\frac{4k-3}{k+1} = 2$ we have

$$4k-3 = 2k+2$$
$$4k-2k = 2+3$$
$$2k = 5 \Rightarrow k = \tfrac{5}{2}$$

From $\frac{-9k+5}{k+1} = -5$ we have

$$-9k+5 = -5k-5$$
$$-9k+5k = -5-5$$
$$-4k = -10$$
$$k = \tfrac{-10}{-4} = \tfrac{5}{2}$$

Thus required ratio is $\frac{5}{2}$:1 i.e. 5 : 2 (internally).

Remark :

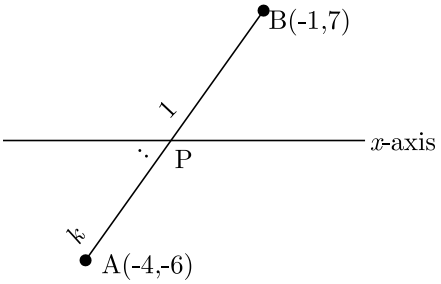
If we get different values of k from the two equations, it means that point P does not lie on the line AB, and the question of finding the ratio will not arise.

Comp 2019

34. In what ratio does the x-axis divide the line segment joining The point (-4,-6) and (-1,7)? Also find the coordinates of the point of division.

Sol :

Let the line segment joining the points A (-4,-6) and B (-1,7) be divided by the x-axis in the ratio k : 1 at the point P i.e. AP : PB = k : 1,



Coordinates of P are

$$P(x,y) = \left(\frac{k\times x_2+x_1}{k+1}, \frac{k\times y_2+y_1}{k+1}\right)$$
$$= \left(\frac{k\times(-1+(-4)}{k+1}, \frac{k\times 7+(-6)}{k+1}\right)$$
$$= \left(\frac{-k-4}{k+1}, \frac{7k-6}{k+1}\right)$$

But P lies on the x-axis, therefore, y-coordinate of P = 0. Therefore

$$\frac{7k-6}{k+6} = 0$$

$$7k-6 = 0 \Rightarrow k = \tfrac{6}{7}$$

The required ratio is $\frac{6}{7}$: 1 i.e 6 : 7

The coordinates of the point of division P are

$$\left(\frac{-\frac{6}{7}-4}{\frac{6}{7}+1}, 0\right) \text{ i.e } \left(-\frac{34}{13}, 0\right).$$

Comp 2018

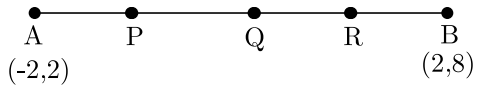
35. Determine the ratio in which the line $2x+y=4$ divides the line segment joining the points A(2, -2) and B(3, 7).

Sol :

We have $2x+y-4=0$ ...(i)

Let the given line (i) divide the line segment joining the points A (2,–2) and B (3,7) in the ratio k : 1 at the point P *i.e.* AP : PB = k : 1

NODIA



Since Q is mid point of AB, we have

$$Q(x,y)=\left(\frac{-2+2}{2},\frac{2+8}{2}\right)$$
$$=(0,\ 5)$$

Since P is mid point of AQ, we have

$$P(x,y)=\left(\frac{-2+0}{2},\ \frac{2+5}{2}\right)$$
$$=(-1,\ \tfrac{7}{2})$$

Since R is mid point of QB, we have

$$R(x,y)=\left(\frac{0+2}{2},\frac{5+8}{2}\right)$$
$$=(1,\ \tfrac{13}{2})$$

Hence, the coordinates of the points of division are $P(-1,\ \frac{7}{2})$, $Q(0,\ 5)$ and $R(1,\ \frac{13}{2})$.

Main 2007

49. The mid-point of the line segment joining $(2a,4)$ and $(-2,2b)$ is $(1,2a+1)$. Find the values of a and b.

Sol :

The mid-point of the line segment joining $A(2a,4)$ and $B(-2,2b)$,

$$M(x,y)=\left(\frac{x_2+x_1}{2},\ \frac{y_2+y_1}{2}\right)$$
$$=\left(\frac{2a-2}{2},\frac{4+2b}{2}\right)$$
$$=(a-1,2+b)$$

But coordinate of mid-point is given $(1,2a+1)$.

$$(1,2a+1)=(a-1,2+b)$$

Comparing the coordinates is given sides, we get

$$1=a-1 \text{ and } 2a+1=2+b$$
$$a=2 \text{ and } 2a=1+b$$

Now, $2a=1+b$

$$2(2)=1+b \qquad [a=2]$$
$$4=1+b$$
$$b=4-1=3$$

Hence, the values of a and b are 2 and 3, respectively.

Main 2007

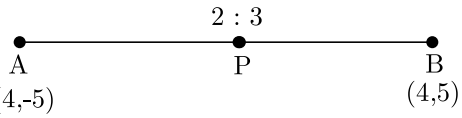
50. If the line joining the points $A(4,-5)$ and $B(4,5)$ is divided by the point P, such that $\frac{AP}{AB}=\frac{2}{5}$, then find the coordinates of P.

Sol :

We have $\frac{AP}{AB}=\frac{2}{5}$

$$\frac{AP}{AB-AP}=\frac{2}{5-2}$$
$$\frac{AP}{BP}=\frac{2}{3}$$

Thus, the point P divides the line segment joining the point A (4,–5) and B (4, 5) in the ratio 2 : 3 internally.



Using section formula, the coordinates of P are

$$P(x,y)=\left(\frac{m\times x_2+n\times x_1}{m+n},\ \frac{m\times y_2+n\times y_1}{m+n}\right)$$
$$=\left(\frac{2\times 4+3\times 4}{2+3},\frac{2\times 5+3\times(-5)}{2+3}\right)$$
$$=\left(\frac{8+12}{5},\ \frac{10-15}{5}\right)$$
$$=\left(\frac{20}{5},\ \frac{-5}{5}\right)$$
$$=(4,-1)$$

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Comp 2007

51. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, –3) and (3, 4). Find its centroid.

Sol :

Let $P(1,\ 1)$, $Q(2,\ -3)$ and $R(3,\ 4)$ be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let $A\ (x_1,\ y_1)$, $B\ (x_2,\ y_2)$ and $C(x_3,\ y_3)$ be the vertices of triangle ABC. Then,

Since $P(1,\ 1)$ is the mid-point of AB, we have

$$\frac{x_1+x_2}{2}=1,\ \frac{y_1+y_2}{2}=1$$
$$x_1+x_2=2 \text{ and } y_1+y_2=2 \qquad (1)$$

Since $Q(2,\ -3)$ is the mid-point of BC, we have

$$\frac{x_2+x_3}{2}=2,\ \frac{y_2+y_3}{2}=-3$$
$$x_2+x_3=4 \text{ and } y_2+y_3=-6 \qquad (2)$$

Since $R(3,\ 4)$ is the mid-point of AC, we have

$$\frac{x_1+x_3}{2}=3 \text{ and } \frac{y_1+y_3}{2}=4$$
$$x_1+x_3=3 \text{ and } y_1+y_3=8 \qquad (3)$$

From (1), (2) and (3), we get

Now $x = \dfrac{1(8)+2(5)}{3} = 6$

$y = \dfrac{1(10)+2(7)}{3} = 8$

Thus $P(x_1, y_1)$ is $P(6,8)$. Since Q is the mid point of PB, we have

$x_1 = \dfrac{6+8}{2} = 7$

$y_1 = \dfrac{8+10}{2} = 9$

Thus $Q(x_2, y_2)$ is $Q(7,9)$

Main 2003

58. Prove that (4, –1), (6, 0), (7, 2) and (5, 1) are the vertices of a rhombus. Is .it a square?

Sol :

A parallelogram whose adjacent sides are equal is always a rhombus.
Let the given points be A,B,C and D respectively.
Thus,
Coordinates of the mid-point of AC are

$\left(\dfrac{4+7}{2}, \dfrac{-1+2}{2}\right)\left(\dfrac{11}{12}, \dfrac{1}{2}\right)$

Coordinates of the mid-point of BD are

$\left(\dfrac{6+5}{2}, \dfrac{0+1}{2}\right) = \left(\dfrac{11}{2}, \dfrac{1}{2}\right)$

Thus, AC and BD have the same mid-point. Hence, $ABCD$ is a parallelogram.
Now,

Now $AB = \sqrt{(6-4)^2+(0+1)^2}$

$= \sqrt{5}$,

$BC = \sqrt{(7-6)^2+(2-0)^2}$

$= \sqrt{5}$

Here $AB = BC$

So, $ABCD$ is a parallelogram whose adjacent sides are equal. Hence, $ABCD$ is a rhombus.

Now, $AC = \sqrt{(7-4)^2+(2+1)^2}$

$= 3\sqrt{2}$

and $BD = \sqrt{(6-5)^2+(0-1)^2}$

$= \sqrt{2}$

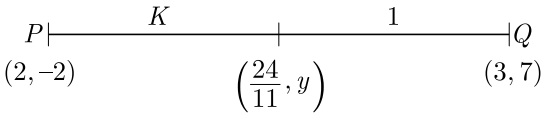
Clearly, $AC \neq BD$, $ABCD$ is not a square.

Comp 2003

59. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divides the line segment joining the points $P(2,-2)$ and $Q(3,7)$? Also find the value of$y$.

Sol :

As per question, line diagram is shown below.



Let $P\left(\frac{24}{11}, y\right)$ divides the segment joining the points $P(2,-2)$ and $Q(3,7)$ in ratio $k:1$.
Using intersection formula $x = \dfrac{mx_2 + nx_1}{m+n}$ we have

$\dfrac{3k+2}{k+1} = \dfrac{24}{11}$

$33k + 22 = 24k + 24$

$9k = 2 \Rightarrow k = \dfrac{2}{9}$

Hence, $y = \dfrac{-18+14}{11} = -\dfrac{4}{11}$

Main 2002

60. If the line segment joining the points $A(2,5)$ and $B(-5,-2)$ divided by the point P such that $\frac{AP}{AB} = \frac{3}{7}$, find the coordinates of P.

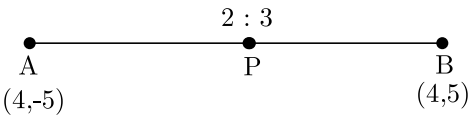
Sol :

We have $\dfrac{AP}{AB} = \dfrac{3}{7}$

$\dfrac{AP}{AB - AP} = \dfrac{3}{7-3}$

$\dfrac{AP}{BP} = \dfrac{3}{4}$

Thus, the point P divides the line segment joining the point $A(2,5)$ and $B(-5,-2)$ in the ratio 3 : 4 internally.



Using section formula, the coordinates of P are

$P(x,y) = \left(\dfrac{m \times x_2 + n \times x_1}{m+n}, \dfrac{m \times y_2 + n \times y_1}{m+n}\right)$

$= \left(\dfrac{3 \times (-5) + 4 \times 2}{3+4}, \dfrac{3 \times (-2) + 4 \times 5}{3+4}\right)$

$= \left(\dfrac{-15+8}{7}, \dfrac{-6+20}{7}\right)$

$= \left(\dfrac{-7}{7}, \dfrac{14}{7}\right)$

$= (4, -1)$

Comp 2002

61. If the point C (–1, 2) divides internally the line segment joining the points A (2, 5) and B(x, y) in the ratio 3 : 4, find the value of $x^2 + y^2$.

Sol :

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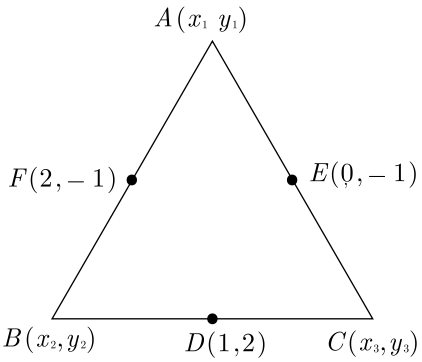
Let $A\ (x_1,\ y_1)$, $B\ (x_2,\ y_2)$ and $C(x_3,\ y_3)$ be the vertices of ΔABC. Let $D\ (1,\ 2)$, $E\ (0,\ -1)$, and $F\ (2,\ -1)$ be the mid-points of sides BC, CA and AB respectively.



Since D is the mid-point of BC.

$$\frac{x_2+x_3}{2}=1$$

and $$\frac{y_2+y_3}{2}=2$$

$$x_2+x_3=2$$

and $$y_2+y_3=4 \qquad ...(1)$$

Similarly, E and F are the mid-points of CA and AB respectively.

$$\frac{x_1+x_3}{2}=0$$

and $$\frac{y_1+y_3}{2}=-1$$

$$x_1+x_3=0$$

and $$y_1+y_3=-2 \qquad ...(2)$$

and $$\frac{x_1+x_2}{2}=2$$

and $$\frac{y_1+y_2}{2}=-1$$

$$x_1+x_2=4$$

and $$y_1+y_2=-2 \qquad ...(3)$$

Adding (1), (2) and (3), we get

$$(x_2+x_3)+(x_1+x_3)+(x_1+x_2)=2+0+4$$

and $$(y_2+y_3)+(y_1+y_3)+(y_1+x_2)=4-2-2$$

We get $$x_1+x_2+x_3=3$$

$$y_1+y_2+y_3=0 \qquad (4)$$

From (1) and (4), we get

$$x_1+2=3 \Rightarrow x_1=1$$

and $$y_1+4=0 \Rightarrow y_1=-4$$

So, the coordinates of A are (1, –4)

From (2) and (4), we get

$$x_2+0=3 \Rightarrow x_2=1$$

and $$y_2-2=0 \Rightarrow y_1=-4$$

So, coordinates of B are (3, 2)

From (3) and (4), we get

$$x_3+4=3 \Rightarrow x_3=-1$$

and $$y_3-2=0 \Rightarrow y_3=2$$

So, coordinates of C are (–1, 2)

Hence the vertices of the triangle ABC are $A(1,-4)$, B (3, 2) and C (–1, 2).

Comp 2005

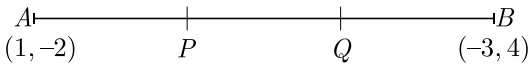
56. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Sol :

Let $P(x_1,y_1), Q(x_2,y_2)$ divides AB into 3 equal parts.

Thus P divides AB in the ratio of 1:2.

As per question, line diagram is shown below.



Now $$x_1=\frac{1(-3)+2(1)}{1+3}=\frac{-3+2}{3}=\frac{-1}{3}$$

$$y_1=\frac{1(4)+2(-2)}{1+2}=\frac{4-4}{3}=0$$

Co-ordinates of P is $(-\frac{1}{3},0)$.

Here Q is mid-point of PB.

Thus $$x_2=\frac{-\frac{1}{3}+(-3)}{2}$$

$$=\frac{-10}{6}=\frac{-5}{3}$$

$$y_2=\frac{0+4}{2}=2$$

Thus co-ordinates of Q is $(-\frac{5}{2},2)$.

Comp 2004

57. Find the co-ordinates of the points which divide the line segment joining the points $(5,7)$ and $(8,10)$ in 3 equal parts.

Sol :

Let $P(x_1,y_2)$ and $Q(x_2,y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.

Using section formula we have,

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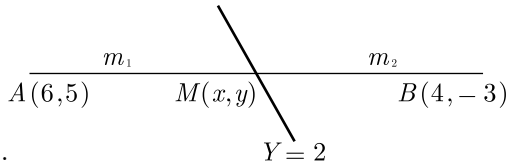


Using section formula we get

$$P(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$
$$=\left(\frac{m\times 4+n\times 6}{m+n},\frac{m\times(-3)+n\times 5}{m+n}\right)$$
$$=\left(\frac{4m+6n}{m+n},\frac{-3m+5n}{m+n}\right)$$

Since, line $y=2$ divides AB at point $P(x,y)$, so y-coordinate of P is 2.

Thus
$$2=\frac{-3m+5n}{m+n}$$
$$2(m+n)=-3m+5n$$
$$2m+2n=-3m_1+5m_2$$
$$2m_1+3m_1=5m_2-2m_2$$
$$5m_1=3m_2$$
$$\frac{m_1}{m_2}=\frac{3}{5}$$

So, the line $y=2$ divide the line joining $A(6,5)$ and $B(4,-3)$ in the ratio $3:5$.

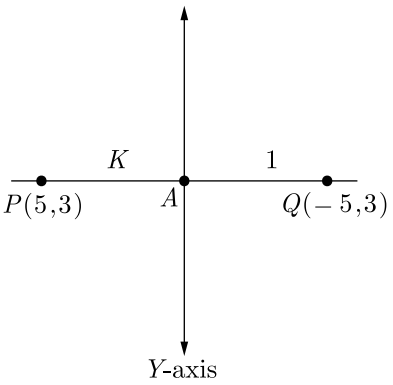
FOUR MARK QUESTIONS

Main 2020

65. In what ratio is the line joining P (5, 3) and $Q(-5,3)$ divided by the Y-axis? Also find the coordinates of the point of intersection.

Sol :

Let line joining $P(5, 3)$ and $Q(-5, 3)$ divided by the Y-axis in the ratio $k:1$ at the point A.



Coordinates of A

$$A(x,\ y)=\left(\frac{-5k+5}{k+1},\frac{3k+3}{k+1}\right)$$

Since, A lies on the Y-axis, therefore, x-coordinates of A must be 0.

$$\frac{-5k+5}{k+1}=0$$
$$-5k+5=0$$
$$-5k=-5$$
$$k=1$$

Thus required ratio is $k:1$ i.e $=1:1$

The coordinates of the point of intersection A are

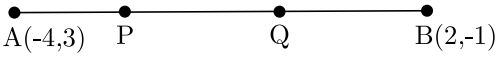
$$\left(\frac{-5\times 1+5}{1+1},\frac{3\times 1+3}{1+1}\right)=(0,3)$$

Main 2018

66. Find the coordinates of the points of trisection of the line segment joining the points A(-4, 3) and B(2, -1).

Sol :

Let P and Q be the points of trisection of the line segment AB, then



$$AP=PQ=QB\Rightarrow 2AP=PB$$
$$\frac{AP}{PB}=\frac{1}{2}$$

Thus P divides AB in the ratio 1 : 2.

Coordinates of P are

$$P(x,y)=\left(\frac{m\times x_2+n\times x_1}{m+n},\ \frac{m\times y_2+n\times y_1}{m+n}\right)$$
$$=\left(\frac{1\times 2+2\times(-4)}{1+2},\frac{1\times(-1)+2\times 3}{1+2}\right)$$
$$=\left(\frac{2-8}{3},\frac{-1+6}{3}\right)$$
$$=(-2\ \ \tfrac{5}{3}).$$

Now $PQ=QB$

Since $AP=PQ=QB$

$$AQ=2QB$$

Thus Q divides AB in the ratio 2:1.

Coordinates of Q are

$$Q(x,y)=\left(\frac{2\times 2+1\times(-4)}{2+1},\frac{2\times(-4)+1\times 3}{2+1}\right)$$
$$=(0,\ \tfrac{1}{3})$$

REMARK :

As Q is the mid-point of PB. So, the coordinates of Q can also be obtained by using mid-point formula.

Since Q is the mid-point of PB.

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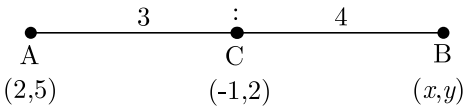
As C (-1, 2) divides the line segment joining the points $A(2, 5)$ and B (x,y) in the ratio (3 : 4) internally, so using the section formula we have



$$x = \frac{m \times x_2 + n \times x_1}{m+n}$$

$$-1 = \frac{3 \times x + 4 \times 2}{3+4}$$

$$-7 = 3x + 8$$

$$3x = -5 - 7 = -15$$

$$x = -\tfrac{15}{3} = -5$$

and $$y = \frac{m \times y_2 + n \times y_1}{m+n}$$

$$2 = \frac{3 \times y + 4 \times 5}{3+4}$$

$$14 = 3y + 20$$

$$3y = 14 - 20 = -6$$

$$y = -\tfrac{6}{3} = -2$$

Now $$x^2 + y^2 = (-5)^2 + (-2)^2$$

$$= 25 + 4$$

$$= 29$$

Comp 2001

62. Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

Sol :

Suppose the line $3x + y - 9 = 0$ divides the line segment joining A (1, 3) and B (2, 7) in the ratio $k : 1$ at point C. Then, the coordinates of C are

$$C(x,\ y) = \left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$$

But, C lies on $3x + y - 9 = 0$. Therefore point C must satisfy the line.. Thus

$$3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$$

$$6k + 3 + 7k + 3 - 9k - 9 = 0$$

$$4k - 3 = 0$$

$$k = \frac{3}{4}$$

So, the required ratio is 3 : 4 internally.

Main 2001

63. $A(10,5)$, $B(6,-3)$ and $C(2,1)$ are the vertices of a ΔABC. L is the mid-point of AB and M is the mid-point of AC. Write down the coordinates of L and M. Show that $LM = \frac{1}{2}BC$.

Sol :

Given, vertices of a triangle are $A(10,5)$, $B(6,-3)$ and $C(2,1)$.

[$\because$ mid-point coordinates $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$]

Coordinates of L = Mid-point of AB

$$= \left(\frac{10+6}{2}, \frac{5-3}{2}\right)$$

$$= \left(\frac{16}{2}, \frac{2}{2}\right) = (8,1)$$

Coordinates of M = Mid-point of AC

$$= \left(\frac{10+2}{2}, \frac{5+1}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{6}{2}\right) = (6,3)$$

Now, length,

$$LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6-8)^2 + (3-1)^2}$$

$$= \sqrt{(-2)^2 + (2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2} \text{ units} \quad \ldots(1)$$

and length,

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2-6)^2 + (1+3)^2}$$

$$= \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+16}$$

$$BC = 4\sqrt{2}$$

$$\tfrac{1}{2}BC = 2\sqrt{2} \quad \ldots(2)$$

From eq (1) and (2) we have

$$\tfrac{1}{2}BC = LM$$ Hence proved.

Main 2000

64. Calculate the ratio in which the line joining $A(6,5)$and $B(4,-3)$ is divided by the line $y = 2$.

Sol :

Let the line joining A and B is divided by the line $y = 2$ in the ratio $m_1 : m_2$. Let the coordinate of intersection point be $P(x,y)$

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$$x_1+x_2+x_2+x_3+x_1+x_3=2+4+6$$

and $y_1+y_2+y_2+y_3+y_1+y_3=2-6+8$

$$x_1+x_2+x_3=6 \text{ and } y_1+y_2+y_3=2 \quad ...(iv)$$

The coordinates of the centroid of ΔABC are

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\ \frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{6}{3},\ \frac{2}{3}\right)$$

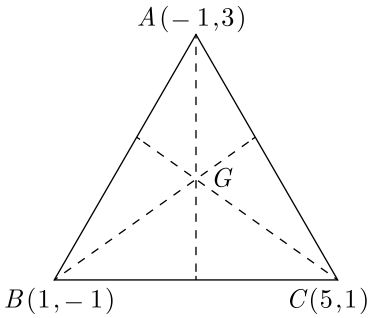
$$=(2,\ \tfrac{2}{3})$$

Main 2006

52. Find the coordinates of the centroid of a triangle, whose vertices are $A(-1,3)$, $B(1,-1)$ and $C(5,1)$.

Sol :

Given, vertices's of the triangle are $A(-1,3)$, $B(1,-1)$ and $\begin{matrix}C(5,1)\\B(2,-7)\end{matrix}$.



Let the coordinates of centroid be $G(x,y)$.

Thus $G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

Hence $x=\frac{-1+1+5}{3}=\frac{5}{3}$

and $y=\frac{3-1+1}{3}=\frac{3}{3}=1$

The required coordinates of centroid are $(\frac{5}{3},1)$.

Comp 2006

53. Find the coordinates of the point which divide the line segment joining $A(2,-3)$ and $B(-4,-6)$ into three equal parts.

Sol :

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.
As per question, line diagram is shown below.
P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1.
By section formula

$$x_1=\frac{mx_2+nx_1}{1+2} \text{ and } y=\frac{my_2+ny_1}{m+n}$$

$$P(x_1,y_1)=\left(\frac{1(-4)+2(2)}{2+1},\frac{2(-6)+1(-3)}{2+1}\right)$$

$$=\left(\frac{-4+4}{3},\frac{-6-(-6)}{3}\right)=(0,-4)$$

$$Q(x_2,y_2)=\left(\frac{2(-4)+1(2)}{2+1},\frac{2(-6)+1(-3)}{2+1}\right)$$

$$=\left(\frac{-8+2}{3},-\frac{12+(-3)}{3}\right)$$

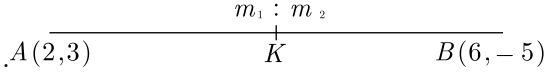
$$=(-2,-5)$$

Main 2006

54. The line segment joining $A(2,3)$ and $B(6,-5)$ is intersected by the x-axis at the point K. Write the coordinates of the point K. Hence, find the ratio in which K divides AB.

Sol :

Since, the line segment joining A and B is intersected by the x-axis at point K, so its coordinates will be $(x,0)$



Let K divides AB in the ratio $m:n$.
Using section formula coordinates of K are

$$K(x,\ 0)=\left(\frac{6m+2n}{m+n},\frac{-5m+3n}{m+n}\right)$$

Since, y-coordinates of K is zero, we have

$$\frac{-5m+3n}{m+n}=0$$

$$-5m+3n=0$$

$$5m=3n$$

$$\frac{m}{n}=\frac{3}{5}$$

Hence, the required ratio is $3:5$.

Now, $x=\frac{6m+2n}{m+n}$

$$=\frac{6\times 3+2\times 5}{3+5}$$

$$=\frac{18+10}{8}=\frac{28}{8}=\frac{7}{2}$$

So, coordinates of point K is $(\frac{7}{2},\ 0)$.

Main 2005

55. If the coordinates of the mid-points of the sides of a triangle are $(1,2),(0,\ -1)$ and $(2,\ -1)$. Find the coordinates of its vertice s.

Sol :

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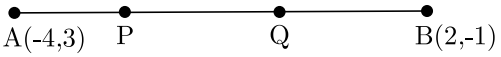


Coordinates of Q are

$$Q(x,y)=\left(\frac{-2+2}{2},\ \frac{\frac{5}{3}+(-1)}{2}\right)$$
$$=\left(0,\ \frac{\frac{2}{3}}{2}\right)$$
$$=\left(0,\ \frac{1}{3}\right)$$

NODIA

Comp 2017

67. The line segment joining $P(-4,5)$ and $Q(3,2)$ intersects the y–axis at R. PM and QN are perpendiculars from P and Q on the x-axis. Find:
(i) the ratio $PR:RQ$
(ii) the coordinates of R
(1ili) the area of quadrilateral $PMNQ$

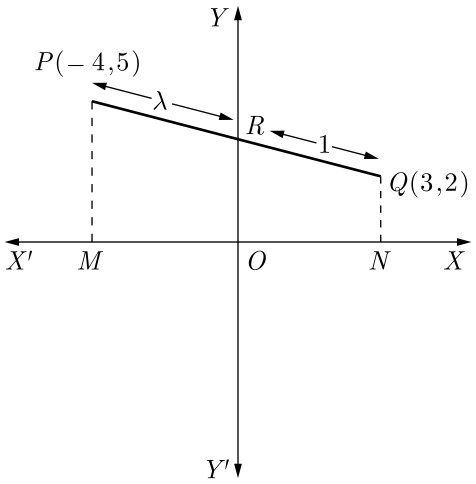
Sol :

(i) Ratio $PR:RQ$
Suppose PQ is divided by y–axis in the ratio $\lambda:1$. Then, the coordinates of R are

$$R(x,y)=\left(\frac{3\times\lambda+1\times-4}{\lambda+1},\frac{2\times\lambda+1\times5}{\lambda+1}\right)$$
$$=\left(\frac{3\lambda-4}{\lambda+1},\frac{2\lambda+5}{\lambda+1}\right)$$

As the point R lies on y–axis, so its x–coordinates is zero.

Thus $$\frac{3\lambda-4}{\lambda+1}=0$$
$$3\lambda-4=0$$
$$\lambda=\frac{4}{3}$$



So, the required ratio is $\frac{4}{3}:1$ or, $4:3$
(ii) the coordinates of R
Substituting the value of λ in $\left(\frac{3\lambda-4}{\lambda+1},\frac{2\lambda+5}{\lambda+1}\right)$, we obtain the coordinates of R as

$$R(x,y)=\left(0,\frac{2\times\frac{4}{3}+1\times5}{\frac{4}{3}+1}\right)$$
$$=\left(0,\ \frac{8+15}{4+3}\right)$$
$$=\left(0,\ \frac{23}{7}\right)$$

(iii) the area of quadrilateral $PMNQ$
Clearly, quadrilateral $PMNQ$ is a trapezium.
Area of quadrilateral $PMNQ$,

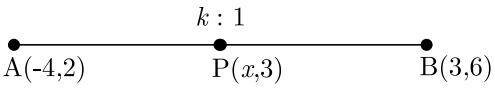
$$\square=\frac{1}{2}(PM+NQ)\times MN$$
$$=\frac{1}{2}(5+2)\times7 \text{ sq. units}=\frac{49}{2}\text{ sq. units}$$

Main 2014

68. Calculate the ratio in which the line joining $A(-4,2)$ and $B(3,6)$ is divided by $P(x,3)$. Also, find (i) x. (ii) length of AP.

Sol :

Let $P(x,3)$ divides the line segment joining the points $A(-4,2)$ and $B(3,6)$ in the ratio $k:1$.



Using the section formula, we have

$$P(x,y)=\left(\frac{kx_2+x_1}{k+1},\ \frac{ky_2+y_1}{k+1}\right)$$
$$=\left(\frac{3\times k+1\times-4}{k+1},\frac{k\times6+1\times2}{k+1}\right)$$
$$=\left(\frac{3k-4}{k+1},\frac{6k+2}{k+1}\right)$$

But coordinates of P is given $(x,3)$.

$$(x,3)=\left(\frac{3k-4}{k+1},\frac{6k+2}{k+1}\right) \quad ...(i)$$

Equating y-coordinates from both sides, we get

$$3=\frac{6k+2}{k+1}\text{ and }x=\frac{6k+2}{k+1}$$

From $3=\frac{6k+2}{k+1}$ we have

$$3(k+1)=6k+2$$
$$3k+3=6k+2$$
$$6k-3k=3-2$$
$$3k=1\Rightarrow k=\frac{1}{3}$$

Hence, the required ratio is $\frac{1}{3}$ i.e. $1:3$ (internally).

Substituting $k=\frac{1}{3}$ in $x=\frac{6k+2}{k+1}$ we have

$$x=\frac{3\times\frac{1}{3}-4}{\frac{1}{3}+4}$$

$$= \frac{3-4\times 3}{1+3}$$

$$= \frac{3-12}{4} = \frac{-9}{4}$$

Length $AP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{\left(-\frac{9}{4}+4\right)^2+(3-2)^2}$$

$$= \sqrt{\left(\frac{-9+16}{4}\right)^2+(1)^2}$$

$$= \sqrt{\left(\frac{7}{4}\right)^2+1^2}$$

$$= \sqrt{\frac{49}{16}+1}$$

$$= \sqrt{\frac{49+16}{16}}$$

$$= \frac{\sqrt{65}}{4} \text{ units}$$

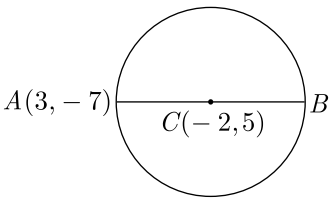
Main 2013

69. AB is a diameter of a circle with centre $C(-2,5)$ and $A(3,-7)$. Find
(i) the length of radius AC.
(ii) the coordinates of B.

Sol :

Let $A(3,-7) = (x_1, y_1)$

and $C(-2,5) = (x_2, y_2)$



Length of radius,

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-2-3)^2+(5+7)^2}$$

$$= \sqrt{(-5)^2+(12)^2}$$

$$= \sqrt{25+144}$$

$$= \sqrt{169} = 13 \text{ units}$$

Let the coordinates of B be (x_3, y_3). Since, C is the centre of circle, C is the mid-point of diameter AB.

$$\text{Coordinates of } C = \left(\frac{3+x_3}{2}, \frac{-7+y_3}{2}\right)$$

But coordinates of C is given $(-2,5)$.

$$(-2,5) = \left(\frac{3+x_3}{2}, \frac{-7+y_3}{2}\right)$$

Comparing the coordinates from both sides,

$$-2 = \frac{3+x_3}{2} \text{ and } 5 = \frac{-7+y_3}{2}$$

$$-2\times 2 = 3+x_3 \text{ and } 5\times 2 = -7+y_3$$

$$x_3 = -4-3 \text{ and } y_3 = 10+7$$

$$x_3 = -7 \text{ and } y_3 = 17$$

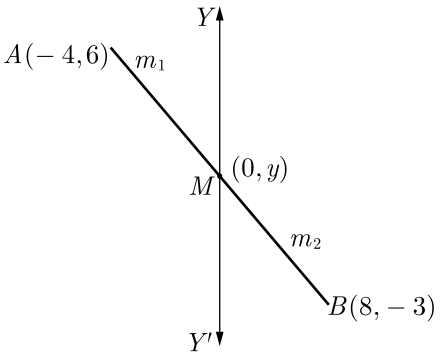
Hence, the coordinates of B are $(-7,17)$.
(i) Length of radius is 13 units
(ii) Coordinates of B are $(-7,17)$

Main 2012

70. Given a line segment AB joining the points $A(-4,6)$ and $B(8,-3)$. Find
(i) the ratio in which AB is divided by the y -axis.
(ii) the coordinates of the point of intersection.
(iii) the length of AB.

Sol :

Given, coordinates of A and B are $(-4,6)$ and $(8,-3)$. Let AB is divided by the y-axis at point $P(x,y)$ in the ratio $k : 1$.



Using section formula we get

$$P(x,y) = \left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}\right)$$

$$= \left(\frac{8k-4}{k+1}, \frac{-3k+6}{k+1}\right)$$

Since point $P(x,y)$ is on $y-$ axis, x-coordinates of $P(x,y)$ is 0.

Thus $0 = \frac{8k-4}{k+1}$

$$0 = 8k-4$$

$$0 = 2k-1$$

$$k = \tfrac{1}{2}$$

Hence, the required ratio is $k:1 = \frac{1}{2}:1$ i.e. $1 : 2$ (internally).
From section formula we get

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$$y = \frac{-3k+6}{k+1}$$

Substituting $k = \frac{1}{2}$ in above we get

$$y = \frac{-3 \times \frac{1}{2} + 6}{\frac{1}{2} + 1}$$

$$= \frac{-3 + 6 \times 2}{1 + 2}$$

$$= \frac{-3 + 12}{3} = 3$$

Hence, coordinates of point of intersection are $(0,3)$.

(iii) Length of AB

Let $(x_1, y_1) = A(-4,6)$ and $(x_2, y_2) = B(8,-3)]$

Length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(8+4)^2 + (-3-6)^2}$$

$$= \sqrt{(12)^2 + (-9)^2}$$

$$= \sqrt{144 + 81}$$

$$= \sqrt{225} = 15 \text{ units}$$

(i) AB is divided by the y-axis in ration 1 : 2

(ii) Coordinates of the point of intersection are $(0,3)$.

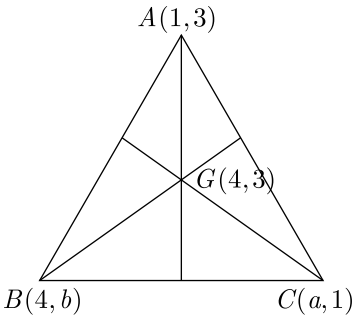
(iii) Length of AB is 15 units.

Main 2011

71. ABC is a triangle and $G(4,3)$ is the centroid of the triangle. If $A(1,3)$, $B(4,b)$ and $C(a,1)$, then the find a and b. Find the length of side BC.

Sol :

Given, vertices and centroid of a triangle are $A(1,3)$, $B(4,b)$, $C(a,1)$ and $G(4,3)$ respectively.



We know coordinates of centroid

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Here, $(x_1, y_1) = A(1,3)$,

$(x_2, y_2) = B(4,b)$

and $(x_3, y_3) = C(a,1)$

Centroid of the triangle

$$G(x,y) = \left(\frac{1+4+a}{3}, \frac{3+b+1}{3}\right)$$

$$= \left(\frac{5+a}{3}, \frac{4+b}{3}\right)$$

But centroid is given $G(4,3)$.

Thus $(4,3) = \left(\frac{5+a}{3}, \frac{4+b}{3}\right)$

Comparing x and y-coordinates from both sides, we get

$$4 = \frac{5+a}{3} \text{ and } 3 = \frac{4+b}{3}$$

$$4 \times 3 = 5 + a \text{ and } 3 \times 3 = 4 + b$$

$$12 = 5 + a \text{ and } 9 = 4 + b$$

$$a = 12 - 5 \text{ and } b = 9 - 4$$

$$a = 7 \text{ and } b = 5$$

Coordinates of B and C are $(4,5)$ and $(7,1)$.

Now, the length of side

$$BC = \sqrt{(7-4)^2 + (1-5)^2}$$

$$= \sqrt{(3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5 \text{ units}$$

Comp 2011

72. Find the coordinates of points which trisect the line segment joining (1, –2) and (–3, 4).

Sol :

Let P and Q be the points of trisection of the line segment AB, then

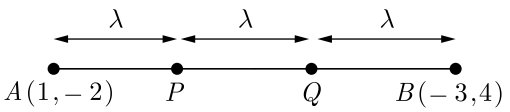


Fig. 11.20

$$AP = PQ = QB \Rightarrow 2AP = PB$$

$$\frac{AP}{PB} = \frac{1}{2}$$

Thus P divides AB in the ratio 1 : 2. Similarly Q divides AB in the ratio 2 : 1.

Coordinates of P are

$$P(x,y) = \left(\frac{m \times x_2 + n \times x_1}{m+n}, \frac{m \times y_2 + n \times y_1}{m+n}\right)$$

$$= \left(\frac{1 \times (-3) + 2 \times 1}{1+2}, \frac{1 \times 4 + 2 \times (-2)}{1+2}\right)$$

$$= \left(\frac{-3+2}{3}, \frac{4-4}{3}\right)$$

$$= \left(-\frac{1}{3} \quad 0\right)$$

Coordinates of Q are

$$Q(x,y) = \left(\frac{m \times x_2 + n \times x_1}{m+n}, \frac{m \times y_2 + n \times y_1}{m+n}\right)$$

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$$= \left(\frac{2\times(-3)+1\times 1}{2+1},\ \frac{2\times 4+1\times(-2)}{2+1}\right)$$

$$= \left(\frac{-6+1}{3},\ \frac{8-2}{3}\right)$$

$$= \left(\frac{-5}{3}\quad \frac{6}{2}\right)$$

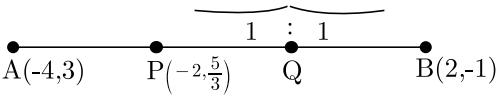
$$= \left(-\tfrac{5}{3}\quad 3\right)$$

REMARK

As Q is the mid-point of BP. So, the coordinates of Q can also be obtained by using mid-point formula.

Now $PQ = QB$

Since Q is the mid-point of PB



Coordinates of Q are

$$Q(x,y) = \left(\frac{-\frac{1}{3}-3}{2},\ \frac{0+4}{2}\right)$$

$$= \left(\frac{-\frac{10}{3}}{2},\ \frac{4}{2}\right)$$

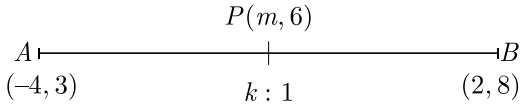
$$= \left(-\tfrac{5}{3},\ \ 2\right)$$

Main 2009

73. Find the ratio in which the point $p(m,6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$.Also find the value of m.

Sol :

As per question, line diagram is shown below.



Let the ratio be $k:1$.

Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \qquad (1)$$

$$6 = \frac{8k+3}{k+1} \qquad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3 \Rightarrow k = \frac{3}{2}$$

Thus ratio is $\frac{3}{2}:1$ or $3:2$.

Substituting value of k in (1) we have

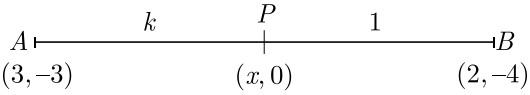
$$m = \frac{2(\frac{3}{2})+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

Comp 2009

74. Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Sol :

We know that y co-ordinate of any point on the $x-$ axis will be zero. Let $(x,\ 0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$. Using section formula for y co-ordinate we have

$$0 = \frac{1(-3)+k(7)}{1+k}$$

$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

$$x = \frac{1(3)+k(-2)}{1+k} = \frac{3-2\times\frac{3}{7}}{1+\frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2},0\right)$.

Main 2008

75. If $A(-4,3)$ and $B(8,-6)$.

(i) Find the length of AB.

(ii) In what ratio is the joining AB, divided by the x-axis?

Sol :

We have $A = (-4,3)$ and $B = (8,-6)$

(i) Length of AB

Length $AB = \sqrt{(8+4)^2+(-6-3)^2}$

$$= \sqrt{(12)^2+(-9)^2}$$

$$= \sqrt{144+81}$$

$$= \sqrt{225} = 15 \text{ units}$$

Let AB is divided by the y-axis at point $P(x,y)$ in the ratio $k:1$.

Using section formula we get

$$P(x,y) = \left(\frac{kx_2+x_1}{k+1},\ \frac{ky_2+y_1}{k+1}\right)$$

$$= \left(\frac{8k-4}{k+1}, \frac{-6k+3}{k+1}\right)$$

Since point $P(x,y)$ is on $x-$ axis, y-coordinates of $P(x,y)$ is 0.

Thus $0 = \frac{-6k+3}{k+1}$

$$0 = -6k+3$$

$$0 = 2k - 1$$

$$k = \tfrac{1}{2}$$

Hence, the required ratio is $k:1 = \frac{1}{2}:1$ i.e. $1 : 2$ (internally). Hence, the line joining AB is divided by the x-axis in the ratio $1 : 2$.

Main 2008

76. In what ratio does the x-axis divide the line segment joining the points $(-4,-6)$ and $(-1,7)$? Find the coordinates of the point of division.

Sol :

Let x-axis divides the line-segment joining $(-4,-6)$ and $(-1,7)$ at the point $P(x,y)$ in the ratio $1:k$.

Now, the coordinates of point of division P,

$$(x,y) = \frac{1(-1)+k(-4)}{k+1}, \frac{1(7)+k(-6)}{k+1}$$

$$= \frac{-1-4k}{k+1}, \frac{7-6k}{k+1}$$

Since P lies on x axis, therefore $y = 0$, which gives

$$\frac{7-6k}{k+1} = 0$$

$$7 - 6k = 0$$

$$k = \frac{7}{6}$$

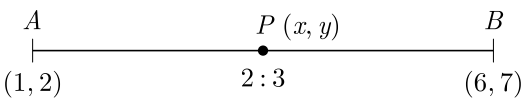
Hence, the ratio is $1:\frac{7}{6}$ or, $6:7$ and the coordinates of P are $(-\frac{34}{13},\ 0)$.

Comp 2008

77. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$.

Sol :

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP:PB = 2:3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2\times 6 + 3\times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2\times 7 + 3\times 2}{2+3} = \frac{14+6}{5} = 4$$

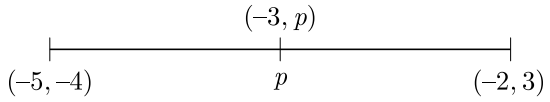
Thus $$P(x,y) = (3,4)$$

Main 2007

78. Find the ratio in which the point $(-3,p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$.Hence find the value of p.

Sol :

As per question, line diagram is shown below.



Let $X(-3,p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.

The co-ordinates of p are $\left[\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right]$

But co-ordinates of P are $(-3,p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k = 2$$

and $$\frac{3k-4}{k+1} = p$$

Substituting $k = 2$ gives

$$p = \frac{2}{3}$$

Hence ratio of division is $2:1$ and $p = \frac{2}{3}$

Comp 2007

79. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Sol :

Let $P(x_1,y_1), Q(x_2,y_2)$ divides AB into 3 equal parts.

Thus P divides AB in the ratio of 1:2.

As per question, line diagram is shown below.

Now $$x_1 = \frac{1(-3)+2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4)+2(-2)}{1+2} = \frac{4-4}{3} = 0$$

Co-ordinates of P is $(-\frac{1}{3}, 0)$.

Here Q is mid-point of PB.

Thus $$x_2 = \frac{-\frac{1}{3}+(-3)}{2}$$

$$= \frac{-10}{6} = \frac{-5}{3}$$

$$y_2 = \frac{0+4}{2} = 2$$

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Thus co-ordinates of Q is $(-\frac{5}{2}, 2)$.

Main 2006

80. If (a,b) is the mid-point of the segment joining the points $A(10,-6)$ and $B(k,4)$ and $a-2b=18$, find the value of k and the distance AB.

Sol :

We have $A(10,-6)$ and $B(k,4)$.
If $P(a,b)$ is mid-point of AB, then we have

$$(a,b)=\left(\frac{k+10}{2},\frac{-6+4}{2}\right)$$

$$a=\frac{k+10}{2} \text{ and } b=-1$$

From given condition we have

$$a-2b=18$$

Substituting value $b=-1$ we obtain

$$a+2=18 \Rightarrow a=16$$

$$a=\frac{k+10}{2}=16 \Rightarrow k=22$$

$$P(a,b)=(16,1)$$

$$AB=\sqrt{(22-10)^2+(4+6)^2}$$

$$=2\sqrt{61} \text{ units}$$

Comp 2006

81. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $(8,-9)$ and $(2,1)$. Also find the co-ordinates of the point of division.

Sol :

Let a point $P(x,y)$ on line $2x+3y-5=0$ divides AB in the ratio $k:1$.

Now $$x=\frac{2k+8}{k+1}$$

and $$y=\frac{k-9}{k+1}$$

Substituting above value in line $2x+3y-5=0$ we have

$$2\left(\frac{2k+8}{k+1}\right)+3\left(\frac{k-9}{k+1}\right)-5=0$$

$$4k+16+3k-27-5k-5=0$$

$$2k-16=0$$

$$k=8$$

Thus ratio is $8:1$.
Substituting the value $k=8$ we get

$$x=\left(\frac{2\times 8+8}{8+1}\right)=\frac{8}{3}$$

$$y=\left(\frac{8-9}{8+1}\right)=-\frac{1}{9}$$

Thus $$P(x,y)=\left(\frac{8}{3},-\frac{1}{9}\right)$$

Main 2005

82. Find the ratio in which the line joining points $(a+b,b+a)$ and $(a-b,b-a)$ is divided by the point (a,b).

Sol :

Let $A(a+b,b+a), B(a-b,b-a)$ and $P(a,b)$ and P divides AB in $k:1$, then we have

$$a=\frac{k(a-b)+1(a+b)}{k+1}$$

$$a(k+1)=k(a-b)+a+b$$

$$ak+a=ak-bk+a+b$$

$$bk=b$$

$$k=1$$

Thus (a,b) divides $A(a+b,b+a)$ and $B(a-b,b-a)$ in 1:1 internally.

Hence, value of k is 2.

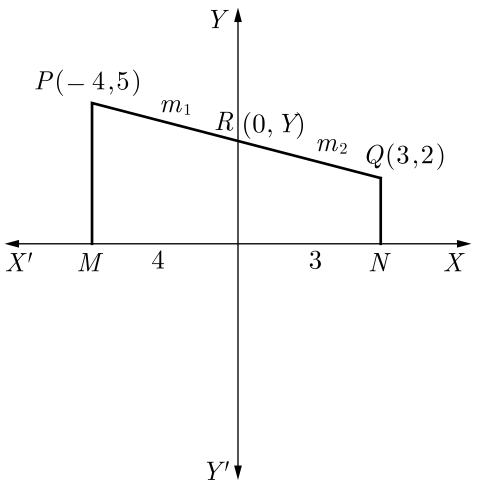
Main 2004

83. The line joining $P(-4,5)$ and $Q(3,2)$ intersect the y-axis at R. PM and QN are perpendicular from P and Q on the x-axis. Find
(i) the ratio $PR : PQ$.
(ii) the coordinates of R.
(iii) the area of the quadrilateral $PMNQ$.

Sol :

Let the line PQ intersects the y-axis at R in the ratio $k : 1$.

Thus $$PR : PQ = k : 1$$



By using section formula, x-coordinate of R,
Using section formula we get

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$$R(x,y)=\left(\frac{kx_2+x_1}{k+1},\ \frac{ky_2+y_1}{k+1}\right)$$
$$=\left(\frac{3k-4}{k+1},\frac{2k+5}{k+1}\right)$$

Thus $x=\frac{3k-4}{k+1}$ and $y=\frac{2k+5}{k+1}$

Since, the line PQ intersects the y-axis at R, the coordinate of R will be $(0,y)$

Thus $R(0,\ y)=\left(\frac{3k-4}{k+1},\frac{2k+3}{k+1}\right)$

$$0=\frac{3k-4}{k+1}$$
$$0=3k-4$$
$$3k=4$$
$$k=\tfrac{4}{3}$$

Hence, the required ratio is $k:1=\frac{4}{3}:1$ i.e. $4:3$ (internally). Hence, the line $P(-4,5)$ and $Q(3,2)$ is divided by the x-axis in the ratio $4:3$.

Hence $\frac{PR}{QR}=\frac{4}{3}$

$$\frac{QR}{PR}=\frac{3}{4}$$
$$\frac{QR+PR}{PR}=\frac{3+4}{4}$$
$$\frac{QR+PR}{PR}=\frac{3+4}{4}$$
$$\frac{PQ}{PR}=\frac{7}{4}$$
$$\frac{PR}{PQ}=\frac{4}{7}$$
$$PR:PQ=4:7$$

Hence, the required ratio is $4:7$.

Now substituting $k=\frac{4}{3}$ in $y=\frac{2k+5}{k+1}$ we have

$$y=\frac{2\times\frac{4}{3}+5}{\frac{4}{3}+1}$$
$$=\frac{2\times4+3\times5}{4+3}$$
$$=\frac{8+15}{7}=\frac{23}{7}$$

Hence, the coordinates of R are $(0,\ \frac{23}{7})$.

(iii) Since, quadrilateral $PMNQ$ is a trapezium.

$$\text{Area}=\frac{1}{2}\text{ (Sum of parallel sides }\times\text{ Height)}$$
$$=\frac{1}{2}(PM+NQ)\times MN$$
$$=\frac{1}{2}\times(5+2)\times7$$
$$=\frac{49}{2}=24.5\text{ sq units}$$

(i) Ratio $PR:PQ$ is $4:3$
(ii) Coordinates of R are $(0,\ \frac{23}{7})$.
(iii) Area of the quadrilateral $PMNQ$ is 24.5 sq units.

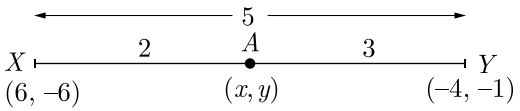
FIVE MARK QUESTIONS

Main 2000

84. Point A lies on the line segment XY joining $X(6,-6)$ and $Y(-4,-1)$ in such a way that $\frac{XA}{XY}=\frac{2}{5}$. If point A also lies on the line $3x+k(y+1)=0$, find the value of k.

Sol :

As per given information in question we have drawn the figure given below.



We use section formula for point $A(x,y)$.
Here, $m_1=2$, $m_2=3$, $x_1=6$, $x_2=-4$, $y_1=-6$ and $y_2=-1$

Now $x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$
$$=\frac{2\times(-4)+3(6)}{2+3}$$
$$=\frac{-8+18}{5}=\frac{10}{5}=2$$

and $y=\frac{m_1y_2+m_2y_2}{m_1+m_2}$
$$=\frac{2\times(-1)+3(-6)}{2+3}$$
$$=\frac{-2-18}{5}=\frac{-20}{5}=-4$$

Hence, coordinates of point A is $(2,-4)$.
Since point A also lies on the line $3x+k(y+1)=0$, its coordinates must satisfies this line.

Thus $3(2)+k(-4+1)=0$
$$6+(-3k)=0$$
$$3k=6$$
$$k=2$$

END

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CHAPTER 13

EQUATION OF A STRAIGHT LINE

MULTIPLE CHOICE QUESTIONS

Main 2024

1. Points $A(x, y)$, $B(3, -2)$ and $C(4, -5)$ are collinear. The value of y in terms of x is:

(a) $3x - 11$ (b) $11 - 3x$

(c) $3x - 7$ (d) $7 - 3x$

Sol :

Thus equation of line passing through the point $B(3, -2)$ and $C(4, -5)$,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 2 = \frac{-5 + 2}{4 - 3}(x - 3)$$

$$y + 2 = -3(x - 3)$$

$$y + 2 = -3x + 9$$

$$y = -3x + 7$$

Point $A(x, y)$, also must satisfy this equation of line.

Thus (d) is correct option.

SQP 2024

2. Which of the following equation represent a line passing through origin?

(a) $3x - 2y + 5 = 0$

(b) $2x - 3y = 0$

(c) $x = 5$

(d) $y = -6$

Sol :

In $2x - 3y = 0$, constant term is zero, thus this line pass through origin.

Comp 2023

3. The equation of a straight line is $5x - 4y - 8 = 0$. What is the y-intercept of the line ?

(a) 2 (b) 8

(c) -2 (d) 4

Sol :

The equation of the line is

$$5x - 4y - 8 = 0$$

It can be written as

$$-4y = -5x + 8$$

$$y = -\frac{5}{4}x - 2$$

Comparing it with $y = mx + c$, we get $c = -2$.

Thus (c) is correct option.

4. What is the slope of a line whose inclination is 60°.

(a) $\sqrt{3}$ (b) 1

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

Sol :

Let m be the slope of the line, then

$$m = \tan 60^\circ \text{ (From Trigonometry)}$$

$$= \sqrt{3}$$

Thus (a) is correct option.

5. Which of the following is the equation of the straight line whose inclination is 45° and whose y-intercept is –3.

(a) $x - y - 1 = 0$ (b) $-x + y + 3 = 0$

(c) $x - y - 3 = 0$ (d) $x - y + 1 = 0$

Sol :

Let m be the slope of the line, then

$$m = \tan 45^\circ = 1$$

Also y-intercept is –3 i.e. $c = -3$

The equation of the line is

$$y = mx + c$$

$$y = 1 \cdot x + (-3)$$

i.e. $x - y - 3 = 0$

Thus (c) is correct option.

6. Which of the following is the line with x-intercept 4 and passing through the point $(-2, 5)$.

(a) $5x + 6y - 20 = 0$ (b) $5x + 6y + 10 = 0$

(c) $5x - 6y + 20 = 0$ (d) $5x - 6y + 10 = 0$

Sol :

Here x-intercept is 4 and y coordinates on x-axis is always zero. Thus line passes through point $(4, 0)$.

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This line pas also through $(-2,5)$.

Thus equation of line passing through the point $(4,0)$ and $(-2,5)$,

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0=\frac{5-0}{-2-4}(x-4)$$

$$y=\frac{5}{-6}(x-4)$$

$$-6y=5x-20$$

$$5x+6y-20=0$$

Thus (a) is correct option.

7. A line passes through (x_1,y_1) and (h,k) . If slope of the line is m, then

(a) $y_1-k=m(h-x_1)$
(b) $k-y_1=,m(h-x_1)$
(c) $h-x_1=m(y_1-k)$
(d) $h-x_1=m(k-y_1)$

Sol :

If one point and slope is given, then equation of line is $k-y_1=m(h-x_1)$.

Thus (b) is correct option.

8. Which of the following is the equation of a line with x-intercept -3 and passing through the point $(-2,5)$?

(a) $5x+y-15=0$ (b) $x-5y+15=0$
(c) $x+5y-15=0$ (d) $5x-y+15=0$

Sol :

Here x-intercept is -3 and y coordinates on x-axis is always zero. Thus line passes through point $(-3,0)$.

Now equation of the line passing through points $(-3,0)$ and $(-2,5)$,

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0=\frac{5-0}{-2+3}(x+3)$$

$$y=\frac{5}{1}(x+3)$$

$$y=5x+15$$

$$5x-y+15=0$$

Thus (d) is correct option.

9. Which of the following is the slope of a line perpendicular to AB if $A(-4,\ 2)$ and $B(5,\ 8)$?

(a) $\frac{1}{2}$ (b) $-\frac{3}{2}$
(c) $\frac{1}{5}$ (d) $\frac{3}{2}$

Sol :

Slope of AB, $m_1=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{8-2}{5-(-4)}=\frac{6}{9}=\frac{2}{3}$$

Slope of a line perpendicular to AB,

$$m_2=-\frac{1}{m_1}=\frac{-3}{2}$$

Thus (b) is correct option.

10. Which of the following is the slope of a line perpendicular to AB if $A(-1,\ 6)$ and $B(3,\ -2)$?

(a) $\frac{1}{2}$ (b) $-\frac{5}{3}$
(c) $\frac{1}{10}$ (d) $\frac{3}{2}$

Sol :

Slope of AB, $m_1=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{-2-6}{3+1}=\frac{-8}{4}=-2$$

Slope of a line perpendicular to AB,

$$m_2=-\frac{1}{m_1}=-\frac{1}{-2}=\frac{1}{2}$$

Thus (a) is correct option.

11. Select the equation of a straight with slope –2 and which intersects x-axis at a distance of 3 units to the left of origin.

(a) $2x-y-6=0$ (b) $2x+y+6=0$
(c) $2x-y-3=0$ (d) $2x-y+3=0$

Sol :

Here slope of the line, $m=-2$

Since the line intersects x-axis at a distance of 3 units to the left of origin, it passes through the point (–3, 0).

The equation of the line passing through the point (–3, 0) and with slope –2 is

$$y-y_1=m(x-x_1)$$

$$y-0=(-2)(x-(-3))$$

$$y=-2(x+3)$$

i.e $2x+y+6=0$

Thus (b) is correct option.

12. Which of the following is the slope of the line passing through the points (7, 7) and (5, 1)?

(a) 2 (b) 3

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or $y = 1 \cdot x - \frac{7}{3}$

Comparing it with $y = mx + c$, we get $c = -\frac{7}{3}$.
Thus (c) is correct option.

THREE MARKS QUESTIONS

Main 2023

34. ABC is a triangle whose vertices are $A(1, -1)$, $B(0, 4)$ and $C(-6, 4)$. D is the mid point of BC. Find the:
(a) coordinates of D.
(b) equation of the median AD.

Sol :

We have, $A(1, -1)$, $B(0, 4)$ and $C(-6, 4)$

$B(0,4)$ $\quad$ $D(x, y)$ $\quad$ $C(-6,4)$

Since, D is the mid-point of BC,

$$D(x, y) = \left(\frac{0 + (-6)}{2}, \frac{4 + 4}{2}\right)$$

$$= \left(\frac{-6}{2}, \frac{8}{2}\right)$$

$$= (-3, 4)$$

Hence, the coordinates of D are $(-3, 4)$.

$A(1, -1)$

B $\quad$ $D(-3, 4)$ $\quad$ C

Equation of a line passing through two points namely (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = (x - x_1)\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Here, $(x_1, y_1) = A(1, -1)$

and $(x_2, y_2) = D(-3, 4)$

Hence, the equation of median AD is

$$y - (-1) = (x - 1)\left(\frac{4 - (-1)}{-3 - 1}\right)$$

$$y + 1 = \frac{(x - 1) \times 5}{(-4)}$$

$$-4(y + 1) = 5(x - 1)$$

$$-4y - 4 = 5x - 5$$

$$5x + 4y - 1 = 0$$

Comp 2023

35. Find the equation of diagonal BD of rhombus $ABCD$ where $A(4, 5)$ and $C(6, -9)$.

Sol :

As per given question we have drawn figure below.

$A(4, 5)$

B $\quad$ D

$C(6, -9)$

Slope of AC $\quad m_1 = \frac{5 + 9}{4 - 6} = \frac{14}{-2} = -7$

Since diagonals of a rhombus bisect each other at right angles, BD is perpendicular to AC.

Slope of BD $\quad m_2 = -\frac{1}{m_1}$

$$= -\frac{1}{-7} = \frac{1}{7}$$

Midpoint of AC $\quad M = \left(\frac{4 + 6}{2}, \frac{5 - 9}{2}\right)$

$$= (5, -2)$$

Mid point M is also mid point of diagonal of BD. Thus equation of line having slope $\frac{1}{7}$ and passing through $(5, -2)$,

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{1}{7}(x - 5)$$

$$7y + 14 = x - 5$$

$$7y = x - 19$$

$$x - 7y - 19 = 0$$

Main 2020

36. Find the value of p if the lines, $5x - 3y + 2 = 0$ and $6x - py + 7 = 0$ are perpendicular to each other. Hence find the equation of a line passing through $(-2, -1)$ and parallel to $6x - py + 7 = 0$.

Sol :

We have $\quad 5x - 3y + 2 = 0 \quad (1)$

and $\quad 6x - py + 7 = 0 \quad (2)$

Writing line (1) in $y = mx + c$ form we have

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Sol :

Slope of PQ, $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-(-3)}{7-5}$

$= \frac{3+3}{7-5} = \frac{6}{2} = 3$

Slope of a line perpendicular to PQ,

$m_2 = -\frac{1}{m_1} = -\frac{1}{3} = -\frac{1}{3}$

Thus (a) is correct option.

20. Which of the following is the equation of the line through (1, 3) making an intercept of 5 on the y-axis?

(a) $2x - y - 6 = 0$ (b) $2x + y + 6 = 0$

(c) $2x + y - 5 = 0$ (d) $2x - y + 5 = 0$

Sol :

Since the y-intercept of the line is 5, its equation ,

$y = mx + c$

$y = mx + 5$

where m is the unknown slope.

As the line passes through the point (1, 3), we get

$3 = m \cdot 1 + 5$

$m = -2$

Substituting this value of m in (1), we get

$y = -2x + 5$

$2x + y - 5 = 0$

which is the required equation.

Thus (c) is correct option.

21. Which of the following is the equation of the line with x-intercept 4 and passing through the point $(-2, 5)$?

(a) $6x + 5y - 20 = 0$

(b) $5x + 6y - 20 = 0$

(c) $6x + 5y - 10 = 0$

(d) $5x + 6y - 10 = 0$

Sol :

Here x-intercept is 4 and y coordinates on x-axis is always zero. Thus line passes through point $(4, 0)$.

This line pas also through $(-2, 5)$.

Thus equation of line passing through the point $(4, 0)$ and $(-2, 5)$,

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{5-0}{-2-4}(x-4)$

$y = \frac{5}{-6}(x-4)$

$-6y = 5x - 20$

$5x + 6y - 20 = 0$

Thus (b) is correct option.

22. Which of the following is the equation of the line passing through the point $(4, 5)$ and having the x -intercept 6 units?

(a) $6x + 5y - 20 = 0$

(b) $5x + 6y - 20 = 0$

(c) $3x + 2y - 20 = 0$

(d) $3x - 2y + 20 = 0$

Sol :

Since line is having x-intercept 6 units, i.e it pass through point (6, 0) because $y-$ coordinate at $x-$ is zero.

Now equation of line passing through $(4, 5)$ and (6, 0) will be

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 4 = \frac{0-5}{6-4}(x-4)$

$y - 4 = \frac{-3}{2}(x-4)$

$2y - 8 = -3x + 12$

$3x + 2y - 8 - 12 = 0$

$3x + 2y - 20 = 0$

Thus (c) is correct option.

23. The vertices of a ΔABC are $A(3, 8)$, $B(-1, 2)$ and $C(6, -6)$. Which of the following is the slope of a line perpendicular to BC?

(a) $-\frac{7}{8}$ (b) $\frac{5}{3}$

(c) $\frac{7}{8}$ (d) $\frac{3}{2}$

Sol :

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Given vertices of a ΔABC are $A(3, 8), B(-1, 2)$ and $C(6, -6)$

Slope of BC, $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-6-2}{6-(-1)} = \frac{-8}{7}$

Slope of line perpendicular to BC,

$m_2 = -\frac{1}{m_1} = -\frac{7}{8}$

(c) 4 (d) -3

Sol :

Slope of the line,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-7}{5-7} = \frac{-6}{-2} = 3$$

Thus (b) is correct option.

13. Which of the following is the slope of the line passing through the points $(-4, 3)$ and $(5, -6)$?

(a) -1 (b) 3

(c) 4 (d) -3

Sol :

Slope of the line,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-3}{5+4}$$

$$= \frac{-9}{9} = -1$$

Thus (a) is correct option.

14. A straight line passes through the points $P(2, -5)$ and $Q(4,3)$. What is the slope of the line PQ ?

(a) 2 (b) 3

(c) 4 (d) -3

Sol :

Given points are $P\,(2, -5)$ and $Q\,(4, 3)$.
The slope of the line PQ

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-(-5)}{4-2}$$

$$= \frac{8}{2} = 4$$

Thus (c) is correct option.

15. The gradient of the line joining $P(6, k)$ and $Q(1-3k, 3)$ is $\frac{1}{2}$. What is the value of k ?

(a) -11 (b) 7

(c) 7 (d) -3

Sol :

Gradient or slope of the line joining $P(6, k)$ and $Q(1-3k, 3)$ is $\frac{1}{2}$, thus

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2}$$

$$\frac{3-k}{1-3k-6} = \frac{1}{2}$$

$$6 - 2k = 1 - 3k - 6$$

$$3k - 2k = -5 - 6$$

$$k = -11$$

Thus (a) is correct option.

16. Which of the following is the slope of the line passing through the points (4, 5) and (7, 1)?

(a) $-\frac{3}{5}$ (b) $-\frac{5}{3}$

(c) $\frac{3}{8}$ (d) $-\frac{4}{3}$

Sol :

Slope of the line,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-5}{7-4} = \frac{-4}{3}$$

Thus (d) is correct option.

17. Which of the following is the slope of the line passing through the (6, 0) and $(-4, 1)$?

(a) $-\frac{3}{5}$ (b) $-\frac{5}{3}$

(c) $\frac{1}{10}$ (d) $\frac{3}{2}$

Sol :

Slope of the line,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-0}{-4-6} = \frac{1}{-10} = \frac{-1}{10}$$

Thus (b) is correct option.

18. Which of the following is the slope of a line perpendicular to PQ if $P(5, -3)$ and $Q(7, 3)$?

(a) $-\frac{7}{8}$ (b) $-\frac{5}{3}$

(c) $\frac{3}{4}$ (d) $\frac{3}{2}$

Sol :

Slope of PQ, $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{5-2}{6-1}$$

$$= \frac{3}{5}$$

Slope of a line perpendicular to PQ,

$$m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{3}{5}} = -\frac{5}{3}$$

Thus (b) is correct option.

19. Which of the following is the slope of a line perpendicular to PQ if $P(5, -3)$ and $Q(7, 3)$?

(a) $-\frac{3}{5}$ (b) $-\frac{5}{3}$

(c) $-\frac{1}{3}$ (d) $\frac{3}{2}$

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$8 - x = 4$ or $x = 4$

Thus (d) is correct option.

29. The line $y - x + 2 = 0$ divides the line joining $(3, -1)$ and $(8, 9)$ in the ratio $\lambda : 1$. where λ is

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$
(c) $\frac{5}{2}$ (d) $\frac{3}{2}$

Sol :

Suppose the line $y - x + 2 = 0$ divides the line segment joining $A(3, -1)$ and $B(8, 9)$ in the ratio $\lambda : 1$ at point P, then the coordinates of the point P are $\left(\frac{8\lambda+3}{\lambda+1}, \frac{9\lambda-1}{\lambda+1}\right)$.

But P lies on $y - x + 2 = 0$, therefore

$$\left(\frac{9\lambda-1}{\lambda+1}\right) - \left(\frac{8\lambda+3}{\lambda+1}\right) + 2 = 0$$

$$9\lambda - 1 - 8\lambda - 3 + 2\lambda + 2 = 0$$

$$3\lambda - 2 = 0$$

or $$\lambda = \frac{2}{3}$$

So, the required ratio is $\frac{2}{3}:1$ i.e. 2:3 (internally), since here λ is positive.

Thus (a) is correct option.

30. Two opposite vertices of a rectangle are (1,3) and (5,1). If the rest two vertices lie on the line $y - x + \lambda = 0$, then λ is equal to

(a) 1 (b) -1
(c) 2 (d) 3

Sol :

Diagonals of a rectangle bisect each other.

Mid-point of (1, 3) and (5, 1) satisfies

$$y - x + \lambda = 0 \quad \text{...(i)}$$

Mid-point of (1,3) and (5,1)

$$\left(\frac{1+5}{2}, \frac{3+1}{2}\right) = \left(\frac{6}{2}, \frac{4}{2}\right) = (3,2)$$

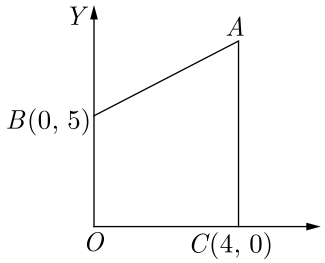
Point (3,2)satisfies Eq. (i)

$$2 - 3 + \lambda = 0$$

$$\lambda = 1$$

Thus (a) is correct option.

31. In the given diagram, $AC \perp x$-axis. $B = (0, 5)$ and $C(4, 0)$. Which of the following is the equation of AC ?



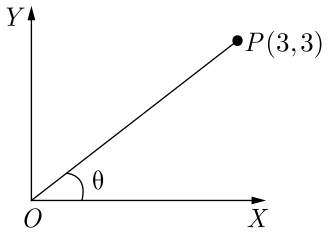
(a) $y = -4$ (b) $y = 4$
(c) $x = 4$ (d) $x = -4$

Sol :

Since AC is perpendicular to x-axis, it is parallel to y-axis. It passes through (4, 0). Thus equation is $y = 4$.

Thus (b) is correct option.

32. What is the measure of angle θ in the figure.



(a) 0^o (b) 30^o
(c) 45^o (d) 60^o

Sol :

Here θ is the slope of line OP where $O(0,0)$ and $P(3,3)$.

Slope of OP $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-0}{3-0} = 1$$

Now $$\tan 45^\circ = 1$$

$$\theta = 45^\circ$$

Thus (c) is correct option.

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33. The equation of a straight line is $3x - 3y - 7 = 0$. What is the y-intercept of the line ?

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$
(c) $-\frac{7}{3}$ (d) $\frac{3}{2}$

Sol :

The equation of the line is

$$3x - 3y - 7 = 0$$

It can be written as

$$-3y = -3x + 7$$

Thus (a) is correct option.

24. The equation of a line is $2x+3y=6$. It intersects the y-axis at A. Which of the following are the co-ordinates of A ?

(a) (0, 2) (b) (0, 3)
(c) (2, 0) (b) (3, 0)

Sol :

We have $2x+3y=6$

Above line intersect y-axis at A. At y-axis, x coordinate is always zero. Hence let point A, be $(0,\ a)$.

Point A must satisfy the given line. Thus substituting point A in given equation of line we have

$$0+3a=6$$
$$a=2$$

Thus point A is (0, 2)

25. The slope of a line joining $P(6,k)$ and $Q(1-3k,3)$ is $\frac{1}{2}$. Which of the following is the value of k ?

(a) -11 (b) 11
(c) 8 (d) 4

Sol :

Given points are $P(6,k)$ and $Q(1-3k,3)$.

Slope m of the line joining the points $P(6,k)$ and $Q(1-3k,3)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$\frac{1}{2}=\frac{3-k}{1-3k-6} \qquad m=\tfrac{1}{2}$$
$$\frac{1}{2}\ \frac{3-k}{-3k-5}$$
$$-3k-5=6-2k$$
$$-3k+2k=6+5$$
$$-k=11$$
$$k=-11$$

Thus (b) is correct option.

26. Select the equation of a line with inclination 135° and passing through the point (2, 3).

(a) $x-y-1=0$ (b) $-x+y+1=0$
(b) $x+y+1=0$ (c) $x-y+1=0$

Sol :

We have $\theta=135^\circ$

Slope $m=\tan 45^\circ=1$

This line passes through point (2, 3).

Equation of the line passing through the point $(2,3)$ and having slope $m=1$,

$$y-y_1=m(x-x_1)$$
$$y-3=1(x-2)$$
$$y-3=x-2$$
$$x-y+1=0$$

Thus (d) is correct option.

27. Select the equation of the line with inclination 60° and passing through the point $(-4,5)$.

(a) $\sqrt{3}x+y+4\sqrt{3}-5=0$
(b) $\sqrt{3}x+y-4\sqrt{3}+5=0$
(c) $\sqrt{3}x-y+4\sqrt{3}+5=0$
(d) $\sqrt{3}x+y+4\sqrt{3}+5=0$

Sol :

We have $\theta=60^\circ$

Slope $m=\tan 60^\circ=\sqrt{3}$

This line passes through point $(-4,5)$.

Equation of the line passing through the point $(-4,5)$ and having slope $m=\sqrt{3}$,

$$y-y_1=m(x-x_1)$$
$$y-5=\sqrt{3}(x+4)$$
$$y=\sqrt{3}x+4\sqrt{3}+5$$
$$\sqrt{3}x-y+4\sqrt{3}+5=0$$

Thus (c) is correct option.

28. Lines through the points $(-2,6)$ and (4,8) is perpendicular to the line through the points $(8,12)$ and $(x,\ 24)$. Then, the value of x is

(a) 2 (b) 6
(c) 8 (d) 4

Sol :

Slope of the line through the points (- 2,6) and (4, 8) is

$$m_1=\frac{8-6}{4-(-2)}=\frac{2}{6}=\frac{1}{3}$$

Slope of the line through the points (8, 12) and $(x,\ 24)$ is

$$m_2=\frac{24-12}{x-8}=\frac{12}{x-8}$$

Since, two lines are perpendicular,

So, $m_1m_2=-1$

$$\frac{1}{3}\times\frac{12}{x-8}=-1$$

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$$-3y = -5x - 2$$
$$y = \frac{5}{3}x + \frac{2}{3}$$

Writing line (2) in $y = mx + c$ form we have

$$-py = -6x - 7$$
$$y = \frac{6}{p}x + \frac{7}{p}$$

Comparing with $y = mx + c$ we get

$$m_1 = \frac{5}{3} \text{ and } m_2 = \frac{6}{p}$$

Since, given lines are perpendicular,

$$m_1 \times m_2 = -1$$
$$\frac{5}{4} \times \frac{6}{p} = -1$$
$$\frac{10}{p} = -1$$
$$p = -10$$

When $p = -10$, then slope of the line 2 is

$$m_2 = \frac{6}{p} = \frac{6}{-10} = -\frac{3}{5}$$

The slope of line parallel to $6x - py + 7 = 0$ is $m_2 = -\frac{3}{5}$.

The equation of a line passing through (–2, –1) and having slope $m_2 = -\frac{3}{5}$ is

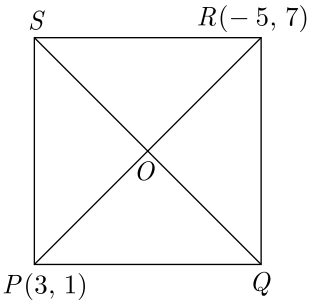
$$y - y_1 = m_2(x - x_1)$$
$$y + 1 = -\frac{3}{5}(x + 2)$$
$$5y + 5 = -3x - 6$$
$$3x + 5y + 11 = 0$$

Comp 2020

37. Find the equation of diagonal QS of square $PQRS$ where $P(3,1)$ and $R(-5,7)$.

Sol :

As per given question we have drawn figure below.



In square $PQRS$, diagonals PR and QS bisect each other at O at 90°. O is mid point of PR and QS Since O is mid point of PR, coordinates of O.

$$O(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{3-5}{2}, \frac{1+7}{2}\right)$$
$$= \left(-\frac{2}{2}, \frac{8}{2}\right)$$
$$= (-1, 4)$$

Slope of PR $\quad m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{7-1}{-5-3}$$
$$= \frac{6}{-8} = -\frac{3}{4}$$

Slope of QS $\quad m_2 = -\frac{1}{m_1}$

$$= -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$$

Thus equation of line QS having slope $\frac{4}{3}$ and passing through $(-1, 4)$,

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{4}{3}(x + 1)$$
$$3y - 12 = 4x + 4$$
$$3y = 4x + 4 + 12$$
$$3y = 4x + 16$$
$$4x - 3y + 16 = 0$$

Main 2019

38. M and N are two points on the X-axis and Y-axis, respectively. $P(3, 2)$ divides the line segment MN in the ratio 2 : 3.

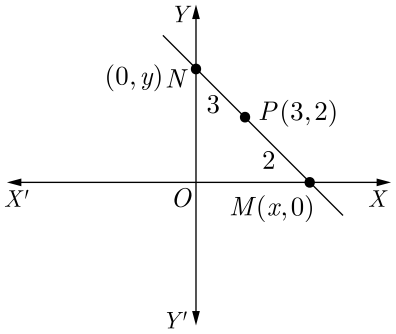
Find

(i) the coordinates of M and N.

(ii) slope of the line MN.

Sol :

Let the coordinates of M and N be $(x, 0)$ and $(0, y)$ as shown below.



Using internal section formula,

$$P(x, 2) = \left(\frac{3x + 0}{2 + 3}, \frac{0 + 2y}{2 + 3}\right)$$

NODIA

$$= \frac{-2-3}{3-(-1)}$$

$$= -\frac{5}{4}$$

Slope of parallel lines are equal, thus slope of line parallel to AC will be $-\frac{5}{4}$.

Now, Equation of the line through G and parallel to AC

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{5}{4}(x - 2)$$

$$4y - 4 = -5x + 10$$

$$5x + 4y - 14 = 0$$

Comp 2017

44. Line $4x - 5y = 20$ meets x-axis at A. Find the equation of a line passing through A and perpendicular to line $2x + 3y = 6$.

Sol :

We have $4x - 5y = 20$ L_1

Above line meet at x-axis at A. At x-axis, y coordinates is always zero. Thus let point A be $(a, 0)$. This point A must satisfy the line. Thus substituting point A in line L_1 we have

$$4x - 5y = 20 \quad L_1$$

$$4a - 0 = 20$$

$$a = 5$$

Thus point A is $(5, 0)$

Now we have $2x + 3y = 6$ L_2

Rearranging line L_2 in $y = mx + c$ form,

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + \frac{6}{3}$$

Slope of L_2 $m_2 = -\frac{2}{3}$

Slope of line perpendicular to L_2,

$$m_3 = -\frac{1}{m_2} = \frac{3}{2}$$

Equation of line, having slope $\frac{3}{2}$ and passing through point $(5, 0)$,

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{2}(x - 5)$$

$$2y = 3x - 15$$

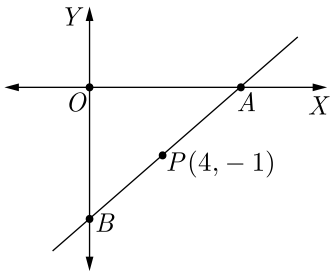
$$3x - 2y - 15 = 0$$

Main 2016

45. A line AB meets x-axis at A and y-axis at B, $P(4, -1)$ divides AB in the ratio $1 : 2$.

(i) Find the coordinates of A and B.

(ii) Find the equation of the line through P and perpendicular to AB.



Sol :

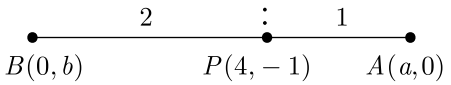
Let the coordinates of the points A and B be $(a, 0)$ and $(0, b)$ respectively.

As the point P (4, –1) divides the line segment AB in the ratio 1 : 2 i.e. $\frac{AP}{PB} = \frac{1}{2}$, the coordinates of the point P are

$$P(x, y) = \left(\frac{m \times x_2 + n \times x_1}{m+n}, \frac{m \times y_2 + n \times y_1}{m+n}\right)$$

$$= \left(\frac{1 \times 0 + 2a}{1+2}, \frac{1 \times b + 2 \times 0}{1+2}\right)$$

$$= \left(\frac{2a}{3}, \frac{b}{3}\right).$$



But P is (4, –1) therefore

$$\frac{2a}{3} = 4 \text{ and } \frac{b}{3} = -1$$

$$a = 6 \text{ and } b = -3.$$

Hence, the coordinates of the points A and B are (6, 0) and (0, –3) respectively.

(ii) Slope m_1 of the line AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3-0}{0-6}$$

$$= \frac{-3}{-6} = \frac{1}{2},$$

Slope m_2 of a line perpendicular to AB,

$$m_2 = -\frac{1}{m_1}$$

$$= -2$$

The equation of the line through P (4, –1) and having slope -2 is

$$y - y_1 = m(x - x_1)$$

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$$-\frac{12}{5a} = -1$$

$$5a = 12$$

$$a = \frac{12}{5}$$

Comp 2018

42. $A(7,-5)$, $B(5,3)$ and $C(-9,1)$ form a triangle. Find the

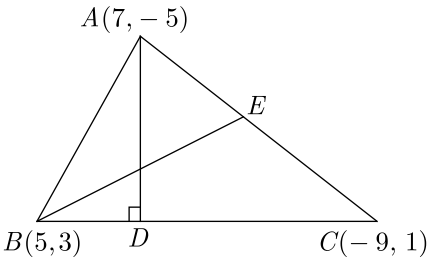
(i) slope of BC.

(ii) equation of altitude through A.

(iii) equation of median through B

Sol :

As per given question we have drawn figure below.



Here $A(7,-5)$, $B(5,3)$ and $C(-9,1)$ are the vertices of a triangle ABC, $AD \perp BC$, BE is median to AC.

Slope of BC $\quad m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{1-3}{-9-5}$$

$$= \frac{-2}{-14} = \frac{1}{7}$$

Line AD is parallel to BC. Thus slope of AD,

$$m_2 = -\frac{1}{m_1} = -\frac{7}{1}$$

$$= -7$$

Equation of AD,

$$y - y_1 = m(x - x_1)$$

$$y + 5 = -7(x-7)$$

$$y + 5 = -7x + 49$$

$$7x + y = 49 - 5$$

$$7x + y = 44$$

Since E is mid point of AC, coordinates of E,

$$E(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{7-9}{2}, \frac{-5+1}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{-4}{2}\right)$$

$$= (-1,-2)$$

Equation of median BE i.e equation of line passing thorouh $B(5,3)$ and $E(-1,-2)$,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 2 = \frac{3+2}{5+1}(x+1)$$

$$y + 2 = \frac{5}{6}(x+1)$$

$$6y + 12 = 5x + 5$$

$$6y = 5x + 5 - 12$$

$$6y = 5x - 7$$

$$5x - 6y - 7 = 0$$

Main 2017

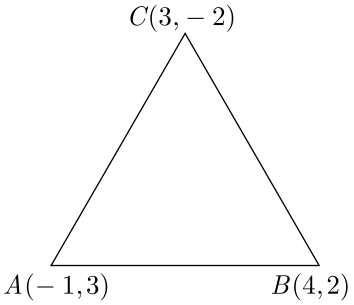
43. $A(-1,3)$, $B(4,2)$ and $C(3,-2)$ are the vertices of a triangle.

(i) Find the coordinates of the centroid G of the triangle.

(ii) Find the equation of the line through G and parallel to AC.

Sol :

Given $A(-1,3)$, $B(4,2)$ and $C(3,-2)$ are the vertices of the traingle.



NODIA

(i) Coordinates of the centroid G of the triangle. Coordinates of centroid of a triangle, whose vertices's are (x_1,y_1), (x_2,y_2) and (x_3,y_3), is given by

$$G(x,y) = G\left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2}\right)$$

Coordinates of centroid G of ΔABC are

$$G(x,y) = G\left(\frac{-1+3+4}{3}, \frac{3-2+2}{3}\right)$$

$$= (2,1)$$

(ii) Equation of the line through G and parallel to AC.

Slope m_1 of line AC,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y-6=2(x+1)$$
$$y=2x+8$$
$$2x+y+8=0$$

Main 2014

49. Find the value of a for which the following points $A(a,3)$, $B(2,1)$ and $C(5,a)$ are collinear. Hence, find the equation of the line.

Sol :

Given, point $A(a,3)$, $B(2,1)$ and $C(5,a)$ are collinear.

So, Slope of AB = Slope of BC

$$\frac{1-3}{2-a}=\frac{a-1}{5-2} \qquad m=\frac{y_2-y_1}{x_2-x_1}$$
$$\frac{-2}{2-a}=\frac{a-1}{3}$$
$$-2\times 3=(a-1)(2-a)$$
$$-6=2a-a^2-2+a$$
$$a^2-3a-4=0$$
$$a^2-4a+a-4=0$$
$$a(a-4)+1(a-4)=0$$
$$(a+1)(a-4)=0$$
$$a=-1,\ 4$$

So, the given points become $A(-1,3)$, $B(2,1)$ and $C(5,-1)$ or $A(4,3)$, $B(2,1)$ and $C(5,4)$.

Case I : When collinear points are $A(-1,3)$, $B(2,1)$ and $C(5,-1)$.

Equation of line passing through $A(-1,3)$ and $B(2,1)$ is given by

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-3=\frac{1-3}{2+1}(x+1)$$
$$y-3=-\frac{2}{3}(x+1)$$
$$3(y-3)=-2(x+1)$$
$$3y-9=-2x-2$$
$$3y+2x=7$$

Case II : When collinear points are $A(4,3)$, $B(2,1)$ and $C(5,4)$.

Equation of line passing through $A(4,3)$ and $B(2,1)$ is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-3=\frac{1-3}{2-4}(x-4)$$
$$y-3=\frac{-2}{-2}(x-4)$$
$$y-3=1(x-4)$$
$$y-3=x-4$$
$$y=x-1$$

Hence, the equations of the line is

$$3y+2x=7 \text{ or } y=x-1.$$

Note : You can take any two points out of A, B and C. e.g. B and C or C and A to find the equation of line.

Comp 2014

50. Point A is on the y-axis whose ordinate is 4. Point B is $(-2,6)$. Find the equation of the perpendicular bisector of AB.

Sol :

Point A is on y-axis and its ordinate is 4. Every point of y-axis has x coordinate zero. Thus Co-ordinates of A are (0, 4).

We have another point $B(-2,6)$.

Slope of AB, $$m_1=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{6-4}{-2-0}=\frac{2}{-2}=-1$$

Slope of its perpendicular bisector,

$$m_2=-\frac{1}{m_1}=-\frac{1}{-1}=1$$

Coordinates of mid point of AB,

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{0-2}{2},\frac{4+6}{2}\right)$$
$$=\left(-\frac{2}{2},\frac{10}{2}\right)$$
$$=(-1,5)$$

Equation of perpendicular bisector of AB

$$y-y_1=m_2(x-x_1)$$
$$y-5=1(x+1)$$
$$y-5=x+1$$
$$y=x+1+5$$
$$x-y+6=0$$

Main 2013

51. In ΔABC, if $A(3,5)$, $B(7,8)$ and $C(1,-10)$, then find the equation of the median through A.

Sol :

Given vertices equation are $A(3,5)$, $B(7,8)$ and $C(1,-10)$.

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$$y-(-1)=-2(x-4)$$
$$y+1=-2x+8$$
$$2x+y-7=0$$

Main 2016

46. The slope of a line joining $P(6,k)$ and $Q(1-3k,3)$ is $\frac{1}{2}$. Find
(i) k.
(ii) mid-point of PQ, using the value of k found in (i).

Sol :

Given points are $P(6,k)$ and $Q(1-3k,3)$.
Slope m of the line joining the points $P(6,k)$ and $Q(1-3k,3)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$\frac{1}{2}=\frac{3-k}{1-3k-6} \qquad m=\tfrac{1}{2}$$
$$\frac{1}{2}\ \frac{3-k}{-3k-5}$$
$$-3k-5=6-2k$$
$$-3k+2k=6+5$$
$$-k=11$$
$$k=-11$$

Mid point of $P(6,k)$ and $Q(1-3k,3)$ is

$$M(x,\ y)=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{6+1-3k}{2},\ \frac{k+3}{2}\right)$$
$$=\left(\frac{7-3(-11)}{2},\ \frac{-11+3}{2}\right)$$
$$=\left(\frac{7+33}{2},\ \frac{-8}{2}\right)$$
$$=\left(\frac{40}{2},\ \frac{-8}{2}\right)$$
$$=(20,-4)$$

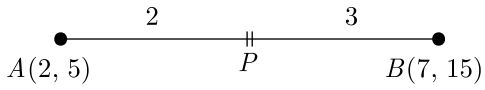
Hence, mid-point of PQ is $(20,-4)$.

Comp 2016

47. Point P divides the joining of line segment $A(2,5)$ and $B(7,15)$ in the ratio $2:3$.
(i) Find the co-ordinates of point P.
(ii) Find the equation of a line with gradient $-\frac{5}{3}$ and passing through P.

Sol :

As per given question we have drawn figure below.



Using internal section formula,

$$P(x,y)=\left(\frac{m\times x_2+n\times x_1}{m+n},\ \frac{m\times y_2+n\times y_1}{m+n}\right)$$
$$=\left(\frac{2\times 7+3\times 2}{5},\ \frac{2\times 15+3\times 5}{5}\right)$$
$$=\left(\frac{14+6}{5},\ \frac{30+15}{5}\right)$$
$$=\left(\frac{20}{5},\ \frac{45}{5}\right)$$
$$=(4,\ 9)$$

Equation of line having gradient or slope $-\frac{5}{3}$ and passing through point $(4,\ 9)$,

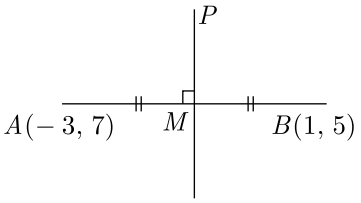
$$y-y_1=m(x-x_1)$$
$$y-9=\frac{-5}{3}(x-4)$$
$$3y-27=-5x+20$$
$$5x+3y-47=0$$

Comp 2015

48. Find the equation of the perpendicular bisector of the line segment joining $A(-3,7)$ and $B(1,5)$.

Sol :

As per given question we have drawn figure below.



Here PM is perpendicular bisector of AB.
Midpoint of AB, $M=\left(\frac{-3+1}{2},\frac{7+5}{2}\right)$

$$=(-1,6)$$

Slope AB $\quad m_1=\frac{5-7}{1+3}=\frac{-2}{4}=\frac{-1}{2}$

Slope of line PM perpendicular to AB,

$$m_2=-\frac{1}{m_1}$$
$$=-\frac{1}{-\frac{1}{2}}=2$$

Equation of line PM, having slope 2 and passing through point $(-1,\ 6)$,

$$y-y_1=m(x-x_1)$$

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$$y = \frac{x}{2} - \frac{5}{4}$$

Comparing with $y = m_2 x + c$, we get

$$m_2 = \frac{1}{2}$$

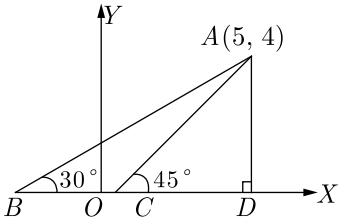
According to the question, the lines are perpendicular, therefore

$$m_1 m_2 = -1$$
$$\frac{b-3}{6} \times \frac{1}{2} = -1$$
$$b - 3 = -12$$
$$b = -12 + 3 = -9$$

Hence, the value of b is -9.

Comp 2012

54. Find the equation of lines AB, AC and AD in the given figure where $A(5,4)$. Also find the coordinates of C.



Sol :

Slope of AB $\quad m_1 = \tan 30^\circ$

$$= \frac{1}{\sqrt{3}}$$

Equation of line AB, having slope $\frac{1}{\sqrt{3}}$ and passing through point $A(5,4)$,

$$(y - y_1) = m(x - x_1)$$
$$y - 4 = \frac{1}{\sqrt{3}}(x - 5)$$
$$\sqrt{3}\,y = x - 5 + 4\sqrt{3}$$
$$x - \sqrt{3}y - 5 + 4\sqrt{3} = 0 \qquad L_1 : \ AB$$

Slope of AC $\quad m_2 = \tan 45^\circ$

$$= 1$$

Equation of line AC, having slope 1 and passing through point $A(5,4)$,

$$(y - y_1) = m(x - x_1)$$
$$y - 4 = 1(x - 5)$$
$$y = x - 1$$
$$x - y - 1 = 0 \qquad L_2 : \ AC$$

Since AD is perpendicular to x-axis, its slope is not defined. This line is parallel to y-axis and x coordinate of every point is 5. Thus equation of this line is $x = 5$.

Point C is intersection of line AC and x. Thus y coordinate of point C must be zero. Substituting $y = 0$ in equation of line AC,

$$x - 0 - 1 = 0$$
$$x = 1$$

Thus we get point $C(1,0)$.

Main 2010

55. The equation of a line is $3x + 4y - 7 = 0$. Find the

(i) slope of the line.

(ii) equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$.

Sol :

(i) Slope of the line.

Given, equation of line is

$$3x + 4y - 7 = 0 \qquad (1)$$
$$4y = -3x + 7$$
$$y = -\frac{3}{4}x + \frac{7}{4}$$

Comparing with $y = mx + c$, we get

slope $\quad m_1 = -\frac{3}{4}$

Hence, slope of line is $-\frac{3}{4}$.

(ii) Line perpendicular to the given line

Slope of the required line, which is perpendicular to the line having slope m_1, is

$$m_1 = -\frac{1}{m} \qquad m_1 m_2 = -1$$
$$= -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$$

Also, given lines are

$$x - y + 2 = 0 \qquad (2)$$

and $\quad 3x + y - 10 = 0 \qquad (3)$

Adding Eqs. (2) and (3), we get

$$4x - 8 = 0$$
$$4x = 8$$
$$x = 2$$

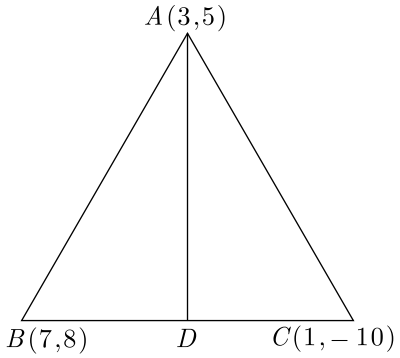
Substituting $x = 2$ in Eq. (2), we get

$$2 - y + 2 = 0$$
$$y = 4$$

So, point of intersection of lines (2) and (3) is $(2,4)$.

Now, equation of line passing through $(2,4)$ and having slope $\frac{4}{3}$ is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{4}{3}(x - 2)$$



Coordinates of mid-point of BC is

$$D(x,\ y) = D\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$$

$$= D\left(\frac{7+1}{2}, \frac{8-10}{2}\right)$$

$$= D\left(\frac{8}{2}, \frac{-2}{2}\right)$$

$$= D(4,-1)$$

Now, equation of median through A i.e. passing through A and D is given by

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-5 = \frac{-1-5}{4-3}(x-3)$$

$$y-5 = \frac{-6}{1}(x-3)$$

$$y-5 = -6x+18$$

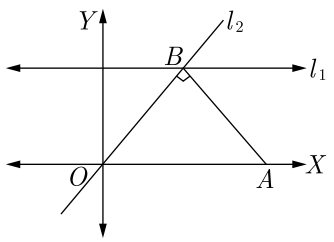
$$y+6x = 5+18$$

$$y+6x = 23$$

Hence, the required equation of the median through A is $y+6x=23$.

Comp 2014

52. In the given figure equation of L_1 is $y=3$ and equation of L_2 is $x=2y$.
(i) Write down the co-ordinates of B.
(ii) Find the equation of AB perpendicular to L_2.
(iii) Write down the co-ordinates of A.



Sol :

B is the point of intersection of L_1 and L_2.

where $y=3$ and $x=2y$

Thus $x=2\times 3 = 6$

Thus point B is $B(6,3)$.

Writing the line $x=2y$ in $y=mx+c$ form we have

$$y = \frac{1}{2}x$$

Slope of L_2 $m_2 = \frac{1}{2}$

Slope of line AB perpendicular to L_2,

$$m_1 = -\frac{1}{m_2} = -\frac{1}{\frac{1}{2}} = -2$$

Equation of line, having slope -2 and passing through point $B(6,3)$,

$$y-y_1 = m(x-x_1)$$

$$y-3 = -2(x-6)$$

$$y-3 = -2x+12$$

$$2x+y-15=0 \qquad L_1$$

Point A is on x-axis and every point of x-axis has y coordinate zero. Let $(a, 0)$ be the co-ordinates of A. Since point A lie on line AB, it must satisfy the line AB. Thus substituting point A in equation of line we have

$$2a+0-15=0$$

$$a = \frac{15}{2} = 7.5$$

Thus point is $A(7.5,0)$.
(i) The co-ordinates of B are $B(6,3)$
(ii) Equation of line AB is $2x+y-15=0$
(iii) The co-ordinates of A are $A(7.5,0)$.

Main 2012

53. The line through $A(-2,3)$ and $B(4,b)$ is perpendicular to the line $2x-4y=5$. Find the value of b.

Sol :

Given, points are $A(-2,3)$ and $B(4,b)$.
Slope of line joining the points A and B is

$$m_1 = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{b-3}{4+2}$$

$$m_1 = \frac{b-3}{6}$$

Also, given equation of line is

$$2x-4y=5$$

$$4y = 2x-5$$

$$\frac{4y}{4} = \frac{2x}{4} - \frac{5}{4}$$

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Main 2009

58. Find the equation of a line with x-intercept $= 5$ and passing through the point $(4, -7)$.

Sol :

Since x-intercept is 5, the line passes through the point (5, 0).
The slope of the line passing through the points (4, –7) and (5, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - (-7)}{5 - 4}$$
$$= 7$$

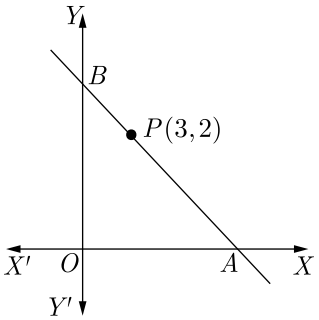
The equation of the line passing through (5, 0) and with slope 7 is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 7(x - 5)$$
$$y = 7x - 35$$
$$7x - y - 35 = 0$$

Hence, the equation of the required line is $7x - y - 35 = 0$.

Comp 2009

59. A line passes through the point $P(2, 3)$ and meets the coordinates axes at the points A and B (as shown in the adjoining figure).



If $2PA = 3PB$, find
(i) the coordinates of A and B.
(ii) the equation of the line AB.

Sol :

Let $OA = a$ and $OB = b$, then the coordinates of points A and B are $(a, 0)$, $(0, b)$ respectively.

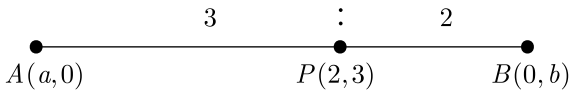
Given $2PA = 3PB$

$$\frac{PA}{PB} = \frac{3}{2}$$

$$PA : PB = 3 : 2$$

The point P divides the line segment AB in the ratio 3 : 2.
The coordinates of the point P are

$$P(x, y) = \left(\frac{m \times x_2 + n \times x_1}{m + n}, \frac{m \times y_2 + n \times y_1}{m + n}\right)$$
$$= \left(\frac{3 \cdot 0 + 2 \cdot a}{3 + 2} \quad \frac{3 \cdot b + 2 \cdot 0}{3 + 2}\right)$$
$$= \left(\frac{2a}{5}, \frac{3b}{5}\right)$$



But the coordinates of the point P are (2, 3)

$$\frac{2a}{5} = 2 \text{ and } \frac{3b}{5} = 3$$

$$a = 5 \text{ and } b = 5.$$

The coordinates of the point A and B are (5, 0) and (0, 5) respectively.
(ii) Slope of line AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 0}{0 - 5} = -1$$

The line AB passes through the point A (5, 0) and has slope $m = -1$.
The equation of above line is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -1(x - 5)$$
$$x + y - 5 = 0.$$

Main 2007

60. Find the equation of the line parallel to the line $3x + 2y = 8$ and passing through the point $(0,1)$.

Sol :

Given equation of line is

$$3x + 2y = 8$$
$$2y = -3x + 8$$
$$y = -\frac{3}{2}x + \frac{8}{2}$$

Comparing with $y = mx + c$, we get

$$m = -\tfrac{3}{2}$$

Since, the required line is parallel to given line, so slope will be same i.e. $-\frac{3}{2}$.
Now, required equation of a line, passing through $(0,1)$ and having slope $-\frac{3}{2}$, is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{-3}{2}(x - 0)$$

NODIA

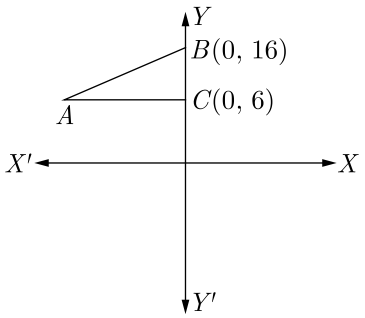
$$3(y-4) = 4x-8$$
$$3y-12 = 4x-8$$
$$3y = 4x-8+12$$
$$3y = 4x+4$$

Hence, equation of line perpendicular to the given line and passing through the intersection of given lines is $3y = 4x+4$

or $\quad 4x-3y+4 = 0.$

Comp 2010

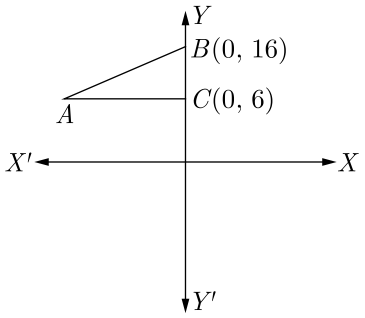
56. In the given figure AC is parallel to x-axis.



(i) Find the equation of AC.
(ii) If slope of $AB = 2$, find the equation of AB.
(iii) Find the co-ordinates of A.
(iv) Find the area of ΔABC.

Sol :

We have redrawn the given figure as below.



In the figure, AC is parallel to the x-axis thus its equation will be $y = k$. Since point C is (0, 6), value of k is 6. Thus equation of AC is $y = 6$.

Slope of $AB \quad m_1 = 2$

Equation of line AB, having slope 2 and point $B(0, 16)$,

$$y-y_1 = m(x-x_1)$$
$$y-16 = 2(x-0)$$
$$y-16 = 2x$$
$$2x-y+16 = 0$$

At every point of line AC, y is 6. Thus let point A be $(a, 6)$. Since this point lie on line AC, it must satisfy it. Substituting $(a, 6)$ in the equation of line AC we have

$$2a-6+16 = 0$$
$$2a+10 = 0$$
$$a+5 = 0$$
$$a = -5$$

Coordinates of A are $(-5, 6)$

$$\text{Area of } \Delta ABC = \frac{1}{2} AC \times BC$$
$$= \frac{1}{2}(0+5) \times (16-6)$$
$$= \frac{1}{2} \times 5 \times 10$$
$$= 25 \text{ sq. units}$$

(i) Equation of line AC is $y = 6$.
(ii) Equation of line AB is $2x-y+16 = 0$.
(iii) Coordinates of A are $(-5, 6)$.
(iv) Area of ΔABC is 25 sq units.

Main 2009

57. Find the value of p for which the lines $2x+3y-7 = 0$ and $4y-px-12 = 0$ are perpendicular to each other.

Sol :

We have $\quad 2x+3y-7 = 0 \quad \ldots\ldots(1)$

Converting into the form $y = mx+c$ we have

$$3y = -2x+7$$
$$y = -\frac{2}{3}x+\frac{7}{3}$$

The slope of the line (1) is $m_1 = -\frac{2}{3}$.

Now $\quad 4y-px-12 = 0 \quad \ldots\ldots(2)$

Converting into the form $y = mx+c$ we have

$$4y = px+12$$
$$y = \frac{p}{4}x+3$$

The slope of the line (2) is $m_2 = \frac{p}{4}$.

Since the given lines are perpendicular to each other,

$$m_1 m_2 = -1$$
$$\left(-\frac{2}{3}\right) \cdot \left(\frac{p}{4}\right) = -1$$
$$-\frac{p}{6} = -1$$
$$p = 6$$

Hence, the value of p is 6.

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$$(3,2)=\left(\frac{3x}{5},\frac{2y}{5}\right)$$

$$\frac{3x}{5}=3 \text{ and } \frac{2y}{5}=2$$

$$x=5 \text{ and } y=5$$

Hence, the coordinates of M and N are $(5, 0)$ and $(0, 5)$, respectively.

Slope of line MN,

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{5-0}{0-5}=\frac{5}{-5}=-1$$

Main 2019

39. The vertices of a ΔABC are $A(3, 8)$, $B(-1, 2)$ and $C(6, -6)$. Find the

(i) slope of BC.

(ii) equation of a line perpendicular to BC and passing through A.

Sol :

Given vertices of a ΔABC are $A(3,8), B(-1,2)$ and $C(6,-6)$

Slope of BC, $$m_1=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-6-2}{6-(-1)}=\frac{-8}{7}$$

Slope of line perpendicular to BC,

$$m_2=-\frac{1}{m_1}=-\frac{7}{8}$$

Equation of line passing through $A(3, 8)$ and perpendicular to BC is

$$y-y_1=m_2(x-x_1)$$

$$y-8=\frac{7}{8}(x-3)$$

$$8y-64=7x-21$$

$$7x-8y+43=0$$

Comp 2019

40. The equation of a line is $2x+3y=6$. It intersects the y-axis at A.

(i) Write the co-ordinates of A.

(ii) Find the equation of a line through A and perpendicular to the given line.

Sol :

We have $$2x+3y=6$$

Above line intersect y-axis at A. At y-axis, x coordinate is always zero. Hence let point A, be $(0, a)$.

Point A must satisfy the given line. Thus substituting point A in given equation of line we have

$$0+3a=6$$

$$a=2$$

Thus point A is $(0, 2)$

Rearranging the line $2x+3y=6$ in $y=mx+c$ form we have

$$3y=-2x+6$$

$$y=-\frac{2}{3}x+\frac{6}{3}$$

Slope $$m_1=\frac{-2}{3}$$

Slope of line perpendicular to $2x+3y=6$,

$$m_2=-\frac{1}{m_1}=\frac{3}{2}$$

Equation of line passing through $A(0, 2)$ and having slope $\frac{3}{2}$

$$y-y_1=m(x-x_1)$$

$$y-2=\frac{3}{2}(x-0)$$

$$2y=3x+4$$

$$3x-2y+4=0$$

Main 2018

41. If the straight lines $3x-5y=7$ and $4x+ay+9=0$ are perpendicular to one another, find the value of a.

Sol :

Given equation of straight lines are

$$3x-5y=7$$

$$3x-7=5y$$

$$5y=3x-7$$

$$y=\frac{3}{5}x-\frac{7}{5} \quad (1)$$

and $$4x+ay+9=0$$

$$ay=-4x-9$$

$$y=-\frac{4}{a}x-\frac{9}{a} \quad (2)$$

Comparing Eq. (1) by $y=m_1x+c_1$ and Eq. (2) by $y=m_2x+c_2$, we get

$$m_1=\frac{3}{5} \text{ and } m_2=-\frac{4}{a}$$

Since, given straight lines are perpendicular to each other

$$m_1\cdot m_2=-1$$

$$\frac{3}{5}\times\left(-\frac{4}{a}\right)-\frac{12}{5a}$$

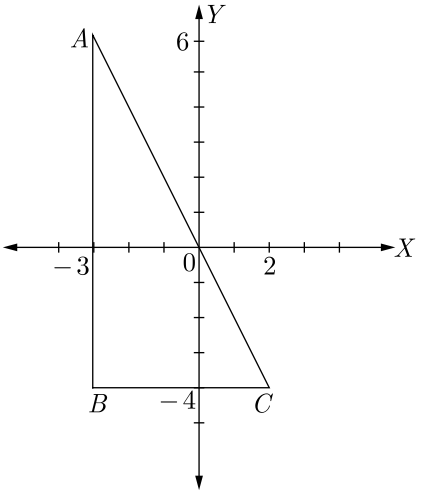
NODIA

$$2(y-1) = -3(x-0)$$
$$2y - 2 = -3x$$
$$3x + 2y = 2$$

which is the required equation of a line.

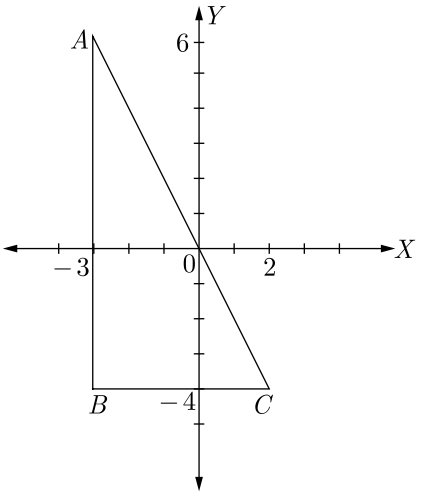
Comp 2007

61. From the given figure, write
(i) the co-ordinates of A, B and C
(ii) the equation of AC
(iii) the x-intercept of AB
(iv) the y-intercept of BC



Sol :

As per given question we have redrawn figure below.



From given figure we easily get $A(-3,6)$, $B(-3,-4)$ and $C(2,-4)$.
Equation of line AC, passing through point $A(-3,6)$ and $C(2,-4)$ we have

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 6 = \frac{-4-6}{2+3}(x+3)$$
$$= \frac{-10}{5}(x+3)$$
$$= -2(x+3)$$
$$y - 6 = -2x - 6$$
$$y = -2x$$
$$2x + y = 0$$

From figure we easily get that x-intercept of AB is -3.
Similarity we easily get that y-intercept of BC is -4.
(i) Co-ordinates of A, B and C are $A(-3,6)$, $B(-3,-4)$ and $C(2,-4)$.
(ii) Equation of AC is $2x + y = 0$.
(iii) x-intercept of $AB = -3$
(iv) y-intercept of $BC = -4$

Main 2006

62. If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, then find the value of p.

Sol :

We have $y = 3x + 7$ .....(1)
The slope of the line (1) is $m_1 = 3$.
Now $2y + px = 3$ .....(2)
Converting into the form $y = mx + c$ we have

$$2y = -px + 3$$
$$y = -\frac{p}{2}x + \frac{3}{2}$$

The slope of the line (2) is $m_2 = -\frac{p}{2}$.
Since the given lines are perpendicular to each other,

$$m_1 m_2 = -1$$
$$(3)\cdot\left(-\frac{p}{2}\right) = -1$$
$$-3p = -2$$
$$p = \frac{2}{3}$$

Hence, the value of p is $\frac{2}{3}$.

Comp 2006

63. Find the equation of the line through $A(2,3)$ and $B(8,-6)$. Also find
(i) If AB meets x-axis at P, find the co-ordinates of P.
(ii) Hence, find the ratio $AP : PB$.

Sol :

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substituting $(5, 0)$ in $3x-4y+\lambda=0$ we have

$$3\times5-0+\lambda=0$$

$$15+\lambda=0$$

$$\lambda=-15$$

Substituting $\lambda=-15$ in $3x-4y+\lambda=0$ we get

$$3x-4y-15=0$$

(i) Co-ordinates of P are $(-3,-2)$, of Q are $(5,0)$ and of R are $(5, 4)$.

(ii) $3x-4+1=0$ and $x=5$

(iii) $3x-4y-15=0$

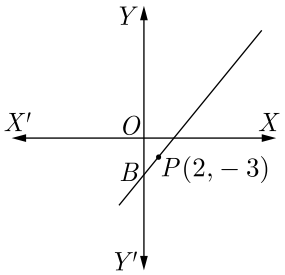
Main 2010

75. A and B are two points on the x-axis and y-axis, respectively. If $P(2,-3)$ is the mid-point of AB, then find the

(i) coordinates of A and B.

(ii) slope of line AB.

(iii) equation of line AB.



Sol :

(i) Coordinates of A and B

Since point A lies on $x-$axis, its $y-$ coordinate will be zero. Let point be $A(a,0)$. Similarly A lies on $y-$axis, its $x-$ coordinate will be zero. Let point be $B(0,b)$.

So, coordinates of mid-point of A and B are,

$$P(x,y)=P\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=P\left(\frac{a+0}{2},\frac{0+b}{2}\right)$$

$$=P\left(\frac{a}{2},\frac{b}{2}\right)$$

But given coordinates of mid-point are $(2,-3)$.

Thus $\quad(2,-3)=\left(\frac{a}{2},\frac{b}{2}\right)$

Equating the x and y-coordinates of both sides, we get

$$2=\frac{a}{2}\text{ and }-3=\frac{b}{2}$$

$$a=4\text{ and }b=-6$$

Hence, coordinates of A are $(4,0)$ and B are $(0,-6)$.

(ii) Slope of line AB.

Now, slope of line AB,

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-6-0}{0-4}$$

$$=\frac{-6}{-4}=\frac{3}{2}$$

(iii) Equation of line AB

Equation of line passing through the point $A(4,0)$ and having slope $\frac{3}{2}$ is given by

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0=\frac{3}{2}(x-4)$$

$$2y=3x-12$$

$$3x-2y=12$$

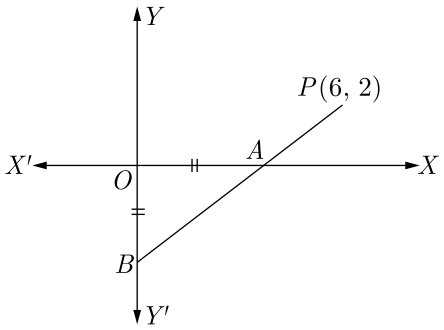
Hence, the equation of line AB is $3x-2y=12$.

Comp 2010

76. From the given figure write the

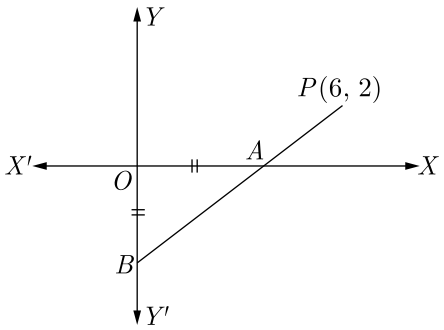
(i) equation of the line through points P, A and B where $OA=OB$.

(ii) co-ordinates of A and B.



Sol :

As per given question we have redrawn figure below.



Since line is equally inclined to x and y-axis

$$OA=OB$$

$$\angle OAB=45^\circ$$

Hence, the coordinates of point P are (1, 3)

(c) Slope of line AB

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-0}{0-4} = -1$$

Product of slopes of two perpendicular lines is 1.

Slope of line perpendicular to AB,

$$m_2 = \frac{1}{m_1}$$

$$= \frac{1}{-1} = -1$$

Equation of the line passing through $P(1,3)$ and perpendicular to AB having $m_2 = -1$ is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1(x-1)$$

$$y - 3 = 1 - x$$

$$x + y - 4 = 0$$

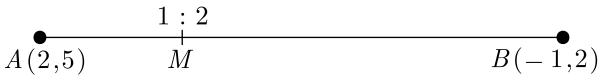
Main 2018

66. $A(2,5)$, $B(-1,2)$ and $C(5,8)$ are the vertices of a ΔABC, M is a point of AB such that $AM : MB = 1 : 2$. Find the coordinate of M . Hence find the equation of the line passing through the points C and M.

Sol :

We have, $A(2,5)$, $B(-1,2)$ and $C(5,8)$.

Since, $AM : MB = 1 : 2$



Using section formula coordinates of M is

$$M(x,y) = M\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) \quad m:n = 1:2$$

$$= M\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= M\left(\frac{1\times(-1) + 2\times 2}{1+2}, \frac{1\times 2 + 2\times 5}{1+2}\right)$$

$$= M\left(\frac{-1+4}{3}, \frac{2+10}{3}\right)$$

$$= M\left(\frac{3}{3}, \frac{12}{3}\right)$$

$$= M(1,4)$$

Now, we have $M(1,4)$ and $C(5,8)$. Equation of line joining C and M is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{8-4}{5-1}(x-1)$$

$$y - 4 = \frac{4}{4}(x-1)$$

$$y - 4 = x - 1$$

$$y = x - 3$$

Main 2015

67. Three vertices of a parallelogram $ABCD$ taken in order are $A(3,6)$, $B(5,10)$ and $C(3,2)$. Find

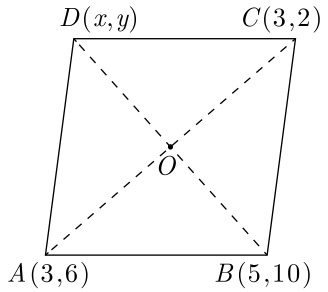
(i) the coordinates of the fourth vertex D.

(ii) length of diagonal BD.

(iii) equation of side AB of the parallelogram $ABCD$.

Sol :

Given, $ABCD$ is a parallelogram with $A(3,6)$, $B(5,10)$ and $C(3,2)$.



(i) Coordinates of the fourth vertex D

Let the coordinates of vertex D be (x,y). Diagonals of a parallelogram bisect each other. Thus

Mid-point of BD = Mid-point of AC

$$\left(\frac{5+x}{2}, \frac{10+y}{2}\right) = \left(\frac{3+3}{2}, \frac{6+2}{2}\right)$$

$$\left(\frac{5+x}{2}, \frac{10+y}{2}\right) = (3,4)$$

Equating the x and y-coordinates of both sides, we get

$$\frac{5+x}{2} = 3 \text{ and } \frac{10+y}{2} = 4$$

$$5 + x = 6 \text{ and } 10 + y = 8$$

$$x = 1 \text{ and } y = -2$$

Thus, the coordinates of the point D are $(1,-2)$.

(ii) Length of diagonal BD

Using distance formula length of diagonal BD,

$$BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-1)^2 + (10+2)^2}$$

$$= \sqrt{4^2 + 12^2}$$

$$= \sqrt{16 + 144}$$

$$= \sqrt{160}$$

$$= 4\sqrt{10} \text{ units}$$

NODIA

(iii)Equation of side AB of the parallelogram

Equation of AB is given by

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\begin{bmatrix}(x_1,y_1)=(3,6)\\(x_2,y_2)=(5,10)\end{bmatrix}$$

$$(y-6)=\frac{10-6}{5-3}(x-3)$$

$$y-6=\frac{4}{2}(x-3)$$

$$y-6=2x-6$$

$$2x-y=0$$

which is the required equation of line.

Main 2015

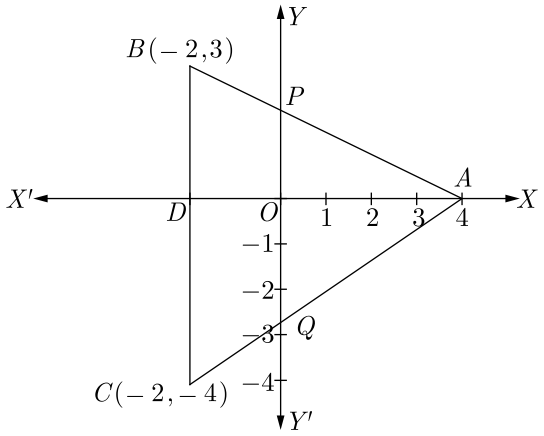
68. In the given figure, ABC a triangle and BC is parallel to the Y-axis. AB and AC intersects the Y-axis at P and Q, respectively.

(i) Write the coordinates of A.

(ii) Find the length of AB and AC.

(iii) Find the ratio in which Q divides AC.

(iv) Find the equation of the line AC.



Sol :

As the point A lies on x-axis and $QA = 4$. therefore, the coordinates of the point A are (4, 0) The coordinates of points A, B and C are (4, 0), (–2, 3) and (–2, –4) respectively.

Length $AB=\sqrt{(-2-4)^2+(3-0)^2}$

$$=\sqrt{36+9}$$

$$=\sqrt{45}$$

$$=3\sqrt{5}\text{ units}$$

Length $AC=\sqrt{(-2-4)^2+(-1-0)^2}$

$$=\sqrt{36+16}$$

$$=\sqrt{52}$$

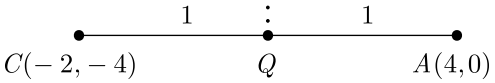
$$=2\sqrt{13}\text{ units}$$

Let point Q divide line segment AC in the ratio $k:1$ i.e. $\frac{AQ}{QC}=\frac{k}{1}$.

Using section formula coordinates of Q are,

$$Q(x,y)=\left(\frac{k\times x_2+x_1}{k+1},\ \frac{k\times y_2+y_1}{k+1}\right)$$

$$=\left(\frac{-2k+4}{k+1},\frac{-4k+0}{k+1}\right)$$



As point Q lies on y-axis, its x-coordinate is 0.

Thus $$\frac{4-2k}{1+k}=0$$

$$4-2k=0$$

$$k=2$$

Hence, the point Q divides the line segment AC in ratio 2 : 1.

(iii) Slope of the line AC

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-4-0}{-2-4}$$

$$=\frac{-4}{-6}=\frac{2}{3}.$$

The equation of line AC having slope $\frac{2}{3}$ is

$$y-y_1=m(x-x_1)$$

$$y-0=\frac{2}{3}(x-4)$$

$$3y=2x-8$$

$$2x-3y=8$$

Hence, the equation of the line AC is

$$2x-3y=8$$

Comp 2015

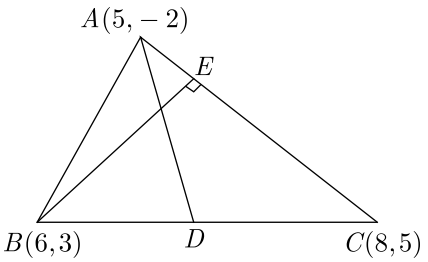
69. In ΔABC, $A(5,-2)$, $B(6,3)$ and $C(8,5)$. Find the equation of

(i) the median through A.

(ii) the altitude through B.

Sol :

As per given question we have drawn figure below.



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Here ΔABC, $A(5,-2)$, $B(6,3)$ and $C(8,5)$ are the vertices of a triangle ABC, $BE \perp AC$, AD is median to BC.

Since D is mid point of BC, coordinates of D,

$$D(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{6+8}{2}, \frac{3+5}{2}\right)$$

$$= \left(\frac{14}{2}, \frac{8}{2}\right)$$

$$= (7,4)$$

Equation of median AD i.e equation of line passing thorough $A(5,-2)$ and $D(7,\ 4)$,

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y+2 = \frac{4+2}{7-5}(x-5)$$

$$= \frac{6}{2}(x-5)$$

$$= 3(x-5)$$

$$y+2 = 3x-15$$

$$y = 3x-15-2$$

$$= 3x-17$$

$$3x-y-17 = 0$$

Slope of AC $\quad m_1 = \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{5+2}{8-5} = \frac{7}{3}$$

Slope of BE perpendicular to AC,

$$m_2 = -\frac{1}{m_1}$$

$$= -\frac{1}{\frac{7}{3}} = -\frac{3}{7}$$

Equation of altitude BE, having slope $-\frac{3}{7}$ and passing through point $B(6,\ 3)$,

$$y - y_1 = m_2(x-x)$$

$$y-3 = -\frac{3}{7}(x-6)$$

$$7y-21 = -3x+18$$

$$3x+7y = 18+21$$

$$3x+7y-39 = 0$$

(i) Equation of median through A is $3x-y-17=0$

(ii) Equation of altitude through B is $3x+7y-39=0$

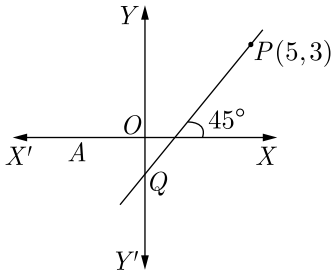
Main 2012

70. The line through $P(5,3)$ intersects y-axis at Q.

(i) Write the slope of the line.

(ii) Write the equation of the line.

(iii) Find the coordinates of Q.



Sol :

(i) Slope of the line.

From the given figure, $\theta = 45^\circ$

Thus slope of the line PQ

$$PQ = m = \tan 45^\circ = 1$$

(ii) Equation of the line

The equation of line passing through $P(5,3)$ and having slope 1 is

$$y - y_1 = m(x-x_1)$$

$$y-3 = 1(x-5)$$

$$y-3 = x-5$$

$$y-x = -5+3$$

$$y = x-2 \qquad \ldots(1)$$

which is the required equation of line.

(iii) Coordiantes of Q

Since, Q lie on y-axis, therefore x-coordinates of Q will be 0. Putting $x=0$ in the Eq. (1), we get

$$y = 0-2 = -2$$

Hence, the coordinates of Q are $(0,-2)$.

Comp 2012

71. Point A divides the line segment $M(3,-2)$ and $N(10,19)$ in the ratio 3 : 4. Find

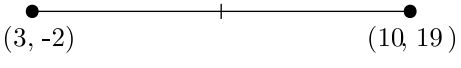
(i) the co-ordinates of point A.

(ii) the equation of a line passing through A and parallel to $2x+3y=1$.

(ii) the equation of a line passing through A and perpendicular to $2x+3y=1$.

Sol :

As per given question we have drawn figure below.



Using internal section formula,

$$A(x,y) = \left(\frac{m\times x_2 + n\times x_1}{m+n}, \frac{m\times y_2 + n\times y_1}{m+n}\right)$$

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$$= \left(\frac{3\times 10+4\times 2}{3+4}, \frac{3\times 19+4\times(-2)}{3+2}\right)$$
$$= \left(\frac{30+12}{7}, \frac{57-8}{7}\right)$$
$$= \left(\frac{42}{7}, \frac{49}{7}\right)$$
$$= (6,7)$$

Thus coordinate of point A is $(6,7)$.

We have $2x+3y=1$ L_1

Rearranging line L_1 in $y=mx+c$ form,

$$3y = -2y+1$$
$$y = \frac{-2x}{3}+\frac{1}{3}$$

Slope of L_1 $m_1 = -\frac{2}{3}$

Slope of line parallel to L_1,

$$m_2 = m_1 = -\frac{2}{3}$$

Equation of line, having slope $-\frac{2}{3}$ and passing through point $(6,7)$,

$$y-y_1 = m(x-x_1)$$
$$y-7 = \frac{-2}{3}(x-6)$$
$$3y-21 = -2x+12$$
$$2x+3y-33=0$$

Slope of line perpendicular to L_1,

$$m_3 = -\frac{1}{m_1} = \frac{3}{2}$$

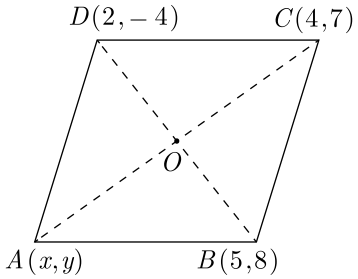
Equation of line, having slope $\frac{3}{2}$ and passing through point $(6,7)$,

$$y-y_1 = m(x-x_1)$$
$$y-7 = \frac{3}{2}(x-6)$$
$$2y-14 = 3x-18$$
$$3x-2y-4=0$$

(i) $(6,7)$
(ii) $2x+3y-33=0$
(iii) $3x-2y-4=0$

Main 2011

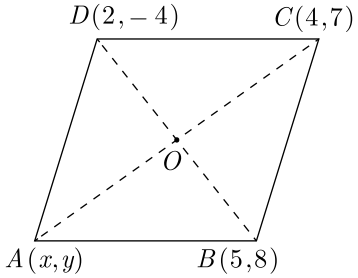
72. $ABCD$ is a parallelogram, where $A(x,y)$, $B(5,8)$, $C(4,7)$ and $D(2,-4)$. Find
(i) Coordinates of A.
(ii) equation of diagonal BD.



Sol :

(i) Coordinates of A

Diagonals of a parallelogram bisect each other.



Thus using mid point formula we have

Mid-point of BD = Mid-point of AC

$$\left(\frac{2+5}{2}, \frac{-4+8}{2}\right) = \left(\frac{x+4}{2}, \frac{y+7}{2}\right)$$
$$\left(\frac{7}{2}, \frac{4}{2}\right) = \left(\frac{x+4}{2}, \frac{y+7}{2}\right)$$

Equating the x and y-coordinates both sides, we get

$$\frac{7}{2} = \frac{x+4}{2} \text{ and } 4 = y+7$$
$$x = 7-4 \text{ and } y = 4-7$$
$$x = 3 \text{ and } y = -3$$

Hence, the coordinates of A are $(3,-3)$.

(ii) Equation of diagonal BD.

Equation of diagonal BD having point $B(5,8)$ and $D(2,-4)$,

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-8 = \frac{-4-8}{2-5}(x-5)$$
$$y-8 = \frac{-12}{-3}(x-5)$$
$$y-8 = 4(x-5)$$
$$y-8 = 4x-20$$
$$y = 4x-20+8$$
$$y = 4x-12$$

which is the required equation of line.

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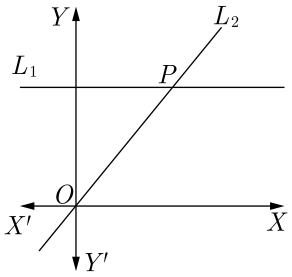
Main 2011

73. Given equation of line L_1 is $y = 4$.

(i) Write the slope of line L_2, if L_2 is the bisector of $\angle O$.

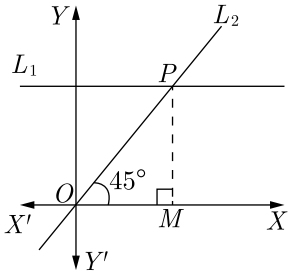
(ii) Write the coordinates of point P.

(iii) Find the equation of L_2.



Sol :

From P, we draw $MP \perp OX$ as shown below. As the equation of line L_1 is $y = 4$, $MP = 4$.



(i) Slope of line L_2 if L_2 is the bisector of angle O.

Since line L_2 is the bisector of $\angle O$, therefore,

$$\angle MOP = \frac{1}{2}\angle O$$

$$= \frac{1}{2} \times 90^\circ$$

$$= 45^\circ$$

Thus inclination of line L_2 is 45°.

Slope of line L_2,

$$m_2 = \tan 45^\circ$$

$$= 1$$

(ii) Coordinates of the point P.

Since $\Delta\, OMP$ is an isosceles right angled triangle, we have

$$OM = MP = 4$$

The coordinates of point P are (4, 4).

(iii) Equation of L_2.

The line L_2 has slope 1 and passes through origin (0, 0), therefore, the equation of line L_2 is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 1\,(x - 0)$$

$$x - y = 0$$

$$y = x$$

Hence, the equation of L_2 is $y = x$.

Comp 2011

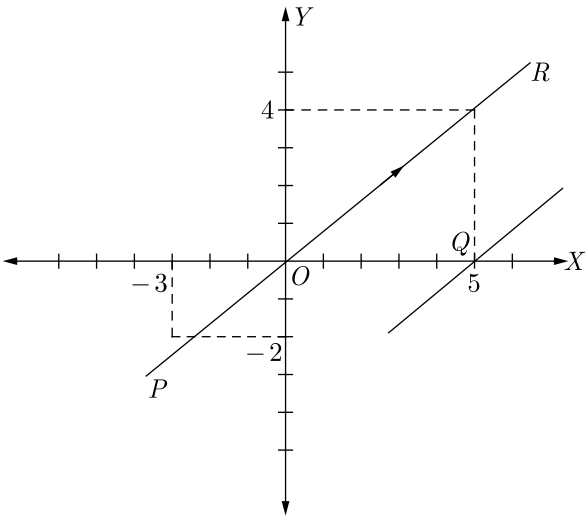
74. From the given figure write the

(i) co-ordinates of P, Q and R

(ii) equations of PR and QR

(iii) equation of a line through Q and parallel to PR.

NODIA



Sol :

In the given figure, we can easily see that co-ordinates of P are $(-3, -2)$, of Q are (5, 0) and of R are (5, 4)

Equation of PR having point $P(-3, -2)$ and $R(5, 4)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 2 = \frac{4 + 2}{5 + 3}(x + 3)$$

$$= \frac{6}{8}(x + 3)$$

$$y + 2 = \frac{3}{4}(x + 3)$$

$$4y + 8 = 3x + 9$$

$$4y = 3x + 1$$

$$3x - 4 + 1 = 0 \qquad L_1 : PR$$

Since QR is perpendicular to x-axis and is parallel to y-axis. x coordinate of every point on this line is 5. Thus its equation is $x = 5$.

Now an general equitation of line parallel to PR,

$$3x - 4y + \lambda = 0$$

Since this line passes through point $Q(5, 0)$,

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Equation of line passing thorough $A(2,3)$ and $B(8,-6)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-3=\frac{-6-3}{8-2}(x-2)$$

$$=\frac{-9}{6}(x-2)$$

$$y-3=\frac{-3}{2}(x-2)$$

$$2y-6=-3x+6$$

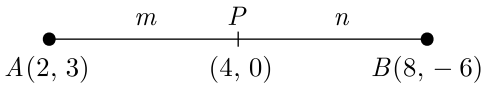
$$3x+2y=12$$

y coordinate of any point on x-axis is zero. Let point P be $(a,0)$. Since AB meets x-axis at P, it must satisfy the equation of line AB. Thus substituting $(a,0)$ in line AB

$$3a+0=12$$

$$a=4$$

Thus point P is $P(4,0)$.
Let point P divide the line AB in ratio $\frac{m}{n}$.



Since y-coordinate of P is zero, we have

$$\frac{-6m+3n}{m+n}=0$$

$$-6m=-3n$$

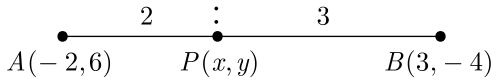
$$\frac{m}{n}=\frac{3}{6}=1:2$$

Main 2001

64. Write down the equation of the line, whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining $A(-2,6)$ and $B(3,-4)$ in the ratio of $2:3$.

Sol :

Let the point $P(x,y)$ divides the line joining A and B in the ratio $2:3$.



Using section formula we have

$$P(x,y)=P\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

$$=P\left[\frac{2\times3+3\times(-2)}{2+3},\frac{2\times(-4)+3\times6}{2+3}\right]$$

$$=P\left(\frac{6-6}{5},\frac{-8+18}{5}\right)$$

$$=P\left(0,\frac{10}{5}\right)$$

$$=P(0,2)$$

Thus $x=0$ and $y=2$

Now, the equation of a line passing through $P(0,2)$ and having $m=\frac{3}{2}$ is

$$y-y_1=m(x-x_1)$$

$$y-2=\frac{3}{2}(x-0)$$

$$2(y-2)=3(x)$$

$$2y-4=3x$$

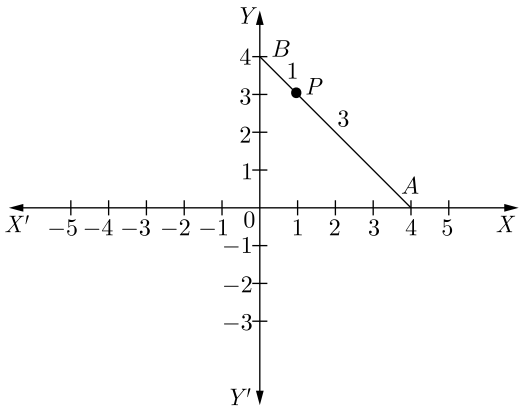
$$3x-2y+4=0$$

Thus equation of line is $3x-2y+4=0$

FOUR MARK QUESTIONS

Main 2023

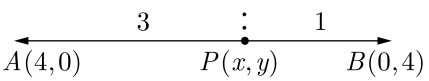
65. A and B are two points on the x-axis and y-axis respectively.



(a) Write down the coordinates of A and B.
(b) P is a point on AB such that $AP:PB=3:1$. Using section formula find the coordinates of point P.
(c) Find the equation of a line passing through P and perpendicular to AB.

Sol :

(a) From the graph, coordinates of A and B are (4, 0) and (0, 4) respectively.
(b) Using section formula,



$$P(x,y)=\left(\frac{3\times0+1\times4}{3+1},\frac{3\times4+1\times0}{3+1}\right)$$

$$=\left(\frac{4}{4},\frac{12}{4}\right)=(1,3)$$

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Thus $\angle PAX$ is also $45°$ and slope of AB

$$m_1 = \tan 45° = 1$$

Equation of line PB, having slope 1 and passing through point $P(6,2)$,

$$y - y_1 = m(x - x)$$
$$y - 2 = 1(x - 6)$$
$$y - 2 = x - 6$$
$$y = x - 4$$
$$x - y - 4 = 0 \qquad L_1 : \ PB$$

Since, A is on x-axis. Let A be $(a, 0)$. Similar and B is on y-axis. Let B be $(0, b)$. Both point must satisfy the equation of line PB.

Substituting $(a, 0)$ in equation of line PB we have

$$a - 0 - 4 = 0$$
$$a = 4$$

Thus point is $A(4, 0)$

Substituting $(0, b)$ in equation of line PB we have

$$0 - b - 4 = 0$$
$$b = -4$$

Thus point is $B(0, -4)$

Main 2008

77. Points A and B have coordinates $(7, -3)$ and $(1,9)$, respectively. Find

(i) the slope of AB.

(ii) the equation of the perpendicular bisector of the line segment AB.

(iii) the value of p, if $(-2, p)$ lies on it.

Sol :

Given, coordinates of A and B are $(7, -3)$ and $(1,9)$.

(i) Slope of AB.

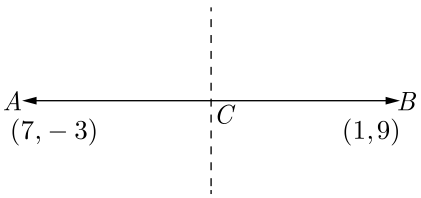
Slope m_1 of line AB,

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{9 + 3}{1 - 7}$$
$$= \frac{12}{-6} = -2$$

(ii) Equation of the perpendicular bisector of the line segment AB

Mid-point of AB using mid point formula

$$C(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= C\left(\frac{7 + 1}{2}, \frac{-3 + 9}{2}\right)$$
$$= C\left(\frac{8}{2}, \frac{6}{2}\right)$$
$$= C(4,3)$$



Let m_2 be the slope of line perpendicular to AB. If two lines are perpendicular then

$$m_1 m_2 = -1$$

Here $$-2 \times m_2 = -1$$

$$m_2 = \frac{-1}{-2} = \frac{1}{2}$$

Now, equation of perpendicular bisector, which is passing through $(4,3)$ and having slope $\frac{1}{2}$, is

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \tfrac{1}{2}(x - 4)$$
$$2(y - 3) = x - 4$$
$$2y - 6 = x - 4$$
$$x - 2y + 2 = 0$$

(iii) Value of p, if $(-2, p)$ lies on it.

If point $(-2, p)$ lies on the line $x - 2y + 2 = 0$, then we get

$$(-2) - 2(p) + 2 = 0$$
$$-2 - 2p + 2 = 0$$
$$-2p = 0 \Rightarrow p = 0$$

Hence, the value of p is 0.

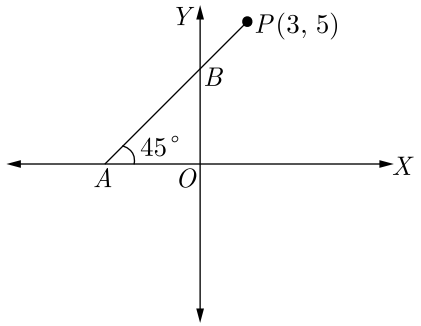
NODIA

Comp 2008

78. From the given figure write the

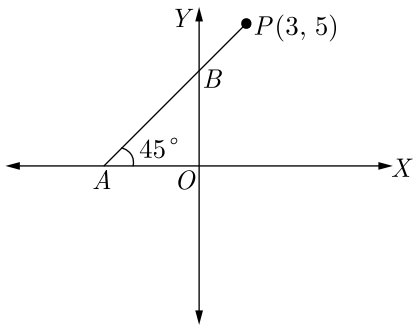
(i) equation of the line PAB.

(ii) co-ordinates of A and B.



Sol :

As per given question we have redrawn figure below.



Slope of line AP $\quad m_1 = \tan 45^\circ = 1$

Equation of line PBA, having slope 1 and passing through point $P(3,5)$,

$$y - y_1 = m(x - x)$$
$$y - 5 = 1(x - 3)$$
$$y - 5 = x - 3$$
$$y - 2 = x$$
$$x - y + 2 = 0 \qquad L : PBA$$

Since, A is on x-axis. Let A be $(a, 0)$. Similar and B is on y-axis. Let B be $(0, b)$. Both point must satisfy the equation of line PBA.

Substituting $(a, 0)$ in equation of line PBA we have

$$a - 0 + 2 = 0$$
$$a = -2$$

Thus point is $A(-2, 0)$

Substituting $(0, b)$ in equation of line PBA we have

$$0 - b + 2 = 0$$
$$b = 2$$

Thus point is $B(0, 2)$.

(i) Equation of the line PAB is $x - y + 2 = 0$

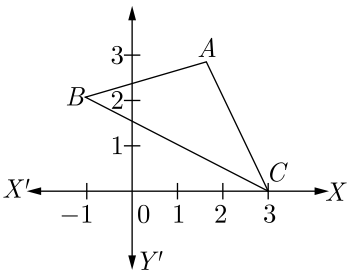
(ii) Co-ordinates of A and B are $A(-2, 0)$ and $B(0, 2)$.

Main 2005

79. In the adjoining figure, write

(i) the coordinates of A, B and C.

(ii) the equation of the line through A and perpendicular to BC.



Sol :

(i) Coordinates of A, B and C.

From the given graph, the coordinates of A, B and C are $(2,3)$, $(-1,2)$ and $(3,0)$.

(ii) Equation of the line through A and perpendicular to BC.

Slope m_1 of line BC,

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - 2}{3 + 1}$$
$$= \frac{-2}{4} = -\frac{1}{2}$$

Let m_2 be the slope of line perpendicular to BC , then

$$m_1 m_2 = -1$$
$$\frac{-1}{2} \times m_2 = -1$$
$$m_2 = 2$$

Equation of line, passing through $A(2,3)$ and having slope $m_2 = 2$, is

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 2(x - 2)$$
$$y - 3 = 2x - 4$$
$$y - 2x + 1 = 0$$

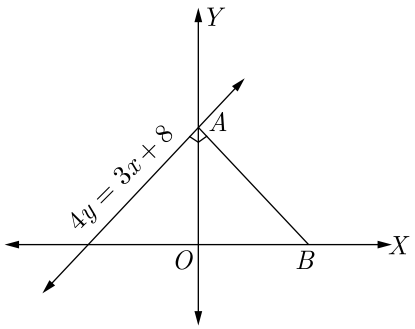
Comp 2005

80. In the given figure line $4y = 3x + 8$ meets y-axis at A. Find

(i) the co-ordinates of A

(ii) the equation of a line AB perpendicular to $4y = 3x + 8$ and passing through A.

(iii) the co-ordinates of B.



Sol :

We have $\quad 4y = 3x + 8 \qquad L_1$

Above line meet at y-axis at A. At y-axis, x coordinates is always zero. Thus let point A be $(0, a)$. This point A must satisfy the line. Thus substituting point A in line L_1 we have

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$$4a = 0 + 8$$

$$a = 2$$

Thus point is $A(0,\ 2)$.

Rewriting line L_1 in $y = mx + c$ form we have

$$y = \frac{3}{4}x + \frac{8}{4}$$

Slope of line L_1, $\quad m_1 = \frac{3}{4}$

Slope of any line perpendicular to L_1,

$$m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$$

Equation of line BC, having slope $-\frac{4}{3}$ and passing through point $A(0,\ 2)$,

$$y - y_1 = m(x - x)$$

$$y - 2 = \frac{-4}{3}(x - 0)$$

$$3y - 6 = -4x$$

$$4x + 3y - 6 = 0 \qquad L_2$$

Point B lie on x-axis and at x-axis, y coordinates is always zero. Thus let point B be $(b,\ 0)$. This point B also lie on line AB. Thus it must satisfy the equation of line. Thus substituting point B in equation of line AB we have

$$4b + 0 - 6 = 0$$

$$b = \tfrac{6}{4} = 1.5.$$

This point is $B(1.5, 0)$.

(i) Co-ordinates of A are $A(0,\ 2)$

(ii) Equation of a line is $4x + 3y - 6 = 0$

(iii) Co-ordinates of B are $B(1.5, 0)$.

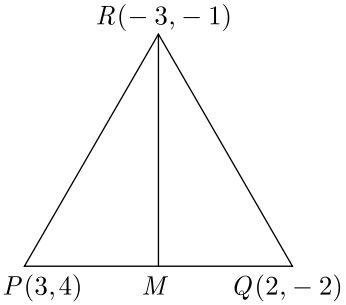
Main 2004

81. $P(3,4)$, $Q(7, -2)$ and $R(-2, -1)$ are the vertices of ΔPQR. Write down the equation of the median of the triangle through R.

Sol :

The vertices's of ΔPQR are P (3, 4), Q (7, –2) and R (–2, –1).

Let M be the mid-point of PQ, then RM is the median through R.



Coordinates of M are $\left(\frac{3+7}{2}, \frac{4+(-2)}{2}\right)$ i.e. $(5, 1)$

Slope of RM $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{1 - (-1)}{5 - (-2)} = \frac{2}{7}$$

The equation of the line RM having slope $\frac{2}{7}$ is

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{2}{7}(x - (-2))$$

$$7y + 7 = 2x + 4$$

$$2x - 7y - 3 = 0,$$

which is the equation of the median through R.

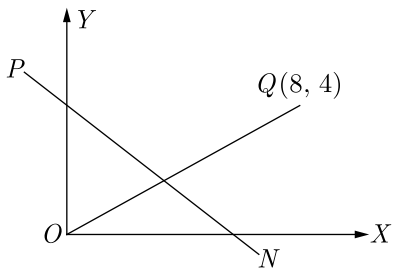
Comp 2004

82. In the given figure find

(i) the coordinates of point P where line $2x + y = 6$ meets y-axis.

(ii) the equation of line OQ where O is the origin and $Q(8,4)$

(iii) State with reason. if the two line are perpendicular to each other.



Sol :

We have $\qquad 2x + y = 6 \qquad L_1$

Writing above equation in $y = mx + c$ form,

$$y = -2x + 6$$

In above equation we can easily see that y intercept of above line is 6. Thus y coordinate of point P must be 6 and point will be $P(0,\ 6)$. We can get this point in another way as follows :

Above line meet at y-axis at P. At y-axis, x coordinates is always zero. Thus let point P be $(0,\ p)$. This point P must satisfy the line. Thus substituting point P in line L_1 we have

$$0 + p = 6$$

$$p = 6$$

Thus point is $P(0,\ 6)$.

Equation of line OQ, passing through point $O(0, 0)$ and $Q(8,\ 4)$ we have

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

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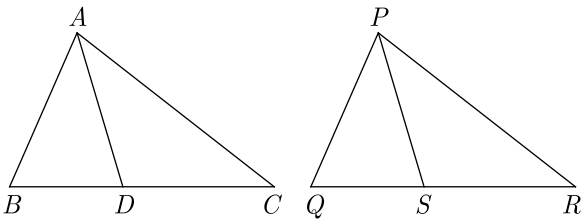
CHAPTER 14

SIMILARITY OF TRIANGLES

MULTIPLE CHOICE QUESTIONS

Main 2024

1. In the given diagram, $\triangle ABC \sim \triangle PQR$. IF AD and PS are bisectors of $\angle BAC$ and $\angle QPR$ respectively then:



(a) $\triangle ABC \sim \triangle PQS$ (b) $\triangle ABD \sim \triangle PQS$
(c) $\triangle ABD \sim \triangle PSR$ (d) $\triangle ABC \sim \triangle PSR$

Sol :

Here $\triangle ABC \sim \triangle PQR$, thus

$$\angle A = \angle P$$

Then $\frac{1}{2}\angle A = \frac{1}{2}\angle P$

Thus $\angle BAD = \angle QPS$

and $\angle B = \angle Q$

Now in $\triangle ABD$ and $\triangle PQS$,

Thus $\angle BAD = \angle QPS$

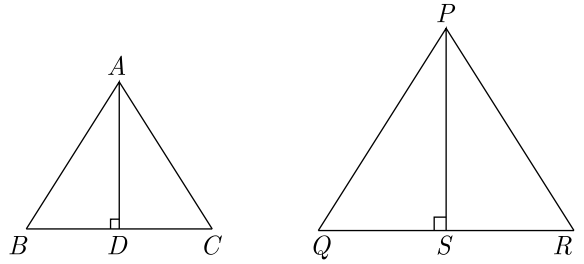
and $\angle B = \angle Q$

By AA similarity criterion we get

$$\triangle ABD \sim \triangle PQS$$

SQP 2024

2. In the given diagram, $\triangle ABC \sim \triangle PQR$ and $\frac{AD}{PS} = \frac{3}{8}$. The value of $AB : PQ$ is:



(a) 8 : 3 (b) 3 : 5
(c) 3 : 8 (d) 5 : 8

Sol :

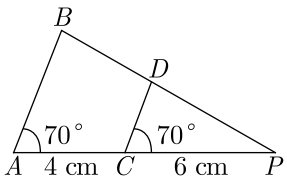
If $\triangle ABC \sim \triangle PQR$, then

$$\frac{AD}{PS} = \frac{AB}{PQ} = \frac{AC}{PR} = \frac{3}{8}.$$

This $AB : PQ$ is also 3 : 8
Thus (c) is correct option.

Main 2023

3. In the given figure $\angle BAP = \angle DCP = 70°$, $PC = 6$ cm and $CA = 4$ cm, then $PD : DB$ is :



(a) 5 : 3 (b) 3 : 5
(c) 3 : 2 (d) 2 : 3

Sol :

In $\angle PDC$ and ΔPBA,

$$\angle DCP = \angle BAP = 70°$$

Due to common angle

$$\angle DPC = \angle BPA$$

By AA similarity criterion we get

$$\Delta PDC \sim \Delta PBA$$

Corresponding sides of similar triangle are proportional,

Thus $$\frac{PC}{PA} = \frac{PD}{PB}$$

$$\frac{PC}{AC + CP} = \frac{PD}{PD + DB}$$

$$\frac{6}{4+6} = \frac{PD}{PD + DB}$$

$$6(PD + DB) = 10PD$$

$$6DB = 4PD$$

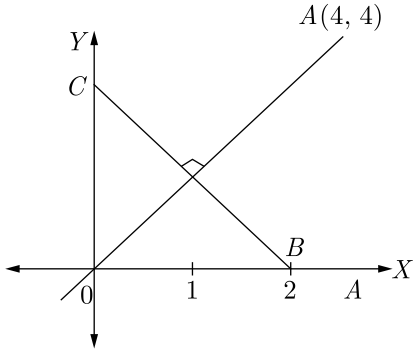
$$\frac{PD}{DB} = \frac{6}{4} = \frac{3}{2}$$

Thus (c) is correct option.

NODIA

Comp 2002

85. From the given figure write the
(i) slope and equation of OA.
(ii) co-ordinates of B.
(iii) equation of $BC \perp$ to OA.
(iv) the co-ordinates of C.



Sol :

Slope of OA passing through point $O(0,\ 0)$ and $A(4,\ 4)$ is

$$m_1 = \frac{4-0}{4-0} = 1$$

Equation of line OA, having slope 1 and passing through point $O(0,0)$,

$$y - y_1 = m(x - x)$$
$$y - 0 = 1(x - 0)$$
$$y = x$$
$$x - y = 0 \qquad L_1 : OA$$

From given figure we can easily see that point B is $B(2,0)$.
Slope of any line perpendicular to OA,

$$m_2 = -\frac{1}{m_1} = -\frac{1}{1} = -1$$

Equation of line BC, having slope -1 and passing through point $B(2,0)$,

$$y - y_1 = m(x - x)$$
$$y - 0 = -1(x - 2)$$
$$y = -x + 2$$
$$x + y - 2 = 0 \qquad L_1 : BC$$

Point C lie on the y axis and this is y intercept of line BC. From equation of line BC $y = -x + 2$ we see that y intercept is 2. Thus point C will be $(0,2)$
(i) Slope of OA is 1 and equation of line is $y = x$
(ii) Co-ordinates of B are $B(2,0)$
(iii) Equation of BC is $x + y - 2 = 0$
(iv) Co-ordinates of C are $(0,2)$.

Main 2002

86. Find the equation of a line passing through the point $(-2,3)$ and having the x-intercept 4 units.

Sol :

Since line is having x-intercept 4 units, i.e it pass through point (4, 0) because $y-$ coordinate at $x-$ is zero.
Now equation of line passing through $(-2,3)$ and (4, 0) will be

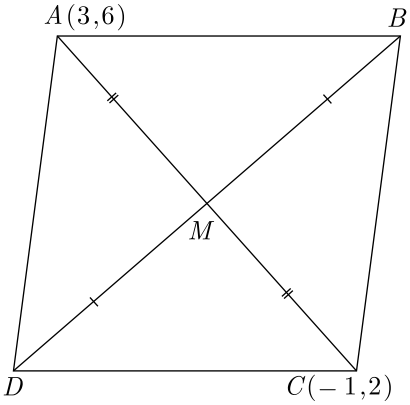
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 3 = \frac{0-3}{4+2}(x+2)$$
$$y - 3 = \frac{-3}{6}(x+2)$$
$$y - 3 = -\frac{1}{2}(x+2)$$
$$2y - 6 = -x - 2$$
$$x + 2y = 4$$

Main 2000

87. $ABCD$ is a rhombus. The coordinates of the points A and C are $(3,6)$ and $(-1,2)$ respectively. Write down the equation of the diagonal BD.

Sol :

Diagram of rhombus bisect each other at 90° is given below.



So, M = Mid-point of AC

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left[\frac{3 + (-1)}{2}, \frac{6+2}{2}\right]$$
$$= (1,4)$$

Now, the diagonal BD is a line passing through $(1,4)$ and perpendicular to AC.
The slope m_1 of AC

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$$y-0=\frac{4-0}{8-0}(x-0)$$

$$y=\frac{4}{8}x$$

$$y=\frac{1}{2}x$$

$$2y=x$$

$$x-2y=0 \qquad L_2:\ OQ$$

Slope of line L_1, $m_1=-2$

Slope of line L_2, $m_2=\frac{1}{2}$

Now $m_1\times m_2=-2\times\frac{1}{2}=-1$

Since $m_1\times m_2=-1$, line L_1 and line L_2 are perpendicular.

(i) Coordinates of point P where line $2x+y=6$ meets y-axis are $(0,6)$

(ii) Equation of line OQ is $x-2y=0$

(iii) The two line are perpendicular to each other.

Main 2003

83. Find the value of k for which the lines $kx-5y+4=0$ and $4x-2y+5=0$ are perpendicular to each other.

Sol :

We have $4x-2y+5=0$ .....(1)

Converting into the form $y=mx+c$ we have

$$2y=4x+5$$

$$y=2x+\frac{5}{2}$$

The slope of the line (1) is $m_1=2$.

Now $kx-5y+4=0$ .....(2)

Converting into the form $y=mx+c$ we have

$$5y=kx+4$$

$$y=\frac{k}{5}x+\frac{4}{5}$$

The slope of the line (2) is $m_2=\frac{k}{5}$.

Since the given lines are perpendicular to each other,

$$m_1m_2=-1$$

$$(2)\cdot\left(\frac{k}{5}\right)=-1$$

$$2k=-5$$

$$k=-\frac{5}{2}$$

Hence, the value of k is $-\frac{5}{2}$.

Main 2002

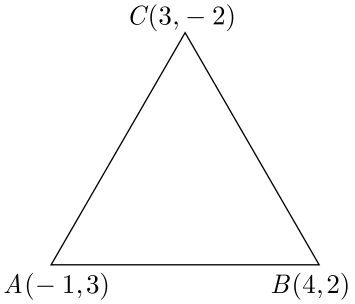
84. $A(1,4)$, $B(3,2)$ and $C(7,5)$ are the vertices's of a ΔABC. Find

(i) the coordinates of the centroid G of ΔABC.

(ii) the equation of a line through G and parallel to AB.

Sol :

Given $A(1,4)$, $B(3,2)$ and $C(7,5)$ are the vertices's of a ΔABC.



(i) Coordinates of the centroid G of the triangle.

Coordinates of centroid of a triangle, whose vertices's are (x_1,y_1), (x_2,y_2) and (x_3,y_3), is given by

$$G(x,y)=G\left(\frac{x_1+x_2+x_3}{2},\frac{y_1+y_2+y_3}{2}\right)$$

Coordinates of centroid G of ΔABC are

$$G(x,y)=G\left(\frac{1+3+7}{3},\frac{4+2+5}{3}\right)$$

$$=\left(\frac{11}{3},\frac{11}{3}\right)$$

(ii) Equation of the line through G and parallel to AC.

Slope m_1 of line AB,

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{2-4}{3-1}$$

$$=-1$$

Slope of parallel lines are equal, thus slope of line parallel to AC will be $\frac{1}{6}$.

Now, equation of the line through G and parallel to AC,

$$y-y_1=m(x-x_1)$$

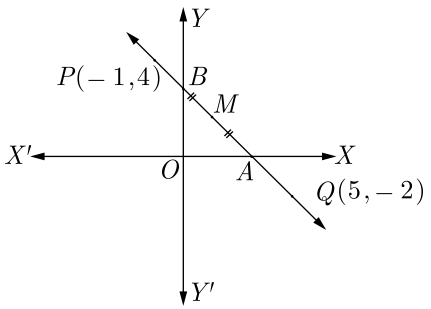
$$y-\frac{11}{3}=-1\left(x-\frac{11}{3}\right)$$

$$y-\frac{11}{3}=-x+\frac{11}{3}$$

$$3y-11=-3x+11$$

$$3x+3y=22$$

NODIA



The slope of the line PQ

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2-4}{3-(-1)} = -1$$

The line passes through $P(-1,4)$ and has slope –1.

Its equation is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x - (-1))$$

$$x + y - 3 = 0.$$

(ii) Coordinates of A and B.

The line PQ intersects x-axis at A, submitting $y = 0$ in equation of line we have

$$x + 0 - 3 = 0$$

$$x = 3$$

The coordinates of A are (3, 0)

The line PQ intersects y-axis at B, submitting $x = 0$ in equation of line we have

$$0 + y - 3 = 0$$

$$y = 3$$

The coordinates of B are (0, 3)

(iii) Coordinates of M.

Since M is mid-point of the segment AB, its coordinates are

$$\left(\frac{3+0}{2}, \frac{0+3}{2}\right) \text{ i.e. } \left(\frac{3}{2}, \frac{3}{2}\right)$$

END

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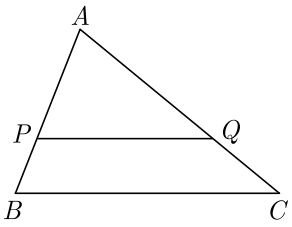
$$QR = 6 \text{ cm}$$

Thus (d) is correct option.

16. In the given figure, P and Q are points on the sides AB and AC respectively of a triangle ABC . PQ is parallel to BC and divides the triangle ABC into 2 parts, equal in area. The ratio of $PA : AB =$



(a) $1 : 1$ (b) $(\sqrt{2}-1):\sqrt{2}$
(c) $1:\sqrt{2}$ (d) $(\sqrt{2}-1):1$

Sol :

As PQ is parallel to BC

$$\Delta ABC \sim \Delta APQ$$
$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta APQ)} = \left(\frac{AB}{AP}\right)^2$$
$$\frac{2}{1} = \left(\frac{AB}{AP}\right)^2$$
$$\frac{\sqrt{2}}{1} = \frac{AB}{AP}$$
$$AP : AB = 1 : \sqrt{2}$$

Thus (c) is correct option.

17. It is given that $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{4}$. Then $\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)}$ is equal to

(a) 16 (b) 3
(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

Sol :

Since, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)} = \frac{AR^2}{AC^2}$$
$$= \frac{QR^2}{BC^2} = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$$

Thus (a) is correct option.

18. If $\Delta ABC \sim \Delta APQ$ and $\text{ar}(\Delta APQ) = 4\text{ar}(\Delta ABC)$, then the ratio of BC to PQ is

(a) $2 : 1$ (b) $1 : 2$
(c) $1 : 4$ (d) $4 : 1$

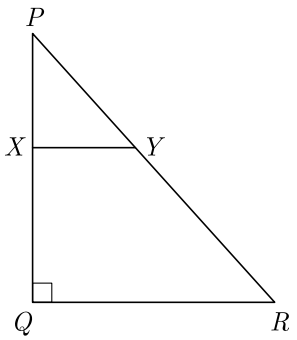
Sol :

Since, $\Delta ABC \sim \Delta APQ$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta APQ)} = \frac{BC^2}{PQ^2}$$
$$\frac{\text{ar}(\Delta ABC)}{4\text{ar}(\Delta ABC)} = \frac{BC^2}{PQ^2}$$
$$\frac{1}{4} = \frac{BC^2}{PQ} \Rightarrow \frac{BC}{PQ} = \frac{1}{2}$$

Thus (b) is correct option.

19. In the given figure, PQR is a triangle right angled at Q and $XY || QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1:2$. The length of PR will be



(a) 12 cm (b) $2\sqrt{3}$ cm
(c) $6\sqrt{3}$ cm (d) 18 cm

Sol :

Since $XY \parallel OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$
$$\frac{1}{2} = \frac{PY}{PR - PY}$$
$$= \frac{4}{PR - 4}$$
$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

Thus (a) is correct option.

20. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is

(a) 5.4 cm (b) 7.8 cm
(c) 2.7 cm (d) 3.9 cm

Sol :

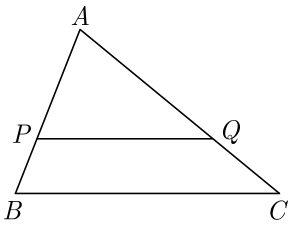
Ratio of the perimeter of two similar triangles is equal to the ratio of corresponding sides.

Thus $$\frac{25}{15} = \frac{9}{\text{side}}$$

NODIA

Comp 2023

4. In the given figure, P and Q are points on the sides AB and AC respectively of a triangle ABC . PQ is parallel to BC and divides the triangle ABC into 2 parts, equal in area. What is the ratio of $AP : AB$?



(a) $2 : 1$ (b) $1 : \sqrt{2}$
(c) $\sqrt{2} : 1$ (d) $1 : 2$

Sol :

As PQ is parallel to BC

$$\Delta ABC \sim \Delta APQ$$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta APQ)} = \left(\frac{AB}{AP}\right)^2$$

$$\frac{2}{1} = \left(\frac{AB}{AP}\right)^2$$

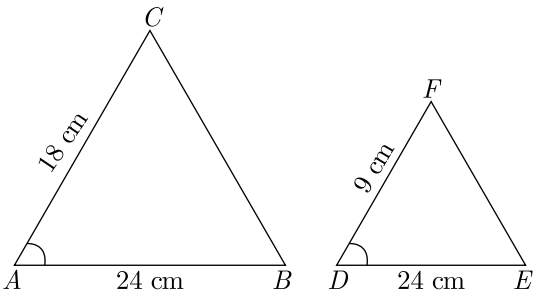
$$\frac{\sqrt{2}}{1} = \frac{AB}{AP}$$

$$AP : AB = 1 : \sqrt{2}$$

Thus (b) is correct option.

Main 2022 Sem I

5. In the given figure $AB = 24$ cm, $AC = 18$ cm, $DE = 12$ cm, $DF = 9$ cm and $\angle BAC = \angle EDF$, Then $\Delta ABC \sim \Delta DEF$ by the condition :



(a) AAA (b) SAS
(c) SSS (d) AAS

Sol :

As we know, if one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles ae similar.
Thus (b) is correct option.

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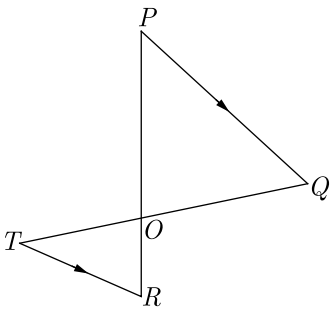
6. In the given figure PQ is parallel to TR, then by using condition of similarity :



(a) $\frac{PQ}{RT} = \frac{OP}{OT} = \frac{OQ}{OR}$

(b) $\frac{PQ}{RT} = \frac{OP}{OR} = \frac{OQ}{OT}$

(c) $\frac{PQ}{RT} = \frac{OR}{OP} = \frac{OQ}{OT}$

(d) $\frac{PQ}{RT} = \frac{OP}{OR} = \frac{OT}{OQ}$

Sol :

As $PQ \,||\, RT$

Due to alternate interior angles we have

$$\angle T = \angle Q,$$

$$\angle R = \angle P$$

By AA similarity criterion we get

$$\Delta PQO \sim \Delta RTO$$

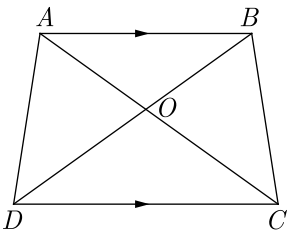
Corresponding sides of similar triangle are proportional,

$$\frac{PQ}{RT} = \frac{OP}{OR} = \frac{OQ}{OT}$$

Thus (b) is correct option.

Main 2022 Sem I

7. If $ABCD$ is a trapezium with AB parallel to DC. Then the triangle similar to ΔAOB is :



(a) ΔADB (b) ΔACB
(c) ΔCOD (d) ΔCOB

Sol :

As we have $AB \,||\, DC$ then,

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Due to alternate interior angles,

$$\angle ABO = \angle CDO$$

$$\angle BAQ = \angle DCO$$

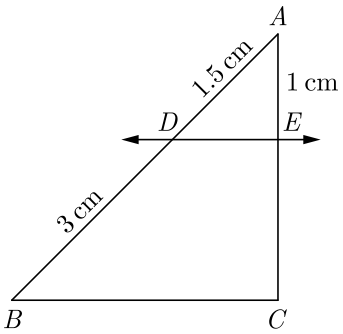
Due to vertically opposite angles,

$$\angle AOB = \angle COD$$

Thus $\Delta AOB \sim \Delta COD$

Thus (c) is correct option.

8. In the given figure, $DE \parallel BC$. The value of EC is



(a) 1.5 cm (b) 3 cm
(c) 2 cm (d) 1 cm

Sol :

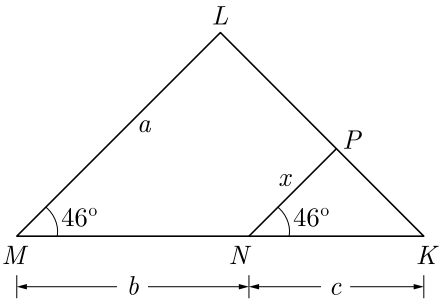
Since, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}$$

Thus (c) is correct option.

9. In the given figure, x is



(a) $\frac{ab}{a+b}$ (b) $\frac{ac}{b+c}$
(c) $\frac{bc}{b+c}$ (d) $\frac{ac}{a+c}$

Sol :

In ΔKPN and ΔKLM, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46^\circ$$

Thus by AA criterion of similarity,

$$\Delta KNP \sim \Delta KML$$

Thus $\frac{KN}{KM} = \frac{NP}{ML}$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

Thus (b) is correct option.

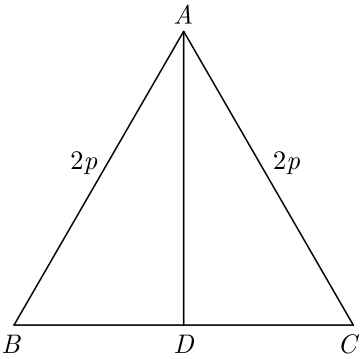
10. ΔABC is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

(a) $\sqrt{3}$ (b) $\sqrt{3}\,p$
(c) $2p$ (d) $4p$

Sol :

We have $AB = BC = CA = 2p$

and $AD \perp BC$



In $\Delta ADB, AB^2 = AD^2 + BD^2$

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}\,p$$

Thus (b) is correct option.

11. Which of the following statement is false?

(a) All isosceles triangles are similar.
(b) All quadrilateral are similar.
(c) All circles are similar.
(d) None of the above

Sol :

Isosceles triangle is a triangle in which two side of equal length. Thus two isosceles triangles may not be similar. Hence statement given in option (a) is false.

Thus (a) is correct option.

12. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is

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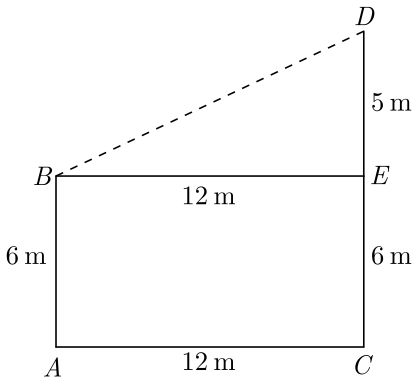
(a) 12 m (b) 14 m
(c) 13 m (d) 11 m

Sol :

Let AB and CD be the vertical poles as shown below.



We have $AB = 6\text{ m}$, $CD = 11\text{ m}$

and $AC = 12\text{ m}$

$$DE = CD - CE$$
$$= (11 - 6)\text{ m} = 5\text{ m}$$

In right angled, ΔBED,

$$BD^2 = BE^2 + DE^2 = 12^2 + 5^2 = 169$$
$$BD = \sqrt{169}\,\text{m} = 13\text{ m}$$

Hence, distance between their tops is 13 m.
Thus (c) is correct option.

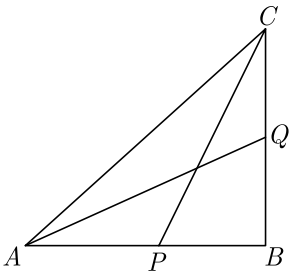
13. In a right angled ΔABC right angled at B, if P and Q are points on the sides AB and BC respectively, then
(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$
(c) $AQ^2 + CP^2 = AC^2 + PQ^2$
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

Sol :

In right angled ΔABQ and ΔCPB,

$$CP^2 = CB^2 + BP^2$$

and $$AQ^2 = AB^2 + BQ^2$$



$$CP^2 + AQ^2 = CB^2 + BP^2 + AB^2 + BQ^2$$
$$= CB^2 + AB^2 + BP^2 + BQ^2$$
$$= AC^2 + PQ^2$$

Thus (c) is correct option.

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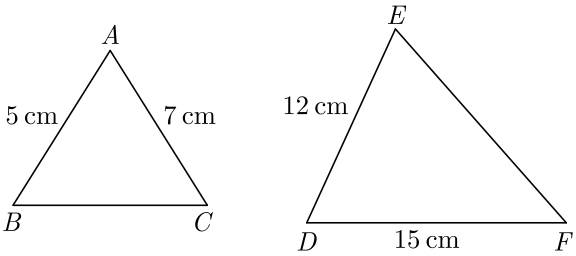
14. It is given that, $\Delta ABC \sim \Delta EDF$ such that $AB = 5\text{ cm}$, $AC = 7$ cm, $DF = 15$ cm and $DE = 12\text{ cm}$ then the sum of the remaining sides of the triangles is
(a) 23.05 cm (b) 16.8 cm
(c) 6.25 cm (d) 24 cm

Sol :

We have $\Delta ABC \sim \Delta EDF$



Now $$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

Taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$
$$= 16.8\text{ cm}$$

Taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$
$$= 6.25\text{ cm}$$

Now, sum of the remaining sides of triangle,

$$EF + BC = 16.8 + 6.25 = 23.05\text{ cm}$$

Thus (a) is correct option.

15. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If $BC = 4.5\text{ cm}$, then the length of QR is
(a) 4 cm (b) 4.5 cm
(c) 3 cm (d) 6 cm

Sol :

Since, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$$
$$\frac{9}{16} = \frac{(4.5)^2}{QR^2}$$
$$QR^2 = \frac{16 \times (4.5)^2}{9}$$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2-6}{-1-3}$$
$$= \frac{-4}{-4} = 1$$

Let m_2 be the slope of BD. Since AC and BD are perpendicular, we have

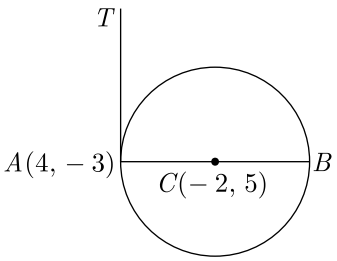
$$m_1 m_2 = -1$$
$$1 \times m_2 = -1$$
$$m_2 = -1$$

So, by the point slope form, the equation of BD is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = (-1)(x-1)$$
$$y - 4 = -x + 1$$
$$x + y = 5$$

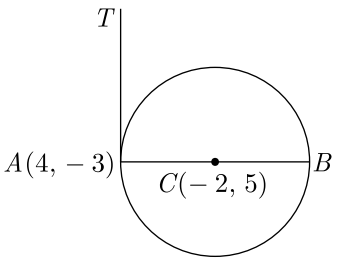
Comp 2000

88. In the given figure AB is diameter of a circle with center $C(2,5)$, If A is $(4,-3)$, find
(i) the coordinates of B
(ii) equation of tangent AT.



Sol :

As per given question we have redrawn figure below.



We have $C = (-2,5)$ and $A(4,-3)$

Let point B be $B(x,y)$. Since centre C is always mid point of diameter AB we have

$$C(-2,5) = \left(\frac{x+4}{2}, \frac{y-3}{2}\right)$$

We get $$\frac{x+4}{2} = -2 \Rightarrow x = -8$$

and $$\frac{y-3}{2} = 5 \Rightarrow y = 13$$

Thus we get point $B(-8,13)$.

Slope of AC, $$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5+3}{-2-4}$$
$$= \frac{8}{-6} = \frac{-4}{3}$$

Here tangent AT is always perpendicular to radius AC. Thus slope of AT,

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-\frac{4}{3}} = \frac{3}{4}$$

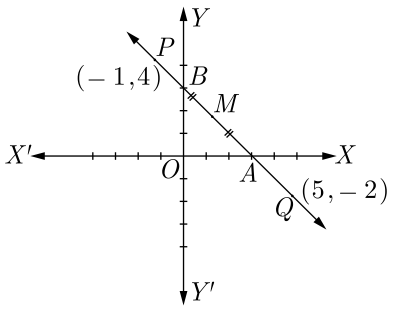
Equation of line, having slope $\frac{3}{4}$ and passing through point $A(4,-3)$,

$$(y - y_1) = m(x - x_1)$$
$$y + 3 = \frac{3}{4}(x-4)$$
$$4y + 12 = 3x - 12$$
$$4y = 3x - 24$$
$$3x - 4y - 24 = 0$$

FIVE MARK QUESTIONS

Main 2003

89. A straight line passes through the point $P(-1,4)$ and $Q(5,-2)$. It intersects the coordinates axes at points A and B. If M is the mid-point of the segment AB, then find
(i) the equation of the line.
(ii) the coordinates of A and B.
(iii) the coordinates of M.



Sol :

(i) Equation of the line
Given points are P (–1, 4) and Q (5, –2).

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$$\text{side} = \frac{9 \times 15}{25} = 5.4 \text{ cm}$$

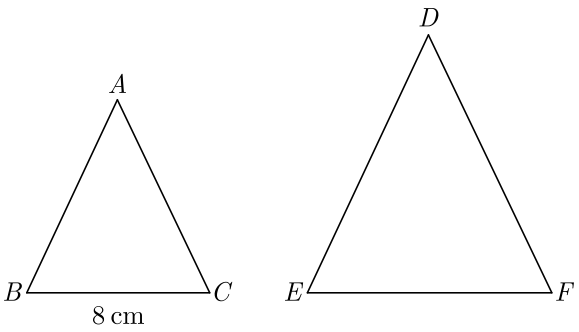
Thus (a) is correct option.

21. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8$ cm then find EF.

(a) 8 cm (b) 10 cm
(c) 16 cm (d) 12 cm

Sol :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

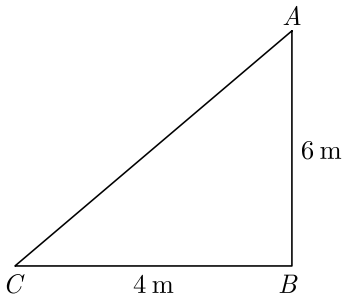
Thus (c) is correct option.

22. A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

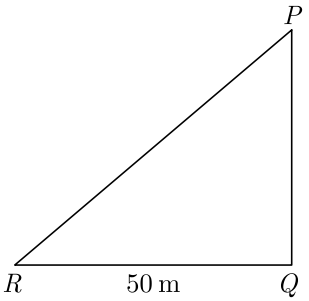
(a) 48 m (b) 150 m
(c) 16 m (d) 75 m

Sol :

Let AB be height of tree and BC its shadow.



Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal i.e $\Delta ABC \sim PQR$



Thus
$$\frac{AB}{BC} = \frac{PQ}{QR}$$

$$\frac{6}{4} = \frac{PQ}{50}$$

$$PQ = \frac{50 \times 6}{4}$$

$$= 75 \text{ m}$$

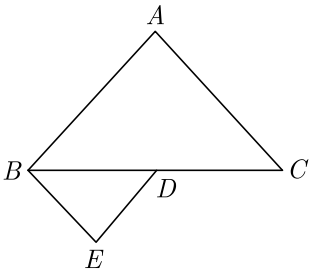
Thus (d) is correct option.

23. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(a) 4 : 1 (b) 1 : 4
(c) 2 : 1 (d) 1 : 2

Sol :

From the given information we have drawn the figure as below.



$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2}$$

$$= \frac{(BC)^2}{\left(\frac{1}{2}BC\right)^2}$$

$$= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

Thus (a) is correct option.

24. In $\Delta ABC, DE \| BC$, what is the value of x ?

NODIA

Area of ΔPST, $\Delta_{PST} = \frac{1}{2} \times PS \times TS$

$= \frac{1}{2} \times 2 \times 2k = 2k$

Area of ΔPQR, $\Delta_{PQR} = \frac{1}{2} PQ \times QR$

$= \frac{1}{2} \times 5 \times 5k = \frac{25k}{2}$

Now, area of quadrilateral $SRQT$

$$\square_{SRQT} = \Delta_{PQR} - \Delta_{PST} \quad \Delta_{PST}$$
$$= \frac{25}{2}k - 2k = \frac{21k}{2}$$

Area of ΔPQR : Area of quadrilateral $SRQT$

$$= \frac{\frac{25}{2}k}{\frac{21}{2}k} = \frac{25}{21}$$

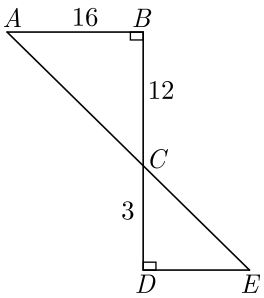
Comp 2019

41. In given figure AB and ED are perpendiculars to BD. AE meets BD at C. If $AB = 16$ cm, $BC = 12$ cm and $CD = 3$ cm :

(i) Show that $\Delta ABC \sim \Delta EDC$

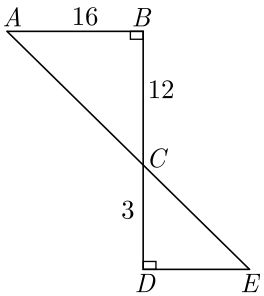
(ii) Find the lengths of DE and CE.

(iii) Find area ΔABC : area ΔEDC.



Sol :

We have redrawn the given figure as below.



(i) $\Delta ABC \sim \Delta EDC$

In ΔABC and ΔEDC,

Due to right angle,

$$\angle B = \angle D$$

Due to vertical opposite angle

$$\angle ACB = \angle ECD$$

By AA-axiom similarity we have

$$\Delta ABC \sim \Delta EDC$$

(ii) Lengths of DE and CE.

Now $AC = \sqrt{12^2 + 16^2}$

$= 20$ cm

If two triangles are similar, then their corresponding sides are proportional.

$$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$$

Substituting $AB = 16$ cm, $BC = 12$ cm and $CD = 3$ cm we have

$$\frac{16}{ED} = \frac{12}{3} = \frac{20}{EC}$$

From above equation we get

$$ED = \frac{16 \times 3}{12} = 4 \text{ cm}$$

$$EC = \frac{20 \times 3}{12} = -5 \text{ cm}$$

(iii) Area ΔABC : Area ΔEDC.

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area } \Delta ABC}{\text{Area } \Delta EDC} = \left(\frac{BC}{DC}\right)^2 = \left(\frac{12}{3}\right)^2$$
$$= \frac{16}{1} = 16:1$$

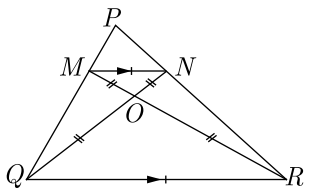
Main 2018

42. In ΔPQR, MN is parallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$.

(i) Find $\frac{MN}{QR}$.

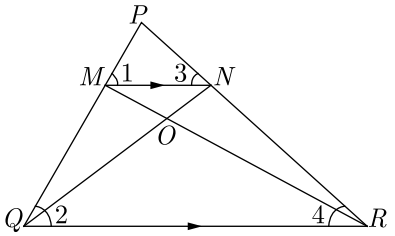
(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find-area of ΔOMN ∶ area of ΔORQ.



Sol :

We have redrawn the figure below.



(i) $\frac{MN}{QR}$

Since $MN \parallel QR$, due to corresponding angles

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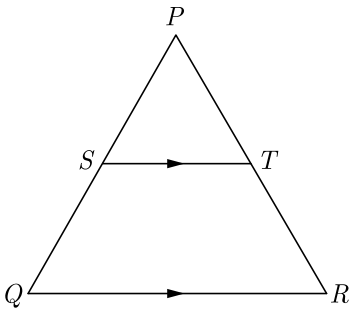


(a) 12 cm (b) 21 cm
(c) 14 cm (d) 10.5 cm

Sol :

We have $\frac{PS}{SQ} = \frac{3}{5}$

$$\frac{PS}{PS+SQ} = \frac{3}{3+5}$$

$$\frac{PS}{PQ} = \frac{3}{8}$$

We also have, $ST \| QR$, thus by BPT we get

$$\frac{PS}{PQ} = \frac{PT}{PR}$$

$$PT = \frac{PS}{PQ} \times PR = \frac{3 \times 28}{8}$$

$$= 10.5 \text{ cm}$$

Thus (d) is correct option.

30. The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratio of the areas of triangles is

(a) 4 : 3 (b) 3 : 4
(c) 9 : 16 (d) 16 : 9

Sol :

Ratio of the square of 2 similar triangles is equal to the square of the ratio of corresponding sides.

$$\frac{\text{Area of triangle 1}}{\text{Area of triangle 2}} = \frac{(\text{corresponding side of triangle 1})^2}{(\text{corresponding side of triangle 2})^2}$$

$$= \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Thus (c) is correct option.

31. The perimeters of two similar triangles ΔABC and ΔPQR are 35 cm and 45 cm respectively, then the ratio of the areas of the two triangles is

(a) 49 : 81 (b) 81 : 49
(c) 9 : 7 (d) 7 : 9

Sol :

Ratio of areas of two similar triangle is equal to the ratio of the squares of their corresponding sides.

Ratio of the perimeter of two similar triangle is the same as the ratio of their corresponding sides. Therefore in result ratio of areas of two similar triangle is equal to the ratio of the squares of their perimeters.

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{35^2}{45^2} = \frac{7^2}{9^2} = \frac{49}{81}$$

Thus (a) is correct option.

32. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If $BC = 4.5$ cm, then the length of QR will be

(a) 8 cm (b) 10 cm
(c) 6 cm (d) 12 cm

Sol :

Since, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\frac{9}{16} = \frac{(4.5)^2}{QR^2}$$

$$QR^2 = \frac{16 \times (4.5)^2}{9}$$

$$QR = 6 \text{ cm}$$

Thus (c) is correct option.

33. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then what is the ratio $\frac{\text{ar}\,\Delta ABC}{\text{ar}\,\Delta PQR}$?

(a) 9 : 1 (b) 1 : 9
(c) 3 : 1 (d) 1 : 3

Sol :

We have $\frac{AB}{PQ} = \frac{1}{3}$

If $\Delta ABC \sim \Delta PQR$, then we have

$$\frac{\text{ar}\,\Delta ABC}{\text{ar}\,\Delta PQR} = \left(\frac{AB}{PQ}\right)^2$$

$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Thus (b) is correct option.

34. If ratio of corresponding sides of two similar triangles is 5:6, then ratio of their areas will be

(a) 25 : 36 (b) 5 : 6
(c) 36 : 25 (d) 6 : 5

Sol :

Let the triangles be ΔABC and ΔDEF

NODIA

(ii) $\Delta APB \sim \Delta ABC$

In ΔAPB and ΔABC,

Due to common angle,

$$\angle A = \angle A$$

Due to right angle,

$$\angle APB = \angle ABC = 90^\circ$$

By AA-axiom similarity we have

$$\Delta APB \sim \Delta ABC$$

(iii) Lengths of BP and AP.

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{AP}{AB} = \frac{PB}{BC} = \frac{AB}{AC}$$

$$\frac{AP}{9} = \frac{PB}{12} = \frac{9}{15}$$

$$AP = \frac{9 \times 9}{15} = \frac{27}{5} = 5.4 \text{ cm}$$

and $$PB = \frac{9}{15} \times 12 = \frac{36}{5} = 7.2 \text{ cm}$$

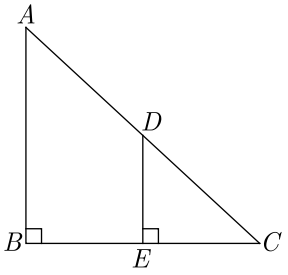
Main 2013

48. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\Delta ABC \sim \Delta DEC$.

(ii) If $AB = 6$ cm, $DE = 4$ cm and $AC = 15$ cm, then calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC.



Sol :

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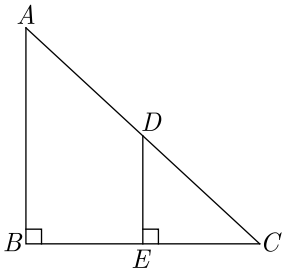


Given $AB \perp BC$ and $DE \perp BC$

(i) $\Delta ABC \sim \Delta DEC$

In ΔABC and ΔDEC due to $AB \perp BC$ and $DE \perp BC$ we have

$$\angle ABC = \angle DEC = 90^\circ$$

Due to common angles,

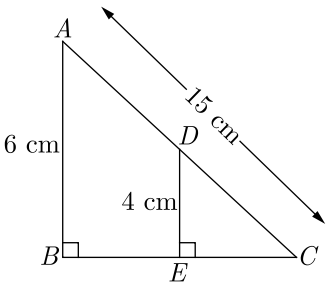
$$\angle ACB = \angle DCE$$

By AA-axiom similarity we have

$$\Delta ABC \sim \Delta DEC \quad \text{Hence proved.}$$

(ii) CD.

We have redrawn the figure below.



Corresponding sides of similar triangle are proportional,

Here $\Delta ABC \sim \Delta DEC$ so we have

$$\frac{AB}{DE} = \frac{AC}{DC}$$

Substituting $AB = 6$ cm, $DE = 4$ cm and $AC = 15$ cm we have

$$\frac{6}{4} = \frac{15}{DC}$$

$$DC = \frac{4 \times 15}{6}$$

$$= 10 \text{ cm}$$

(iii) Ratio of the area of ΔABC : area of ΔDEC.

Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Here $\Delta ABC \sim \Delta DEC$ so we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DEC)} = \frac{(AB)^2}{(DE)^2}$$

$$= \frac{(6)^2}{(4)^2} = \frac{36}{16} = \frac{9}{4}$$

Thus ratio is $9 : 4$.

Main 2012

49. In the given figure, ΔABC and ΔAMP are right angled at B and M respectively. Given, $AC = 10$ cm, $AP = 15$ cm and $PM = 12$ cm.

(i) Prove that $\Delta ABC \sim \Delta AMP$.

(ii) Find AB and BC.

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m^2. Find the floor area of that in the model.

Sol :

Here, the model of the building is made with scale factor 1 : 50.

$$k = 50$$

(i) Height of the actual building

$$= k \text{ [height of the model]}$$
$$= 50 \times 0.8$$
$$= 40 \text{ m}$$

(ii) Floor area of that in the model

$$\text{Floor area in building} = k^2 \text{ Floor area in model}$$
$$\text{Floor area in model} = \frac{1}{k^2} \text{ Floor area in building}$$
$$= \frac{1}{50^2} \times 20$$
$$= \frac{20}{2500}$$
$$= 0.008 \text{ m}^2$$

Main 2019, 2009

39. The model of a building is constructed with scale factor 1 : 30.

(i) If the height of the model is 80 cm, find the actual height of the building in meters.

(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the of the model.

Sol :

Here, the model of the building is made with scale factor 1 : 30.

$$k = 30$$

(i) Actual height of the building in meters

Height of the model = 80 cm

Actual height of the building

$$= k \times \text{Height of the mode)}$$
$$= 30 \times 80 \text{ cm}$$
$$= 2400 \text{ cm}$$
$$= 24 \text{ m}$$

(ii) Volume of the tank on the top of the model

Volume of actual solid,

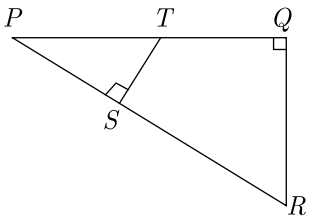
$$= k^3 \times \text{Volume of the model}$$

$$\text{Volume of model} = \frac{1}{k^3} \times \text{Volume of the tank}$$
$$= \frac{1}{30^3} \times 27 \text{ m}^3$$
$$= \frac{27}{27000} \text{ m}^3$$
$$= \frac{1}{1000} \text{ m}^3$$
$$= 0.001 \text{ m}^3$$
$$= 1 \text{ L}$$

Main 2019

40. In the given figure, $\angle PQR = \angle PST = 90^\circ$, $PQ = 5$ cm and $PS = 2$ cm.

NODIA

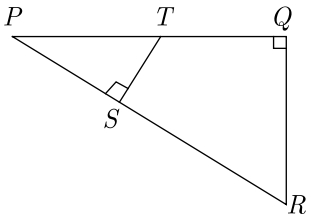


(i) Prove that $\Delta PQR \sim \Delta PST$.

(ii) Find Area of ΔPQR: Area of quadrilateral $SRQT$.

Sol :

We have redrawn the figure below.



Here $\angle PQR = \angle PST = 90^\circ$, $PQ = 5$ cm and $PS = 2$ cm

(i) $\Delta PQR \sim \Delta PST$.

In ΔPQR and ΔPST, given

$$\angle PQR = \angle PST = 90^\circ$$

Due to common angle,

$$\angle QPR = \angle SPT$$

By AA similarity criterion we get

$$\Delta PQR \sim \Delta PST$$

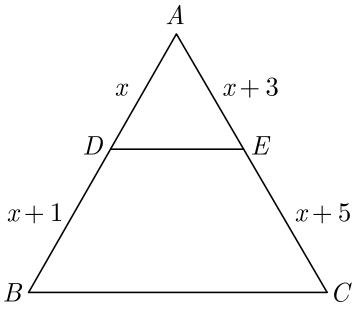
(ii) Area of ΔPQR: Area of quadrilateral $SRQT$.

Corresponding sides of similar triangle are proportional,

Since, $\Delta PQR \sim \Delta PST$, thus

$$\frac{PQ}{PS} = \frac{QR}{TS} = \frac{PR}{PT}$$
$$\frac{5}{2} = \frac{QR}{TS} = \frac{PR}{PT} = k$$ Let

Now $$TS = 2k$$
$$QR = 5k$$



(a) 1 (b) 2
(c) 3 (d) 4

Sol :

In the given figure $DE \| BC$, thus

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

Thus (3) is correct option.

25. It is given that $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{4}$. What is the value of $\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)}$?

(a) 4 : 1 (b) 1 : 4
(c) 16 : 1 (d) 16 : 1

Sol :

Since, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)} = \frac{AR^2}{AC^2}$$

$$= \frac{QR^2}{BC^2} = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$$

Thus (c) is correct option.

26. The areas of two similar triangles are $81\,\text{cm}^2$ and $49\,\text{cm}^2$ respectively. Tatio of their corresponding medians will be

(a) 49 : 81 (b) 81 : 49
(c) 7 : 9 (d) 9 : 7

Sol :

Area of two similar triangles,

$$A_1 = 81\,\text{cm}^2$$

$$A_2 = 49\,\text{cm}^2$$

Ratio of corresponding medians

$$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{81}{49}} = \frac{9}{7}$$

Thus (d) is correct option.

27. Sides of two similar triangles are in the ratio 4 : 9. What is the ratio of areas of these triangles ?

(a) 16 : 81 (b) 81 : 16
(c) 4 : 9 (d) 9 : 4

Sol :

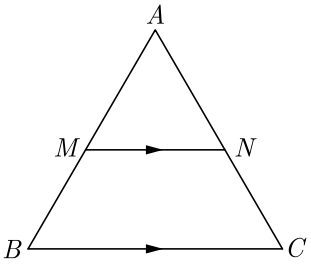
The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Thus $$\frac{A_1}{A_2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

$$A_1 : A_2 = 16 : 81$$

Thus (a) is correct option.

28. In figure, $MN \| BC$ and $AM : MB = 1:2$, then what is $\frac{\text{ar}(\Delta AMN)}{\text{ar}(\Delta ABC)}$?



(a) 9 : 1 (b) 1 : 9
(c) 3 : 1 (d) 1 : 3

Sol :

Here, $AM = k$

$MB = 2k$

$MN \| BC$

Thus $AB = AM + MB$

$= k + 2k = 3k$

$$\frac{\text{ar}(\Delta AMN)}{\text{ar}(\Delta ABC)} = \frac{AM^2}{AB^2} = \frac{k^2}{9k^2} = \frac{1}{9}$$

Thus (b) is correct option.

29. In the given figure, in triangle PQR, $ST \| QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, then PT will be

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$$\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

By AA similarly criterion

$$\Delta PMN \sim \Delta PQR$$

Thus $$\frac{MN}{QR} = \frac{PM}{PQ} \quad \text{...(i)}$$

We have, $$\frac{PM}{MQ} = \frac{2}{3}$$

$$\frac{MQ}{PM} = \frac{3}{2}$$

$$\frac{MQ + PM}{PM} = \frac{3+2}{2}$$

$$\frac{PQ}{PM} = \frac{5}{2}$$

$$\frac{PM}{PQ} = \frac{2}{5} \quad \text{(ii)}$$

Hence, from Eq. (i) and (ii) we get

$$\frac{MN}{QR} = \frac{2}{5}$$

(ii) ΔOMN and ΔORQ are similar

In ΔOMN and ΔORQ,

Due alternate interior angles

$$\angle OMN = \angle ORQ$$

$$\angle ONM = \angle ORQ$$

By AA similarity criteria,

$$\Delta OMN \sim \Delta ORQ$$ Hence Proved

(iii) Area of ΔOMN : area of ΔORQ.

$$\frac{ar(\Delta OMN)}{ar(\Delta ORQ)} = \left(\frac{MN}{QR}\right)^2$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

Thus $ar(\Delta OMN) \cdot ar(\Delta ORQ) = 4 : 25$

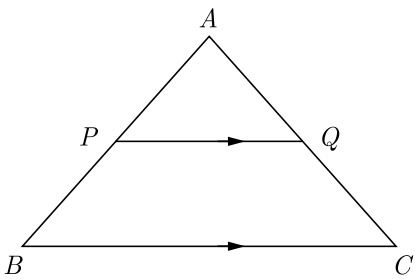
Comp 2018

43. In given triangle ΔABC, line PQ and BC are parallel. If $AP : PB = 2 : 3$, find the :

(i) length of PQ, if $BC = 7.5$ cm.

(ii) area of ΔAPQ : area of ΔABC.

(iii) area of ΔAPQ : area of $PBCQ$.



Sol :

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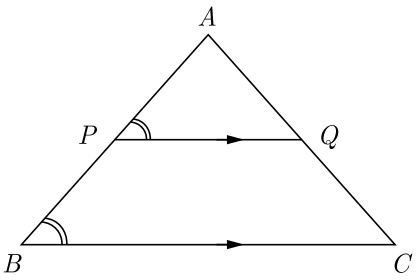

In triangle ΔAPQ and ΔABC,

Due to common angle,

$$\angle A = \angle A$$

Due to corresponding angle,

$$\angle APQ = \angle ABC$$

By AA-axiom similarity we have,

$$\Delta APQ \sim \Delta ABC$$

(i) Length of PQ

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\frac{2}{5} = \frac{PQ}{7.5}$$

$$PQ = \frac{2}{5} \times 7.5 = 3 \text{ cm}$$

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

(ii) Area of ΔAPQ : Area of ΔABC.

$$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta ABC} = \left(\frac{AB}{AB}\right)^2$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

(iii) Area of ΔAPQ : area of $PBCQ$.

$$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta PBCQ} = \frac{\text{Area } \Delta APQ}{\text{Area } \Delta ABC - \text{Area } \Delta APQ}$$

$$= \frac{4}{25 - 4}$$

$$= \frac{4}{21}$$

Main 2017

44. PQR is triangle. S is a point on the side QR of ΔPQR such that $\angle PSR = \angle QPR$. Given $QP = 8$ cm, $PR = 6$ cm and $SR = 3$ cm.

(i) Prove $\Delta PQR \sim \Delta SPR$.

(ii) Find the length of QR and PS.

(iii) $\frac{\Delta PQR}{\Delta SPR}$.

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$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

Thus (a) is correct option.

35. **Assertion :** In the ΔABC, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then ΔABC is a right angle triangle.

Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have, $AB^2 + BC^2 = (24)^2 + (10)^2$

$= 576 + 100$

$= 676$

$= AC^2$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

36. **Assertion :** ABC and DEF are two similar triangles such that $BC = 4$ cm, $EF = 5$ cm and area of $\Delta ABC = 64$ cm^2, then area of $\Delta DEF = 100$ cm^2.

Reason : The areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

Sol :

Reason is true because this is standard result.

Assertion :

Since $\Delta ABC \sim \Delta DEF$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{(4)^2}{(5)^2} = \frac{16}{25}$$

Ratio of areas of two similar triangle is equal to the ratio of the squares of corresponding sides.

$$\frac{64}{\text{area}(\Delta DEF)} = \frac{16}{25}$$

$$\text{area}(\Delta DEF) = \frac{64 \times 25}{16}$$

$$= 4 \times 25 = 100 \text{ cm}^2$$

Assertion is true. But Reason is not correct explanation for Assertion.

Thus (b) is correct option.

37. **Assertion :** $\Delta ABC \sim \Delta DEF$ such that $\text{ar}(\Delta ABC) = 36$ cm^2 and $\text{ar}(\Delta DEF) = 49$ cm^2 then, $AB : DE = 6 : 7$.

Reason : If $\Delta ABC \sim \Delta DEF$, then

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\frac{36}{49} = \frac{AB^2}{DE^2}$$

$$\frac{AB}{DE} = \frac{6}{7}$$

$$AB : DE = 6 : 7$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

THREE MARKS QUESTIONS

Main 2020

38. A model of a high rise building is made to a scale of 1 : 50.

(i) If the height of the model is 0.8 m, find the height of the actual building.

(ii) If the floor area of a flat in the building is 20

$$AB = \frac{5 \times 6}{3}$$

$$= 10 \text{ cm}$$

$$\frac{AC}{AE} = \frac{CD}{EF}$$

$$\frac{2}{5} = \frac{6}{EF}$$

$$EF = \frac{6 \times 5}{2}$$

$$= 15 \text{ cm}$$

(iv) Area ΔECD : Area ΔEAB

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area}(\Delta ECD)}{\text{Area}(\Delta EAB)} = \left(\frac{EC}{EA}\right)^2$$

$$= \left(\frac{3}{2}\right)^2 = \frac{9}{25}$$

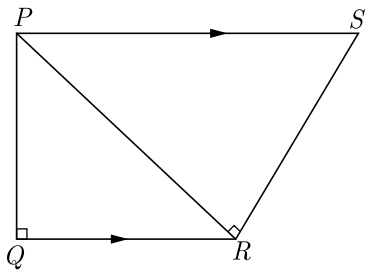
Main 2016

46. In the quadrilateral $PQRS$,
$PS \parallel QR, PQ \perp QR$ and $PR \perp RS$.

(i) $\Delta PQR \sim \Delta SRP$

(ii) If $PQ = 12$ cm, $QR = 9$ cm, find PS and RS.

(iii) Find area of ΔPQR : area of ΔSRP.



Sol :

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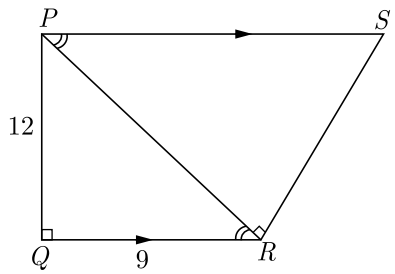


(i) $\Delta PQR \sim \Delta SRP$

In ΔPQR and ΔSRP

Due to right angle,

$$\angle Q = \angle SRP$$

Due to vertical opposite angle

$$\angle QRP = \angle RPS$$

By AA-axiom similarity we have

$$\Delta PQR \sim \Delta SRP$$

(ii) Length of PS and RS.

$$PR = \sqrt{9^2 + 12^2}$$

$$= 15 \text{ cm}$$

If two triangles are similar, then their corresponding sides are proportional.

$$\frac{PQ}{SR} = \frac{QR}{RP} = \frac{PR}{SP}$$

Substituting $PR = RP = 15$, $PQ = 12$ cm, and $QR = 9$ cm we have

$$\frac{12}{SR} = \frac{9}{15} = \frac{15}{SP}$$

$$SR = \frac{12 \times 5}{3} = 20 \text{ cm}$$

$$SP = \frac{15 \times 5}{3} = 25 \text{ cm}$$

(iii) Area of ΔPQR : area of ΔSRP.

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\Delta PQR}{\Delta SRP} = \frac{QR^2}{RP^2}$$

$$= \left(\frac{9}{15}\right)^2 = \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25}$$

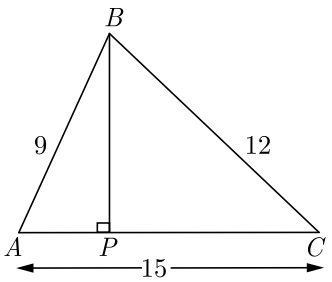
Comp 2016

47. In the given figure, $AB = 9$ cm, $BC = 12$ cm and $AC = 15$ cm. $BP \perp AC$.

(i) What is the measure of $\angle ABC$?

(ii) Prove that $\Delta APB \sim \Delta ABC$

(iii) Find the lengths of BP and AP.



Sol :

(i) Measure of $\angle ABC$?

Here $AB^2 + BC^2 = 9^2 + 12^2$

$$= 15^2$$

$$= AC^2$$

Thus $\angle ABC = 90^\circ$

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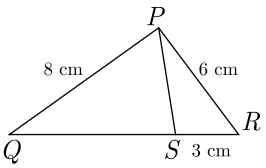

Sol :

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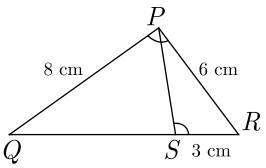


(i) $\Delta PQR \sim \Delta SPR$

In ΔPQR and ΔSPR, we have,

$$\angle PSR = \angle QPR$$

Due to common in both the triangles

$$\angle SRP = \angle PRQ$$

By AA-axiom similarity we have

$$\Delta PQR \sim \Delta SPR$$

(ii) Length of QR and PS.

Since $\Delta PQR \sim \Delta SPR$ we have

$$\frac{QR}{PR} = \frac{RP}{RS}$$

$$\frac{QR}{6} = \frac{6}{3}$$

$$QR = \frac{6 \times 6}{3} = 12 \text{ cm}$$

Also,

$$\frac{PQ}{PS} = \frac{RP}{RS}$$

$$\frac{8}{PS} = \frac{6}{3}$$

$$PS = \frac{8 \times 3}{6} = 4 \text{ cm}$$

(iii) $\frac{\Delta PQR}{\Delta SPR}$.

Since the ratio of areas of two similar triangle is equal to the ratio of the squares of their corresponding sides.

$$\frac{ar(\Delta PQR)}{ar(\Delta SPR)} = \frac{RP^2}{RS^2}$$

$$= \frac{(6)^2}{(3)^2} = \frac{36}{9} = 4$$

Comp 2017

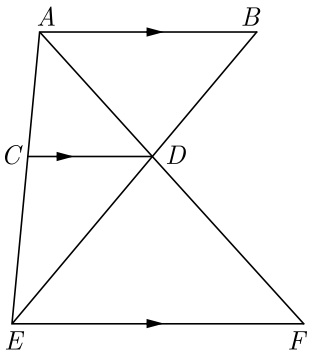
45. In the given figure, $AB \parallel CD \parallel EF$. If $CD = 6$ cm and $AC : CE = 2 : 3$, find :

(i) Shows that $\Delta ECD \sim \Delta EAB$.

(ii) Shows that $\Delta ACD \sim \Delta AEF$.

(iii) AB and EF

(iv) area AECD : area ΔEAB



Sol :

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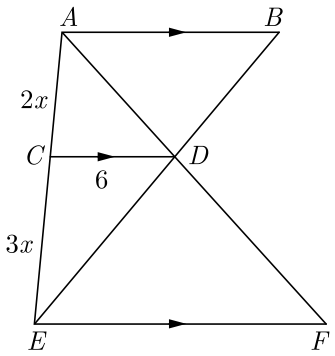

(i) $\Delta ECD \sim \Delta EAB$

In ΔABE and ΔCDE,

Due to common angle,

$$\angle E = \angle E$$

Due to corresponding angle,

$$\angle ECD = \angle EAB$$

By AA-axiom similarity we have

$$\Delta ECD \sim \Delta EAB$$

(ii) $\Delta ACD \sim \Delta AEF$

In ΔACD and ΔAEF we have,

Due to common angle,

$$\angle A = \angle A$$

Due to corresponding angle,

$$\angle ACD = \angle AEF$$

By AA-axiom similarity we have

$$\Delta ACD \sim \Delta AEF$$

(iii) AB and EF

If two triangles are similar, then their corresponding sides are proportional.

Thus

$$\frac{EC}{EA} = \frac{CD}{AB}$$

$$\frac{3x}{5x} = \frac{6}{AB}$$

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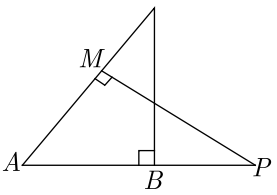


Sol :

(i) $\Delta ABC \sim \Delta AMP$

In ΔABC and ΔAMP,

$$\angle ABC = \angle AMP = 90^\circ$$

Due to common angle

$$\angle BAC = \angle PAM$$

By AA-axiom similarity we have

$\Delta ABC \sim \Delta AMP$ Hence proved.

(ii) AB and BC.

If two triangles are similar, then their corresponding sides are proportional.

Here $\Delta ABC \sim \Delta AMP$ thus we have

$$\frac{AC}{AP} = \frac{BC}{MP} = \frac{AB}{AM}$$

Now $$\frac{AC}{AP} = \frac{BC}{MP}$$

Now submitting $AC = 10$ cm, $AP = 15$ cm, $PM = 12$ cm in above we have

$$\frac{10}{15} = \frac{BC}{12}$$

$$BC = \frac{10 \times 12}{15} = 8 \text{ cm}$$

In ΔABC, right angled at B, using Pythagoras theorem,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{(10)^2 - BC^2}$$
$$= \sqrt{100 - 64}$$
$$= \sqrt{36} = 6 \text{ cm}$$

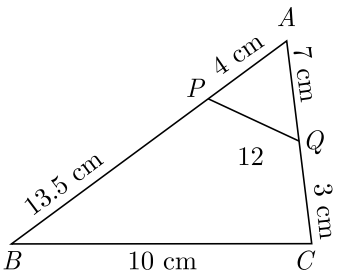
Comp 2012

50. In ΔABC, P and Q are points on AB and AC such that $AP = 4$ cm, $PB = 13.5$ cm, $AQ = 7$ cm and $QC = 3$ cm.

(i) Prove that $\Delta APQ \sim \Delta ACB$.

(ii) If $BC = 10$ cm, find the length of PQ.

(iii) Find the area of ΔAPQ : area of ΔACB.



Sol :

Here $$AB = AP + PB$$
$$= 4 + 13.5$$
$$= 17.5 \text{ cm}$$
$$AC = AQ + QC$$
$$= 7 + 3$$
$$= 10 \text{ cm}$$
$$\frac{AC}{AB} = \frac{10}{17.5} = \frac{2}{3.5} = \frac{2 \times 2}{7} = \frac{4}{7}$$
$$\frac{AP}{AQ} = \frac{4}{7}$$

(i) $\Delta APQ \sim \Delta ACB$.

In ΔAPQ and ΔACB,

Due to common angle,

$$\angle A = \angle A$$
$$\frac{AP}{AQ} = \frac{AC}{AB}$$

By SAS axiom similarity we have

$$\Delta APQ \sim \Delta ACB$$

(ii) Length of PQ.

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{PQ}{BC} = \frac{AP}{AC} = \frac{AB}{AQ} = \frac{4}{10}$$
$$\frac{PQ}{BC} = \frac{AP}{AC}$$

Substituting the values we have

$$\frac{PQ}{10} = \frac{4}{10}$$

$$PQ = 4 \text{ cm}$$

(ii) Area of ΔAPQ : Area of ΔACB

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

Thus $$\frac{\text{Area of } \Delta APQ}{\text{Area of } \Delta ACB} = \frac{AP^2}{AC^2}$$
$$\frac{\text{Area of } \Delta APQ}{\text{Area of } \Delta ACB} = \frac{4^2}{10^2}$$
$$= \frac{16}{100} = \frac{4}{25}$$

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$$DE = 15$$

(iii) $DB : AB$

In ΔABC, since $DE \parallel AC$, by basic proportionality theorem, we get

$$\frac{DE}{AC} = \frac{DB}{AB}$$

$$\frac{15}{24} = \frac{DB}{AB}$$

$$DB : AB = 5 : 8$$

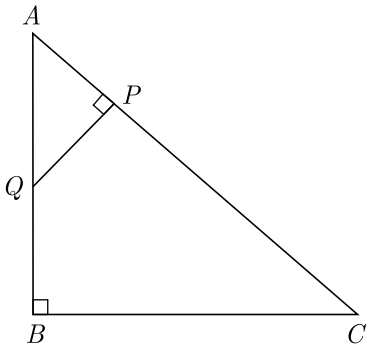
Comp 2023

63. In given triangle ΔABC angle $\angle ABC$ is right angle and $QP \perp AC$:

(i) Show that $\Delta ABC \sim \Delta APQ$.

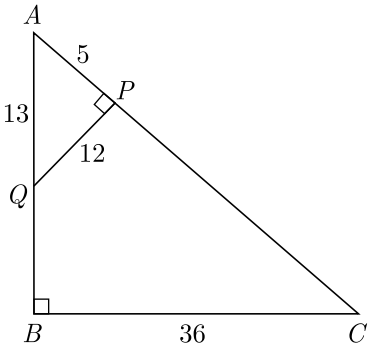
(ii) if $AP = 5, PQ = 12, BC = 36$, find the lengths of AB and AC

(iii) Find area of ΔAPQ : area of ΔABC.



Sol :

We have redrawn the given figure as below.



(i) $\Delta ABC \sim \Delta APQ$

In ΔAPQ and ΔABC,

Due to common angle,

$$\angle A = \angle A$$

Due to right angle,

$$\angle APQ = \angle ABC$$

By AA-axiom similarity we have

$$\Delta APQ \sim \Delta ABC$$

(ii) Length of AB and AC

$$AQ = \sqrt{5^2 + 12^2}$$

$$= 13 \text{ cm}$$

If two triangles are similar, then their corresponding sides are proportional.

$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

Substituting $AP = 5, PQ = 12, BC = 36$ and $AQ = 13$ cm we have

$$\frac{5}{AB} = \frac{12}{36} = \frac{AQ}{AC}$$

$$AB = \frac{5 \times 36}{12} = 15 \text{ cm}$$

In right ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$= 15^2 + 36^2$$

$$= 225 + 1296$$

$$= 1521$$

$$AC = \sqrt{1521}$$

$$= 39 \text{ units}$$

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{area of } \Delta APQ}{\text{area of } \Delta ABC} = \frac{AP^2}{AB^2}$$

$$= \frac{5^2}{15^2}$$

$$= \frac{25}{225} = \frac{1}{9}$$

Area of ΔAPQ : area of $\Delta ABC = 1 : 9$

$$\angle C = \angle A$$

By AA-axiom similarity we have

$$\Delta FCD \sim \Delta FAB$$

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{FC}{FA} = \frac{CD}{AB} = \frac{FD}{FB}$$

$$\frac{6}{6+4} = \frac{y}{5}$$

$$CD = y$$

$$= \frac{6}{10} \times 5 = 3 \text{ cm}$$

(iii) Area ΔCDF : area of ΔABF.

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area}(\Delta CDF)}{\text{Area}(\Delta ABF)} = \frac{CD^2}{AB^2}$$

$$= \frac{3^2}{5^2} = \frac{9}{25}$$

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(iii) area of ΔADF : area of ΔABC.

Since $DE \| BC$ we have

$$DF \| BC$$

By basic theorem of proportionality

$$\frac{AF}{AC} = \frac{AD}{AB} \quad ...(i)$$

Due to common angles,

$$\angle DAF = \angle BAC$$

Thus by SAS similarity we get

$$\Delta ABC \sim \Delta ADF$$

Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Thus $$\frac{ar(\Delta ADF)}{ar(\Delta ABC)} = \frac{(AF)^2}{(AC)^2} = \left(\frac{5}{8}\right)^2$$

$$= \frac{25}{64} \qquad \left[\frac{AF}{AC} = \frac{5}{8}\right]$$

$$ar(\Delta ADF) : ar(\Delta ABC) = 25 : 64$$

Main 2008

53. In ΔABC, $AP : PB = 2 : 3$, PO is parallel to BC and is extended to Q, so that CQ is parallel to BA. Find

(i) area of ΔAPO : area of ΔABC.

(ii) area of ΔAPO : area of ΔCQO.

Sol :

(i) area of ΔAPO : area of ΔABC.

We have $AP : PB = 2 : 3$

$$\frac{AP}{PB} = \frac{2}{3} \quad ...(i)$$

$$\frac{AP}{AP + PB} = \frac{2}{2+3}$$

$$\frac{AP}{AB} = \frac{2}{5} \quad ...(ii)$$

Since $PO \| BC$ we get $PQ \| BC$ and $PB \| QC$

Thus $PBCQ$ is a parallelogram.

Since in a parallelogram opposite sides are equal, we have

$$PB = CQ$$

Substituting above result in eq (1) we have

$$\frac{AP}{CQ} = \frac{2}{3} \quad \text{(iii)}$$

Now in ΔAPO and ΔABC, we have corresponding angles as $PO \| BC$,

$$\angle APO = \angle ABC$$

Due to common angles,

$$\angle PAO = \angle ABC$$

By AA-axiom similarity we have

$$\Delta APO \sim \Delta ABC$$

Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Thus, $$\frac{ar(\Delta APO)}{ar(\Delta ABC)} = \frac{(AP)^2}{(AB)^2}$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25} \qquad \frac{AP}{CQ} = \frac{2}{3}$$

Hence, area of ΔAPO : area of ΔABC is $4 : 25$.

(ii) area of ΔAPO : area of ΔCQO.

In ΔAPO and ΔCQO, due to vertically opposite angles

$$\angle AOP = \angle COQ$$

Due to alternate interior angles we have

$$\angle OAP = \angle OCQ$$

By AA-axiom similarity we have

$$\Delta AOP \sim \Delta COQ$$

Thus $$\frac{ar(\Delta APO)}{ar(\Delta CQO)} = \frac{AP^2}{CQ^2}$$

$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9} \qquad \frac{AP}{CQ} = \frac{2}{3}$$

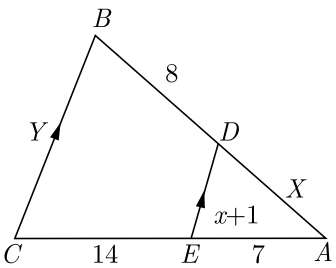
Hence, area of ΔAPO : area of ΔCQO is $4 : 9$.

Comp 2008

54. In ΔABC, line DE and BC are parallel

(i) Prove that $\Delta ADE \sim \Delta ABC$.

(ii) If $AD = x$, $DE = x+1$, $BC = y$, $AE = 7$, $BD = 8$, and $CE = 14$, find x and y.



Sol :

(i) $\Delta ADE \sim \Delta ABC$.

Due to common angle,

$$\angle A = \angle A$$

Due to corresponding angle,

$$\angle ADE = \angle ABC$$

By AA-axiom similarity we have

$$\Delta ADE \sim \Delta ABC \quad \text{(AA similarity)}$$

(ii) x and y.

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

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$$\frac{x}{x+8} = \frac{x+1}{y} = \frac{7}{21}$$

$$3x = x+8$$

or $$\frac{4+1}{y} = \frac{1}{3}$$

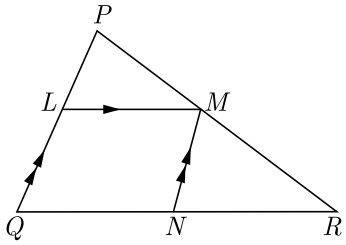
$$2x = 8$$

$$x = 4$$

$$y = 15$$

Comp 2007

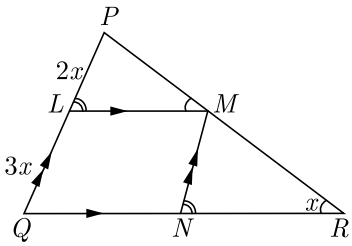
55. In given figure in triangle ΔPQR, line LM and QR are parallel. Line PQ and MN are also parallel. Ratio $PL:LQ$ is $2:3$.



(i) Prove that $\Delta PLM \sim \Delta MNR$.
(ii) Find the ratio : area of ΔPLM : area of ΔMNR
(iii) Find the ratio : area of ΔPLM : area of ΔPQR
(iv) Find the ratio : area of ΔMNR : area of ΔPQR

Sol :

We have redrawn the given figure as below.



Here $LMNQ$ is a parallelogram.
Since $PL:LQ$ is $2:3$, let PL be $2x$ then

$$LQ = 3x = MN$$

In ΔPLM and ΔMNR,

$$\angle PML = \angle MRN$$

$$\angle PLM = \angle MNR$$

By AA-axiom similarity we have

$$\Delta PLM \sim \Delta MNR$$

(ii) Area of ΔPLM : Area of ΔMNR

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area}(\Delta PLM)}{\text{Area}(\Delta MNR)} = \frac{PL^2}{MN^2} = \frac{4}{9}$$

(iii) Area of ΔPLM : area of ΔPQR
From figure it may be easily seen that

$$\Delta PLM \sim \Delta PQR$$

Thus $$\frac{\text{Area}(\Delta PLM)}{\text{Area}(\Delta PQR)} = \left(\frac{PL}{PQ}\right)^2 = \frac{4}{25}$$

(iv) Area of ΔMNR : area of ΔPQR
From figure it may be easily seen that

$$\Delta MNR \sim \Delta PQR$$

Thus $$\frac{\text{Area}(\Delta MNR)}{\text{Area}(\Delta PQR)} = \left(\frac{MN}{PQ}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

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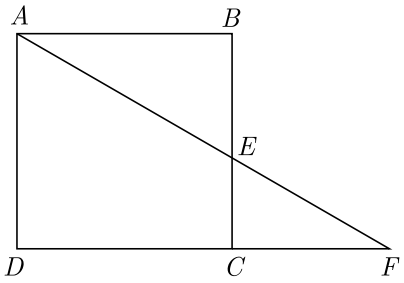
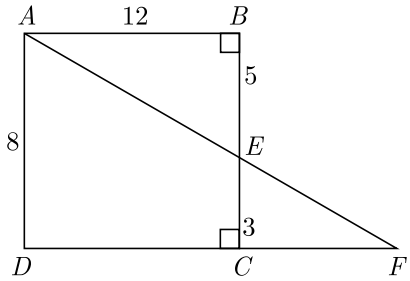
Comp 2006

56. $ABCD$ is a rectangle, $AB = 12$ and $BC = 8$ cm. E is a point on BC such that $BE = 5$ cm. AE produced meets DC produced at F.
(i) Prove that $\Delta ABE \sim \Delta FCE$
(ii) Find the lengths of EF and CF.
(iii) Find area of ΔFEC : area of ΔFAD.



Sol :

We have redrawn the given figure as below.



Here $AD = BC = 8$ cm

$BE = 5$ cm

$EC = 3$ cm

$$AE = \sqrt{5^2 + 12^2} = 13 \text{ cm.}$$

(i) $\Delta ABE \sim \Delta FCE$

In ΔABE and ΔFCE,

Due to both $90°$ angle

$$\angle B = \angle C$$

Due to vertical opposite angle,

$$\angle AEB = \angle FEC$$

By AA-axiom similarity we have

$$\Delta ABE \sim \Delta FCE$$

(ii) Lengths of EF and CF.

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{AB}{FC} = \frac{BE}{CE} = \frac{AE}{FE}$$

$$\frac{12}{FC} = \frac{5}{3} = \frac{13}{EF}$$

$$FC = \frac{12 \times 3}{5} = 7.2 \text{ cm}$$

$$EF = 13 \times \frac{3}{5} = 7.8 \text{ cm}$$

Also $\Delta FEC \sim \Delta FAD$ (AA similarity)

(iii) Area of ΔFEC : area of ΔFAD.

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area}(\Delta FEC)}{\text{Area}(\Delta FAD)} = \frac{EC^2}{AD^2}$$

$$= \frac{3^2}{8^2} = \frac{9}{64}$$

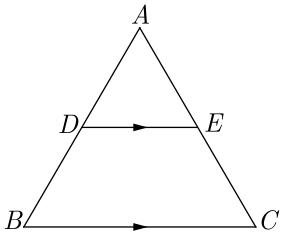
Thus ratio is 9 : 64.

Main 2004

57. In the given figure, $DE \| BC$.

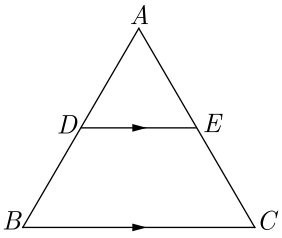
(i) Prove that ΔAPO ∴ and ΔABC are similar.

(ii) Given that $AD = \frac{1}{2}BD$, calculate DE, if $BC = 4.5$ cm.



Sol :

We have redrawn the figure below.



(i) ΔAPO ∴ and ΔABC are similar

In ΔADE and ΔABC, due to common angles

$$\angle DAE = \angle BAC$$

Since $DE \| BC$, due to corresponding angles,

$$\angle ADE = \angle ABC$$

By AA-axiom similarity we have

$\Delta ADE \sim \Delta ABC$ Hence proved.

(ii) DE, if $BC = 4.5$ cm.

If two triangles are similar, then their corresponding sides are proportional.

Here $\Delta ADE \sim \Delta ABC$ thus we have

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\frac{AD + BD}{AD} = \frac{BC}{DE} \quad [AB = AD + BD]$$

$$1 + \frac{BD}{AD} = \frac{BC}{DE} \quad AD = \tfrac{1}{2}BD$$

$$1 + \frac{BD}{\frac{1}{2}BD} = \frac{BC}{DE}$$

$$1 + \frac{1}{\frac{1}{2}} = \frac{BC}{DE}$$

$$1 + 2 = \frac{4.5}{DE} \quad BC = 4.5 \text{ cm}$$

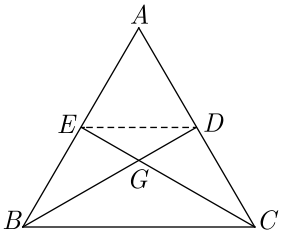
$$DE = \frac{4.5}{3} = 1.5 \text{ cm}$$

Main 2002

58. In the adjoining figure, the medians BD and CE of a ΔABC meet at the point G. Prove that

(i) $\Delta EGD \sim \Delta CGB$.

(ii) $BG = 2GD$.



Sol :

We have redrawn the figure below.

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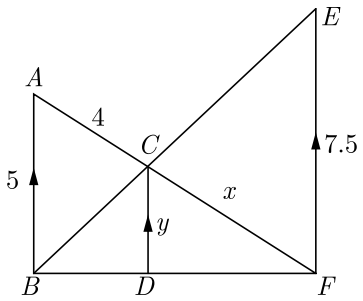


(i) $\Delta FEC \sim \Delta ABC$.
In ΔABC and ΔFEC,
Due to alternate angle

$$\angle A = \angle CFE$$

Due to vertical opposite angle,

$$\angle ACB = \angle FCE$$

By AA-axiom similarity we have

$$\Delta ABC \sim \Delta FEC$$

(ii) x and y.
If two triangles are similar, then their corresponding sides are proportional.

$$\frac{AB}{FE} = \frac{BC}{EC} = \frac{AC}{FC}$$

$$\frac{5}{7.5} = \frac{4}{x}$$

$$5x = 30$$

$$x = 6 \text{ cm}$$

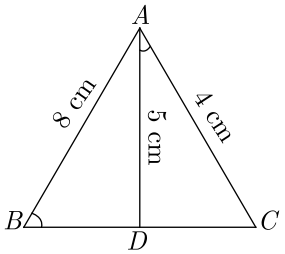
In ΔFCD and ΔFAB,
Due to common angle,

$$\angle F = \angle F$$

Due to corresponding angle,

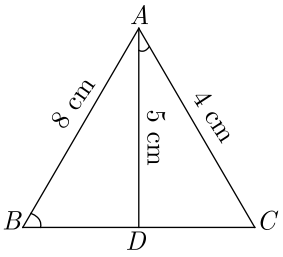
Main 2014

66. In ΔABC, $\angle ABC = \angle DAC$, $AB = 8$ cm, $AC = 4$ cm and $AD = 5$ cm.
(i) Prove that ΔACD is similar to ΔBCA.
(ii) Find BC and CD.
(iii) Find area of ΔACD : area of ΔABC.



Sol :

We have redrawn the figure below.



Here $\angle ABC = \angle DAC$, $AB = 8$ cm,

$AC = 4$ cm, $AD = 5$ cm

(i) ΔACD is similar to ΔBCA.
In ΔACD and ΔBCA, we have

$$\angle ABC = \angle DAC$$

Due to common angles,

$$\angle ACD = \angle BCA$$

Thus by AA axiom of similarity we have

$\Delta ACD \sim \Delta BCA$ Hence proved.

(ii) BC and CD.
If two triangles are similar, then their corresponding sides are proportional.
Since $\Delta ACD \sim \Delta BCA$, we have

$$\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{BA}$$

$$\underset{I}{\frac{4}{BC}} = \underset{II}{\frac{CD}{4}} = \underset{III}{\frac{5}{8}}$$

Taking first and third term we have

$$\frac{4}{BC} = \frac{5}{8}$$

$$BC = \frac{4 \times 8}{5} = \frac{32}{5}$$

$$BC = 6.4 \text{ cm}$$

Taking second and third term we have

$$\frac{CD}{4} = \frac{5}{8}$$

$$CD = \frac{4 \times 5}{8} = \frac{5}{2}$$

$$CD = 2.5 \text{ cm}$$

(iii) area of ΔACD : area of ΔABC.
Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.
Since $\Delta ACD \sim \Delta ABC$ we have

$$\frac{ar(\Delta ACD)}{ar(\Delta ABC)} = \frac{(AC)^2}{(BC)^2}$$

$$= \frac{(4)^2}{(6.4)^2}$$

$$= \frac{(1)^2}{(1.6)^2}$$

$$= \frac{1}{2.56} = \frac{25}{64}$$

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Due to both 90° angle

$$\angle SRP = \angle SPQ$$

By AA-axiom similarity we have

$$\Delta SRP \sim \Delta SPQ$$

(ii) If $QR = 2$ cm, $RS = 4.5$ cm, find PR

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{SR}{SP} = \frac{RP}{PQ} = \frac{SP}{SQ}$$

$$\frac{4.5}{SP} = \frac{SP}{6.5}$$

$$SP^2 = 4.5 \times 6.5$$

$$= 29.25$$

$$PR^2 = SP^2 - RS2$$

$$= 29.25 - 20.25$$

$$= 9$$

$$PR = 3 \text{ cm}$$

(iii) Area ΔPQR : Area ΔSPR.

From figure we can easily see that due to AA similarity

$$\Delta PQR \sim \Delta SPR$$

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

Thus $$\frac{\text{Area}\,(\Delta PQR)}{\text{Area}\,(\Delta SPR)} = \frac{QR^2}{PR^2}$$

$$= \frac{4}{9}$$

Thus required ratio is 4 : 9

Main 2001

61. A model of a ship is made to a scale of 1 ∶ 200.

(i) The lengh of the model is 4 m. Calculate the length of the ship.

(ii) The area of the deck of the ship is 160000 m^2. Find the area of the deck of the model.

(iii) The volume of the model is 200L. Calculate the volume of the ship in m^3.

Sol :

(i) Length of the ship.

Since yhe model of the ship is made to scale 1 ∶ 200.

$$k = 200$$

Actual length of the ship

$$= k \times (\text{Length of the model})$$

$$= (200 \times 4)\, m = 800 \text{ m}$$

(ii) Area of the deck of the ship

$$= k^2 \times (\text{Area of the deck of the model})$$

Area of the deck of the model

$$= \frac{1}{k^2} \times \text{Area of the deck of the ship}$$

$$= \frac{1}{200^2} \times 160000$$

$$= 4 \text{ m}^2$$

(iii) Volume of the ship

Volume of the ship$= k^3 \times$ Volume of the model

$$= (200)^3 \times 200\, L$$

$$= \frac{8000000 \times 200}{1000} \text{m}^3 \qquad 1000L = 1\,\text{m}^3$$

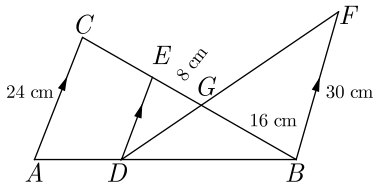
$$= 1600000 \text{ m}^3$$

FOUR MARK QUESTIONS

Main 2023

62. In the given figure, $AC || DE || BF$.

If $AC = 24$ cm, $EG = 8$ cm, $GB = 16$ cm, $BF = 30$ cm



(i) Prove $\Delta GED - \Delta GBF$

(ii) Find DE

(iii) $DB : AB$

Sol :

We have $DE \,||\, BF$

(i) Prove $\Delta GED - \Delta GBF$

In ΔGED and ΔGBF,

Due to vertically opposite angles.

$$\angle EGD = \angle BGF$$

Due to alternate angles,

$$\angle GED = \angle GBF$$

By AA similarity axiom we get

$$\Delta GED \sim \Delta GBF$$

(ii) Find DE

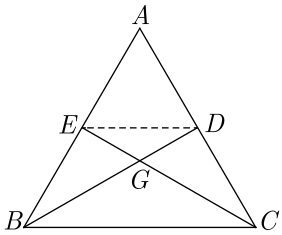
Corresponding sides of similar triangle are proportional,

Since $\Delta GED \sim \Delta GBF$ we have

$$\frac{GE}{GB} = \frac{DE}{BF}$$

$$\frac{8}{16} = \frac{DE}{30}$$

NODIA



Here median BD and CE of a ΔABC meet at G.
(i) $\Delta EGD \sim \Delta CGB$
The median BD and CE of a ΔABC meet at G.
In ΔEGD and ΔCGB, by mid-point theorem

$$ED \parallel BC$$

Due to vertically opposite angles

$$\angle EGD = \angle BGC$$

Due to alternate angle we have

$$\angle EDG = \angle GCB$$

Due to alternate angle we have

$$\angle GED = \angle GCB$$

By AAA axiom of similarity we get

$$\Delta EGD \sim \Delta CGB$$

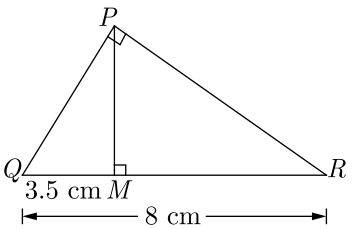
(ii) $BG = 2GD$
Centroid of a ΔABC divides the median in the ratio $2 : 1$.
Since, BD is a median, we have

$$\frac{BG}{GD} = \frac{2}{1}$$

$$BG = 2GD \qquad \text{Hence proved.}$$

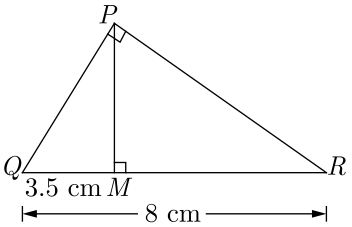
Main 2002

59. In right angled ΔQPR, PM is an altitude. Given that, $QR = 8$ cm and $MQ = 3.5$ cm. Calculate the value of PR.



Sol :

We have redrawn the figure below.



Here ΔQPR is right angled at P and $PM \perp QR$.
In ΔQPR and ΔPMR, due to each right angle

$$\angle QPR = \angle PMR$$

Due to common angles,

$$\angle QRP = \angle PRM$$

Thus by AA axiom of similarity we have

$$\Delta QPR \sim PMR$$

If two triangles are similar, then their corresponding sides are proportional.

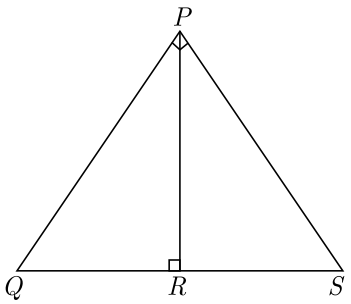
Thus $$\frac{QR}{PR} = \frac{PR}{MR}$$

$$PR^2 = QR \cdot MR$$

$$\begin{aligned} PR^2 &= 8 \times (QR - MQ) \\ &= 8 \times (8 - 3.5) \\ &= 8 \times 4.5 \\ &= 36 \text{ cm} \end{aligned}$$

Comp 2002

60. In $\Delta PQS, PR \perp QS$ and $\angle QPS = 90^\circ$.
(i) Show that $\Delta PQR \sim \Delta SPR$
(ii) If $QR = 2$ cm, $RS = 4.5$ cm, find PR
(iii) Find area ΔPQR : area ΔSPR.



Sol :

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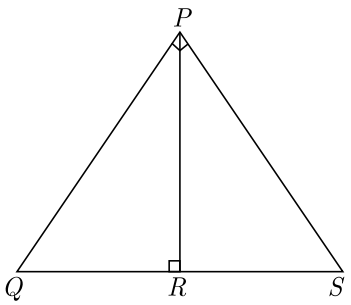

(i) $\Delta PQR \sim \Delta SPR$
Due to common angle,

$$\angle S = \angle S$$

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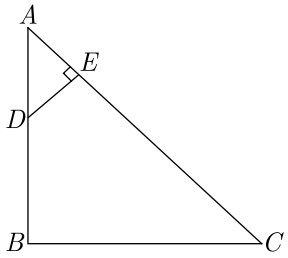
Main 2015

64. ABC is a right angled triangle with $\angle ABC = 90^\circ$. D is any point on AB and DE is perpendicular to AC. Prove that

(i) $\Delta ADE \sim \Delta ACB$.

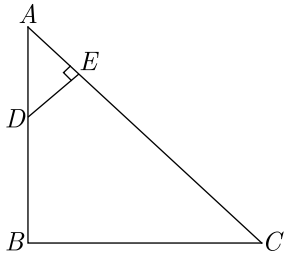
(ii) If $AC = 13$ cm, $BC = 5$ cm and $AE = 4$ cm, find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral $BCED$.



Sol :

We have redrawn the figure below.



(i) $\Delta ADE \sim \Delta ACB$.

In ΔADE and ΔACB, due to each right angle

$$\angle AED = \angle ABC$$

Due to common angles,

$$\angle DAE = \angle BAC$$

Thus by AA axiom of similarity we have

$$\Delta ADE \sim \Delta ACB \quad \text{Hence proved.}$$

(ii) DE and AD.

In ΔABC, $AC = 13$ cm, $BC = 5$ cm

Using Pythagoras theorem in ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{(13)^2 - (5)^2}$$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

If two triangles are similar, then their corresponding sides are proportional.

Since, $\Delta ADE \sim \Delta ACB$,

$$\frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$$

$$\frac{AD}{13} = \frac{DE}{5} = \frac{4}{12} \qquad AE = 4$$

Taking first and third term we have

$$\frac{AD}{13} = \frac{4}{12}$$

$$AD = \frac{13}{3}$$

$$= 4.33 \text{ cm}$$

Taking second and third term we have

$$\frac{DE}{5} = \frac{4}{12}$$

$$DE = \frac{5 \times 4}{3} = \frac{5}{3}$$

$$= 1.66 \text{ cm}$$

(iii) area of ΔADE : area of $\square BCED$.

$$\text{Area } \Delta ADE = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AE \times DE = \frac{1}{2} \times 4 \times \frac{5}{3}$$

$$= \frac{10}{3} = 3.33 \text{ cm}^2$$

$$\text{Area } \Delta ABC = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

Area of quadrilateral $\square BCED$

$$= \text{Area of } \Delta ABC - \text{Area of } \Delta AED$$

$$= 30 - 3.33$$

$$= 26.67 \text{ cm}^2$$

area of ΔADE : area of quadrilateral $\square BCED$

$$= 3.33 : 26.67$$

$$= 1 : 8$$

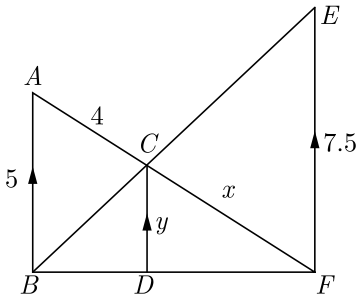
Comp 2015

65. In the given figure, $AB \parallel CD \parallel EF, AB = 5$ cm, $AC = 4$ cm, $EF = 7.5$ cm, $CF = x$ and $CD = y$.

(i) Prove that $\Delta FEC \sim \Delta ABC$.

(ii) Solve for x and y.

(iii) Find area ΔCDF : area of ΔABF.



Sol :

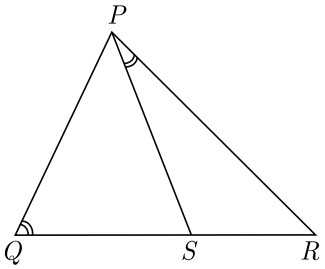
We have redrawn the given figure as below.

NODIA

$$\text{Ratio} = 4:25$$

Comp 2011

51. In ΔPQR, S is a point on QR such that $\angle Q = \angle SPR$

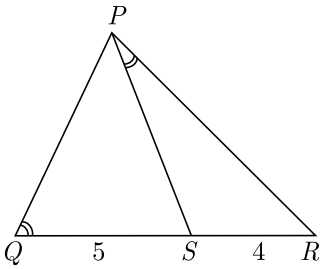


(i) Prove that $\Delta PQR \sim \Delta SPR$

(ii) If $QS = 5$ cm, $SR = 4$ cm, find the length of PR

Sol :

We have redrawn the given figure as below.



(i) $\Delta PQR \sim \Delta SPR$

In ΔPQR and ΔSPR,

Given angle, $\angle Q = \angle SPR$

Due to common angle,

$$\angle R = \angle R$$

By AA-axiom similarity we have

$$\Delta PQR \sim \Delta SPR$$

(ii) Length of PR

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR}$$

$$\frac{9}{PR} = \frac{PR}{4}$$

$$PR^2 = 36$$

$$PR = 6 \text{ cm}$$

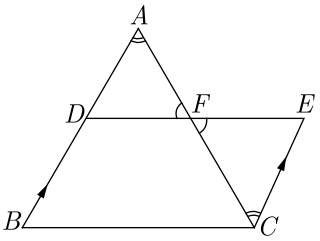
Main 2009

52. In the given figure, ABC and CEF are two triangles, where BA is parallel to CE and $AF : AC = 5 : 8$.

(i) Prove that $\Delta ABC \sim \Delta CEF$.

(ii) Find AD, if $CE = 6$ cm.

(iii) If DE is parallel to BC, find area of ΔADF : area of ΔABC.



Sol :

Given $BA \,||\, CE$ and $AF : AC = 5 : 8$

(i) $\Delta ADF \sim \Delta CEF$

In ΔADF and ΔCEF due to vertically opposite angles

$$\angle AFD = \angle FCE$$

Since $BA \,||\, CE$, due to alternate interior angles

$$\angle DAF = \angle CFE$$

By AA-axiom similarity we have

$$\angle DAF \sim \Delta CEF \quad \text{Hence proved}$$

(ii) AD, if $CE = 6$ cm.

If two triangles are similar, then their corresponding sides are proportional.

Here $\Delta ADF \sim \Delta CEF$ thus we have

$$\frac{AD}{CE} = \frac{AF}{CF} \qquad (1)$$

We have $$\frac{AF}{AC} = \frac{5}{8}$$ Given

$$\frac{AC}{AF} = \frac{8}{5}$$

$$\frac{AC - AF}{AF} = \frac{8-5}{5}$$

$$\frac{CF}{AF} = \frac{3}{5}$$

$$\frac{AF}{CF} = \frac{5}{3} \qquad (2)$$

From eq (1) and (2) we have

$$\frac{AD}{CE} = \frac{5}{3} \qquad (1)$$

Substituting $CE = 6$ cm we have

$$\frac{AD}{6} = \frac{5}{3}$$

$$AD = \frac{5 \times 6}{3}$$

$$AD = 5 \times 2$$

$$= 10 \text{ cm}$$

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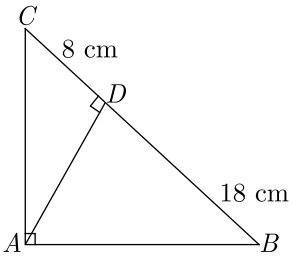
$= 25 : 64$

Main 2011

67. In the adjoining figure, ABC is a right angled triangle with $\angle BAC = 90^\circ$.
(i) Prove that $\Delta ADB \sim \Delta CDA$.
(ii) If $BD = 18$ cm, $CD = 8$ cm, find AD.
(iii) Find the ratio of the area of ΔADB is to area of ΔCDA.



Sol :

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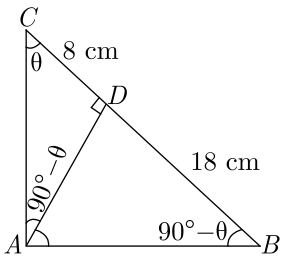


Here $\angle BAC = 90^\circ$

Let $\angle DAB = \theta$

(i) $\Delta ADB \sim \Delta CDA$.
In ΔADB and ΔCDA,

$$\angle DAB = \angle DCA = \theta$$

and $\angle CAD = \angle ABD = 90^\circ - \theta$

Thus by AA axiom of similarity we have

$\Delta ADB \sim \Delta CDA$ Hence proved.

(ii) If $BD = 18$ cm, $CD = 8$ cm, find AD.
If two triangles are similar, then their corresponding sides are proportional.

Since $\Delta ADB \sim \Delta CDA$ we have

$$\frac{AD}{CD} = \frac{BD}{AD}$$

$$AD^2 = BD \times CD$$

Substituting $BD = 18$ cm, $CD = 8$ cm, we have

$$AD^2 = 18 \times 8 = 144$$

$$AD = 12 \text{ cm}$$

(iii) ratio of the area of ΔADB to area of ΔCDA
Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Since $\Delta ADB \sim \Delta CDA$ we have

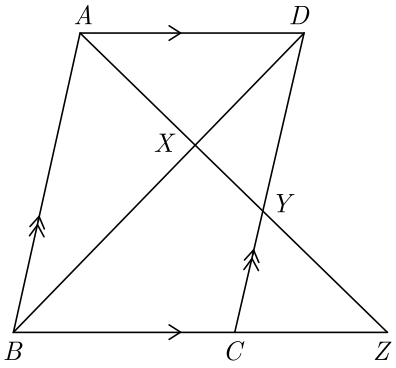
$$\frac{ar(\Delta ADB)}{ar(\Delta CDA)} = \frac{(AD)^2}{(CD)^2}$$

$$= \frac{(12)^2}{(8)^2} = \frac{144}{64} = \frac{9}{4}$$

Hence area of ΔADB : area of ΔCDA is $9 : 4$.

Comp 2011

68. In the given figure, $ABCD$ is a parallelogram. BC is extended to a point Z such that AZ meets diagonal DB at X and side DC at Y and $3DX = XB$:
(i) Prove that $\Delta AXD \sim \Delta ZXB$.
(ii) Name a triangle similar to ΔDXY
(ii) Calculate the ratio of area of ΔAXD : area of ΔZXB.
(iii) If $AB = 6$ cm, $AD = 4$ cm, find the lengths of BZ and DY.



Sol :

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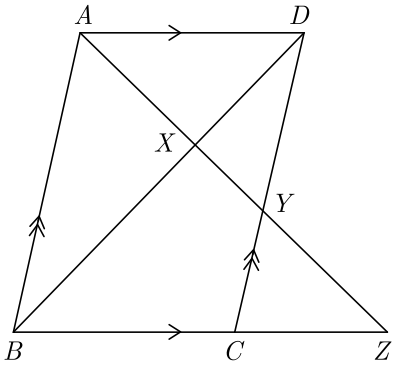


$$3DX = XB$$

$$\frac{DX}{XB} = \frac{1}{3}$$

(i) $\Delta AXD \sim \Delta ZXB$.
In ΔAXD and ΔZXB,
Due to vertical opposite angle

NODIA

$$\angle AXD = \angle ZXB$$

Due to alternate angle

$$\angle DAX = \angle BZX$$

By AA-axiom similarity we have

$$\Delta AXD \sim \Delta ZXB$$

(ii) Triangle similar to ΔDXY

Triangle ΔABX similar to ΔDXY.

(iii) Ratio of area of ΔAXD : area of ΔZXB.

If two triangles are similar, then their corresponding sides are proportional.

$$\frac{XA}{XZ} = \frac{AD}{ZB} = \frac{XD}{XB} = \frac{1}{3} \qquad (1)$$

Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area}(\Delta AXD)}{\text{Area}(\Delta ZXB)} = \left(\frac{XD}{XB}\right)^2$$

$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

(iii) Lengths of BZ and DY.

Since $\Delta ABX \sim \Delta YDX$ we can write

$$\frac{AB}{YD} = \frac{BX}{XD} = \frac{AX}{YX} \qquad (2)$$

From (1) and (2) we have

$$\frac{AB}{YD} = \frac{XD}{XB} = \frac{1}{\frac{1}{3}}$$

Substituting $AB = 6$ cm we have

$$\frac{6}{YD} = \frac{3}{1}$$

$$DY = 2 \text{ cm}$$

Again from (1) and (2) we have

$$\frac{AD}{ZB} = \frac{1}{3}$$

Substituting $AD = 4$ cm we have

$$\frac{4}{ZB} = \frac{1}{3}$$

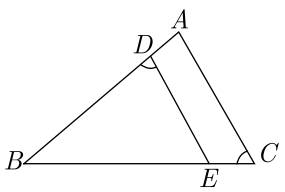
$$ZB = 12 \text{ cm}$$

Main 2010

69. In figure, ABC is a triangle with $\angle EDB = \angle ACB$. Prove that $\Delta ABC \sim \Delta EBD$. If $BE = 6$ cm, $EC = 4$ cm, $BD = 5$ cm and area of $\Delta BED = 9$ cm^2. Calculate the

(i) length of AB.

(ii) area of ΔABC.



Sol :

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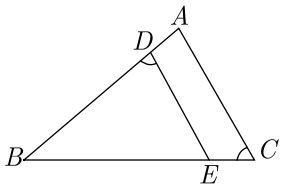

In ΔABC and ΔEBD,

$$\angle ACB = \angle EDB$$

Due to common angles,

$$\angle ABC = \angle EBD$$

Thus by AA axiom of similarity we have

$$\Delta ABC \sim \Delta EBD$$

(i) Length of AB.

We have $BE = 6$ cm, $EC = 4$ cm, $BD = 5$ cm and $ar(\Delta BED) = 9$ cm^2

If two triangles are similar, then their corresponding sides are proportional.

Since $\Delta ABC \sim \Delta EBD$ we have

$$\frac{AB}{EB} = \frac{BC}{BD} = \frac{AC}{ED}$$

Taking first and third term we have

$$\frac{AB}{EB} = \frac{BC}{BD}$$

$$\frac{AB}{EB} = \frac{BE + EC}{BD}$$

$$\frac{AB}{6} = \frac{6+4}{5}$$

$$AB = \frac{6 \times 10}{5} = 12 \text{ cm}$$

(ii) Area of ΔABC.

Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Since $\Delta ABC \sim \Delta EBD$ we have

$$\frac{ar(\Delta ABC)}{ar(\Delta EBD)} = \frac{(AB)^2}{(EB)^2}$$

$$\frac{ar(\Delta ABC)}{9} = \frac{(12)^2}{(6)^2}$$

$$\frac{ar(\Delta ABC)}{9} = \frac{144}{36} = 4 \qquad ar(\Delta BED) = 9 \text{ cm}^2$$

$$ar(\Delta ABC) = 9 \times 4 = 36 \text{ cm}^2$$

Comp 2010

70. In the given figure, $PQ \,||\, ST, PR = 4$ cm and $RT = 6$ cm

(i) Prove that $\Delta PQR \sim \Delta TSR$

(ii) Find the value of x if $PQ = x + 2$ and $ST = 2x + 1$

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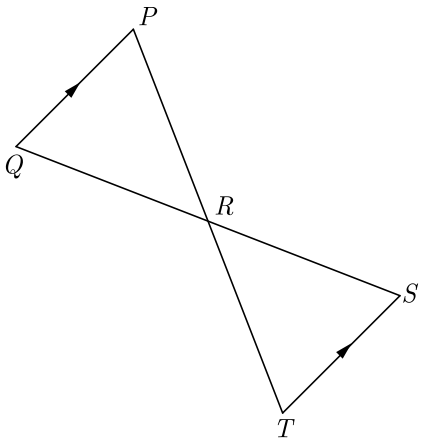
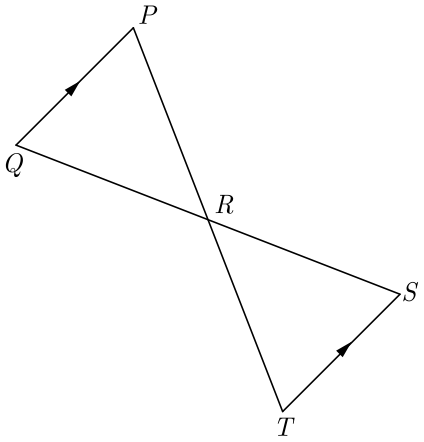


Sol :

We have redrawn the given figure as below.



Due to alternate angle

$$\angle P = \angle T$$

Due to alternate angle

$$\angle Q = \angle S$$

Due to vertical opposite angle,

$$\angle PRQ = \angle TRS$$

By AA-axiom similarity we have

$$\Delta PQR \sim \Delta TSR$$

(ii) Value of x if $PQ = x+2$ and $ST = 2x+1$

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{PQ}{TS} = \frac{QR}{SR} = \frac{PR}{TR}$$

$$\frac{x+2}{2x+1} = \frac{2}{3}$$

$$3x+6 = 4x+2$$

$$4 = x$$

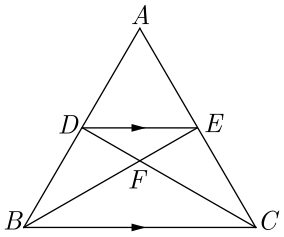
Main 2007

71. In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{BD} = \frac{3}{2}$.

(i) Determine the ratio $\frac{AD}{AB}$ and $\frac{DE}{BC}$.

(ii) Prove that ΔDEF is similar to ΔCBF. Also, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of ΔDEF and ΔCFB?



Sol :

(i) Ratio $\frac{AD}{AB}$ and $\frac{DE}{BC}$.

We have $$\frac{AD}{DB} = \frac{3}{2}$$

$$\frac{AD}{AD+DB} = \frac{3}{3+2}$$

$$\frac{AD}{AB} = \frac{3}{5}$$

In ΔADE and ΔABC, due to common angle,

$$\angle DAE = \angle BAC \quad []$$

Since $DE || BC$, due to corresponding angles,

$$\angle ADE = \angle ABC$$

Thus by AA axiom of similarity we have

$$\Delta ADE \sim \Delta ABC$$

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{DE}{BC} = \frac{AD}{AB}$$

$$\frac{DE}{BC} = \frac{3}{5} \quad \text{...(i)}$$

(ii) ΔDEF is similar to ΔCBF

In ΔDEF and ΔCBF, as $DE || BC$, due to alternate interior angle

$$,\angle FED = \angle FBC$$

Due to vertically opposite angles,

$$\angle DFE = \angle BFC$$

Thus by AA axiom of similarity we have

$$\Delta DEF \sim \Delta CBF$$

If two triangles are similar, then their corresponding sides are proportional.

Thus $$\frac{FE}{FB} = \frac{DE}{BC}$$

$$\frac{FE}{FB} = \frac{3}{5} \qquad \frac{DE}{BC} = \frac{3}{5}$$

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(c) Two-third (d) None of these

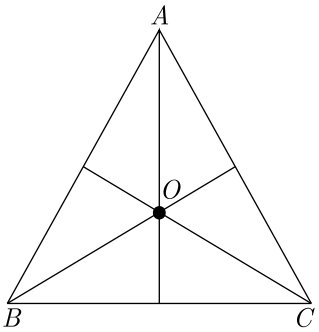
Sol :

Chords which are equidistant from the centre are equal.
Thus (b) is correct option.

8. The intersecting point of medians of triangle is called:
(a) Centroid (b) Incentre
(c) Orthocentre (d) Circumcentre

Sol :

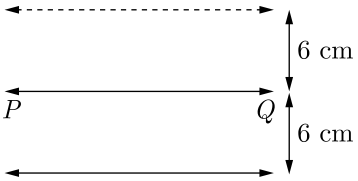
Here, O is called centroid.



Thus (a) is correct option.

9. Locus of a point which remaining at a distance of 6 cm from line PQ is:
(a) Two parallel lines below the line PQ at a distance of 6 cm.
(b) Two parallel lines on both sides of PQ at a distance 6 cm.
(c) Two parallel lines above PQ at a distance of 6 cm.
(d) None of these.

Sol :



Thus (b) is correct option.

10. Locus of a point whose distance from origin is always equal to is:
(a) $a^2+b^2=2$ (b) $a^2-b^2=4$
(c) $a^2+b^2=4$ (d) $a^2-b^2=2$

Sol :

Let $O(0,0)$ and $Q(a,\ b)$ be the co-ordinates of Q of the required locus then

$$(a-0)^2+(b-0)^2=2^2 \qquad (r=2)$$
$$a^2+b^2=4$$

Thus (c) is correct option.

11. A point Q is always at the distance of 4 units from (3, 2). Equation of locus of a point Q, will be
(a) $x^2+y^2+6x+4y-3=0$
(b) $x^2+y^2+6x-4y-3=0$
(c) $x^2+y^2-6x-4y-3=0$
(d) $x^2+y^2-6x-4y+3=0$

Sol :

Let $Q(x,\ y)$ be point then using distance formula,

$$(x-3)^2+(y-2)^2=(4)^2$$
$$x^2+9-6x+4+y^2-4y=16$$
$$x^2+y^2-6x-4y-3=0$$

Thus (c) is correct option.

12. A point P moves two units above the x-axis at a constant distance. The locus of point P is:
(a) $x=2$ (b) $y=-2$
(c) $y=2$ (d) $x=-2$

Sol :

Locus of a point that moves k unite above the x-axis is given by

$$y=k$$

Here, $k=2$

$$y=2$$

Thus (c) is correct option.

13. A point P moves 3 units below the x-axis at a constant distance. The locus of point P is:
(a) $y=3$ (b) $x=-3$
(c) $x=3$ (d) $y=-3$

Sol :

Point that moves k units below the x-axis is

$$y=-k$$

Here, $k=3$

$$y=-3$$

Thus (d) is correct option.

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to square of the ratio of their corresponding sides.

Thus, $\frac{ar(\Delta AOQ)}{ar(\Delta POB)} = \frac{(OQ)^2}{(PO)^2}$

$$\frac{ar(\Delta AOQ)}{120} = \left(\frac{9}{6}\right)^2$$

$$ar(\Delta AOQ) = \left(\frac{9}{6}\right)^2 \times 120$$

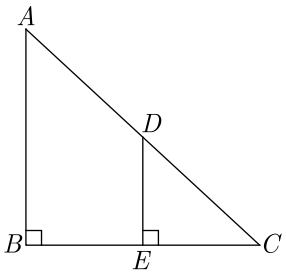
$$= \frac{81 \times 120}{36}$$

$$= 270 \text{ cm}^2$$

Hence, the area of ΔAOQ is 270 cm^2.

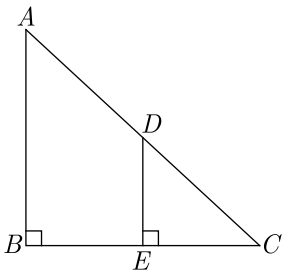
Main 2006

74. In the given figure, AB and DE are perpendicular to BC. If $AB = 9$ cm, $DE = 3$ cm and $AC = 24$ cm, calculate AD.



Sol :

We have redrawn the figure below.



Here $AB \perp BC$ and $DE \perp BC$

In ΔABC and ΔDEC,

$$\angle B = \angle E = 90^\circ$$

Thus $\angle ABC = \angle DEC$

Due to common angles,

$$\angle ACB = \angle DCE$$

Thus by AA axiom of similarity we have

$$\Delta ABC \sim \angle DEC$$

If two triangles are similar, then their corresponding sides are proportional.

Thus $\frac{AB}{DE} = \frac{AC}{DC}$

Substituting $AB = 9$ cm, $DE = 3$ cm and $AC = 24$ cm we have

$$\frac{9}{3} = \frac{24}{DC}$$

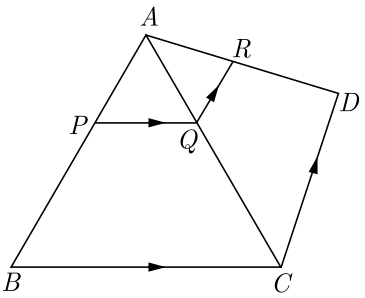
$$DC = \frac{24 \times 3}{9} = 8 \text{ cm}$$

Now, $AD = AC - DC$

$$= 24 - 8 = 16 \text{ cm}$$

Comp 2006

75. In given figure, in triangle ΔABC, $PQ \parallel BC$ and in ΔACD, $QR \parallel CD$.

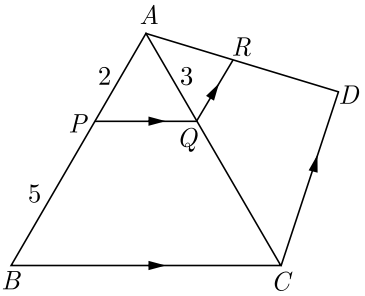


(i) Prove that $\Delta AQR \sim \Delta ACD$.

(ii) If $AP = 2, PB = 5, AQ = 3$ and $QR = 5$, calculate the lengths of QC and CD.

Sol :

We have redrawn the given figure as below.



(i) $\Delta AQR \sim \Delta ACD$.

Due to common angle,

$$\angle A = \angle A$$

Due to corresponding angle,

$$\angle AQR = \angle ACD$$

By AA-axiom similarity we have

$$\Delta AQR \sim \Delta ACD$$

(ii) Lengths of QC and CD.

Using basic proportionality theorem

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{3}{QC} = \frac{2}{5}$$

$$QC = \frac{3 \times 5}{2} = 7.5 \text{ cm}$$

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Due to $\Delta AQR \sim \Delta ACD$ we have

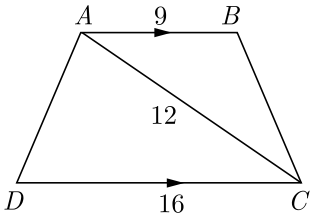
$$\frac{AQ}{AC} = \frac{QR}{CD}$$

$$\frac{3}{10.5} = \frac{5}{CD}$$

$$CD = \frac{5 \times 10.5}{3} = 17.5 \text{ cm}$$

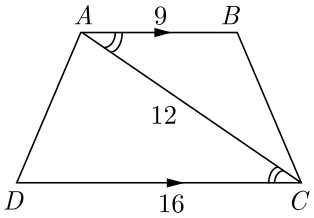
Comp 2004

76. In given figure $ABCD$ is a trapezium, line AB and CD are parallel, $AB = 9, AC = 12$ and $CD = 16$.
(i) Prove that $\Delta ABC \sim \Delta CAD$
(ii) If $AD = 10$, find BC.



Sol :

We have redrawn the given figure as below.



(i) $\Delta ABC \sim \Delta CAD$
In ΔABC and ΔCAD,
Due to alternate angle,

$$\angle BAC = \angle ACD$$

Now $$\frac{AB}{AC} = \frac{9}{12} = \frac{3}{4}$$

and $$\frac{AC}{BC} = \frac{12}{16} = \frac{3}{4}$$

Thus $$\frac{AB}{AC} = \frac{AC}{BC}$$

By SAS axiom similarity we have

$$\Delta ABC \sim \Delta CAD$$

(ii) If $AD = 10$, BC

We have $$\frac{BC}{AB} = \frac{AB}{AC}$$

$$\frac{BC}{10} = \frac{9}{12}$$

$$BC = \frac{10^5 \times 9^3}{12_2}$$

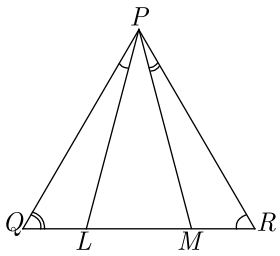
$$= \frac{15}{2} = 7.5 \text{ cm}$$

Main 2003

77. In a ΔPQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that
(i) $\Delta PQL \sim \Delta RPM$.
(ii) $QL \times RM = PL \times PM$.
(iii) $PQ^2 = QR \cdot QL$.

Sol :

As per given information we have drawn the figure below.



Here $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$
From diagram we can get easily that

$$\angle LPQ = \angle MRP \text{ and } \angle LQP = \angle RPM$$

(i) $\Delta PQL \sim \Delta RPM$
In ΔPQL and ΔRPM,

$$\angle LPQ = \angle MRP$$

$$\angle LQP = \angle RPM$$

Thus by AA axiom of similarity we have

$$\Delta PQL \sim \Delta RPM$$

(ii) $QL \cdot RM = PL \cdot PM$
If two triangles are similar, then their corresponding sides are proportional.
Since $\Delta PQL \sim \Delta RPM$ we have

$$\frac{QL}{PM} = \frac{PL}{RM}$$

$$QL \times RM = PL \times PM$$

(iii) $PQ^2 = QR \cdot QL$
In ΔPQL and ΔRQP, we have

$$\angle QPL = \angle PRM$$

Due to common angle,

$$\angle PQL = \angle RQP$$

Thus by AA axiom of similarity we have

$$\Delta PQL \sim \Delta RQP$$

Thus, $$\frac{PQ}{QR} = \frac{QL}{PQ}$$

$$PQ^2 = QR \cdot QL$$ Hence proved.

Comp 2003

78. In given triangle ΔABC, $\angle B$ is right angle and $PQ \perp AC$.

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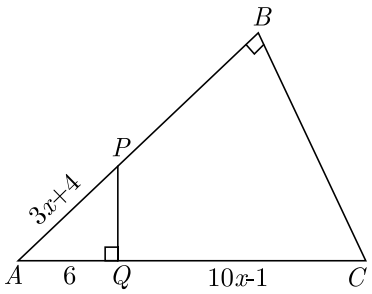
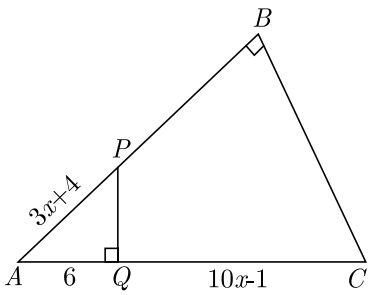
(i) Prove $\Delta ABC \sim \Delta AQP$..
(ii) If $AB = 15$, $AQ = 6$, $AP = 3x + 4$ and $QC = 10x - 1$, find x, AP and AC.



Sol :

We have redrawn the given figure as below.



(i) $\Delta ABC \sim \Delta AQP$..

In ΔAQP and ΔABC,

Due to common angle,

$$\angle A = \angle A$$

Due to both 90° angle

$$\angle Q = \angle B$$

By AA-axiom similarity we have

$$\Delta AQP \sim \Delta ABC$$

(ii) x, AP and AC.

If two triangles are similar, then their corresponding sides are proportional.

Thus
$$\frac{AQ}{AB} = \frac{QP}{BC} = \frac{AP}{AC}$$

$$\frac{6}{15} = \frac{3x+4}{10x-1+6}$$

$$2(10x+5) = 5(3x+4)$$

$$20x + 10 = 15x + 20$$

$$5x = 10$$

$$x = 2$$

$$AP = 3x + 4$$

$$= 3 \times 2 + 4 = 10 \text{ cm}$$

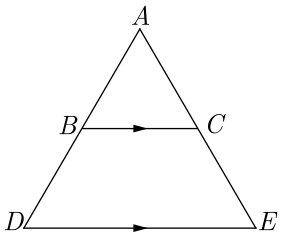
$$\frac{AP}{AC} = \frac{2}{5}$$

$$\frac{10}{AC} = \frac{2}{5}$$

$$AC = 25 \text{ cm}$$

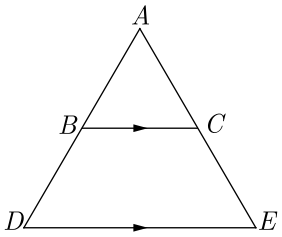
Main 2000

79. In the figure alongside, BC is parallel to DE . Area of $\Delta ABC = 25$ cm2, area of trapezium $BCED = 24$ cm2 and $DE = 14$ cm. Calculate the length of BC.



Sol :

We have redrawn the figure below.



Here $BC \| DE$ and $DE = 14$ cm

$$ar(\Delta ABC) = 24 \text{ cm}^2$$

$$ar(\square BCED) = 24 \text{ cm}^2$$

$$ar(\Delta ADE) = ar(\Delta ABC) + ar(\square BCED)$$

$$= 25 + 24$$

$$= 49 \text{ cm}^2$$

In ΔABC and ΔADE due to common angle,

$$\angle BAC = \angle DAE$$

Since $BC \| DE$, due to corresponding angle

$$\angle ABC = \angle ADE$$

Thus by AA axiom of similarity we have

$$\Delta ABC \sim \Delta ADE$$

Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

$$\frac{ar(\Delta ABC)}{ar(ADE)} = \frac{BC^2}{DE^2}$$

$$\frac{25}{49} = \frac{(BC)^2}{(14)^2}$$

$$\frac{25}{49} = \frac{(BC)^2}{196}$$

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(iii) Ratio of the areas of ΔDEF and ΔCFB?
Ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.
Since, $\Delta DEF \sim \Delta CFB$ we have

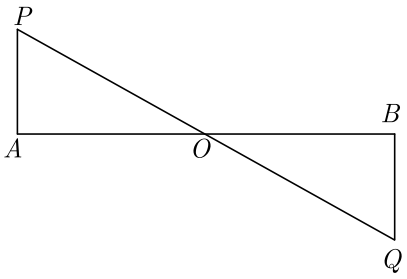
$$\frac{ar(\Delta DEF)}{ar(\Delta CFB)} = \frac{DE^2}{BC^2}$$

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25} \quad \text{[from Eq. (i)]}$$

Hence $ar(\Delta DEF) : ar(\Delta CFB) = 9 : 25$

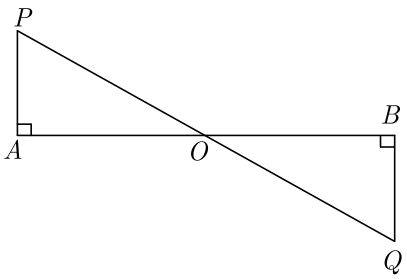
Comp 2007

72. In the figure, PA and QB are drawn perpendicular to AB. If $PO:OQ = 3:2$ and the area of $\Delta QOB = 120\ \text{cm}^2$. Calculate.
(i) Prove that $\Delta AOP \sim \Delta BOQ$.
(ii) AO if $AB = 7.5$ cm.
(iii) area of ΔPOA



Sol :

We have redrawn the given figure as below.



(i) $\Delta AOP \sim \Delta BOQ$.
Due to both 90° angle

$$\angle A = \angle B$$

Due to vertical opposite angle

$$\angle AOP = \angle BOQ$$

By AA axiom similarity we have

$$\Delta AOP \sim \Delta BOQ$$

(ii) AO if $AB = 7.5$ cm.
Let $AO = x$ then $OB = 7.5 - x$
If two triangles are similar, then their corresponding sides are proportional.
Thus

$$\frac{OA}{OB} = \frac{PO}{QO} = \frac{AP}{BP}$$

$$\frac{x}{7.5 - x} = \frac{3}{2}$$

$$2x = 22.5 - 3x$$

$$5x = 22.5$$

$$x = 4.5$$

Thus $AO = 4.5$ cm
(iii) Area of ΔPOA
Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{area } \Delta POA}{\text{area } \Delta QOB} = \frac{PO^2}{QO^2}$$

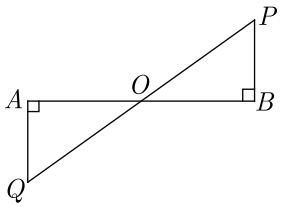
$$= \frac{3^2}{2^2} = \frac{9}{4}$$

$$\text{Area } \Delta POA = \frac{9}{4} \times 120$$

$$= 270\ \text{cm}^2$$

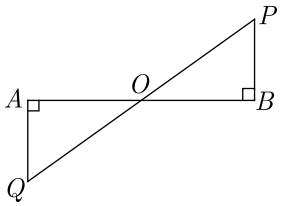
Main 2006

73. In the figure given alongside, PB and QA are perpendicular to the line segment AB. If $PO = 6$ cm, $QO = 9$ cm and the area of $\Delta POB = 120$ cm2, find the area of ΔAOQ.



Sol :

We have redrawn the figure below.



We have $PO = 6$ cm and $QO = 9$ cm
$PB \perp AB$ and $QA \perp AB$]

$$ar(\Delta POB) = 120\ \text{cm}^2$$

In ΔAOQ and ΔBOP, we have

$$\angle QAO = \angle OBP = 90^\circ$$

Due to vertically opposite angles,

$$\angle AOQ = \angle POB$$

Thus by AA axiom of similarity we have

$$\Delta AOQ \sim \Delta POB$$

Ratio of the areas of two similar triangles is equal

NODIA

CHAPTER 15

LOCUS

MULTIPLE CHOICE QUESTIONS

1. A point move in a plane about another point in such a way that its distance from it is constant. Locus of point is called:
(a) Axe (b) Angle
(c) Circle (d) None of these

Sol :

Locus of point is called circle.
Thus (c) is correct option.

2. The locus of a point equidistant from two point p and q in the same plane is called:
(a) Straight line
(b) Circle
(c) Angle
(d) Perpendicular bisector

Sol :

Locus of point is called perpendicular bisector.
Thus (d) is correct option.

3. If the locus of a moving point which is always equidistant from the points $(2, -1)$ and $(3, 2)$ then it represents the curve:
(a) Hyperbola (b) Ellipse
(c) Straight line (d) Circle

Sol :

Let (a, b) be the co-ordinates of a point Q on the required locus.

WE have $A = (2, -1)$ and $B = (3, 2)$

Using distance formula we have

$$QA^2 = (a-2)^2 + (b+1)^2$$

and $$QB^2 = (a-3)^2 + (b-2)^2$$

$$QA = QB$$

$$QA^2 = QB^2$$

$$(a-2)^2 + (b+1)^2 = (a-3)^2 + (b-2)^2$$

and we get $a + 3b = 4$

which is the straight line equation.
Thus (c) is correct option.

4. A point moves in such a manner that 2 times of its abscissa is greater by 4 than times of Its ordinate then the equation of its locus is:
(a) $2x - 3y = 4$ (b) $2x + 3y = 4$
(c) $3x - 2y = 4$ (d) $3x + 2y = 4$

Sol :

As abscissa represents x-axis and ordinate represents y-axis.

$$2x - 3y = 4.$$

Thus (a) is correct option.

5. Geometry made by a locus of a swimmer maintaining the same distant from a pole in water is:
(a) Circle (b) Triangle
(c) Rectangle (d) Square

Sol :

Locus of point is called circle.
Thus (a) is correct option.

6. Which of the following statement is correct:
(a) A point Q moving in a plane about a fixed line as on are with same centre.
(b) A moving point Q in a plane about another point in such a way that its distance is constant.
(c) A point equidistant from two fixed non-parallel AB and CD in the same plane.
(d) A point Q moving in a plane about another point in such a way that its distance from a fixed line AB is constant.

Sol :

According to locus postulate.
Thus (a) is correct option.

7. Chords which are ..... from the centre are equal.
(a) Unequal (b) Equidistant

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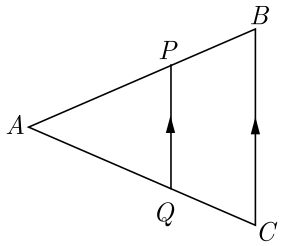
$$(BC)^2 = \frac{25 \times 196}{49}$$

$$(BC)^2 = 100$$

$$BC = 10 \text{ cm}$$

Comp 2000

80. In the following figure
(i) Prove that $\Delta APQ \sim \Delta ABC$.
(ii) If $AP : PB = 4:3$, find PQ if $BC = 14$ cm.
(iii) Find area ΔAPQ : area ΔABC.
(iv) Find area ΔAPQ : area $\square PBCQ$.



Sol :

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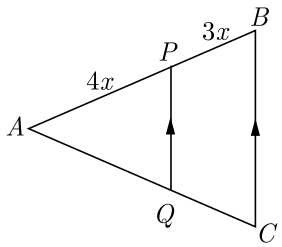


In ΔAPQ and ΔABC,
Due to common angle,

$$\angle A = \angle A$$

Due to corresponding angle,

$$\angle P = B$$

By AA-axiom similarity we have

$$\Delta APQ = \Delta ABC$$

(ii) If $AP : PB = 4:3$, find PQ if $BC = 14$ cm.
If two triangles are similar, then their corresponding sides are proportional.

Thus
$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{4}{7} = \frac{PQ}{14}$$

$$PQ = 8 \text{ cm}$$

(iii) area ΔAPQ : area ΔABC
Since areas of two similar triangles are proportional to the square of their corresponding sides. Thus

$$\frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$= \frac{4^2}{7^2} = \frac{16}{49}$$

(iv) area ΔAPQ : area $\square PBCQ$

We have
$$\frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta ABC)} = \frac{16}{49}$$

$$\frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta ABC) - \text{Area}(\Delta APQ)} = \frac{16}{49 - 16}$$

$$\frac{\text{Area}(\Delta APQ)}{\text{Area}(\square PBCQ)} = \frac{16}{33}$$

END

NODIA

14. A point P is equidistant from the points (–2, 0) and (3, 2). The locus of point P is:
(a) $10x + 4y + 9 = 0$
(b) $10x + 4y + 8 = 0$
(c) $10x + 4y - 9 = 0$
(d) $10x - 4y + 9 = 0$

Sol :

Let $P(x, y)$ be the given point on the locus.

and $A = (-2, 0)$ and $B = (3, 2)$

then $PA^2 = PB^2$

$$(x+2)^2 + (y-0)^2 = (x-3)^2 + (y-2)^2$$

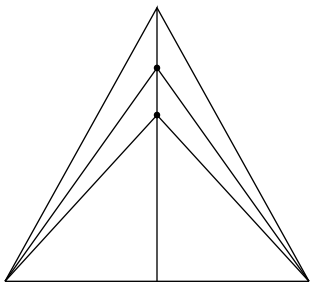
Solving, we get

$$10x + 4y - 9 = 0.$$

Thus (c) is correct option.

15. The locus of vertices of all isosceles triangles having a common base will be the of the common base of the triangles:
(a) Perpendicular
(b) Perpendicular Bisector
(c) Bisector
(d) None of the above

Sol :



Thus (d) is correct option.

16. 17. The locus of a point Pn space which is always at a distance of 5 cm from a fixed point will be the:
(a) Sphere
(b) Centre of circle
(c) Centre of sphere
(d) None of the above

Sol :

Because in space we see 3-axis i.e., x-axis, y-axis and z-axis.
Thus (d) is correct option.

17. A point Q is at the distance of 5 units from (3, 1). Equation of locus of point D is:
(a) $x^2 + y^2 - 6x + 4y - 15 = 0$
(b) $x^2 + 6x + 4y - 15 = 0$
(c) $x^2 + y^2 - 6x - 2y - 15 = 0$
(d) $x^2 + y^2 + 6x - 2y - 15 = 0$

Sol :

Let $Q(x, y)$ be the point
Using distance formula we have

$$(x-3)^2 + (y-1)^2 = (5)^2$$

$$x^2 + 9 - 6x + y^2 + 1 - 2y = 25$$

$$x^2 + y^2 - 6x - 2y - 15 = 0$$

Thus (c) is correct option.

18. A point P is allowed to travel in space. What is the locus of P so that it always remains at a constant distance from a fixed point C.
(a) Sphere (b) Cube
(c) Circle (d) Line

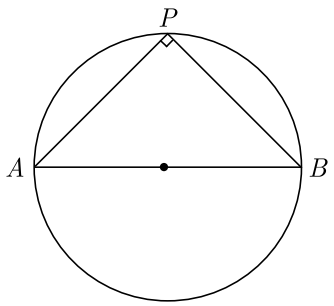
Sol :

The point P is moving in the space and it is at a constant distance from a fixed point C. Its locus is a sphere.
Thus (a) is correct option

19. AB is a fixed line. Which of the following is the locus of the point P so that $\angle APB = 90^o$.
(a) Sphere (b) Cube
(c) Circle (d) Line

Sol :

AB is a fixed line and P is a point such that. $\angle APB = 90^o$



The locus of P will be the circle whose diameter is AB. We know that angle in a semi-circle is always equal to 90^o.

$$\angle APB = 90^o$$

Thus (c) is correct option.

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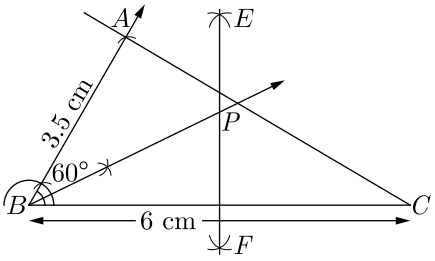
inside the triangle, which are equidistant from B and C.

Step 7 : The intersection point of angle bisector and EF is the required point P, which is equidistant from AB, BC, B and C. On measuring the length of PB is 3.5 cm.



Main 2009

37. Using ruler and compass, construct

(i) a ΔABC in which $AB = 5.5$ cm, $BC = 3.4$ cm and $CA = 4.9$ cm.

(ii) the locus of points equidistant from A and C.

(iii) a circle touching AB at A and passing through C.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $AC = 4.9$ cm.

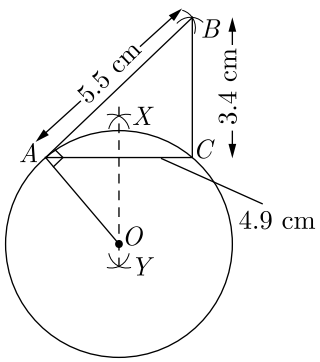
Step 2 : Taking A as centre mark an arc of length 5.5 cm and taking B as centre mark an arc of length 3.4 cm, which intersect the previous arc at point B.

Step 3 : Join AB and BC, which is the required ΔABC.

Step 4 : Draw perpendicular bisector XY of AC, which is the required locus of points equidistant from A and C.

Step 5 : Draw perpendicular to AB at A, which intersect the line XY at O.

Step 6 : Taking O as centre and AO or OC as radius, draw a circle, which touching AB at A and passing through C.



Main 2008

38. A straight line AB is 8 cm long. Locate by construction the locus of a point which is

(i) equidistant from A and B.

(ii) always 4 cm long from the line AB.

(iii) Mark two points X and Y, which are 4 cm from AB equidistant from A and B. Name the figure $AXBY$.

Sol :

Steps of Construction :

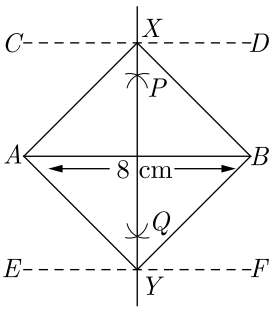
Step 1 : Draw a line segment $AB = 8$ cm.

Step 2 : Draw perpendicular bisector PQ of AB, which is the locus of a point which is equidistant from A and B.

Step 3 : Draw CD and EF both parallel to opposite side of AB and each at a distance of 4 cm from AB. Thus, CD and EF are the required locus.

Step 4 : Extends PQ on both sides meet the lines CD and EF at X and Y respectively, which are at a distance 4 cm from AB.

Step 5 : Join AX, XB, BY and YA. Hence, the figure formed $AXBY$ is a square.



Main 2006

39. Ruler and compasses only may be used in this question. All construction lines and arcs must be clearly shown, and be of sufficient length and clarity to permit assessment.

(i) Construct a triangle ABC, in which $BC = 6$ cm, $AB = 9$ cm. and $\angle ABC = 60°$.

(ii) Construct the locus of all points, inside ΔABC, which are equidistant from B and C.

(iii) Construct the locus of the vertices of the triangles with BC as base, which are equal in area to ΔABC.

(iv) Mark the point Q, in your construction, which would make ΔQBC equal in area to ΔABC, and isosceles.

(v) Measure and record the length of CQ.

Sol :

Steps of Construction :

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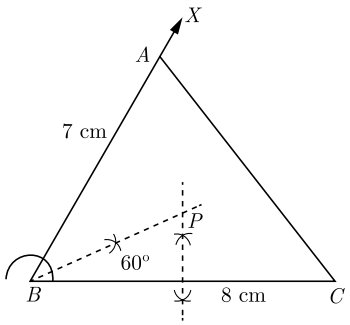
(ii) P is equidistant from AB and BC
(iii) Measure and record the length of PB.

Sol :

Steps of Construction :

Step 1 : Take $BC = 8$ cm a long line segment.
At B, draw a ray BX making an angle of $60°$ with BC.
Step 2 : Cut off $BA = 7$ cm. and join AC.
Step 3 : Draw the perpendicular bisector of BC.
Step 4 : Draw the angle bisector of $\angle B$ which intersect the perpendicular bisector of BC at P. P is the required point.
Step 4 : On measuring the length of $BP = 4.6$ cm (approx.)



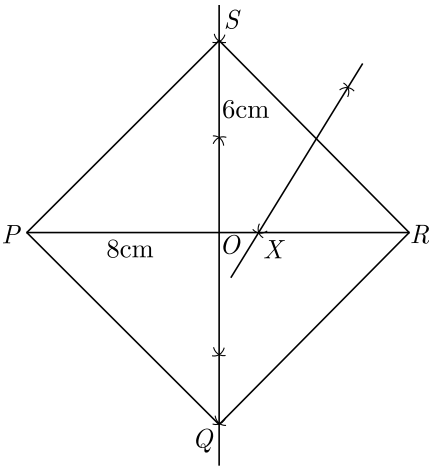
Comp 2019, Main 2003

24. Construct a rhombus $PQRS$ whose diagonals PR, QS are 8 cm and 6 cm respectively. Find by construction a point X equidistant from PQ, PS and equidistant from R, S. Measure XR.

Sol :

Steps of Construction :

Step 1 : Take PR 8 cm and draw the perpendicular bisector of PR intersecting it at O.
Step 2 : From O, out. off $OS = OQ = 3$ cm
Step 3 : Join PQ, QR, RS and SP. $PQRS$ is a rhombus. Whose diagonal are PR and QS. PR is the bisector of $\angle SPQ$.
Step 4 : Draw the perpendicular bisector of SR intersecting PR at X. X is equidistant from PQ and PS and also from S and R.
Step 5 : On measuring length of $XR = 3.2$ cm (approx)



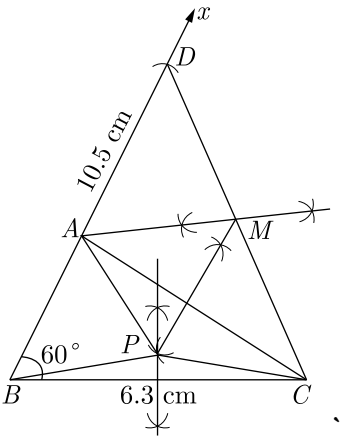
SQP 2018

25. Draw a triangle ABC such that $BC = 6.3$ cm, $AB + AC = 10.5$ cm and $\angle ABC = 60°$. Find a point P such that $PA = PB = PC$.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 6.3$ cm.
Step 2 : At B, draw a ray BX making an angle of $60°$ and cut off $BD = 10.5$ cm.
Step 3 : Join DC.
Step 4 : Draw the perpendicular bisector of CD which intersects BD at A.
Step 5 : Join AC.
Step 6 : Draw the perpendicular bisectors of BC and AC intersecting each other at P.



Then P is the required point which is equidistant from A, B and C.

Comp 2018, SQP 2015

26. Draw a ΔPQR such that $QR = 7.5$ cm, $\Delta PQR = 60°$ and altitude $PN = 4$ cm. Find a point A inside the triangle such that it is equidistant from PR and QR. and also equidistant

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from P and R.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $QR = 7.5$ cm.

Step 2 : At R draw a ray perpendicular and cut off $RS = PN = 4$ cm.

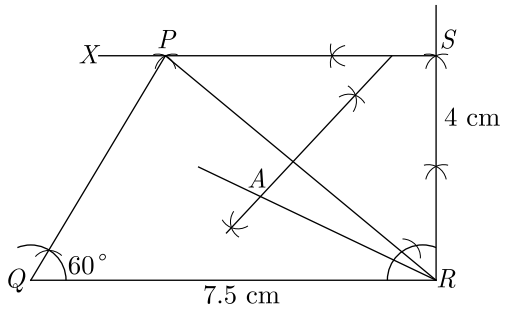
Step 3 : From S, draw a line SX parallel to QR.

Step 4 : At Q, draw a ray making an angle of 60^o which intersects the line SX at P.

Step 5 : Join PR.

Step 6 : Draw the angle bisector of $\angle R$.

Step 7 : Draw the perpendicular bisectors of PR which intersect the angle bisector of $\angle R$ at A.



Then A is the required point which is equidistant from PQ and QR and also from P and R.

Comp 2017

27. Draw a ΔPQR such that $QR = 7$ cm, $PQR = 45°$ and $PQ - PR = 1.8$ cm. Find a point O such that it is equidistant from PQ, QR and PR.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $QR = 7$ cm.

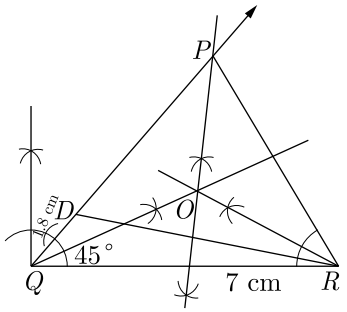
Step 2 : At Q, draw a ray making an angle of $45°$ and cut off $QD = 1.8$ cm.

Step 3 : Join DR.

Step 4 : Draw the perpendicular bisector of DR which intersects QD on producing at P.

Step 5 : Join PR.

Step 6 : Draw the angle bisector of $\angle Q$ and $\angle R$ intersecting each other at O.



Then O is the required point which is equidistant from PQ, QR and PR.

Main 2017

28. Use ruler and compasses only for this question.

(i) Construct ΔABC, where $AB = 3.5$ cm, $BC = 6$ cm and $\angle ABC = 60^o$.

(ii) Construct the locus of points inside the triangle which are equidistant from BA and BC.

(iii) Construct the locus of points inside the triangle which are equidistant from B and C.

(iv) Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB. (2010)

Sol :

Steps of construction :

Step 1 : Draw a line segment $BC = 6$ cm

Step 2 : At, B draw a ray BX making an angle of $60°$ and cut off $BA = 3.5$ cm.

Step 3 : Join AC. The ΔABC is the required triangle.

Step 4 : Draw the bisector BY of $\angle ABC$.

Step 5 : Draw the perpendicular bisector of BC which intersects BY at P. P is the required point P which is equidistant from BC and BA and also equidistant from B and C.

Step 6 : On measuring PB it is 3.4 cm (approx)

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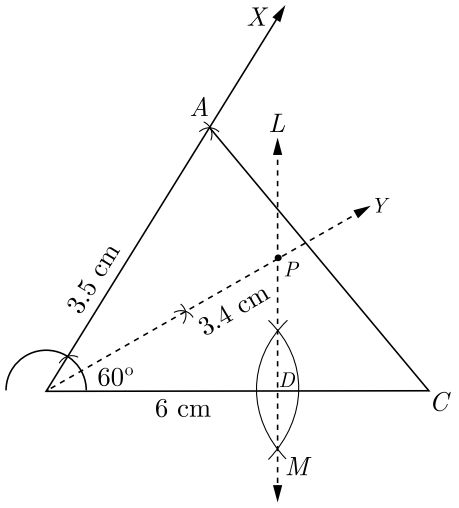


Comp 2016

29. A straight line AB is 8 cm long. Locate by construction the locus of a point which is :

(i) Equidistant from A and B.

(ii) Always 4 cm from the line AB.

(iii) Mark two points X and Y, which are 4 cm from AB and equidistant from A and B. Name the figure $AXBY$.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $AB = 8\,cm$.

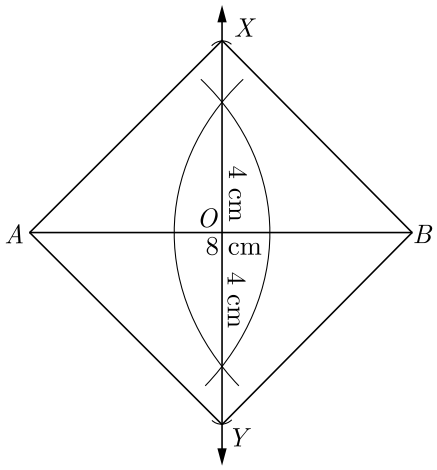
Step 2 : With the help of compasses and ruler, draw the perpendicular bisector I of AB which intersects AB at O.

Step 3 : Then any point on l, is equidistant from A and B.

Step 4 : Cut off $OX = OY = 4$ cm. The X and Y are the required locus, which is equidistant from AB and also from A and B.

Step 5 : Join AX, XB, BY and YA.

The figures so formed $AXBY$, is the shape of a square because its diagonals are equal and bisect each other at right angles.



SQP 2015

30. Draw a line segment AB of length 12 cm. Mark M, the mid-point of AB. Draw and describe the locus of a point which is

(i) at a distance of 3 cm from AB.

(ii) at a distance of 5 cm from the point M. Mark the points P, Q, R, S which satisfy both the above conditions. What kind of quadrilateral is $PQRS$? Compute the area of the quadrilateral $PQRS$.

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Sol :

Steps of Construction :

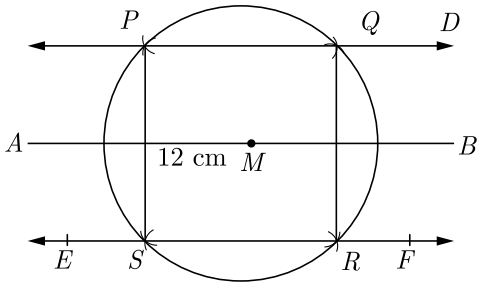
Step 1 : Take a line $AB = 12$ cm

Step 2 : Take M, the mid point of AB.

Step 3 : Draw straight lines CD and EF parallel to AB at a distance of 3 cm.

Step 4 : With centre M and radius 5 cm, draw areas which intersects CD at P and Q and EF at R and S.

Step 5 : Join QR and PS. $PQRS$ is a rectangle where length $PQ = 8$ cm.



Area of rectangle

$$PQRS = PQ \times RS$$

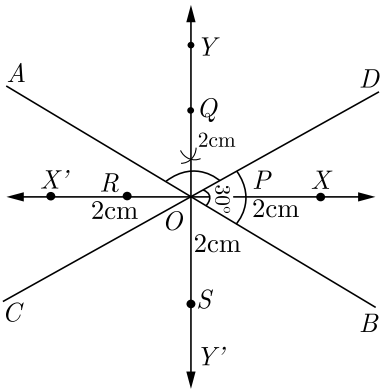
$$= 8 \times 6$$

$$= 48 \text{ cm}^2$$

other at O.

Step 2 : Draw the bisectors of $\angle BOD$ and $\angle AOD$.

Step 3 : With centre O and radius equal to 2 cm. marks points on the bisector of angles at P, Q, R and S respectively.



Hence there are four points which are equidistant from AB and CD and 2 cm from O, the point of intersection of AB and CD.

Main 2000

43. Construct a ΔABC with $AB = 7$ cm, $BC = 8$ cm and $\angle ABC = 60^\circ$, locate by construction the point P, such that

(i) P is equidistant from B and C.

(ii) P is equidistant from AB and BC.

(iii) Measure and record the length of PB.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $AB = 7$ cm.

Step 2 : Draw an $\angle XBA = 60^\circ$ at point B with the help of compass.

Step 3 : From point B, cut $BC = 8$ cm from BX.

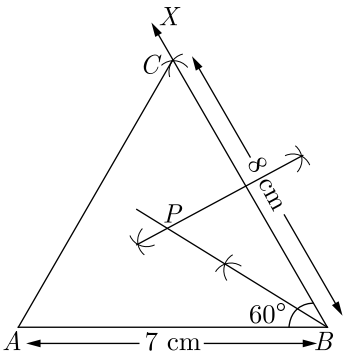
Step 4 : Join A and C, which is the required ΔABC.

Step 5 : Draw perpendicular bisector of line BC . The point situated on this line will be equidistant from B and C.

Step 6 : Draw angular bisector of $\angle ABC$. Any point situated on this angular bisector is equidistant from lines AB and BC.

Step 7 : The intersection point of perpendicular bisector point P, which is equidistant from lines AB and BC and also from points B and C.

Step 8 : Length of PB found to be 4.5 cm.



Comp 2000

44. Using ruler and compasses construct :

(i) a triangle ABC in which $AB = 5.5$ cm, $BC = 3.4$ cm and $CA = 4.9$ cm.

(ii) the locus of points equidistant from A and C.

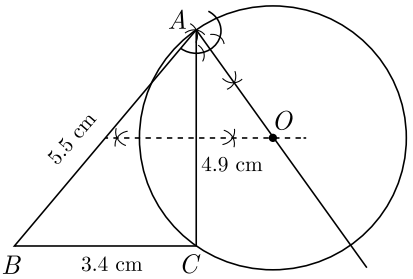
Sol :

Steps of Construction :

Step 1 : Draw $BC = 3.4$ and mark the arcs of 5.5 arid 4.9 cm from B and C. Join A, B and C. ABC is the required triangle.

Step 2 : Draw $\perp$ bisector of AC.

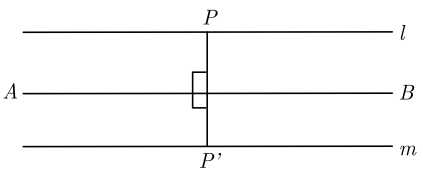
Step 3 : Draw an angle of 90° at AB at A which intersects $\perp$ bisector at O. Draw circle taking O as centre and OA as radius.



45. A point moves such that its distance from a fixed line AB is always the same. What is the relation between AB and the path travelled by P ?

Sol :

Let point P moves in such a way that it is at a fixed distance from the fixed line AB.



It is a set of two lines l and m parallel to AB

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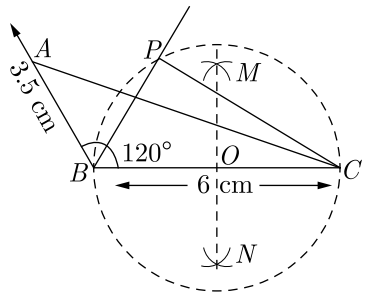
Step 8 : Join BP and CP. On measuring, we get $\angle BCP = 30^\circ$.



Main 2012

34. Without using set square or protractor, construct the quadrilateral $ABCD$ in which $\angle BAD = 45^o$, $AD = AB = 6\,cm$, $BC = 3.6$ cm and $CD = 5$ cm.

(i) Measure $\angle BCD$.

(ii) Locate the point P on BD which is equidistant from BC and CD.

Sol :

Steps of Construction :

Step 1 : Take $AB = 6$ cm long

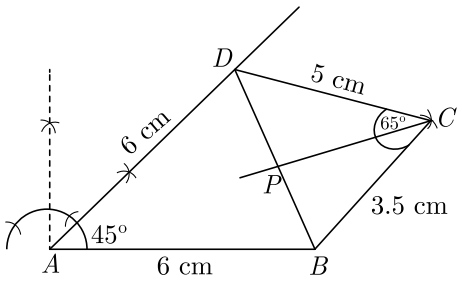
Step 2 : AT A, draw the angle of 450 and cut off $AD = 6$ cm.

Step 3 : With centre D and radius 5 cm and with centre B, and radius 3.5 cm draw two arcs intersecting each other at C.

Step 4 : Join CD and CB and join BD. $ABCD$ is the required quadrilateral.

Step 5 : On measuring $\angle BCD = 65^o$.

Step 6 : Draw the bisector of $\angle BCD$ which intersects BD at P. P is the required point which is equidistant from CD and CB.



Main 2011

35. Without using set square or protractor, construct rhombus $ABCD$ with sides of length 4 cm and diagonal AC of length 5 cm. Measure $\angle ABC$. Find the point R on AD such that $RB = RC$. Measure the length of AR.

Sol :

Steps of Construction :

Step 1 : Take $AB = 4$ cm

Step 2 : With centre A, draw an arc of 5 cm radius and with B draw another arc of radius 4 cm intersecting each other at C.

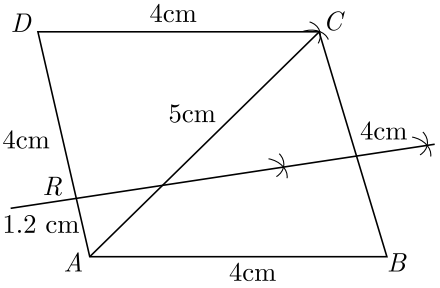
Step 3 : Join AC and BD.

Step 4 : Again with centre A and C, draw two arcs of radius 4 cm intersecting each other on D.

Step 5 : Join AD and CD. $ABCD$ is the required rhombus and on measuring the $\angle ABC$, it is 78^o.

Step 6 : Draw perpendicular bisector of BC intersecting AD at R.

Step 7 : On measuring the length of AR, it is equal to 1.2 cm.



Main 2010

36. Use ruler and compass only for this question.

(i) Construct ΔABC, where $AB = 3.5$ cm, $\angle ABC = 60^\circ$.

(ii) Construct the locus of points inside the triangle, which are equidistant from BA and BC.

(iii) Construct the locus of points inside the triangle which are equidistant from B and C.

(iv) Mark the point P which is equidistant from AB, BC and also equidistant from B and C . Measure and record the length of PB.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 6$ cm.

Step 2 : From the point B, draw $\angle XBC = 60^\circ$.

Step 3 : Taking radius 3.5 cm, with B as centre cut $BA = 3.5$ cm from BX.

Step 4 : Join $A$to C, which is the required ΔABC .

Step 5 : Draw angle bisector of $\angle ABC$, which is the locus of points inside the triangle, which are equidistant from BA and BC.

Step 6 : Draw perpendicular bisector EF of BC, which intersects the angle bisector at point P. Then, EF is the locus of points

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TWO MARKS QUESTIONS

THREE MARKS QUESTIONS

Comp 2023

20. Two straight lines PQ and PK cross each other at P at an angle of 75^o. S is a stone on the road PQ. 800 m from P towards Q. By drawing a figure to scale 1 cm = 100 m, locate the position of a flag staff X, which is equidistant from P and 5, and is also equidistant from the road.

Sol :

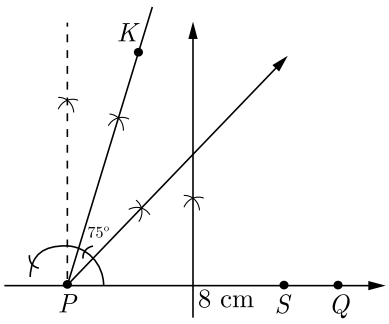
Steps of Construction :

Step 1 : Draw the lines PQ and PK intersecting each other at P making an angle of 75°.

Step 2 : Take a point S on PQ such that $PS = 8$ cm.

Step 3 : Draw the perpendicular bisector of PS.

Step 4 : Draw the angle bisector of $\angle KPS$ intersecting the perpendicular bisector at X. X is the required point which is equidistant from P and S and also from PQ and PK.



Comp 2022

21. By using ruler and compass only, construct a quadrilateral $ABCD$ in which 6.5 cm, $AB = 6.5$ cm, $AD = 4$ cm and $\angle DAB = 75^o$ C is equidistant from the sides AB and AD, also C is equidistant from the points A and B.

Sol :

Steps of Construction :

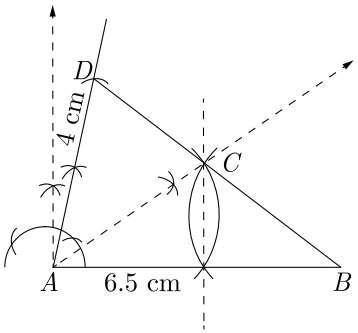
Step 1 : Draw a line segment $AB = 6.5$ cm.

Step 2 : At A, draw a ray making an angle of 75^o and cut off $AD = 4$ cm.

Step 3 : Draw the bisector of $\angle DAB$.

Step 4 : Draw perpendicular bisector of AB intersecting the angle bisector at C.

Step 5 : Join CB and CD. $ABCD$ is the required quadrilateral.



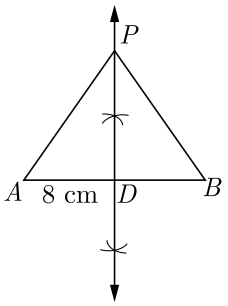
SQP 2019

22. Draw a straight line AB of length 8 cm. Draw the locus of all points which are equidistant from A and B. Prove your statement.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $AB = 8$ cm.



Step 2 : Draw the perpendicular bisector of AB intersecting AB at D. Every point P on it will be equidistant from A and B.

Step 2 : Take a point P on the perpendicular bisector.

Step 3 : Join PA and PB.

Proof :

In ΔPAD and ΔPBD

$PD = PD$ (common)

$AD = BD$ (D is mid-point of AB)

$\angle PDA = \angle PDB$ (each 90^o)

$\Delta PAD \cong PBD$ (SAS)

$PA = PB$ (c.p.c.t.)

Similarly we can prove any other point on the perpendicular bisector of AB, is equidistant from A and B.

Main 2018

23. Construct triangle ABC, with $AB = 7$ cm, $BC = 8$ cm and $\angle ABC = 60^o$. Locate by construction the point P such that :

(i) P is equidistant from B and C and

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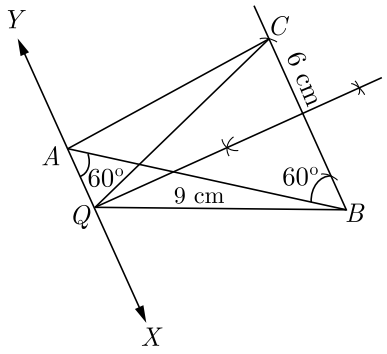
Step 1 : Draw $AB = 9$ cm
Step 2 : At B draw an angle of 60^o and cut off $BC = 6$ cm.
Step 3 : Join AC
Step 4 : Draw perpendicular bisector of BC. All points on it, will be equidistant from B and C.
Step 5 : From A, draw a line XY parallel to BC.
Step 6 : Produce the perpendicular bisector of BC to meet XY in Q.
Step 6 :Join QC and QB.



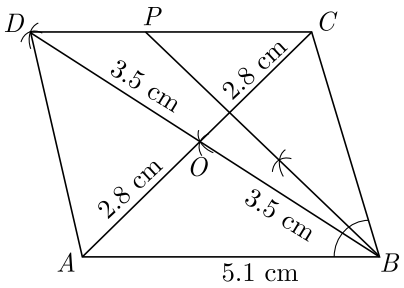
ΔQBC will be the triangle equal in area to ΔABC because these are on the same base BC and between the same parallel lines. On measuring, the length of CQ is 8.2 cm (approx).

Main 2004

40. Without using set square or protractor, construct the parallelogram $ABCD$ in which $AB = 5.1$ cm. the diagonal $AC = 5.6$ cm and the diagonal $BD = 7$ cm. Locate the point P on DC, which is equidistant from AB and BC

Sol :

Steps of Construction :
Step 1 : Take $AB = 5.1$ cm
Step 2 : At A, with readius $\frac{5.6}{2} = 2.8$ cm and at B with radius $\frac{7.0}{2} = 3.5$ cm, draw two arcs intersecting each other at O.
Step 3 : Join AO and produce it to C such that $OC = AD = 2.8$ cm and join BO and produce it to D such that $BO = OD = 3.5$ cm
Step 4 : Join BC, CD, DA. $ABCD$ is a parallelogram.
Step 5 : Draw the angle bisector of $\angle ABC$ intersecting CD at P. P is the required point which is equidistant from AB and BC.

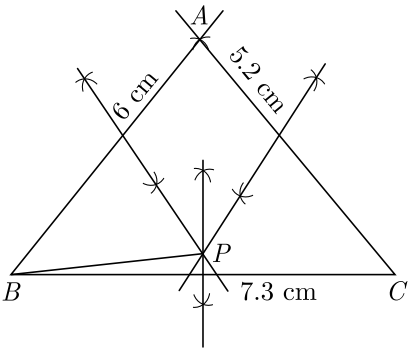


Comp 2003

41. Points A, B and C represent position of three towers such that $AB = 60$ m, $BC = 73$ m and $CA = 52$ m . Taking a scale of 10 m to 1 cm, make an accurate drawing of ΔABC. Find by drawing, the location of a point which is equidistant from A, B and C, and its actual distance from any of the towers.

Sol :

Steps of Construction :
Step 1 : Draw a line segment $BC = 7.3$ cm
Step 2 : With centre B and radius 6 cm and with centre C and radius 5.2 cm, draw two arcs intersecting each other at A.
Step 3 : Joining AB and AC.
Step 4 : Draw the perpendicular bisector of AB, BC and CA respectively, which intersect each other at point P. Join PB. P is equidistant from A, B and C



Step 5 : On measuring $PB = 3.7$ cm. Actual distance is 37 m.

Comp 2002

42. Draw two intersecting lines to include an angle of 30°. Use ruler and compasses to locate points which are equidistant from these lines and also 2 cm away from their point of intersection. How many such points exist ?

Sol :

Steps of Construction :
Step 1 : Two lines AB and CD intersect each

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Main 2014

31. Construct a triangle ABC with $AB = 5.5$ cm, $AC = 6$ cm and $\angle BAC = 105^0$. Hence:

(i) Construct the locus of points equidistant from BA and BC.

(ii) Construct the locus of points equidistant from B and C.

(iii) Mark the point which satisfies the above two loci as P. Measure and write the length of PC.

Sol :

Steps of Construction :

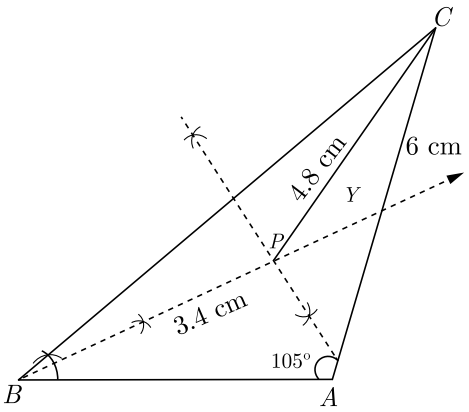
Step 1: Construct the triangle ABC with $AB = 5.5$ cm $\angle BAC = 105^0$ and $AC = 6$ cm.

Step 2 : Draw the bisector of $\angle ABC$. Points which are equidistant from BA and BC lies on the bisector of $\angle ABC$.

Step 3 : Draw perpendicular bisector of BC. Points equidistant from B and C lies on the perpendicular bisector of BC.

Step 4 : The required point P is the point of intersection of the bisector of $\angle ABC$ and the perpendicular bisector of BC.

Step 5 : Required length of $PC = 4.8$ cm.



Comp 2014

32. Without using set-squares or protractor construct :

(i) Triangle ABC, in which $AB = 5.5$ cm, $BC = 3.2$ cm and $CA = 4.8$ cm.

(ii) Draw the locus of a point which moves so that it is always 2.5 cm from B.

(iii) Draw the locus of a point which moves so that it is equidistant from the sides BC and CA.

(iv) Mark the point of intersection of the loci with the letter P and measure PC.

Sol :

Steps of Construction :

Step 1 : Draw $BC = 3.2$ cm long.

Step 2 : With centre B and radius 5.5 cm and with centre C and radius 4.8 cm draw arcs intersecting each other at A.

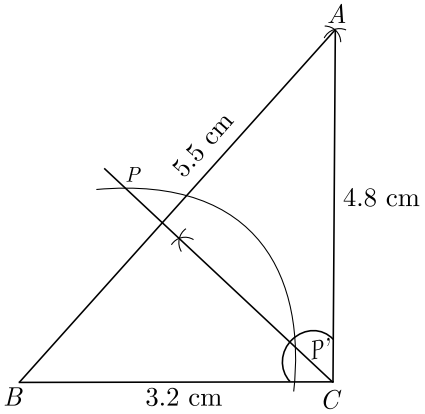
Step 3 : Join AB and AC.

Step 4 : Draw the bisector of $\angle BCA$.

Step 5 : With centre B and radius 2.5 cm, draw an arc intersecting the angle bisector of $\angle BCA$ at P and P'. P and P' are two loci which satisfies the given condition.

Step 6 : On measuring CP and CP' $CP = 3.6$ cm and $CP = 1.1$ cm.

NODIA



Main 2013, 2005

33. Using a ruler and compass only.

(i) Construct a ΔABC with the following data, $AB = 3.5$ cm, $BC = 6$ cm and $\angle ABC = 120^\circ$.

(ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle that is equidistant from AB and BC.

(iii) Measure $\angle BCP$.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 6$ cm.

Step 2 : From the point B, draw $\angle ABC = 120^\circ$.

Step 3 : Taking length 3.5 cm, with B are centre cut $BA = 3.5$ cm from BX.

Step 4 : Join A to C. Thus, we get the required ΔABC.

Step 5 : Draw perpendicular bisector MN of BC, which meet BC at point O.

Step 6 : Draw a circle O as centre and OC or OB as radius.

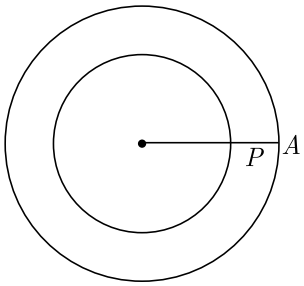
Step 7 : Draw angle bisector of $\angle ABC$, which intersects the circumference of the cirlce at P. Then, point P is equidistant from AB and BC.

drawn on either side of it at equal distance from it.

46. Describe completely the locus of points of mid-point of radii of a circle.

Sol :

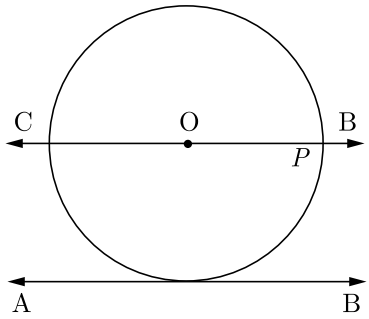
The locus of mid points of the radii of a circle is another concentric circle with radius is half of the radius of the given circle.



47. Describe completely the locus of points of centre of a ball, rolling along a straight line on a level floor.

Sol :

AB is the straight line on the ground and the ball is rolling on it

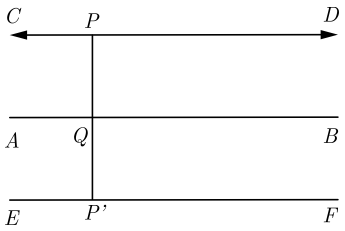


Locus of the centre of the ball is a line parallel to the given line AB.

48. Describe completely the locus of point in a plane equidistant from a given line.

Sol :

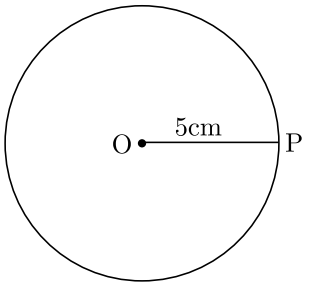
From P, draw a line CD and another line EF from P' parallel to AB.
Thus CD and EF are the lines which are the locus of the point equidistant from AB.



49. Describe completely the locus of point in a plane, at a constant distance of 5 cm from a fixed point (in the plane).

Sol :

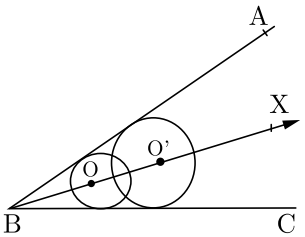
Take a point O and another point P such that $OP = 5\,cm$. with centre O and radius equal to OP, draw a circle. Thus this circle is the locus of point P which is at a distance of 5 cm from O, the given point.



50. Describe completely the locus of points of centre of a circle of varying radius and touching two arms of $\angle ABC$.

Sol :

Draw the bisector BX of $\angle ABC$. This bisector of angle is the locus of the centre of a circle with different radii.
Any point on BX, is equidistant from the arms BA and BC of the $\angle ABC$.



51. Describe completely the locus of points of centre of a circle of radius 2 cm and touching a fixed circle of radius 3 cm with centre O.

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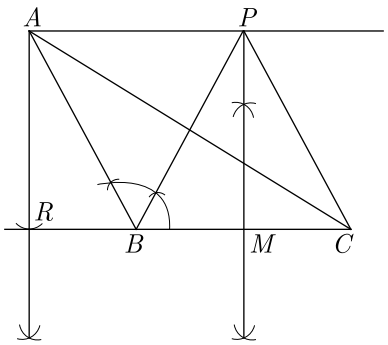
(iii) Mark the point of intersection of the two loci.

Sol :

Step 1 : Draw a line segment $BC = 6$ cm.
Step 2 : At B, draw a ray BX making an angle of $120°$ and cut off $BA = 7.5$ cm.
Step 3 : Join AC. This is required $\triangle ABC$.
Step 4 : Extend B till N. Draw perpendicular AR on BN.
Step 5 : Draw perpendicular bisector of BC. This is locus of point equidistant from B and C.
Step 6 : On above perpendicular cut off $MP = RA$ and draw a line parallel to BC.
Step 7 : P lies on this parallel line and on the perpendicular bisector of BC.



Height of ΔPBC = height of ΔABC
Base BC is same, we get .
Area ΔABC = Area ΔPBC.

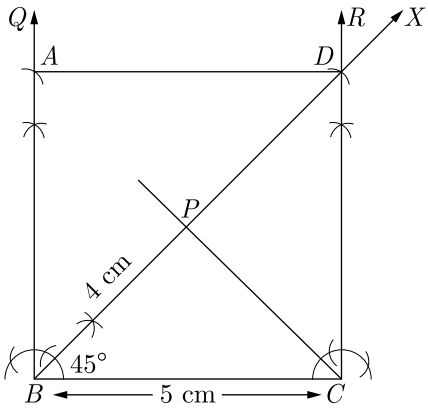
Main 2007

63. Construct a ΔBCP, given $BC = 5$ cm, $BP = 4$ cm and $\angle CBP = 45°$.
(i) Complete the rectangle $ABCD$, such that
(a) P is equidistant from AB and BC.
(b) P is equidistant from C and D.
(ii) Measure and record the length of AB.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 5$ cm.
Step 2 : Draw a ray making $\angle XBC = 45°$
Step 3 : Cut $BP = 4$ cm from BX and join PC. Then, ΔBPC is the required triangle.
Step 4 : Draw $BQ \perp BC$ and $CR \perp BC$. Here, PB is the bisector of $\angle QBC$, so it is equidistant from BC and QB.
Step 5 : Now, cut BQ at A such that $PA = PC$ and cut CR at D such that $PC = PD$.
Step 6 : Join AD. Thus, the rectangle $ABCD$ is the required rectangle.
Step 7 : On measuring the length of $AB = 5.7$ cm. (approx).



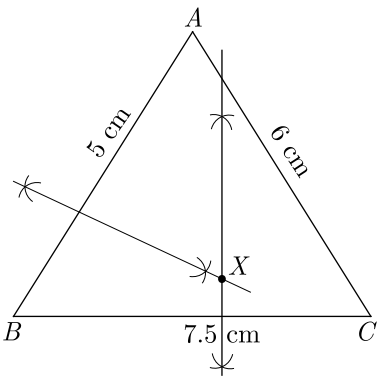
Comp 2007

64. Construct ΔABC in which $AB = 5$ cm, $BC = 7.5$ cm and $AC = 6$ cm. Construct the locus of points which are equidistant from
(i) A and B
(ii) B and C
(iii) Mark a point X, the intersection of the two loci. What is the special name given to the point ?

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 7.5$cm
Step 2 : With centre B and radius 5 cm and with centre C and radius 6 cm draw arcs which intersect each other at A.
Step 3 : Join AB and AC. This is required $\triangle ABC$.
Step 4 : Draw the perpendicular bisector of AB which is locus of equidistant from A and B.
Step 5 : Draw the perpendicular bisector of BC which is locus of equidistant from B and C.
Step 6 : Mark the intersection point of perpendicular bisector of BC and AB as X. X is called circumcentre of ΔABC.



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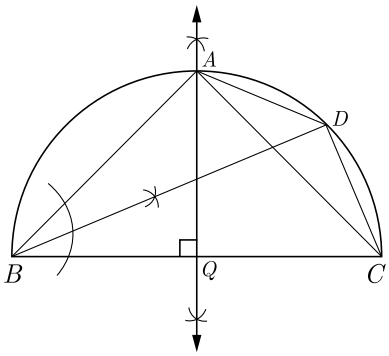


Step 7 : On measuring $ADC = 135^\circ$.

Main 2016

54. Use ruler and compass only for the following question. All construction lines and arcs must be clearly shown.

(i) Construct a ΔABC in which $BC = 6.5$ cm, $\angle ABC = 60^\circ$, $AB = 5$ cm.

(ii) Construct the locus of points at a distance of 3.5 cm from A.

(iii) Construct the locus of points equidistant from AC and BC.

(iv) Mark 2 points X and Y which are at a distance of 3.5 cm from A and also equidistant from AC and BC. Measure XY.

Sol :

We have $BC = 6.5$ cm, $\angle ABC = 60^\circ$ and $AB = 5$ cm.

Steps of Construction :

Step 1 : Draw $BC = 6.5$ cm.

Step 2 : At point B, make $\angle ZBC = 60^\circ$, with compasses.

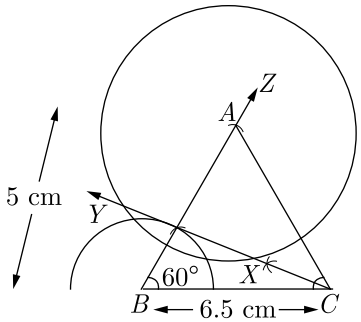
Step 3 : Taking radius 5 cm, with B as centre cut $BA = 5$ cm from BZ.

Step 4 : Join A to C, which is the required triangle.

Step 5 : Draw a circle with A as centre and radius equal to 3.5 cm, which is the locus of a point equidistant from point A.

Step 6 : Draw angle bisector of $\angle ACB$, which is the locus of points equidistant from AC and BC.

Step 7 : Since, point are equidistant from point A and also equidistant from AC and BC, therefore, the required point will be the point of intersection of angle bisector of $\angle ACB$ and circle $XY = 5.2$ cm.



SQP 2016, Main 2005

55. Draw a triangle LMN such that $LM = 3.9$ cm. $MN = 7.2$ cm and $LN = 5.8$ cm.

(i) Find the circumcentre of ΔLMN.

(ii) Find the incentre of ΔLMN.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $MN = 7.2$ cm.

Step 2 : With centre M and radius 3.9 cm and with centre N and raduis 5.8 cm draw, arc's which intersect each other at L.

Step 3 : Join LM and LN. This is required ΔLMN.

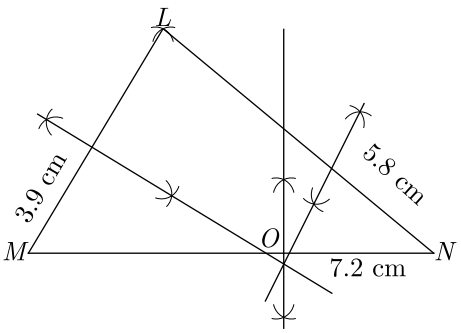
Step 4 : Draw the perpendicular bisector of the sides LM, MN and NL intersect at a point O. The O is the circumentre of the ΔLMN.

Step 5 : Draw a line segment $MN = 7.2$ cm.

Step 6 : With centre M and radius 3.9 cm and with centre N and raduis 5.8 cm draw arc's which intersect each other at L.

Step 7 : Join LM and LN.

Step 8 : Draw the angle bisectors of $\angle L, \angle M$ and $\angle N$ which intersect at I. I is the incentre of the ΔLMN.



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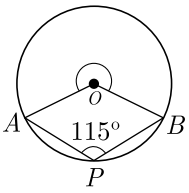
$$x = \frac{240^o}{2} = 120^o$$

Thus (c) is correct option.

5. In the following figure O is the centre of circle. what is the value of x ?



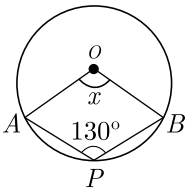
(a) 230^o (b) 150^o
(c) 115^o (d) 140^o

Sol :

$$\text{Reflex } \angle AOB = 2\angle APB$$
$$x = 2 \times 115^o$$
$$= 230^o$$

Thus (a) is correct option.

6. In the following figure O is the centre of circle. What is the value of x ?



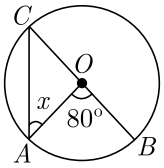
(a) 150^o (b) 150^o
(c) 115^o (d) 100^o

Sol :

$$\text{Reflex } \angle AOB = 2\angle APB$$
$$= 2 \times 130^o$$
$$= 260^o$$
$$x = 360^o - 260^o$$
$$= 100^o$$

Thus (d) is correct option.

7. In the following figure O is the centre of circle. What is the value of x ?



(a) 30^o (b) 40^o
(c) 15^o (d) 50^o

Sol :

Due to radii of same circle,

$$OC = OA$$

Equal side make equal angle. Thus

$$\angle ACO = \angle CAO$$
$$\angle ACB = x$$

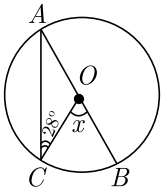
In given figure arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

Thus $\angle AOB = 2\angle ACB$

$$80^o = 2x$$
$$x = \frac{80^o}{2}$$
$$= 40^o$$

Thus (b) is correct option.

8. In the following figure O is the centre of circle. What is the value of x ?



(a) 50^o (b) 40^o
(c) 30^o (d) 20^o

Sol :

In ΔOAC due to radii of same circle

$$OA = OC$$

Equal side make equal angle. Thus

$$\angle OAC = \angle ACO = 28^o$$
$$\angle BAC = \angle ACO = 28^o$$

In the figure, arc BC subtends $\angle BOC$ at the centre and $\angle BAC$ at the remaining part of the circle.

Thus $\angle BOC = 2\angle BAC$

$$x = 2 \times 28^o$$

NODIA

Steps of Construction:

Step 1 : Draw a line segment $AB = 7$ cm

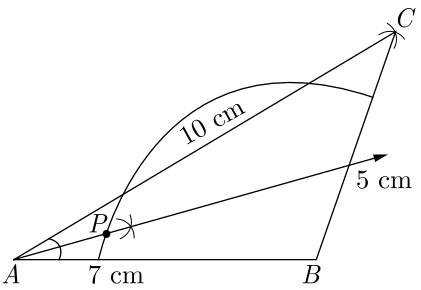
Step 2 : With centre A and radius 10 cm and with centre B and radius 5 cm draw arcs intersecting each other at C.

Step 3 : Join CA and CB. This is required $\triangle ABC$.

Step 4 : Draw the angle bisector of $\angle A$. This is locus of points equidistant from the sides AB and BC.

Step 5 : Draw a circle with centre B and radius 4 cm. Which intersects the angle bisector of $\angle A$ at P.

P is the required locus.



Comp 2010

60. Use ruler and compasses only for this question. Draw a circle of radius 4 cm and mark two chords AB and AC of the circle; of length 6 cm and 5 cm respectively.

(i) Construct the locus of points, inside the circle, that are equidistant from A and C. Prove your construction.

(ii) Construct the locus of points, inside the circle, that are equidistant from AB and AC.

Sol :

Steps of Construction :

Step 1 : With centre O and radius 4 cm draw a circle.

Step 2 : Take a point A on this circle.

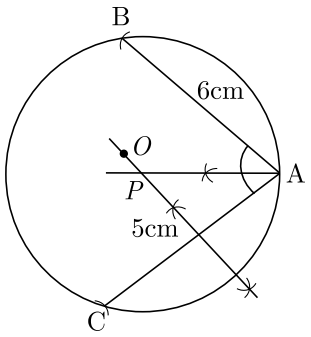
Step 3 : With centre A and radius 6 cm draw an arc cutting the circle at B.

Step 4 : Again with radius 5 cm, draw another arc cutting the circle at C.

Step 5 : Join AB and AC.

Step 6 : Draw the perpendicular bisector of AC. Any point on it, will be equidistant from A and C.

Step 7 : Draw the angle bisector of $\angle A$ intersecting the perpendicular bisector of AC at P. P is the required locus.



Comp 2009

61. Construct an isosceles triangle ABC such that $AB = 6$ cm, $BC = AC = 4$ cm Bisect $\angle C$ internally and mark a point P on this bisector such that $CP = 5$ cm. Find the points Q and R which are 5 cm from P and also 5 cm from the line AB.

Sol :

Steps of Construction

Step 1 : Draw a line $AB = 6$ cm

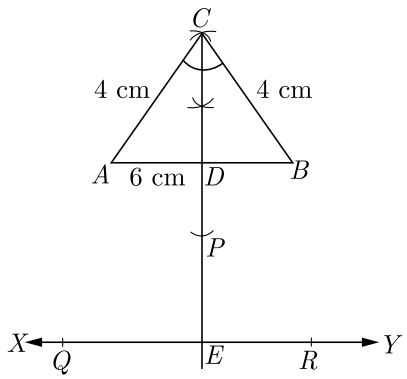
Step 2 : With centre A and B and radius 4 cm, draw two arcs intersecting each other at C.

Step 3 : Join CA and CB

Step 4 : Draw the bisector of $\angle C$ and cut off $CP = 5$cm.

Step 5 : Draw a line XY parallel to AB at a distance of 5 cm.

Step 6 : From P, draw arcs of radius 5 cm each intersecting the line XY at Q and R.



Hence Q and R are the required points.

Comp 2008

62. Construct ΔABC , $BC = 6$ cm, $\angle ABC = 120^{\circ}$, and $AB = 7.5$ cm.

(i) Construct the locus of points equidistant from B and C.

(ii) Draw the locus of a point P such that area of $\Delta PBC =$ area of ΔABC.

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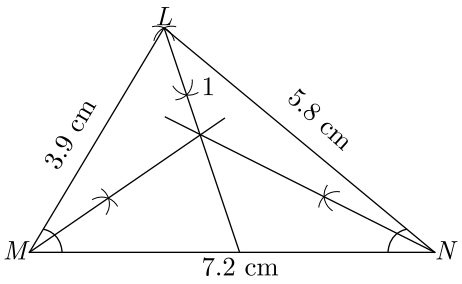


Comp 2015

56. By using ruler and compasses only construct an isosceles triangle ABC in which $BC = 5$ cm, $AB = AC$ and $\angle BAC = 90^o$. Locate the point P such that :

(i) P is equidistant from the sides BC and AC.

(ii) P is equidistant from the points B and C.

Sol :

Steps of Construction :

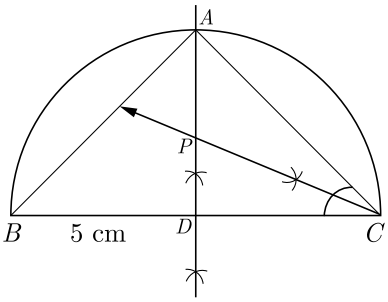
Step 1 : Take $BC = 5.0$ cm and bisect it at D.

Step 2 : Taking BC as diameter, draw a semicircle.

Step 3 : At D, draw a perpendicular intersecting the circle at A.

Step 4 : Join AB and AC.

Step 5 : Draw the angle bisector of C intersecting the perpendicular at P.



P is the required point.

Comp 2013

57. Using ruler and compasses only, construct a quadrilateral $ABCD$ in which $AB = 6$ cm, $BC = 5$ cm, $\angle B = 60^o$, $AD = 5$ cm and D is equidistant from AB and BC. Measure CD.

Sol :

Steps of Construction :

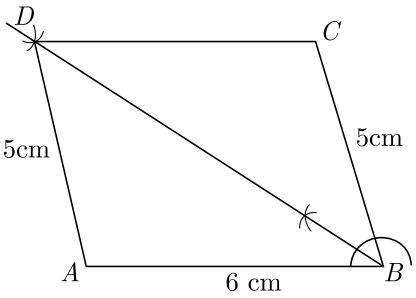
Step 1 : Draw $AB = 6$ cm

Step 2 : At B, draw angle of 60^o and cut off $BC = 5$ cm.

Step 3 : Draw the angle bisector of $\angle B$.

Step 4 : With centre A and radius 5 cm draw an arc which intersect the angle bisector of $\angle B$ at D.

Step 5 : join AD and DC. $ABCD$ is the required quadrilateral.



Step 6 : On measuring CD, it is 5.3 cm (approx).

Comp 2012

58. Construct ΔABC in which $BC = 6.5$ cm, $\angle ABC = 135°$ and $AB = 8$ cm.

(i) Construct the locus of points

(a) which are equidistant from AB and BC,

(b) 2 cm from BC and same side of BC as A.

(ii) Name the point of intersection of the two loci as P.

Sol :

Steps of Construction:

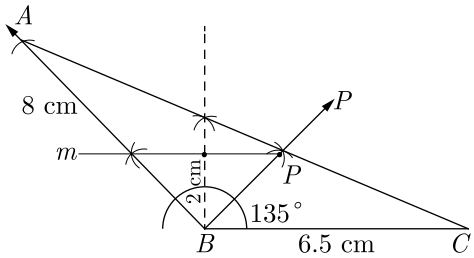
Step 1 : Draw a line segment $BC = 6.5$ cm

Step 2 : At B, draw a ray BX making an angle of $135°$ and cut off $BA = 8$ cm.

Step 3 : Join AC. This is required $\triangle ABC$.

Step 4 : Draw the angle bisector of $\angle B$. This is locus of points equidistant from the sides AB and BC.

Step 5 : Draw a parallel line to BC at a distance of 2 cm on the same side of BC which intersects the angle bisector at P. P is the required locus.



Comp 2011

59. Construct ΔABC, $AB = 7$ cm, $BC = 5$ cm and $AC = 10$ cm.

Draw the locus of a point:

(i) 4 cm from B, and

(ii) equidistant from AB and AC.

Sol :

NODIA

Comp 2006

65. Construct ΔPQR, $\angle R = 135°, \angle Q = 30^o$ and $QR = 5$ cm. Find the point I equidistant from PQ, QR and PR. Give a special name to point I.

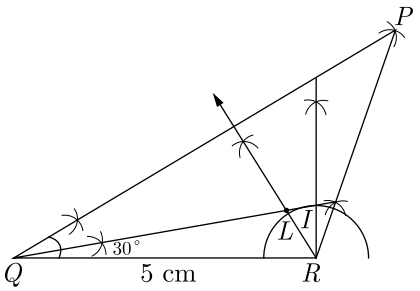
Sol :

Steps of Construction:

Step 1 : Draw a line segment $QR = 5$ cm

Step 2 : At Q draw a ray making an angle of 30° and at R. a ray making an angle of 135^o which meet at P. This is required ΔPQR

Step 3 : Draw the angle bisectors of Q and R intersecting each other at I. I is the required point which is equidistant from PQ, QR and RS. I is the incentre of the ΔPQR.



Comp 2005

66. Construct triangle ABC in which $BC = 7$ cm, $AB = 4$ cm and $\angle ABC = 60^o$.

(i) Construct the locus of points which are equidistant :

(a) from BA and BC

(b) from B and C

(ii) Mark P, the point which is equidistant from AB, BC and also equidistant from B and C.

Sol :

Steps of Construction:

Step 1 : Draw a line segment $BC = 7$ cm.

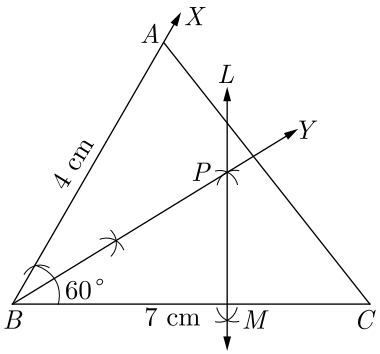
Step 2 : With centre B draw a ray making an angle of 60° and cut off $BA = 4$ cm.

Step 3 : Join AC. This is required $\triangle ABC$.

Step 4 : Draw the angle bisector of $\angle B$. This is locus of points equidistant from the sides AB and BC.

Step 5 : Draw perpendicular bisector of BC. This is locus of equidistant point from B and C.

Step 6 : Above two loci intersect at point a point. Mark this point P.



NODIA

FIVE MARK QUESTIONS

Main 2002

67. Using only a ruler and compass, construct $\angle ABC = 120°$, where $AB = BC = 5$ cm.

(i) Mark two points D and E, which satisfy the condition that they are equidistant from both BA and BC.

(ii) In the above figure, join AE and EC. Describe the figures.

(a) $ABCD$

(b) ABD

(c) ABE

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 5$ cm.

Step 2 : From point B draw $\angle XBC = 120°$.

Step 3 : Taking radius 5 cm, with B as centre cut $BA = 5$ cm from BX.

Step 4 : Draw angular bisector of ABC.

Step 5 : Draw perpendicular bisector of BC.

Step 6 : The intersection point of perpendicular bisector and angle bisector is the required point E, which is equidistant from both BA and BC.

Step 7 : Similarly, we can draw a point D which is equidistant from AB and BC.

Step 8 : Here we have joined AE and EC.

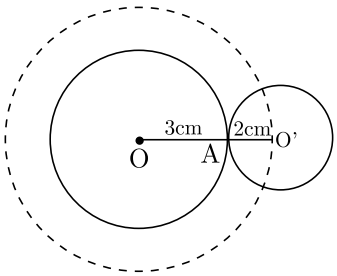
(a) Figure $ABCD$ is a rhombus.

(b) $\angle ABD$ is the bisector of $\angle ABC$.

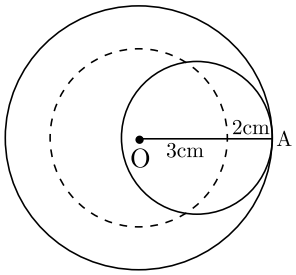
(c) ABE is triangle.

Sol :

If the circle with 2 cm as radius touches the given circle externally then the locus of the centre of the circle will be a concentric circle with radius $3 + 2 = 5$ cm.



If the circle with 2 cm as radius touches the given 1f circle with 3 cm as radius internally, then the locus of the centre of the circle will be a concentric with radius $3 - 2 = 1$ cm.



FOUR MARK QUESTIONS

Main 2020

52. Use ruler and compass for this question. Construct a circle of radius 4.5 cm. Draw a chord. $AB = 6$ cm.

(i) Find the locus of points equidistant from A and B. Mark the point where it meets the circles as D.

(ii) Join AD and find the locus of points which are equidistant from AD and AB. Mark the point where it meets the circle as C.

(iii) Join BC and CD. Measure and write down the length of side CD of the quadrilateral $ABCD$.

Sol :

Steps of Construction :

Step 1 : Draw a circle of radius 4.5 cm.

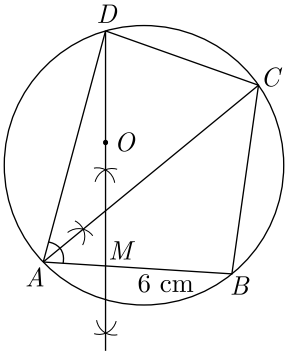
Step 2 : Take a point A on the circumference of the circle, draw an arc of 6 cm, which intersect the circumference of the circle at B and join AB.

Step 3 : Draw a line bisector of AB which intersect at D on the circumference of the circle. Join AD.

Step 4 : Draw angle bisector of $\angle BAD$ which intersect at P and C (Circumference of the cricle). Hence, P is the locus of points equidistant from A and B.

Step 5 : Join BC and DC, then C is the locus of points which are equidistant from AD and AB.

Step 6 : Now, we get a quadrilateral, in which $BC = DC$.



Main 2019, SQP 2013

53. Using ruler and a compass only construct a semi-circle with diameter $BC = 7$ cm.
Locate a point A on the circumference of the semi-circle such that A is equidistant from B and C. Complete the cyclic quadrilateral $ABCD$, such that D is equidistant from AB and BC. Measure $\angle ADC$ and write it down.

Sol :

Steps of Construction :

Step 1 : Draw $BC = 7$ cm.

Step 2 : Draw perpendicular bisector of BC which meets BC at point O.

Step 3 : With O as centre and $OB = OC$ as radius, draw a semi-circle which meets perpendicular bisector of BC at point A.

Step 4 : Since, A lies on the perpendicular bisector of BC, A is equidistant from B and C.

Step 5 : Draw bisector of angle ABC which meets the semi-circle at point D. D lies on bisector of angle ABC.

Step 6 : Join AD and DC. $ABCD$ is the required cyclic quadrilateral.

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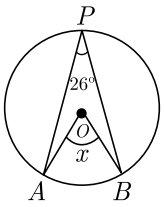
CHAPTER 16

ANGLE PROPERTIES OF CIRCLE

MULTIPLE CHOICE QUESTIONS

1. In the following figure O is the centre of circle. what is the value of x ?



(a) 42^o (b) 52^o
(c) 26^o (d) 64^o

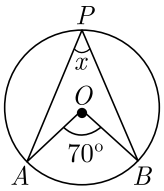
Sol :

In a circle, angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle AOB = 2\angle APB$$
$$x = 2 \times 26^o$$
$$= 52^o$$

Thus () is correct option.

2. In the following figure O is the centre of circle. what is the value of x ??



(a) 35^o (b) 25^o
(c) 45^o (d) 65^o

Sol :

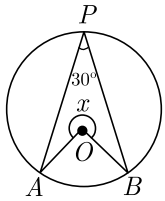
Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus $\angle AOB = 2\angle APB$

$$70^o = 2x$$
$$x = \frac{70^o}{2} = 35^o$$

Thus (a) is correct option.

3. In the following figure O is the centre of circle. what is the value of x ?



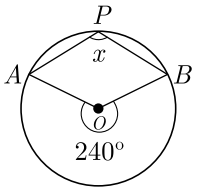
(a) 360^o (b) 330^o
(c) 300^o (d) 270^o

Sol :

$$\angle AOB = 2\angle APB$$
$$= 2 \times 30^o$$
$$= 60^o$$
$$x = \text{Reflex } \angle AOB$$
$$= 360^o - 60^o$$
$$= 300^o$$

Thus (c) is correct option.

4. In the following figure O is the centre of circle. what is the value of x ?

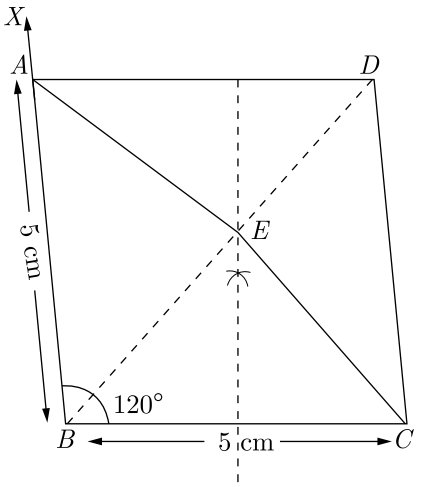


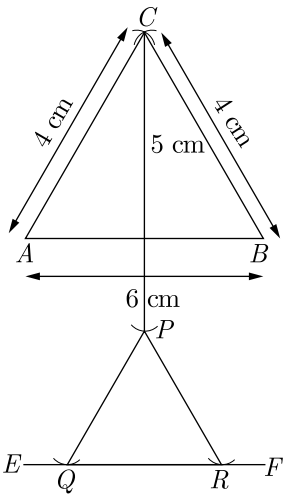
(a) 240^o (b) 260^o
(c) 120^o (d) 140^o

Sol :

$$\angle AOB = 2\angle APB$$
$$240^o = 2x$$

NODIA





Main 2001

68. Construct an isosceles ΔABC, such that $AB = 6$ cm, $BC = AC = 4$ cm. Bisect $\angle C$ inernally and mark a point P on this bisector, such that $CP = 5$ cm. Find the points Q and R, which are 5 cm from P and 5 cm from the line AB.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $AB = 6$ cm.

Step 2 : Taking A and B, as centre draw arcs of length 4 cm which intersect each other at point C.

Step 3 : Join AC and BC. Then, ΔABC is the required triangle.

Step 4 : Draw the angle bisector of $\angle ABC$ and mark the point Pat this bisector, such that $CP = 5$ cm.

Step 5 : Draw the angle EF parallel to AB, which is at a distance of 5 cm.

Step 6 : Taking P as centre draw an arc of semi-circle of radius 5 cm, which intersect the line EF at points Q and R. Then, Q and R are the required points, such that $PQ = PR = 5$ cm.

END

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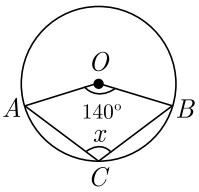
$= 56^o$

Thus $x = 56^o$.

Thus (a) is correct option.

9. In the following figure O is the centre of circle. What is the value of x ?



(a) 120^o (b) 150^o
(c) 110^o (d) 140^o

Sol :

In the figure $\angle AOB = 140^o$

Reflex $\angle AOB = 360^o - 140^o$

$= 220^o$

Arc ACB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle

Thus $\angle AOB = 2\angle ACB$

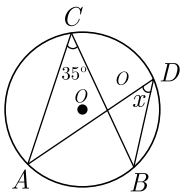
$220^o = 2x$

$x = \frac{220^o}{2}$

$= 110^o$

Thus (c) is correct option.

10. In the following figure O is the centre of circle. What is the value of x ?



(a) 30^o (b) 70^o
(c) 45^o (d) 35^o

Sol :

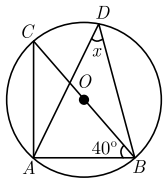
Angle $\angle ACB$ and $\angle ADB$ are in the same segment.

Thus $\angle ADB = ACB$

$x = 35^o$

Thus (d) is correct option.

11. In the following figure O is the centre of circle. What is the value of x ?



(a) 30^o (b) 50^o
(c) 70^o (d) 40^o

Sol :

In ΔABC, $\angle ABC = 40^o$

Due to angle in a semi circle,

$\angle BAC = 90^o$)

Sum of angles of a triangle is always 180^o.

$\angle ACB + \angle BAC + \angle ABC = 180^o$

$\angle ACB + 90^o + 40^o = 180^o$

$\angle ACB + 130^o = 180^o$

$\angle ACB = 50^o$

Angle $\angle ADB$ and $\angle ACB$ are in same segment of the circle.

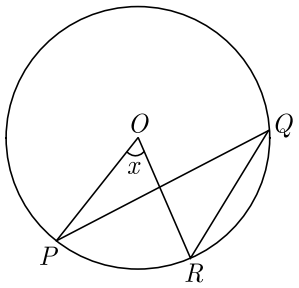
Thus $\angle ADB = \angle ACB$

$x = 50^o$

Thus (b) is correct option.

NODIA

12. What is value of angle x in the given figure if $\angle PQR = 28°$?



(a) 56^o (b) 31^o
(c) 28^o (d) 62^o

Sol :

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus $\angle POR = 2\angle PQR$

$x = 2 \times 28°$

$= 56°$

Thus $\angle EMN + \angle MNE = 90^o$

$\angle PMN + 20^\circ = 90^o$

$\angle PMN = 70^o$

Angles in the same segment are always equal.

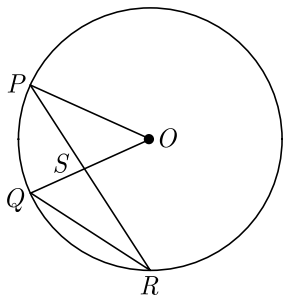
$\angle NQP = \angle PMN$

$= 70^o$

Thus $\angle NQP = 70^o$

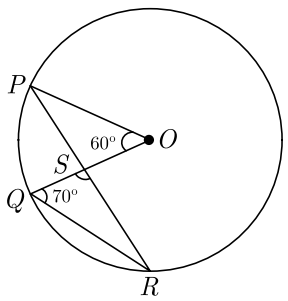
Comp 2019

28. In the given circle with centre O, $\angle POQ = 60^o$ and $\angle QSR = 70^o$.
(i) Find $\angle PRQ$.
(ii) Find $\angle SQR$



Sol :

We have redrawn the given figure as below



(i) $\angle PRQ$.
Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.
Arc PQ subtends $\angle POQ$ at the centre and $\angle PRQ$ at any point on the circumference of the circle.

$\angle POQ = 2\angle PRQ$

$\angle PRQ = \frac{1}{2}\angle POQ$

$= \frac{1}{2} \times 60^o$

$= 30^o$

(ii) $\angle SQR$
The sum of angles of a linear pair is always equal to 180°.

$\angle QSR + \angle SQR + \angle SRQ = 180^o$

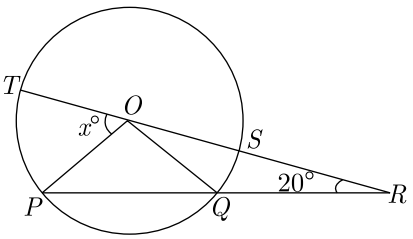
$70^o + \angle SQR + 30^o = 180^o$

$\angle SQR + 100^o = 180^o$

$\angle SQR = 80^o$

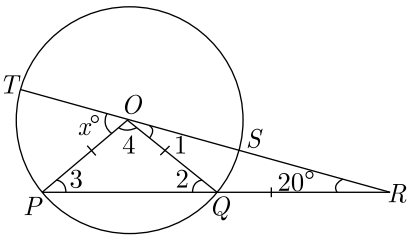
Main 2018

29. In the figure given below, O is the centre of the circle. If $QR = OP$ and $\angle QRP = 20^\circ$. Find the value of 'x' giving reasons.



Sol :

We have redrawn the figure as below.



Given $QR = OP$ ...(1)

Due to raddi of circle

$OP = OQ$ ...(2)

From Eqs. (1) and (2), we get

$OP = OQ = QR$

Since angle opposite to equal sides are equal,

$\angle 1 = \angle ORP$

$\angle 1 = 20^\circ$

Due to external angle

$\angle 2 = \angle 1 + \angle ORP$

$= 20^\circ + 20^\circ = 40^\circ$

Due to raddi of circle, $OP = OQ$, thus

$\angle 3 = \angle 2 = 40^\circ$

Sum of all angles in a triangles is 180°,

$\angle 2 + \angle 3 + \angle 4 = 180^\circ$

$40^\circ + 40^\circ + \angle 4 = 180^\circ$

$\angle 4 = 100^\circ$

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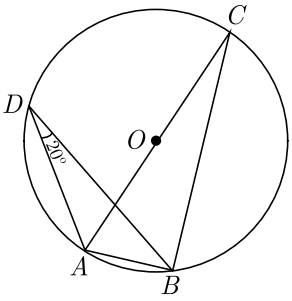


The angle in a semi-circle is always a right angle.

$$\angle ABC = 90^o$$

Angles in the same segment are always equal.

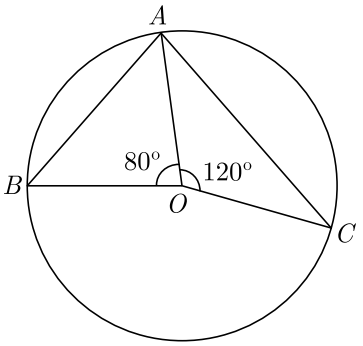
$$\angle ADB = \angle ACB$$
$$= 20^o$$

Sum of all angles of triangle is always 180°.

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ$$
$$90^\circ + 20^\circ + \angle CAB = 180^\circ$$
$$110^\circ + \angle CAB = 180^\circ$$
$$\angle CAB = 70^o$$

Thus (b) is correct option.

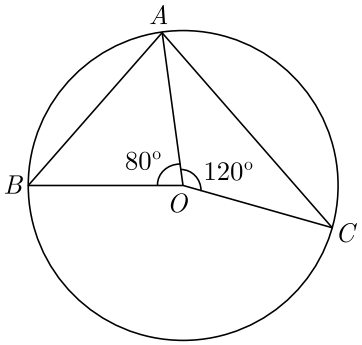
17. In given figure, A, B, C are three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 80° and 120^o respectively. What is the value of angle $\angle BAC$?



(a) 30^o (b) 60^o
(c) 40^o (d) 80^o

Sol :

We have redrawn the figure as below.



Now, $\angle BOC = 360^o - (120^o - 80^o)$

$$= 160^o$$

Since arc BPC makes $\angle BOC$ at the centre and $\angle BAC$ at a point on the remaining part of the circle, we have

$$\angle BOC = 2\angle BAC$$
$$\angle BAC = \tfrac{1}{2}\angle BOC$$
$$= \tfrac{1}{2} \times 160^o$$
$$= 80^o$$

Thus (d) is correct option.

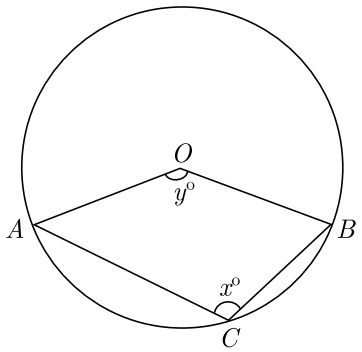
18. C is a point on the minor arc AB of the circle O. Given $\angle ACB = x$ and $\angle AOB = y$ express y in terms of x. What is value of angle x in if $ACBO$ is a parallelogram?

(a) 120^o (b) 150^o
(c) 110^o (d) 140^o

Sol :

As per given in question we have shown the figure below.

NODIA

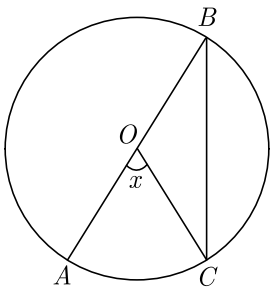


Arc ACB makes Reflex$\angle AOB$ at the centre of the circle and $\angle ACB$ at a point on the remaining part of the circle.

Angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

Thus (a) is correct option.

13. What is value of angle x in the given figure if $\angle ABC = 35°$?



(a) 30^o (b) 50^o
(c) 70^o (d) 40^o

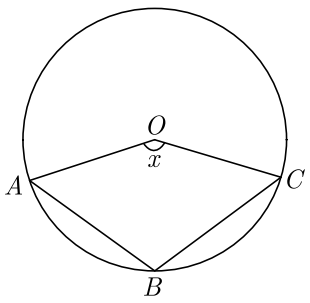
Sol :

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus $$\angle AOC = 2\angle ABC$$
$$x = 2 \times 35°$$
$$= 70°$$

Thus (c) is correct option.

14. What is value of angle x in the given figure if $\angle ABC = 110°$?



(a) 120^o (b) 150^o
(c) 110^o (d) 140^o

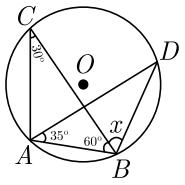
Sol :

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus $$\text{reflex}\angle AOC = 2\angle ABC$$
$$= 2 \times 110°$$
$$= 220°$$
$$\angle AOC = 360° - 220$$
$$x = 140°$$

Thus (d) is correct option.

15. In the following figure O is the centre of circle. What is the value of x ?



(a) 30^o (b) 40^o
(c) 55^o (d) 50^o

Sol :

In the given circle $\angle ACB = 30^o$,
$\angle DAB = 35^o$
$\angle CBA = 60^o$

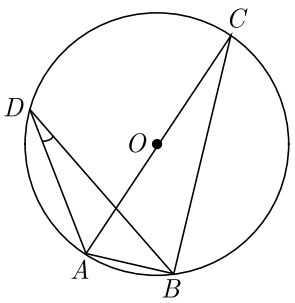
Due to angles in the same segment we have
$$\angle ADB = \angle ACB$$
$$\angle ADB = 30^o$$

Now in ΔABD,
$$\angle DAB + \angle ADB + \angle ABD = 180^o$$
$$35^o + 30^o + 60^o + x = 180^o$$
$$125^o + x = 180^o$$
$$x = 55^o$$

Thus $x = 55^o$.

Thus (c) is correct option.

16. In the circle with centre O, if $\angle ADB = 20°$ what is the value of angle $\angle BAC$?



(a) 30^o (b) 70^o
(c) 60^o (d) 50^o

Sol :

We have redrawn the given figure as below

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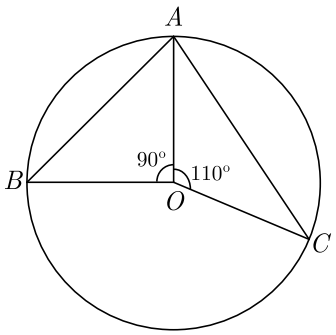
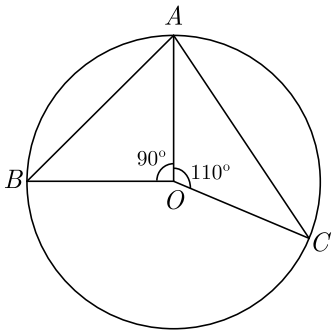
$\angle BAC$?



(a) 80^o (b) 60^o
(c) 40^o (d) 50^o

Sol :

We have redrawn the figure as below.



Here $\angle BOA = 90°$

and $\angle AOC = 110°$

Sum of all angles around a point is always $360°$.

$$\angle BOC + \angle BOA + \angle AOC = 360°$$
$$\angle BOC + 90° + 110° = 360°$$
$$\angle BOC + 200° = 360°$$
$$\angle BAC = 160°$$

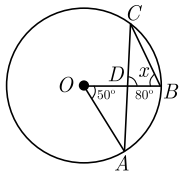
Angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

Now, $\angle BOC = 2\angle BOC$

$$\angle BAC = \tfrac{1}{2}\angle BOC$$
$$= \tfrac{1}{2} \times 160^o$$
$$= 80^o$$

Thus (a) is correct option.

22. In the following figure O is the centre of circle. What is the value of x ?



(a) 30^o (b) 75^o
(c) 40^o (d) 80^o

Sol :

In the given circle $\angle AOB = 50^o$,

$\angle BDC = 80^o$

Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle. Thus

$$\angle AOB = 2\angle ACB$$
$$\angle ACB = \tfrac{1}{2}\angle AOB$$
$$\angle ACB = \tfrac{1}{2} \times 50^o$$
$$\angle DCB = 25^o$$

Now in ΔBCD,

$$\angle BDC + \angle DCB + \angle CBD = 180^o$$
$$80^o + 25^o + x = 180^o$$
$$105^o + x = 180^o$$
$$x = 180^o - 105^o$$
$$= 75^o$$

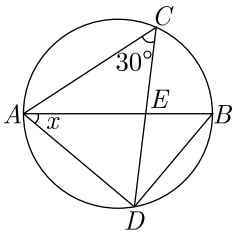
Thus $x = 75^o$.

Thus (b) is correct option.

THREE MARKS QUESTIONS

Comp 2023

23. In the given circle with diameter AB, find the value of x.



Sol :

We redraw the figure as below.

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Thus Reflex $\angle AOB = 2\angle ACB$

$$360^\circ - y = 2x$$

$$y = 360^o - 2x$$

Opposite angles of a parallelogram are equal. If $ACBO$ is a parallelogram, we have

$$x = y$$

$$x = 360^\circ - 2x$$

$$3x = 360^\circ$$

$$x = 120^\circ$$

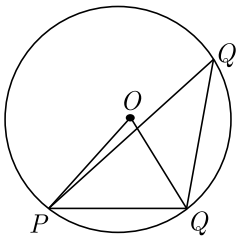
Thus (a) is correct option.

19. In the given figure, O is the centre of a circle. If the length of chord PQ is equal to the radius of the circle, the $\angle PRQ$ is

(a) 60^o (b) 45^o
(c) 30^o (d) 15^o

Sol :

We have redrawn the given figure as below.



Here OQ and OP are radius of circle. PQ is equal to the radius of the circle. Thus ΔOPQ is an equilateral triangle.

Thus $\angle POQ = 60^\circ$

Arc PQ subtends $\angle POQ$ at the centre and $\angle PRQ$ at the remaining part of the circle

$$\angle POQ = \angle PRQ$$

$$\angle PRQ = \tfrac{1}{2}\angle POQ$$

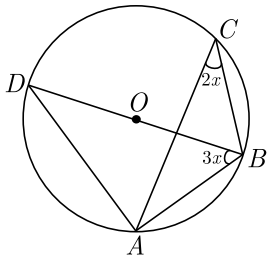
$$= \tfrac{1}{2} \times 60^\circ$$

$$= 30^\circ$$

Thus (c) is correct option.

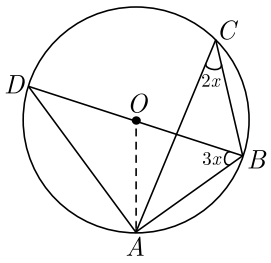
20. In the given figure, if O is centre of the circle then the value of x is

(a) 18^o (b) 20^o
(c) 24^o (d) 36^o



Sol :

We have redrawn the given figure as below. Here we have joined OA.



Due to angle in same segment we have

$$\angle ADB = \angle ACB$$

$$= 2x$$

Arc AB subtends $\angle AOB$ at the centre and $\angle ADB$ at the remaining part of the circle.

Thus $\angle AOB = 2\angle ADB$

$$= 2 \times 2x$$

$$= 4x$$

Due to the radius of same circle,

$$OA = OB$$

Angle opposite to equal sides of a triangle are equal. Thus in ΔOAB,

$$\angle OAB = \angle OBA$$

$$= 3x$$

Sum of all internal angle of a triangle is always $180°$. Thus in ΔOAB,

$$\angle OAB + \angle ABO + \angle AOB = 180^\circ$$

$$3x + 3x + 4x = 180^\circ$$

$$10x = 180^\circ$$

$$x = 18^\circ$$

Thus (a) is correct option.

21. In the given figure A, B, and C are three points on a circle such that the angles subtended by the chords AB and AC at the centre O are $90°$ and $110°$, respectively. What is the value of angle

NODIA

$$\angle BAD = \tfrac{1}{2}\angle BOD$$
$$= \tfrac{1}{2} \times 140^o$$
$$= 70^o$$

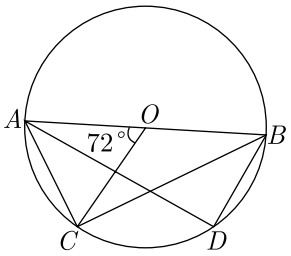
Again, arc DAB makes angle reflex $\angle BOD = (360° - 140°) = 220°$ at the centre and $\angle BCD$ at a point C on the circumference.

Thus $\text{Reflex}\angle BOD = 2\angle BCD$

$$220° = 2\angle BCD$$
$$\angle BCD = \tfrac{1}{2} \times 220°$$
$$= 110^o$$

Main 2020

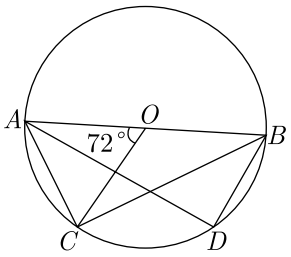
26. In the figure below, O is the centre of the circle and AB is diameter. If $AC = BD$ and $\angle AOC = 72°$. Find



(i) $\angle ABC$
(ii) $\angle BAD$
(iii) $\angle ABD$

Sol :

We have redrawn the figure as below. Here O is the centre of the circle and AB is a diameter.



Here $AC = BD$ and $\angle AOC = 72°$

(i) $\angle ABC$

Angle subtended by an arc at the remaining circumference is half the angle subtended at the centre. Thus

$$\angle ABC = \tfrac{1}{2}\angle AOC$$
$$= \tfrac{1}{2} \times 72°$$
$$= 36°$$

(ii) $\angle BAD$

Equal chords of a circle subtend equal angles at any point on the major (or minor) arcs of the circle. Here $AC = BD$, thus

$$\angle BAD = \angle ABC$$
$$= 36°$$

(iii) $\angle ABD$

Angle in semi-circle is a right angle, thus

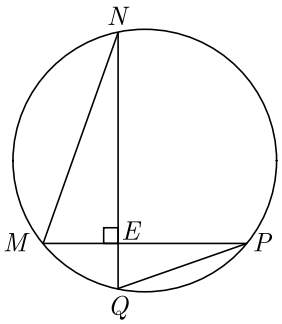
$$\angle ADB = 90°$$

Using angle sum property in triangle ΔABD we have

$$\angle ABD + \angle ADB + \angle BAD = 180°$$
$$\angle ABD + 90° + 36° = 180°$$
$$\angle ABD + 126° = 180°$$
$$\angle ABD = 180° - 126°$$
$$= 54°$$

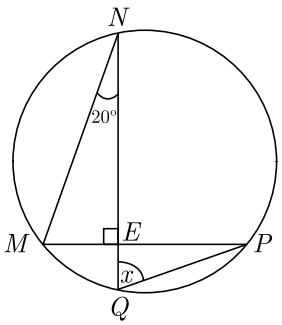
Main 2019

27. In the given circle, chord QN is perpendicular to chord MP and $\angle MNQ = 20°$. Find $\angle NQP$.



Sol :

We have redrawn the given figure as below.



In right ΔMNE, we have

$$\angle NEM = 90^o$$
$$\angle MNE = 20^o$$

Sum of two acute angles in the right angle is always 90^o.

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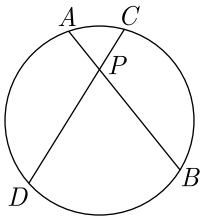
Main 2015

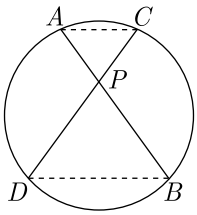
32. AB and CD are two chords of a circle intersecting at P. Prove that

$$AP \times PB = CP \times PD$$



Sol :

We have redrawn the figure as below. Here we have joined AC and BD.



In ΔACP and ΔDBP, due to vertically opposite angles

$$\angle APC = \angle BPD$$

Due to angles is same segment,

$$\angle CAP = \angle PDB$$

By AA similarity criterion,

$$\Delta ACP \sim \Delta DBP$$

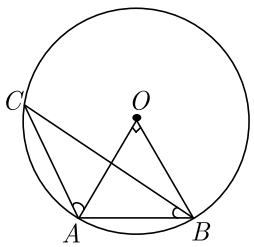
Corresponding sides of similar triangles are always proportional. Thus

$$\frac{PA}{PD} = \frac{PC}{PB}$$

$$PA \cdot PB = PC \cdot PD$$

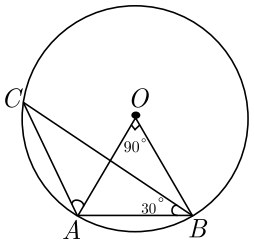
Comp 2015

33. In the given figure, O is the centre of the circle. If $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$, then find $\angle CAO$.



Sol :

We have redrawn the given figure as below.



Due to the radius of same circle,

$$OA = OB$$

Angle opposite to equal sides of a triangle are equal. Thus in ΔABO,

$$\angle OAB = \angle OBA$$

Thus ΔAOB is right angle triangle. Sum of two acute angles in the right angle triangle is always 90°.

$$\angle OAB + \angle OBA = 90^\circ$$
$$\angle OAB + \angle OAB = 90^\circ$$
$$2\angle OAB = 90^\circ$$
$$\angle OAB = \tfrac{1}{2} \times 90^\circ$$
$$= 45^\circ$$

Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of circle.

Thus
$$\angle AOB = 2\angle ACB$$
$$\angle ACB = \tfrac{1}{2}\angle AOB$$
$$= \tfrac{1}{2} \times 90^\circ$$
$$= 45^\circ$$

Sum of all internal angle of a triangle is always 180°. Thus in ΔACB

$$\angle ACB + \angle CBA + \angle BAC = 180^\circ$$
$$45^\circ + 30^\circ + \angle BAC = 180^\circ$$
$$75^\circ + \angle BAC = 180^\circ$$
$$\angle BAC = 105^\circ$$

Now
$$\angle CAO + \angle OAB = \angle BAC$$
$$\angle CAO + 45^\circ = 105^\circ$$
$$\angle CAO = 60^\circ$$

SQP 2015

34. Two circles intersect at A and B. Given that AC and AD are respectively the diameters of the circles. Prove that C, B, D are collinear.

Sol :

As per given in question we have shown the figure below. Here we have joined CB, BD and AB.

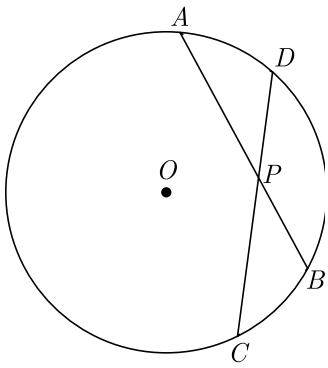
NODIA

Due to straight angle,

$$x + \angle 1 + \angle 4 = 180^\circ$$
$$x + 20^\circ + 100^\circ = 180^\circ$$
$$x + 120^\circ = 180^\circ$$
$$x = 60^\circ$$

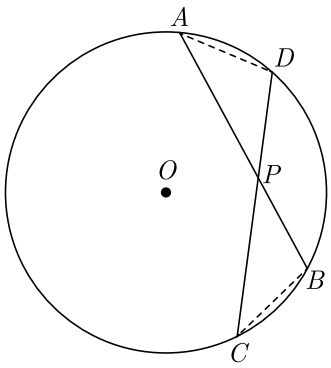
Comp 2017

30. In given figure, AB and CD are two chords of a circle, intersecting each other at P such that $AP = CP$. Show that $AB = CD$.



Sol :

We have redrawn the figure as below. Here we have joined AD and BC.



In $\triangle PAD$ and $\triangle PCB$:
Due to angles in the same segment of arc BD,

$$\angle PAD = \angle PCB$$

Due to vertically opposite angles,

$$\angle APD = \angle CPB$$

So, by AAA criterion of similarity, we have

$$\Delta PAD \sim PCB$$

Corresponding sides of similar triangles are in the same ratio.

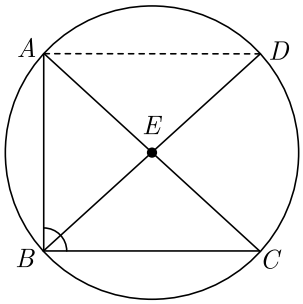
Thus $$\frac{PA}{PC} = \frac{PD}{PB}$$

Due to radius of same circle $PA = PC$, thus

$$\frac{PC}{PC} = \frac{PD}{PB}$$
$$1 = \frac{PD}{PB}$$
$$PB = PD$$
$$AP + PB = AP + PD$$
$$AP + PB = CP + PB \qquad [AP = CP]$$
$$AB = CD \qquad \text{Hence Proved}$$

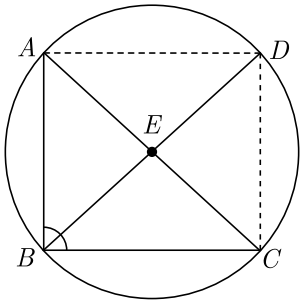
Comp 2016

31. In the given figure, ABC is a triangle inscribed in a circle. Given that BD bisects $\angle ABC$. Prove that $\frac{AB}{BD} = \frac{BE}{BC}$.



Sol :

We have redrawn the figure as below.



Here ΔABC is inscribed. Bisector of $\angle ABC$ meets AC at E and the circle at D. We have joined DC.
In ΔABE and ΔBCD :
Due to angle in the same segment,

$$\angle BAE = \angle BDC$$

Since BD is the bisector of $\angle B$,

$$\angle ABE = \angle DBC$$

By AA Axiom we get

$$\Delta ABE \sim \Delta BCD$$

Corresponding side of similar triangles are in the same ratio. Therefore

$$\frac{AB}{BD} = \frac{BE}{BC} \qquad \text{Hence proved.}$$

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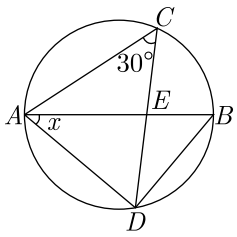


Due to angle in the same segment.

$$\angle ABD = \angle ACD = 30^\circ$$

Since angle in a semi-circle is always 90°,

$$\angle ADB = 90^\circ$$

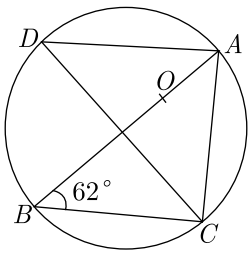
Since sum of all angles in a triangles is 180°, in ΔABD, we have

$$\angle DAB + \angle ABD + \angle ADB = 180^\circ$$
$$x + 90^\circ + 30^\circ = 180^\circ$$
$$x + 120^\circ = 180^\circ$$
$$x = 180^\circ - 120^\circ$$
$$= 60^\circ$$

Hence, the value of x is 60°.

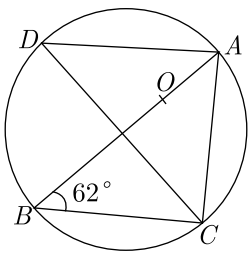
Main 2022 Sem II

24. In the given figure A, B, C and D are points on the circle with centre O. Given $\angle ABC = 62^\circ$. Find :
(i) $\angle ADC$
(ii) $\angle BAC$



Sol :

We have redrawn the figure as below.



(i) $\angle ADC$

Here $\angle ABC = 62^\circ$

Due to angle in the same segment we have

$$\angle ADC = \angle ABC$$
$$\angle ADC = 62^\circ$$

(ii) $\angle BAC$

Here AB is diameter. Since angle in a semi-circle is always 90°, thus

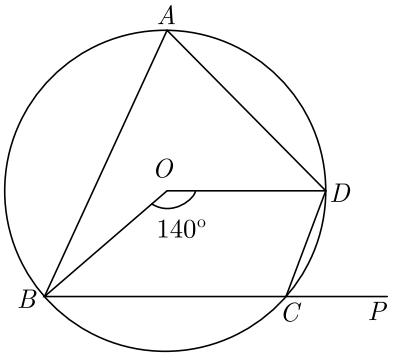
$$\angle ACB = 90^\circ$$

Using angle sum property in triangle ΔABC we have

$$\angle ACB + \angle ABC + \angle BAC = 180^\circ$$
$$\angle BAC + 62^\circ + 90^\circ = 180^\circ$$
$$\angle BAC + 152^\circ + 90^\circ = 180^\circ$$
$$\angle BAC = 180^\circ - 152^\circ$$
$$= 28^\circ$$

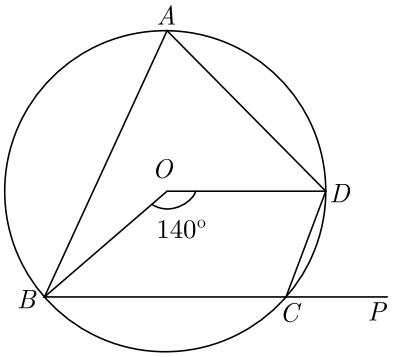
Comp 2022

25. In given figure, O is the centre of the circle. The angle subtended by the arc BCD at the centre is 140°. BC is produced to P.
(i) Find $\angle BAD$
(ii) Find $\angle DCB$



Sol :

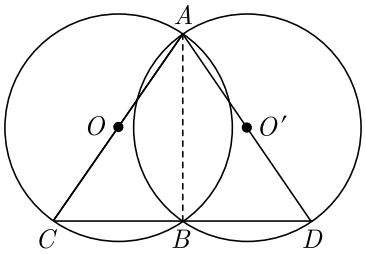
We have redrawn the figure as below.



Since the arc BCD makes $\angle BOD$ at the centre of circle and $\angle BAD$ at a point on the remaining part of the circle, we have

$$\angle BOD = 2\angle BAD$$

NODIA



Here AC is a diameter of the circle with centre at O and AD is a diameter of the circle with centre at O'.

Since the angle in a semi-circle is a right angle, we have

Therefore $\angle ABC = 90°$ (i)

and $\angle ABD = 90°$ (ii)

Adding (i) and (ii), we get

$$\angle ABC + \angle ABD = 90^o + 90^o$$
$$= 180^o$$

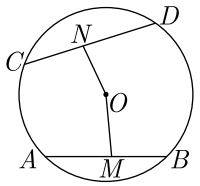
Therefore CBD is a straight line and C, B, D are collinear.

Main 2014

35. In the given figure, O is the centre of the circle, AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. If $AB = 24$ cm, $OM = 5$ cm, $ON = 12$ cm, find the

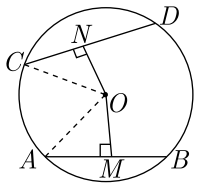
(i) radius of the cirlce.

(ii) length of chord CD.



Sol :

We have redrawn the figure as below.



Here $AB = 24$ cm, $OM = 5$ cm, $ON = 12$ cm

$OM \perp AB$ and $ON \perp CD$

Since perpendicular drawn from center to the chord bisects the chord. Thus M is the mid-point of AB.

$$AM = MB = \frac{AB}{2} = \frac{24}{2} = 12 \text{ cm}$$

(i) Radius of the cirlce.

Let r be the radius of circle.

In right ΔAMO, by Pythagoras theorem,

$$AO^2 = AM^2 + OM^2$$
$$r^2 = (12)^2 + (5)^2$$
$$= 144 + 25$$
$$= 169$$
$$r^2 = (13)^2$$
$$r = 13 \text{ cm}$$

(ii) Length of chord CD.

In right ΔCNO, by Pythagoras theorem

$$CO^2 = ON^2 + CN^2$$
$$r^2 = 12^2 + CN^2 \qquad [r = OC]$$
$$13^2 = 12^2 + CN^2$$
$$CN^2 = 169 - 144$$
$$= 25$$
$$= 5^2$$
$$CN = 5 \text{ cm}$$

Since, $ON \perp CD$, so N is the mid-point of CD.

$$CD = 2CN = 2 \times 5 = 10 \text{ cm}$$

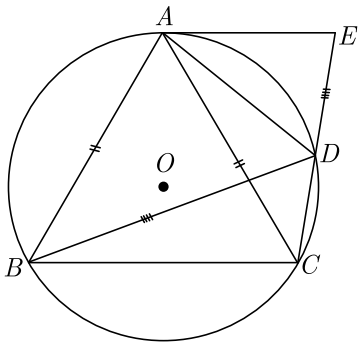
Hence, radius of circle is 13 cm and length of chord is 10 cm.

Comp 2014

36. D is a point on the circumcircle of ΔABC in which $AB = AC$ such that B and OM on the opposite side of line AC. If CD is produced to a point E such that $CE = BD$, prove that $AD = AE$.

Sol :

As per given in question we have shown the figure below.



We have, $AB = AC$

and $CE = BD$

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Due to angles in the same segment,

$$\angle CBE = \angle CAE$$
$$= 30^o$$

(iii) $\angle DAC$

Due to angle in a semi-circle,

$$\angle DAE = 90^o$$
$$\angle DAC + \angle CAE = 90^o$$
$$\angle DAC + 30^o = 90^o$$
$$\angle DAC = 90^o - 30^o$$
$$= 60^o$$

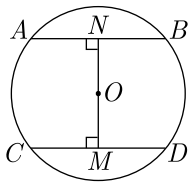
(iv) $\angle AEB$

In ΔABE,

$$\angle ABE + \angle BAE + \angle AEB = 180^o$$
$$(72^o + 30^o) + 30^o + \angle AEB = 180^o$$
$$102^o + 30^o + \angle AEB = 180^o$$
$$132^o + \angle AEB = 180^o$$
$$\angle AEB = 180^o - 132^o$$
$$= 48^o$$

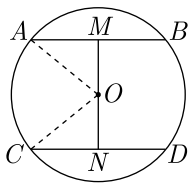
Main 2017

53. AB and CD are two parallel chords of a circle such that $AB = 24$ cm and $CD = 10$ cm. If the radius of the circle is 13 cm, find the distance between the two chords.



Sol :

We have redrawn the given figure as below



We have $OA = OC = 13$ cm, $AB = 24$ cm,

$CD = 10$ cm

A perpendicular to a chord drawn from the centre, always bisects the chord.

Here AB and CD are chord, OM and ON are perpendicular drawn from center. Thus

$$AN = NB = \frac{AB}{2} = \frac{24}{2} = 12 \text{ cm}$$

and $$CM = MD = \frac{CD}{2} = \frac{10}{2} = 5 \text{ cm}$$

In right ΔANO, using Pythagoras theorem,

$$OA^2 = ON^2 + AN^2$$
$$13^2 = ON^2 + 12^2$$
$$ON^2 = 169 - 144 = 25$$
$$ON = 5 \text{ cm}$$

Similarly, in ΔOMC, using Pythagoras theorem,

$$OC^2 = OM^2 + CM^2$$
$$13^2 = OM^2 + 5^2$$
$$OM^2 = 169 - 25$$
$$= 144 = 12^2$$
$$OM = 12 \text{ cm}$$
$$MN = OM + ON$$
$$= 5 + 12 = 17 \text{ cm}.$$

Hence, the value of MN is 17 cm.

SQP 2016. Comp 2012

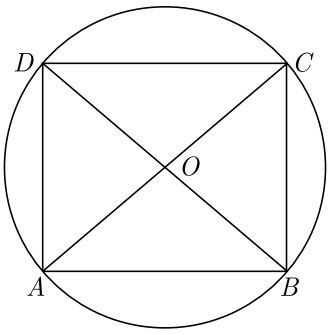
54. AC and BD are chords of a circle that bisect each other. Prove that:

(i) AC and BD are diameters

(ii) $ABCD$ is a rectangle.

Sol :

As per given in question we have shown the figure below.



(i) AC and BD are diameters

In ΔAOB and ΔCOD :

Suppose AC and BD bisect each other at point O.

Then $OA = OC$ (i)

and $OB = OD$ ...(ii)

Due to vertically opposite angles,

$$\angle AOB = \angle COD$$

So, by SAS congruence criterion, we have

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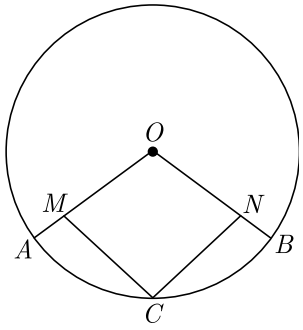
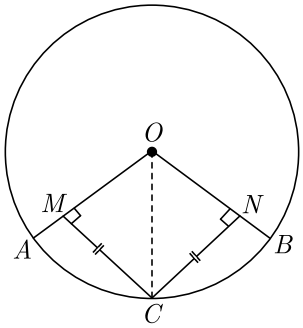
$CM = CN$, prove that C is the midpoint of $\overset{\frown}{AB}$.



Sol :

We have redrawn the figure as below.



Here in the given circle, O is the centre, OA and OB are the two radii of the circle. From a point C on the circle $CM \perp OA$ and $CN \perp OB$ and $CM = CN$. We have joined OC.
In right ΔOMC and ΔONC,

Common $OC = OC$

Given $CM = CN$

Due to RHS axiom we get

$$\Delta OMC \cong \Delta ONC$$

Now due to CPCT we have

$$\angle COM = \angle CON$$

or $\angle COA = \angle COB$

Equal arcs subtends equal angles at the centre.

Thus $\overset{\frown}{AC} = \overset{\frown}{CB}$

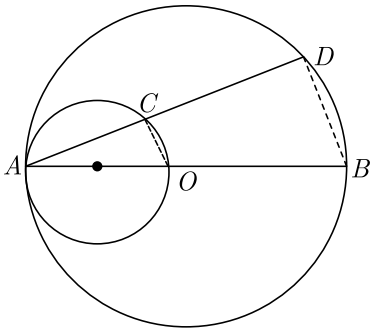
Hence C is the midpoint of $\overset{\frown}{AB}$.

Main 2012

40. A circle with centre O, diameter AB and a chord AD are drawn. Another circle is drawn with AO as diameter to cut AD at C. Prove that $BD = 2OC$.

Sol :

As per given in question we have shown the figure below.



Angle in a semi-circle is a right angle, thus

$$\angle ACO = 90°$$

and $\angle ADB = 90^{o}$

So we have $\angle ACO = \angle ADB$

Thus, in ΔABD , C and O are points on AD and AB respectively such that $CO \,||\, DB$. Therefore, by basic proportionality theorem, we have

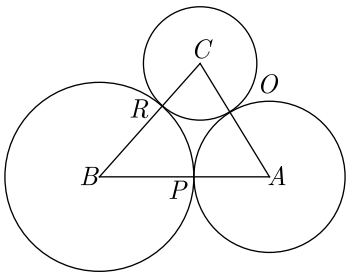
$$\frac{BD}{OC} = \frac{AB}{AO}$$

$$\frac{BD}{OC} = \frac{2AO}{AO}$$

$$BD = 2OC$$

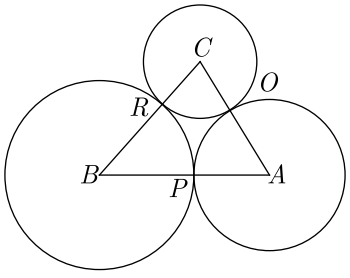
Main 2011

41. ABC is a triangle with $AB = 10$ cm, $BC = 8$ cm and $AC = 6$ cm (not down to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.



Sol :

We have redrawn the figure as below.



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In ΔABD and ΔACE,

Given $AB = AC$

Due to angles in the same segment,

$$\angle ABD = \angle ACE$$

Given $BD = CE$

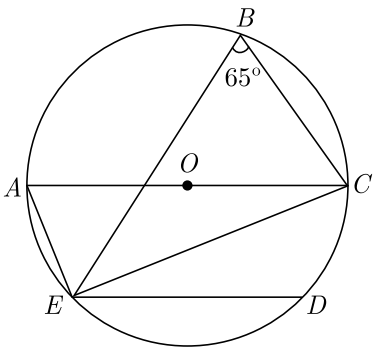
So, by SAS congruence criterion, we have

$$\Delta ABD \cong \Delta ACE$$

$$AD = AE$$

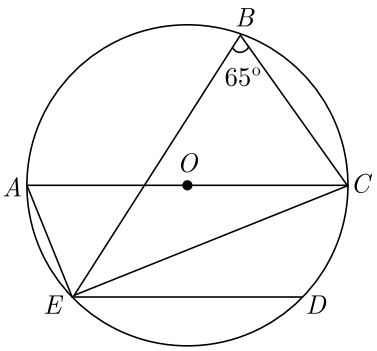
SQP 2013

37. In given figure, chord ED is parallel to the diameter AC of the circle. Given $\angle CBE = 65^o$, find $\angle DEC$.



Sol :

We have redrawn the figure as below.



Consider the arc CDE. Angles $\angle CBE$ and $\angle CAE$ are in the same segment of arc CDE.

Thus $\angle CAE = \angle CBE$

$$\angle CAE = 65^o$$

Since AC is the diameter of the circle and the angle in a semi-circle is a right angle, we have

$$\angle AEC = 90^o$$

Now, in ΔACE, we have

$$\angle ACE + \angle AEC + \angle CAE = 180^o$$

$$\angle ACE + 90^o + 65^o = 180^o$$

$$\angle ACE = 25^o$$

But, $\angle DEC$ and $\angle ACE$ are alternate angles, because $AC || DE$.

Thus $\angle DEC = \angle ACE$

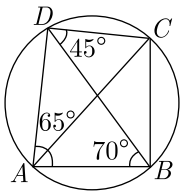
$$= 25^o$$

Main 2013

38. In the given figure, $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$, $\angle BDC = 45^\circ$.

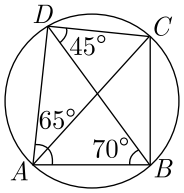
(i) Prove that AC is a diameter of the circle.

(ii) Find $\angle ACB$.



Sol :

We have redrawn the figure as below.



Here $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$,

and $\angle BDC = 45^\circ$

(i) Prove that AC is a diameter of the circle.

Since sum of all angles in a triangles is 180°, in ΔABD, we can write

$$\angle BAD + \angle ABD + \angle ADB = 180^\circ$$

$$65^\circ + 70^\circ + \angle ADB = 180^\circ$$

$$135^\circ + \angle ADB = 180^\circ$$

$$\angle ADB = 180^\circ - 135^\circ$$

$$= 45^\circ \quad ...(1)$$

Now $\angle ADB + \angle BDC = 45^\circ + 45^\circ = 90^\circ$

So, angle in a semi-cricle is 90°.

Hence, AC is a diameter of a circle.

(ii) $\angle ACB$

Since angles in a same segment are equal, from (1) we have

$$\angle ACB = \angle ADB = 45^\circ$$

SQP 2014, Comp 2005

39. In the given figure O is the centre of the circle. Given that $CM \perp OA$ and $CN \perp OB$. If

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We have $OA = OC = 15$ cm, $AB = 24$ cm,

$CD = 18$ cm

A perpendicular drawn to a chord from the centre, always bisects the chord.

Here AB and CD are chord, OM and ON are perpendicular drawn from center. Thus

$$AM = MB = \frac{AB}{2} = \frac{24}{2} = 12 \text{ cm}$$

and $$CN = ND = \frac{CD}{2} = \frac{18}{2} = 9 \text{ cm}$$

In right ΔAMO, using Pythagoras theorem,

$$AO^2 = OM^2 + MA^2$$

$$15^2 = OM^2 + 12^2$$

$$OM^2 = 225 - 144 = 81$$

$$OM = 9 \text{ cm}$$

Similarly, in ΔCNO, using Pythagoras theorem,

$$OC^2 = ON^2 + CN^2$$

$$15^2 = ON^2 + 9^2$$

$$ON^2 = 225 - 81$$

$$= 144 = (12)^2$$

$$ON = 12 \text{ cm}$$

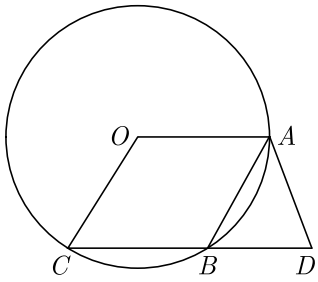
$$MN = OM + ON$$

$$= 9 + 12 = 21 \text{ cm}.$$

Hence, the value of MN is 21 cm.

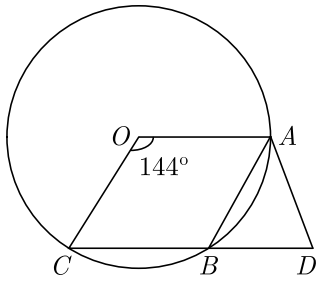
SQP 2010

44. In the circle with centre O, $\angle AOC = 144^\circ$, If $AB = AD$, find $\angle ADB$



Sol :

We have redrawn the given figure as below



In the figure,

$$\text{Reflex } \angle AOC = 360^\circ - 144^\circ$$

$$= 216^\circ$$

Major arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle ABC = \tfrac{1}{2} \text{ Reflex } \angle AOC$$

$$= \tfrac{1}{2} \times 216^\circ$$

$$= 108^\circ$$

Due to linear pair we have

$$\angle ABC + \angle ABD = 180^\circ$$

$$108^\circ + \angle ABD = 180^\circ$$

$$\angle ABD = 72^\circ$$

In ΔABD, $AB = AD$

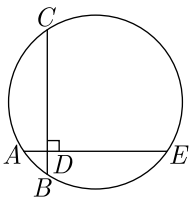
Same side make equal angle in any triangle.

$$\angle ADB = \angle ABD$$

$$= 72^\circ$$

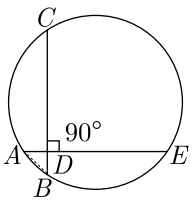
Main 2008

45. In the given figure, AE and BC intersect each other at point D. If $\angle CDE = 90^\circ$, $AB = 5$cm, $BD = 4$ cm and $CD = 9$ cm, find DE.



Sol :

We have redrawn the given figure as below



NODIA

Let the radii of three circles be

$BR = BP = x$, $AO = PA = y$ and

$CR = OC = z$

Sides of a triangle are

$AB = 10$ cm, $BC = 8$ cm and $AC = 6$ cm.

$$x + y = 10 \quad ...(1)$$

$$x + z = 8 \quad ...(2)$$

and $$y + z = 6 \quad ...(3)$$

Adding Eqs. (1), (2) and (3), we get

$$2(x + y + z) = 10 + 8 + 6$$

$$x + y + z = \frac{24}{2}$$

$$x + y + z = 12 \quad ...(4)$$

Subtracting Eq. (1) from Eq. (4), we obtain

$$z = 12 - 10 = 2 \text{ cm}$$

Subtracting Eq. (2) from Eq. (4), we obtain

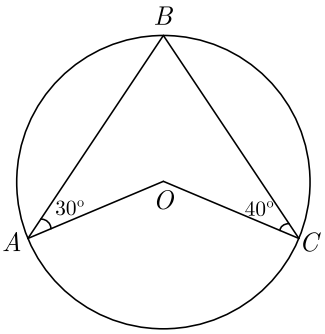
$$y\ y = 12 - 8 = 4 \text{ cm}$$

Subtracting Eq. (3) from Eq. (4), we obtain

$$x = 12 - 6 = 6 \text{ cm}$$

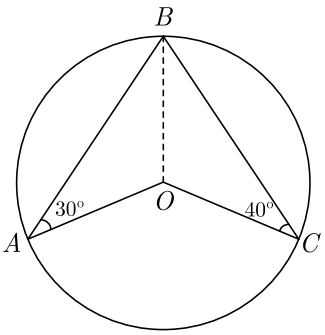
SQP 2011

42. In the given figure calculate the measure of ΔAOC.



Sol :

We have joined BO and redrawn the figure as below.



In ΔAOB :

Due to the radius of same circle,

$$OA = OB$$

Angle opposite to equal sides of a triangle are equal. Thus

$$\angle OBA = \angle OAB$$

$$\angle OBA = 30^{o} \quad ...(i)$$

In ΔBOC :

Due to the radius of same circle,

$$OB = OC$$

Angle opposite to equal sides of a triangle are equal. Thus

$$\angle OBC = \angle OCB$$

$$\angle OBC = 40^{o} \quad (ii)$$

Using eq. (i) and (ii), we have

$$\angle ABC = \angle OBA + \angle OBC$$

$$= 30^{o} + 40^{o}$$

$$= 70^{o}$$

Angle subtended by an arc of a circle at the circle is double the angle subtended by the arc on the circumference. Therefore

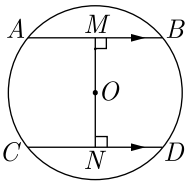
$$\angle AOC = 2\angle ABC$$

$$= 2 \times 70^{o}$$

$$= 140^{o}$$

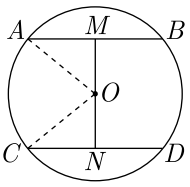
Main 2010

43. In the given figure, AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm, respectively.



Sol :

We have redrawn the given figure as below



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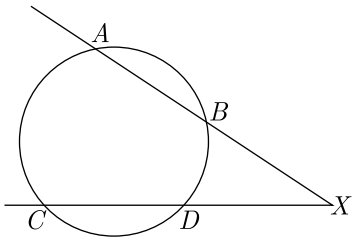
$= 300^\circ - 160^\circ$

$= 140^\circ$ Hence proved.

Main 2000

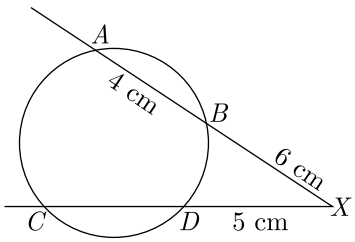
48. In the given figure, chords AB and CD, when extended meet at X. Given $AB = 4$ cm, $BX = 6$ cm, $XD = 5$ cm. Calculate the length of CD.



Sol :

We have $AB = 4$ cm, $BX = 6$ cm, $XD = 5$ cm

We have redrawn the given figure as below



From the given figure, we have

$$XB \cdot XA = XD \cdot XC$$

If two chords intersects internally or externally, then the product of the lengths of the segments are equal. Therefore

$$XB \cdot (XB + BA) = XD \cdot (XD + DC)$$

$$6 \cdot (6 + 4) = 5 \cdot (5 + DC)$$

$$6 \times 10 = 25 + 5DC$$

$$60 = 25 + 5DC$$

$$5DC = 60 - 25$$

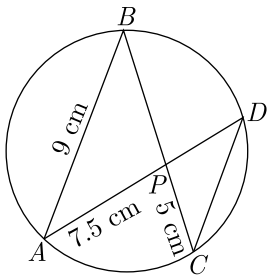
$$DC = \frac{35}{5} = 7 \text{ cm}$$

Hence, the value of DC is 7 cm.

FOUR MARK QUESTIONS

Main 2020

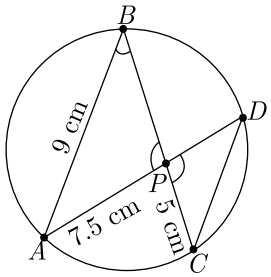
49. In the given figure $AB = 9$ cm, $PA = 7.5$ cm and $PC = 5$ cm. Chords AD and BC intersect at P.



(i) Prove that $\Delta PAB - \Delta PCD$.
(ii) Find the length of CD.
(iii) Find area of ΔPAB : Area of ΔPCD.

Sol :

We have redrawn the figure as below.



Here $AB = 9$ cm, $PA = 7.5$ cm and $PC = 5$ cm

(i) $\Delta PAB - \Delta PCD$.

In ΔPAB and ΔPCD

Angle in the same segment of a circle are equal, thus

$$\angle PBA = \angle PDC$$

Due to vertically opposite angles we have

$$\angle APB = \angle CPD$$

By AA similarity criterion, we have

$$\Delta PAB \sim \Delta PCD$$

(ii) Length of CD

Corresponding sides of similar triangles are always proportional. Since, $\Delta PAB \sim \Delta PCD$ we have

$$\frac{AP}{CP} = \frac{AB}{CD}$$

$$\frac{7.5}{5} = \frac{9}{CD}$$

$$CD = \frac{5 \times 9}{7.5} = 6 \text{ cm}$$

(iii) Area of ΔPAB : Area of ΔPCD.

The ratio of areas of two similar triangles in equal to the square of the ratio of their corresponding sides

$$\frac{\text{area of } \Delta PAB}{\text{area of } \Delta PCD} = \frac{AB^2}{CD^2}$$

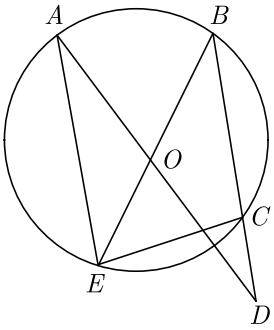
NODIA

$$= \frac{9^{\,2}}{6^2} = \frac{9}{4}$$

area of ΔPAB : area of $\Delta PCD = 9:4$

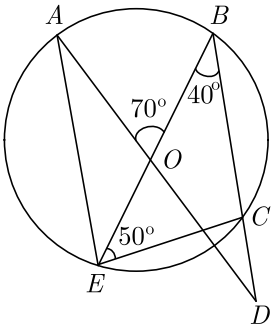
SQP 2020

50. In the given circle with centre O, $\angle AOB = 70^o$ and $\angle BEC = 50^o$.
(i) Find $\angle AEB$
(ii) Find $\angle ADB$
(iii) Find $\angle DAE$.



Sol :

We have redrawn the given figure as below



(i) $\angle AEB$
Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle AOB = \angle AEB$$
$$\angle AEB = \tfrac{1}{2}\angle AOB$$
$$= 35^o$$

(ii) $\angle ADB$
The angle in a semi-circle is always a right angle.

$$\angle ECB = 90^o$$

Due to angle sum property of a triangle,

$$\angle BEC + \angle ECB + \angle CBE = 180^\circ$$
$$50^\circ + 90^\circ + \angle CBE = 180^\circ$$
$$140^\circ + \angle CBE = 180^\circ$$
$$\angle CBE = 40^\circ$$

In ΔBOD, we have

$$\angle OBD + \angle ODB = \angle AOB$$
$$40^\circ + \angle ADB = 70^\circ$$
$$\angle ADB = 30^\circ$$

(iii) $\angle DAE$.
The sum of angles of a linear pair is always equal to 180°.

$$\angle AOE + \angle AOB = 180^\circ$$
$$\angle AOE + 70^\circ = 180^\circ$$
$$\angle AOE = 110^\circ$$

Due to radii of same circle,

$$OA = OE$$

Same side make equal angle in any triangle.

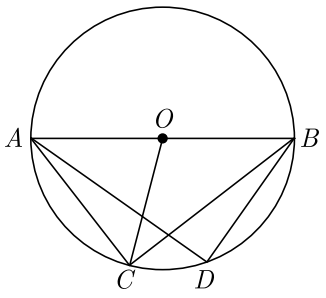
Thus $\angle OAE = \angle OEA$

The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle OAE + \angle OEA + \angle AOE = 180^\circ$$
$$\angle OAE + \angle OAE + 110^\circ = 180^\circ$$
$$2\angle OAE + 110^\circ = 180^\circ$$
$$2\angle OAE = 70^\circ$$
$$\angle OAE = \tfrac{1}{2} \times 70^\circ$$
$$\angle DAE = 35^\circ$$

SQP 2019

51. In the given figure O is the centre of the circle and AB is a diameter. If $AC = BD$ and $\angle AOC = 72^o$, find :
(i) $\angle ABC$
(ii) $\angle BAD$
(iii) $\angle ABD$



Sol :

We have redrawn the given figure as below.

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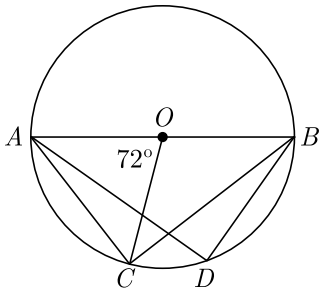


Since $\angle ADB$ is angle in semi-circle

$$\angle ADB = 90^o$$

(i) $\angle ABC$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus $\angle AOC = 2\angle ABC$

$$\angle ABC = \tfrac{1}{2}\angle AOC$$
$$= \tfrac{1}{2} \times 72^\circ$$
$$= 36^o$$

(ii) $\angle BAD$

Given $AC = BD$

Equal arcs make equal angle. Thus

$$\angle BAD = \angle ABC$$
$$= 36^o$$

(iii) $\angle ABD$

In ΔABD

$$\angle ABD + \angle BAD + \angle ADB = 180^o$$
$$\angle ABD + 36^o + 90^o = 180^o$$
$$\angle ABD + 126^o = 180^o$$
$$\angle ABD = 54^o$$

SQP 2018

52. ΔABC has been inscribed in the circle with centre O. E is the midpoint of $\overset{\frown}{BC}$. If $\angle ABC = 72^o$ and $\angle ACB = 48^o$, find

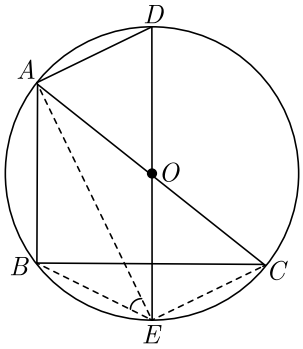
(i) $\angle BAE$

(ii) $\angle CBE$

(iii) $\angle DAC$

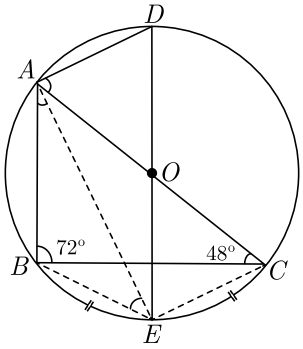
(iv) $\angle AEB$

NODIA



Sol :

We have redrawn the figure as below.



In the given circle ΔABC is inscribed in it Here E is midpoint of $\overset{\frown}{BC}$, EOD is diameter.

Here $\angle ABC = 72^o$,

$\angle ACB = 48^o$

(i) $\angle BAE$

Due to angles in the same segment,

$$\angle BAE = \angle BCE$$

In ΔABC, $\angle ABC = 72^o$

and $\angle ACB = 48^o$

Now in ΔABC,

$$\angle BAC + \angle ABC + \angle ACB = 180^o$$
$$\angle BAC + 72^o + 48^o = 180^o$$
$$\angle BAC + 120^o = 180^o$$
$$\angle BAC = 60^o$$

Due to angles subtended by equal arc,

$$\angle BAE = \angle CAE$$

But $\angle BAE + \angle CAE = 60^o$

$$\angle BAE + \angle BAE = 60^o$$
$$\angle BAE = \tfrac{1}{2} \times 60^o = 30^o$$

(ii) $\angle CBE$

Here we have $\angle CDE = 90^\circ$, $AB = 5$ cm,

$BD = 4$ cm, $CD = 9$ cm

In right ΔADB,

Due to vertically opposite anlges

$$\angle ADB = \angle CDE = 90^\circ$$

Using Pythagoras theorem,

$$AB^2 = AD^2 + BD^2$$

$$(5)^2 = AD^2 + (4)^2$$

$$AD^2 = 25 - 16 = 9$$

$$AD = 3 \text{ cm}$$

If two chords intersect internally or externally, then the product of the lengths of their segment is equal. Thus

$$DA \times DE = DB \times DC$$

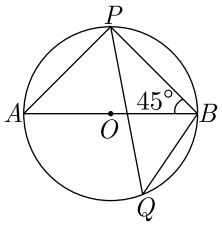
$$3 \times DE = 4 \times 9$$

$$DE = \frac{4 \times 9}{3} = 12 \text{ cm}$$

Hence, the value of DE is 12 cm.

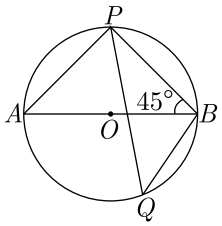
Main 2007

46. In the given figure, O is the centre of the circle and $\angle PBA = 45^\circ$. Calculate the value of $\angle PQB$.



Sol :

We have redrawn the given figure as below



Here $\angle PBA = 45^\circ$

In this give figure, AOB is a straight line, thus

$$\angle AOB = 180^\circ$$

Since, AB is a diameter of circle, so ΔAPB is in semi-circle. Therefore

$$\angle APB = 90^\circ$$

Using angle sum property in triangle ΔAPB we have

$$\angle APB + \angle PBA + \angle BAP = 180^\circ$$

$$90^\circ + 45^\circ + \angle BAP = 180^\circ$$

$$\angle BAP = 180^\circ - 135^\circ$$

$$= 45^\circ$$

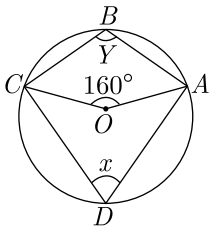
Since angles in a same segment are equal,

$$\angle PQB = \angle BAP = 45^\circ$$

Hence, the value of $\angle PQB$ is 45°.

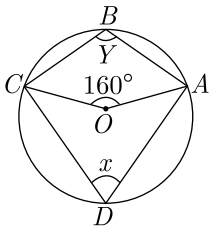
Main 2005

47. In the alongside figure, O is the centre of the circle and $\angle AOC = 160^\circ$. Prove that $3\angle y - 2\angle x = 140^\circ$.



Sol :

We have redrawn the given figure as below



Angle that an arc of a circle, subtend at the centre is twice that angle, which subtend at any point on the remaining part of a circle. Thus

$$\angle AOC = 2 \times \angle ADC$$

$$\angle AOC = 2x$$

But given that $\angle AOC = 160^\circ$, thus we have

$$160^\circ = 2x$$

$$x = \frac{160^\circ}{2} = 80^\circ$$

Similarly using same concept we have

$$2y = 360^\circ - 160^\circ$$

$$y = \frac{360^\circ - 160^\circ}{2}$$

$$= \frac{200^\circ}{2} = 100^\circ$$

Now, $3\angle y - 2\angle x = 2 \times 100^\circ - 2 \times 80^\circ$

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$$\Delta AOB \cong \Delta COD$$

Corresponding side of similar triangles are in the same ratio. Therefore

$$AB = CD$$

$$\overset{\frown}{AB} \cong \overset{\frown}{CD} \quad \text{...(iii)}$$

Similarly, we have

$$\overset{\frown}{BC} \cong \overset{\frown}{DA} \quad \text{...(iv)}$$

Adding (iii) and (iv), we have

$$\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{CD} + \overset{\frown}{DA}$$

$$\overset{\frown}{ABC} = \overset{\frown}{CDA}$$

Thus $\overset{\frown}{AC}$ divides the circle into two equal parts. Hence $\overset{\frown}{AC}$ is a diameter of the circle.

Similarly, we can prove that BD is also a diameter of the circle.

(ii) $ABCD$ is a rectangle.

Since AC and BD are diameters of the circle and $\angle ABC$ and $\angle ADC$ are angle in semi-circle.

Therefore, $\angle ABC = 90^o$

$$= \angle ADC$$

Also, $\angle BCD = \angle BAD = 90^o$

From (iii) and (iv), we have

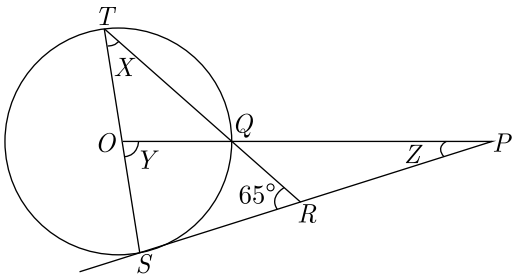
$$AB = CD$$

and $BC = DA.$

Hence, $ABCD$ is a rectangle.

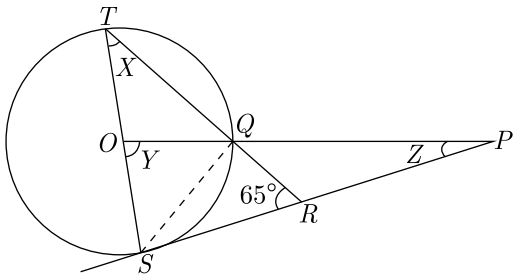
Comp 2011

55. In the figure below, O is the centre of the cirlce and SP is a tangent. If $\angle SRT = 65^\circ$, find the values of x, y and z.



Sol :

We have redrawn the given figure as below



Here $\angle TRS = 65^\circ$

Here we have joined SQ and SP is a tangent. Since tangents at any point of a circle and radius are perpendicular to each other. Therefore

$$\angle OSR = 90^\circ$$

Using angle sum property of a triangle ΔSTR we have

$$\angle STR + \angle TRS + \angle TSR = 180^\circ$$

$$x + 65^\circ + 90^\circ = 180^\circ$$

$$x = 180^\circ - 155^\circ$$

$$= 25^\circ$$

Angle made by a chord at centre is double the angle made by it at remaining part. Thus

$$y = 2x$$

$$= 2 \times 25^\circ = 50^\circ$$

$$y = 2x \Rightarrow y - = 50^\circ$$

Using angle sum property of a triangle ΔOPS we have

$$\angle SOP + \angle OPS + \angle OSP = 180^\circ$$

$$50^\circ + z + 90^\circ = 180^\circ$$

$$z = 180^\circ - 140^\circ$$

$$= 40^\circ$$

Comp 2010

56. In given figure $ABCD$ is a cyclic quadrilateral in the circle with centre O. $\angle ADC = 50^o$ and $\angle AOB = 68^o$.

(i) Find $\angle ACB$

(ii) Find $\angle OBA$.

(iii) Find $\angle BOC$.

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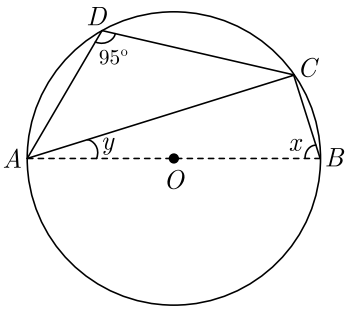


(a) 85^o and 5^o (b) 5^o and 85^o
(c) 70^o and 15^o (d) 15^o and 70^o

Sol :

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

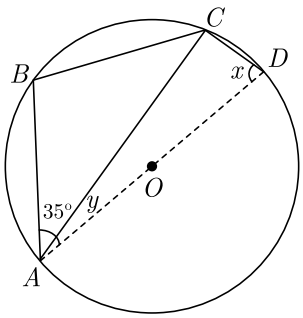
$$\angle ABC + \angle ADC = 180^\circ$$
$$x + 95^\circ = 180^\circ$$
$$x = 85^\circ$$

Since, $\angle ACB$ is in the semi-circle and angle in a semi-circle is right angle. Thus $\Delta\, ACB$ is right angle triangle.
Sum of two acute angle in a right angle triangle is always 90°.

$$\angle CAB + \angle CBA = 90^\circ$$
$$y + x = 90^\circ$$
$$y + 85^\circ = 90^\circ$$
$$y = 5^\circ$$

Thus (a) is correct option.

13. In the given figure O is the centre of the circle and given that $AB = BC$. What are the values of x and y ?



(a) 45^o and 65^o (b) 20^o and 70^o
(c) 70^o and 20^o (d) 65^o and 45^o

Sol :

In ΔABC, given that

$$AB = BC$$

Equal side make equal angle. Thus

$$\angle BAC = \angle BCA$$
$$= 35^o$$

The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle BAC + \angle BCA + \angle ABC = 180^o$$
$$35^o + 35^o + \angle ABC = 180^o$$
$$70^o + \angle ABC = 180^o$$
$$\angle ABC = 110^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

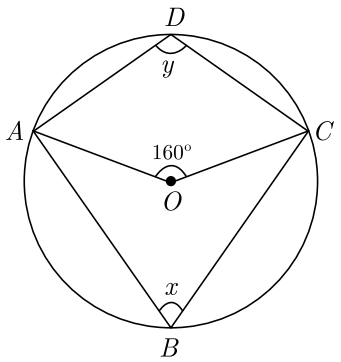
$$\angle ABC + \angle ADC = 180^o$$
$$110^o + x = 180^o$$
$$x = 70^o$$

Since, $\angle ACD$ is in the semi-circle and angle in a semi-circle is right angle. Thus $\Delta\, ACD$ is right angle triangle.
Sum of two acute angle in a right angle triangle is always 90°.

$$x + y = 90^o$$
$$70^o + y = 20^o$$
$$y = 20$$

Thus (c) is correct option.

14. In the figure, O is the centre of the circle and $\angle AOC = 160^\circ$, What is the value of $4\angle y - 5\angle x$



(a) 25^o (b) 35^o
(c) 45^o (d) 0^o

Sol :

Arc ADC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$$\angle AOC = 2\angle ABC$$
$$160^o = 2x$$

NODIA

$$t = 180^o - 2y$$

Now, $z = 360° - t$

$$z = 360° - (180^o - 2y)$$

$$z = 180^o + 2y \quad \text{(i)}$$

Angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

Thus $z = 2x$ (ii)

From (i) and (ii) we have

$$2x = 180° + 2y$$

$$2x - 2y = 180°$$

$$x - y = 90°$$

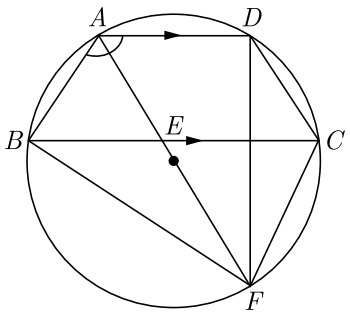
$\angle BAC - \angle OBC = 90^o$ Hence Proved

Comp 2008

58. The adjoining figure shows a quadrilateral $ABCD$ in which $AD \,||\, BC$ inscribed in a circle. The bisector of $\angle BAD$ cuts BC at E and the circle at F. Prove that

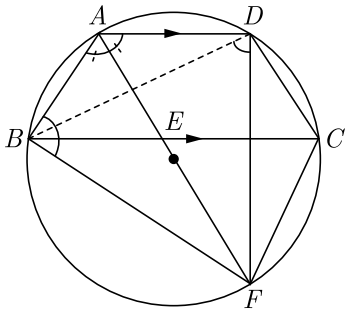
(i) $EF = FC$

(ii) $BF = DF$



Sol :

We have redrawn the figure as below.



Here $ABCD$ is a quadrilateral inscribed in the circle and $AD \,||\, BC$. The bisector of $\angle BAD$, intersects BC at E and the circle at F. We have joined BD.

(i) $EF = FC$

Since AF is the bisector of $\angle A$,

$$\angle DAE = \angle BAE \quad \text{(i)}$$

Since $AD \,||\, BC$, due to corresponding angles,

$$\angle CEF = \angle DAE \quad \text{(ii)}$$

From (i) and (ii) we get

$$\angle CEF = \angle BAE \quad \text{(iii)}$$

Due to angles in the same segment,

$$\angle BCF = \angle BAF$$

$$\angle ECF = \angle BAE \quad \text{(iv)}$$

From (iii) and (iv) we get

$$\angle ECF = \angle CEF$$

Equal opposite side in triangle make equal angles.

Thus $EF = FC$

(ii) $BF = DF$.

Due to angles in the same segment,

$$\angle DAF = \angle DBF \quad \text{(v)}$$

and $\angle BAF = \angle BDF$ (vi)

Since AF is the bisector of $\angle A$,

$$\angle DAF = \angle BAF \quad \text{(vii)}$$

From (iii) and (iv) we get

$$\angle DBF = \angle BDF$$

Equal opposite side in triangle make equal angles.

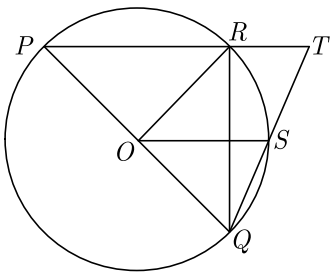
$BF = DF$ Hence proved.

Main 2006

59. In the given circle arc RS subtends $42°$ angle at the centre O in the given figure. PR and QS extended meet at a point T.

(i) Find $\angle RQS$

(ii) Find $\angle RTQ$.



Sol :

We have redrawn the given figure as below.

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$= 40°$

In ΔABD,

The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle ABD + \angle BAD + \angle ADB = 180^o$$
$$40° + \angle BAD + 40° = 180^o$$
$$80° + \angle BAD = 180^o$$
$$\angle BAD = 100°$$

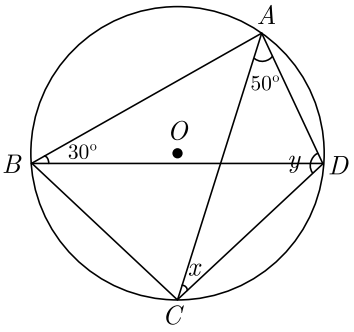
Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

Thus $\angle BAD + \angle BCD = 180^o$

$$100° + \angle BCD = 180^o$$
$$\angle BCD = 80^o$$

Thus (a) is correct option.

7. What are the values of x and y in given figure where O is the centre of the circle.



(a) 45° and 65° (b) 30° and 100°
(c) 100° and 30° (d) 65° and 45°

Sol :

Angles in the same segment of a circle are equal.

Thus $\angle ACD = \angle ABD$

$$x = 30^o$$

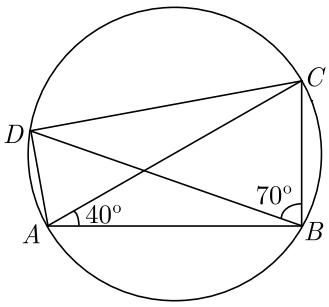
In ΔADC,

$$\angle CAD + \angle ACD + \angle ADC = 180^o$$
$$50^o + x + y = 180^o$$
$$50^o + 30^o + y = 180^o$$
$$80^o + y = 180^o$$
$$y = 100^o$$

Hence $x = 30^o, y = 100^o$

Thus (b) is correct option.

8. In given figure if $\angle DBC = 70°$ and $\angle BAC = 40°$, what is the value of $\angle BCD$?



(a) 70° (b) 60°
(c) 90° (d) 65°

Sol :

Angle $\angle CBD$ and $\angle CAD$ are angles in the same segment of the circle. Angles in the same segment are always equal.

Thus $\angle CAD = \angle CBD$

$$= 70^o$$
$$\angle BAD = \angle BAC + \angle CAD$$

Now $= 40^o + 70^o$

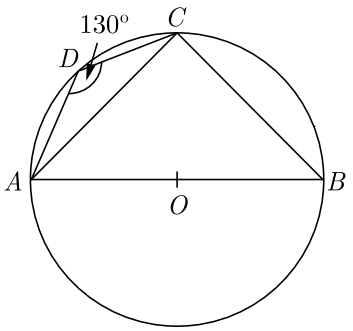
$$= 110^o$$

Since $ABCD$ is a cyclic quadrilateral, we have

$$\angle BAD + \angle BCD = 180^o$$
$$110^o + \angle BCD = 180^o$$
$$\angle BCD = 70^o$$

Thus (a) is correct option.

9. In given figure $ABCD$ is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B, C, D. If $\angle ADC = 130^o$, find $\angle BAC$.



(a) 30° (b) 55°
(c) 40° (d) 20°

Sol :

Since $ABCD$ is a cyclic quadrilateral, the opposite angles of a cyclic quadrilateral are always

and AD are diameter and chord of bigger circle respectively. Also, AO is diameter of smaller circle. We have joined CO and DB.
Now in ΔOCA and ΔBDA,
Since angle in semi-circle is right angle, we have

$$\angle OCA = \angle BDA = 90^\circ$$

Due to common angle we have

$$\angle OAC = \angle BAD$$

By AA axiom of similarity, we get

$$\Delta OCA \sim \Delta BDA$$

Since in similar triangle sides are proportional, thus

$$\frac{OA}{BA} = \frac{OC}{BD}$$

$$\frac{OA}{2OA} = \frac{OC}{BD} \qquad [BA = 2AO]$$

$$BD = 20C \qquad \text{Hence proved.}$$

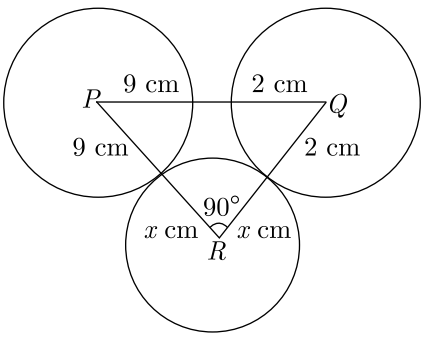
FIVE MARK QUESTIONS

Main 2004

62. P and Q are centres of circles of radii 9 cm and 2 cm, respectively, $PQ = 17$ cm. R is the centre of a circle of radius x cm, which touches the above circles externally. Given that $\angle PRQ = 90^\circ$, write an equation in x and solve it.

Sol :

As per given in question, we have drawn the given figure as below



Here P and Q are the centre of circles, whose radii are 9 cm and 2 cm, respectively.

Also, $PQ = 17$ cm

A circle with centre R and radius x touches above circles. Therefore

$$PR = 9 + x$$

and $$QR = 2 + x.$$

Since PRQ is a right angled triangle, using Pythagoras theorem

$$PQ^2 = PR^2 + QR^2$$

$$(17)^2 = (9 + x)^2 + (2 + x)^2$$

$$289 = 81 + x^2 + 18x + x^2 + 4x + 4$$

$$289 = 2x^2 + 22x + 85$$

$$2x^2 + 22x - 204 = 0$$

$$x^2 + 11x - 102 = 0$$

$$x^2 + 17x - 6x - 102 = 0$$

$$(x - 6)(x + 17) = 0$$

$$x = -17, 6$$

Thus $x = 6$ cm since radius cannot be negative.

END

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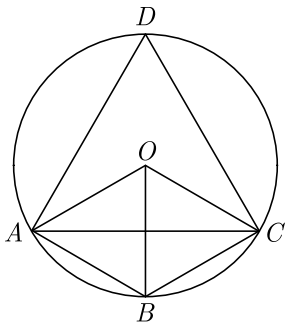
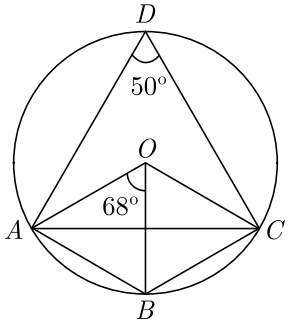


Sol :

We have redrawn the given figure as below.



(i) $\angle ACB$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle AOB = \angle ACB$$

$$\angle ACB = \tfrac{1}{2}\angle AOB$$

$$= 34^o$$

(ii) $\angle OBA$

Due to radii of same circle,

$$OA = OB$$

Same side make equal angle in any triangle.

In $\angle OAB$, $\angle OBA = \angle OAB$

$$= \frac{180^o - 68^o}{2}$$

$$= \frac{112^o}{2} = 56^o$$

(iii) $\angle BOC$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle AOC = 2\angle ADC$$

$$= 2 \times 50^o$$

$$= 100^o$$

Now $\angle BOC + \angle AOB = \angle AOC$

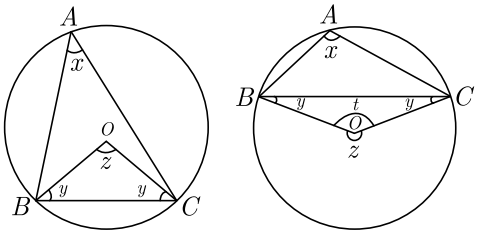
$$\angle BOC = \angle AOC - \angle AOB$$

$$= 100^o - 68^o$$

$$= 32^o$$

Comp 2009

57. In the given figure BC is a chord with centre O. A is a point on an arc BC as shown figure.

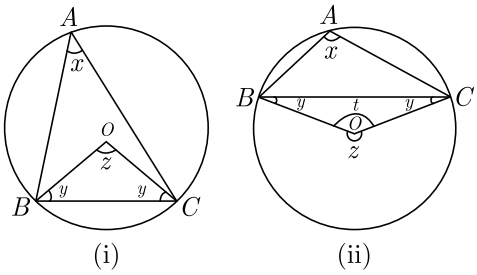


(i) Prove that $\angle BAC + \angle OBC = 90°$, if A is the point on the major arc.

(ii) Prove that $\angle BAC - \angle OBC = 90°$, if A is the point on the minor arc.

Sol :

We have redrawn the figure as below.



The arc BC makes $\angle BOC = z$ at the centre and $\angle BAC = x$ at a point on the circumference.

In both case $z = 2x$

(i) In ΔOBC

Sum of all internal angle of a triangle is always 180°. Thus

$$\angle OBC + \angle OCB + \angle BOC = 180°$$

$$y + z + y = 180^o$$

$$z + 2y = 180^o$$

$$2x + 2y = 180^o$$

$$x + y = 90°$$

$\angle BAC + \angle OBC = 90°$ Hence Proved

(ii) In ΔOBC,

Sum of all internal angle of a triangle is always 180°. Thus

$$\angle OBC + \angle OCB + \angle BOC = 180°$$

$$y + y + t = 180^o$$

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(c) 125^o (d) 120^o

Sol :

Since, $\angle ABD$ is in the semi-circle and angle in a semi-circle is right angle. Thus $\triangle ABD$ is right angle triangle.
Sum of two acute angle in a right angle triangle is always $90°$.

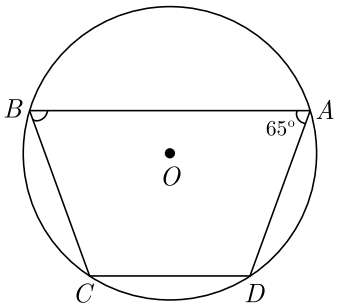
Thus $$\angle BAD + \angle BDA = 90^\circ$$
$$\angle BAD + 30^\circ = 90^\circ$$
$$\angle BAD = 60^\circ$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle A + \angle C = 180^o$$
$$60^\circ + x = 180^o$$
$$x = 120^\circ$$

Thus (d) is correct option.

4. In the given figure $AB \parallel DC$ and O is the centre of the circle. What is the value of the $\angle CBA$?



(a) 45^o (b) 35^o
(c) 55^o (d) 65^o

Sol :

Since $AB \parallel DC$, due to co-interior angle we have

$$\angle B + \angle C = 180^o \quad \text{(i)}$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

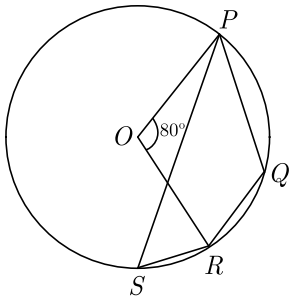
$$\angle C + \angle A = 180^o \quad \text{(ii)}$$

From (i) and (ii) we get

$$\angle B = \angle A$$
$$= 65^\circ$$

Thus (d) is correct option.

5. In the given figure O is the centre of the circle. If $\angle POR = 80^\circ$, what is the value of $\angle PQR$?



(a) 110^o (b) 95^o
(c) 140^o (d) 120^o

Sol :

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

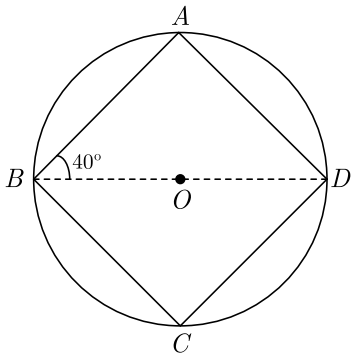
Thus $$\angle POR = \angle PSR$$
$$\angle PSR = \tfrac{1}{2}\angle POR$$
$$= 40^o$$

The opposite angles of a cyclic quadrilateral are always supplementary.

Thus $$\angle PSR + \angle PQR = 180^\circ$$
$$40^\circ + \angle PQR = 180^\circ$$
$$\angle PQR = 140^o$$

Thus (c) is correct option.

6. In the given figure O is the centre of the circle and given that $AB = AD$. What is the value of angle $\angle BCD$?



Q.1.(e) P.No.344

(a) 80^o (b) 60^o
(c) 70^o (d) 40^o

Sol :

In the figure, $AB = AD$

$$\angle ADB = \angle ABD$$

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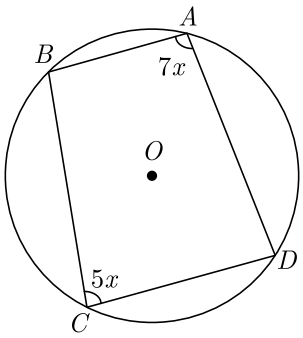
CHAPTER 17

CYCLIC PROPERTIES OF CIRCLE

MULTIPLE CHOICE QUESTIONS

1. What is the value of angle x in the following figure where O is the centre of the circle ?



Q.1.(a) P.No.343

(a) 45^o (b) 15^o
(c) 55^o (d) 25^o

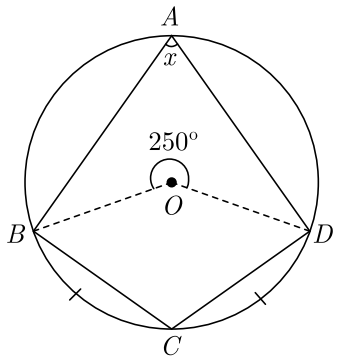
Sol :

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle A + \angle C = 180^\circ$$
$$7x + 5x = 180^\circ$$
$$12x = 180^\circ$$
$$x = 15^o$$

Thus (b) is correct option.

2. What is the value of angle x in the following figure where O is the centre of the circle ?



(a) 45^o (b) 35^o
(c) 55^o (d) 65^o

Sol :

Arc BAD subtends $\angle BOD$ at the centre and $\angle BCD$ at the remaining part of the circle.

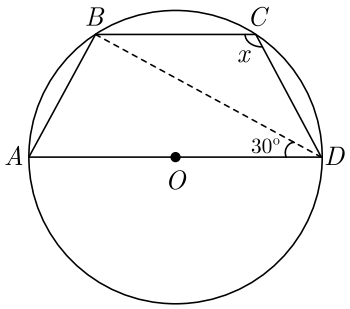
$$\angle BOD = 2\angle BCD$$
$$\angle BCD = \tfrac{1}{2}\angle BOD$$
$$= \tfrac{1}{2} \times 250^o$$
$$= 125^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle A + \angle C = 180^\circ$$
$$x + 125^\circ = 180^\circ$$
$$x = 55^\circ$$

Thus (c) is correct option.

3. In the given figure O is the centre of the circle. What is the value of angle x?



(a) 140^o (b) 150^o

NODIA

supplementary.

Thus $\angle ADC + \angle ABC = 180^o$

$130^o + \angle ABC = 180^o$

$\angle ABC = 50^o$

Since $\angle ACB$ is the angle in a semi-circle, we have

$\angle ACB = 90^o$

Now, in ΔABC, we have

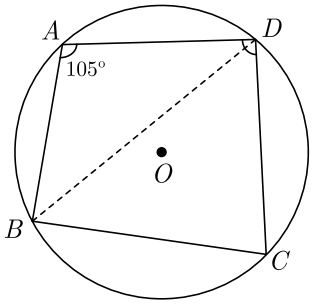
$\angle BAC + \angle ACB + \angle ABC = 180^o$

$\angle BAC + 90^o + 50^o = 180^o$

$\angle BAC = 40^o$

Thus (c) is correct option.

10. In the given figure $AB = AD$ and $CB = CD$ What is the of value of $\angle ADC$ in the following figure where O is the centre of the circle ?



(a) 80^o (b) 60^o

(c) 90^o (d) 40^o

Sol :

In ΔABD, $AB = AD$

$\angle ABD = \angle ADB$ ...(i)

Similarly in ΔBCD,

$BC = CD$

$\angle DBC = \angle BDC$ ...(ii)

Adding (i) and (ii) we get

$\angle ABD + \angle DBC = \angle ADB + \angle BDC$

$\angle ABC = \angle ADC$ (iii)

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$\angle ABC + \angle ADC = 180^\circ$ (iv)

From (iii) and (iv) we have

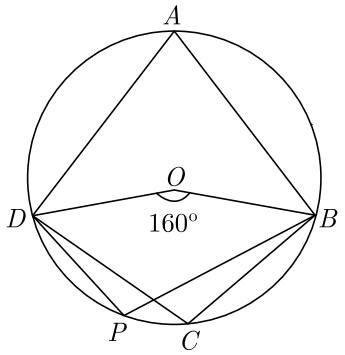
$\angle ADC + \angle ADC = 180^\circ$

$2\angle ADC = 180^o$

$\angle ADC = 90^o$

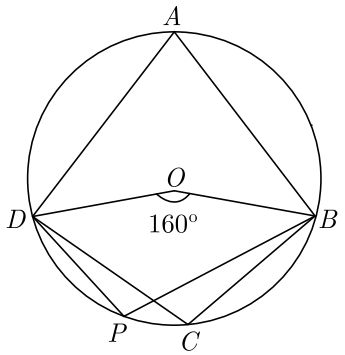
Thus (c) is correct option.

11. In given figure $ABCD$ is a cyclic quadrilateral, O is the centre of the circle. If $\angle BOD = 160^o$, what is the value of $\angle BCD$?



Sol :

We have redrawn the given figure as below.



Consider the arc BCD of the circle. This arc makes angle $\angle BOD = 160^o$ at the centre of the circle and $\angle BAD$ at a point A on the circumference. Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$\angle BOD = 2\angle BAD$

$\angle BAD = \frac{1}{2}\angle BOD$

$= 80^o$

Since $ABCD$ is a cyclic quadrilateral, the opposite angles of a cyclic quadrilateral are always supplementary.

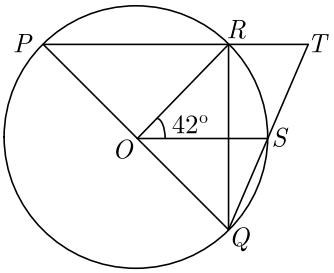
Thus $\angle BAD + \angle BCD = 180^o$

$80^o + \angle BCD = 180^o$

$\angle BCD = 100^o$

12. In the given figure O is the centre of the circle. What are the values of x and y ?

NODIA



(i) $\angle RQS$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus $\angle ROS = 2\angle RQS$

$\angle RQS = \frac{1}{2}\angle ROS$

$= \frac{1}{2} \times 42^o$

$= 21^o$

(ii) $\angle RTQ$

Angle in a semi-circle is always 90^o.

$\angle PRQ = 90^o$

Due to linear pair we have

$\angle QRT = 90^o$

Thus 90^o is right angle triangle. The sum of acute angles of a right triangle is always equal 90^o. Thus

$\angle RQT + \angle RTQ = 90^o$

$\angle RQS + \angle RTQ = 90^o$

$21^\circ + \angle RTQ = 90^o$

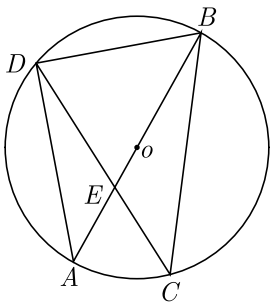
$\angle RTQ = 69^o$

Main 2007

60. In the given figure AB is a diameter of the circle. Given that $\angle AEC = 40^o$ and $\angle BCE = 22^o$.

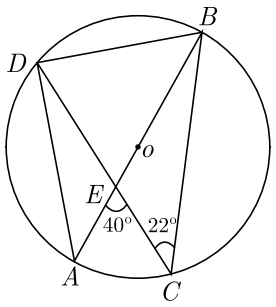
(i) Find $\angle ABD$

(ii) Find $\angle BDE$.



Sol :

We have redrawn the given figure as below.



(i) $\angle ABD$

Since $\angle ADB$ is angle in a semi-circle,

$\angle ADB = 90^o$

Due to angles in the same segment,

$\angle DAB = \angle DCB$

$= 22^o$

Sum of two acute angles in the right angle is always 90^o. In $\triangle ADB$,

$\angle DAB + \angle ABD = 90^o$

$22^\circ + \angle ABD = 90^o$

$\angle ABD = 68^o$

(ii) $\angle BDE$.

Due to vertically opposite angle we have

$\angle BED = \angle BEC$

$= 40^o$

Now in ΔEBD,

$\angle DEB + \angle EBD + \angle BDE = 180^o$

$40^o + 68^o + \angle BDE = 180^o$

$108^o + \angle BDE = 180^o$

$\angle BDE = 72^o$

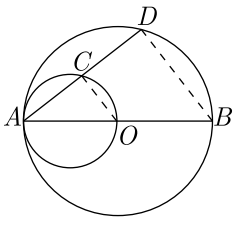
Hence $\angle ABD = 68^o$ and $\angle BDE = 72^o$

Main 2005

61. A circle with centre O, diameter AB and a chord AD si drawn. Another circle is drawn with AO as diameter to cut AD at C. Prove that $BD = 20C$.

Sol :

As per given in question, we have drawn the given figure as below



Here two circles intersect each other at A. AB

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$$x = 80^o$$

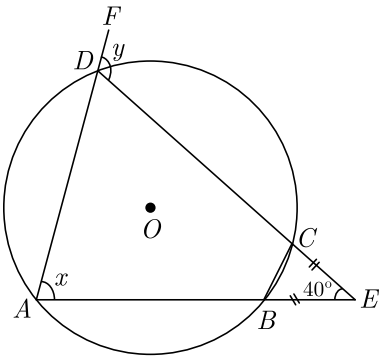
Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle ADC + \angle ABC = 180^o$$
$$y + x = 180^o$$
$$y + 80^\circ = 180^o$$
$$y = 100^o$$

Now $4\angle y - 5\angle x = 4 \times 100^o - 5 \times 80^o$
$= 400^o - 400^o$
$= 0$

Thus (d) is correct option.

15. In the given figure O is the centre of the circle. Given that $\angle CEB = 40^\circ$ and $CE = EB$. What is the value of $x + y$?



(a) 145^o (b) 235^o
(c) 255^o (d) 180^o

Sol :

In ΔBCE, $CE = BE$

Thus $\angle ECB = \angle EBC$

The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle ECB + \angle EBC + \angle CEB = 180^o$$
$$\angle ECB + \angle EBC + 40^\circ = 180^o$$
$$2\angle EBC + 40^\circ = 180^o$$
$$2\angle EBC = 140^o$$
$$\angle EBC = 70^o$$

Exterior angle of a cyclic quadrilateral is equal to interior opposite angle. Thus

$$\angle DAB = \angle ECB$$
$$x = 70^o$$

Similarly $\angle ADC = \angle EBC$
$= 70^o$

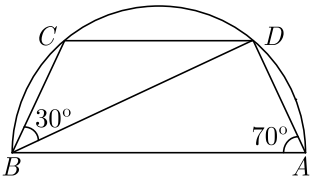
Due to linear pair,

$$\angle ADC + \angle FDC = 180^o$$
$$70^\circ + y = 180^o$$
$$y = 110^o$$

Now $x + y = 70^o + 110^o$
$= 180^o$

Thus (d) is correct option.

16. In given figure C and D are points on the semi-circle described on BA as diameter. Given $\angle BAD = 70^\circ$ and $\angle DBC = 30^o$. What is the value of $\angle BDC$?



(a) 30^o (b) 35^o
(c) 40^o (d) 45^o

Sol :

Since $ABCD$ is a cyclic quadrilateral, the opposite angles of a cyclic quadrilateral are always supplementary.

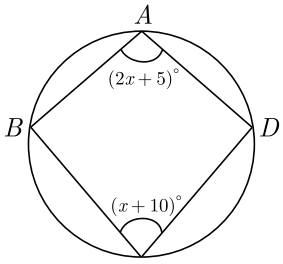
Thus $\angle BCD + \angle BAD = 180^o$
$\angle BCD + 70^o = 180^o$
$\angle BCD = 110^o$

In $\Delta ABCD$, we have

$$\angle CBD + \angle BCD + \angle BDC = 180^o$$
$$30^o + 110^o + \angle BDC = 180^o$$
$$\angle BDC = 40^o$$

Thus (c) is correct option.

17. In the given figure $ABCD$ is a cyclic quadrilateral. If $\angle BAD = 2x + 5$ and $\angle BCD = x + 10$ then x is equal to



NODIA

(d) Statement 1 is false and statement 2 is true

Sol :

Using angle sum property in triangle ΔABQ we

$$\angle BAQ + \angle AQB + \angle ABQ = 180^o$$
$$100^o + 25^o + y = 180^o$$
$$125^o + y = 180^o$$
$$y = 55^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle BAD + \angle BCD = 180^o$$
$$100^o + \angle BCD = 180^o$$
$$\angle BCD = 80^o$$

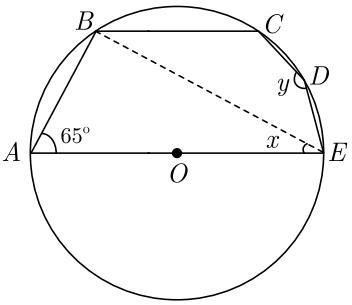
Using angle sum property in triangle ΔBCP we

$$x + y + \angle BCD = 180^o$$
$$x + 55^o + 80^o = 180^o$$
$$x + 135^o = 180^o$$
$$x = 45^o$$

Hence $x = 45^o, y = 55^o$

Thus statement 1 is true and statement 2 is false.
Thus (c) is correct option.

23. In the given figure O is the centre of the circle. Given that $BC \parallel AE$. What is the value of $y - 6x$



Statement 1 : The value $y - 6x$ is 5^o.
Statement 2 : The value of angle y is 65^o and x is 10^o.
Which of the following is valid ?
(a) Both statements are true.
(b) Both statements are false.
(c) Statement 1 is true and statement 2 is false
(d) Statement 1 is false and statement 2 is true

Sol :

Since, $\angle ABC$ is in the semi-circle and angle in a semi-circle is right angle. Thus ΔABC is right angle triangle.
Sum of two acute angle in a right angle triangle is always 90°.

$$\angle BAE + \angle BEA = 90^o$$
$$65^o + x = 90^o$$
$$x = 25^o$$

Since $BC \parallel AE$, due to alternate angle

$$\angle CBE = \angle BEA$$
$$= 25^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $BCDE$ is cyclic quadrilateral, thus

$$\angle CBE + \angle CDE = 180^o$$
$$25^o + y = 180^o$$
$$y = 155^o$$

Now $$y - 6x = 155^\circ - 6 \times 25^\circ$$
$$= 155^\circ - 150^\circ$$
$$= 5^\circ$$

Thus statement 1 is true and statement 2 is false
Thus (c) is correct option.

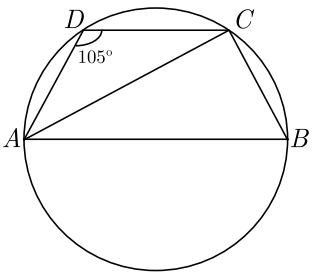
THREE MARKS QUESTIONS

SQP 2023

24. $ABCD$ is a cyclic quadrilateral. Diameter AB is parallel to DC and $\angle ADC = 105^o$.
(i) Find $\angle ABC$.
(ii) Find $\angle ACD$.

Sol :

As per given in question we have drawn the figure below.



The opposite angles of a cyclic quadrilateral are always supplementary.

Thus $$\angle ABC + \angle ADC = 180^\circ$$
$$\angle ABC + 105^o = 180^\circ$$
$$\angle ABC = 75^o$$

The angle in a semi-circle is always a right angle.

$$\angle ACB = 90^o$$

Sum of two acute angles in the right angle is

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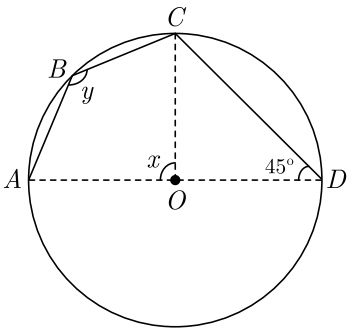


Q.3.(a) P.No.348

(a) 45^o (b) 35^o
(c) 55^o (d) 0^o

Sol :

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle ADC + \angle ABC = 180^o$$
$$45^o + y = 180^o$$
$$y = 135^o$$

Due to radii of same circle,

$$OC = OD$$

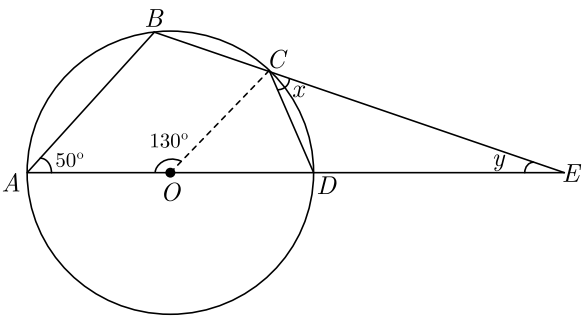
Equal side make equal angle. Thus in ΔOCD

$$\angle OCD = \angle ODC$$
$$= 45^o$$
$$\angle AOC = \angle OCD + \angle ODC$$
$$x = 45^o + 45^o$$
$$= 90^o$$

Thus $y - x = 135^o - 90^o$
$= 45^o$

Thus (a) is correct option.

21. In the given figure O is the centre of the circle. What is the value of $y - x$?



(a) 25^o (b) 35^o
(c) 5^o (d) 0^o

Sol :

Exterior angle of a cyclic quadrilateral is equal to interior opposite angle. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle OCE = \angle BAO$$
$$x = 50^\circ$$

Due to radii of same circle,

$$OC = OD$$

Equal side make equal angle. Thus in ΔOCD

$$\angle OCD = \angle ODC$$

In a triangle, the exterior angle is always equal to the sum of the interior opposite angle.

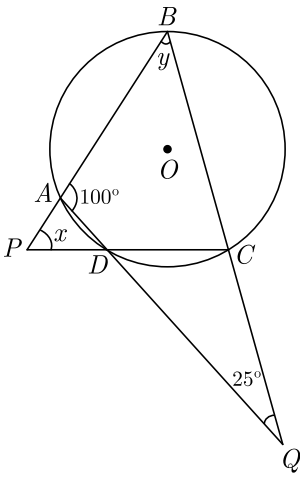
Thus $\angle OCD + \angle ODC = 130^\circ$
$\angle ODC + \angle ODC = 130^\circ$
$2\angle ODC = 130^\circ$
$\angle ODC = 65^o$

Similarly in ΔCDE, $x + y = 65^o$
$50^\circ + y = 65^o$
$y = 15^o$

Now $x - y = 50^\circ - 15^\circ$
$= 35^\circ$

Thus (b) is correct option.

22. In the given figure O is the centre of the circle.



Statement 1 : The value of angle x is 45^o.
Statement 2 : The value of angle y is 75^o.
Which of the following is valid ?
(a) Both statements are true.
(b) Both statements are false.
(c) Statement 1 is true and statement 2 is false

NODIA

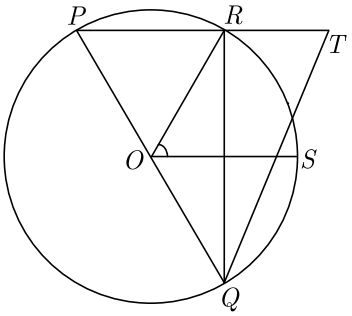
$$\angle CMB = 120^o$$

Due to linear pair,

$$\angle CMD + \angle CMB = 180^o$$
$$\angle CMD + 120^\circ = 180^o$$
$$\angle CMD = 60^o$$

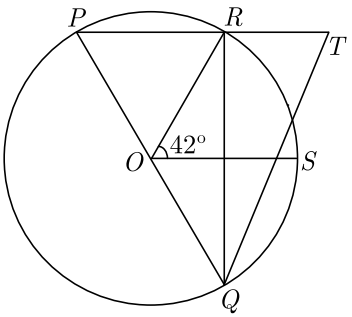
Comp 2019

27. In given figure PQ is a diameter of the circle whose centre is O. Given $\angle ROS = 42^o$, calculate $\angle RTS$.



Sol :

We have redrawn the given figure as below.



Here arc RS makes angle $\angle ROS = 42^o$ at the centre and $\angle RQS$ at a point on the circumference of the circle.

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle ROS = \angle RQS$$
$$\angle RQS = \tfrac{1}{2}\angle ROS$$
$$= \tfrac{1}{2} \times 42^o$$
$$= 21^o$$

Since $\angle QRP$ is angle in a semi-circle, we have

$$\angle QRP = 90^o$$

In ΔQRT, we have

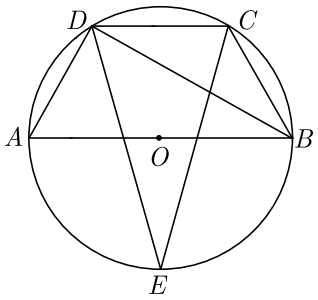
$$\angle QRP = \angle RQT + \angle RTQ$$
$$90^o = \angle RQS + \angle RTS$$
$$90^o = 21^o + \angle RTS$$
$$\angle RTS = 90^o - 21^o$$
$$= 69^o$$

SQP 2019

28. In the given figure O is the centre of the circle. Given that $AB \,||\, DC$ and $\angle ADB = 26^o$.

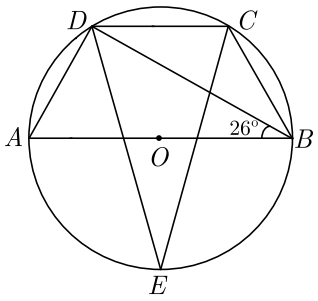
(i) Find $\angle DAB$

(ii) Find $\angle DEC$.



Sol :

We have redrawn the given figure as below.



(i) $\angle DAB$

Due to alternate angle we have

$$\angle BDC = \angle ABD$$
$$= 26^o$$

Here $\angle ADB$ is angle in semi-circle, thus

$$\angle ADB = 90^o$$

Sum of two acute angles in the right angle is always 90^o. In $\Delta\, ADB$,

$$\angle DAB + \angle DBA = 90^o$$
$$\angle DAB + 26^\circ = 90^o$$
$$\angle DAB = 64^o$$

(ii) $\angle DEC$

Now $\angle ADC = \angle ADB + \angle BDC$

$$= 90^\circ + 26^\circ$$
$$= 116^\circ$$

The opposite angles of a cyclic quadrilateral are

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always 90^o.

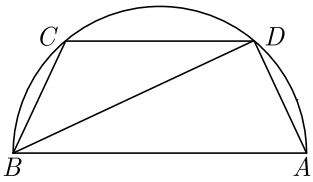
$$\angle CAB + \angle ABC = 90^o$$
$$\angle CAB + 75^\circ = 90^o$$
$$\angle CAB = 15^o$$

Alternate angles are always equal.

$$\angle ACD = \angle CAB$$
$$= 15^o$$

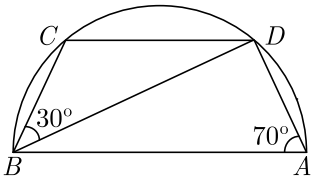
Main 2022

25. In given figure C and D are points on the semi-circle described on BA as diameter. Given $\angle BAD = 70^\circ$ and $\angle DBC = 30^o$.
(i) Find $\angle ABD$.
(ii) Find $\angle BDC$.



Sol :

We have redrawn the given figure as below.



Since $ABCD$ is a cyclic quadrilateral, the opposite angles of a cyclic quadrilateral are always supplementary.

Thus $\angle BCD + \angle BAD = 180^o$
$$\angle BCD + 70^o = 180^o$$
$$\angle BCD = 110^o$$

In $\Delta ABCD$, we have

$$\angle CBD + \angle BCD + \angle BDC = 180^o$$
$$30^o + 110^o + \angle BDC = 180^o$$
$$\angle BDC = 40^o$$

Since $\angle ADB$ is the angle in a semi-circle,

$$\angle ADB = 90^o$$

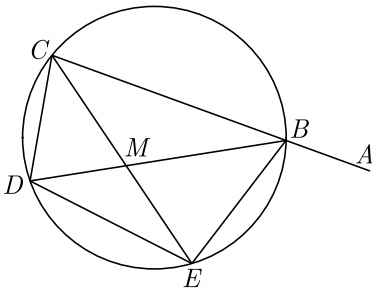
Now, in ΔABD, we have

$$\angle ABD + \angle ADB + \angle BAD = 180^o$$
$$\angle ABD + 90^o + 70^o = 180^o$$
$$\angle ABD = 20^o$$

Hence, $\angle ABD = 20^o$ and $\angle BDC = 40^o$

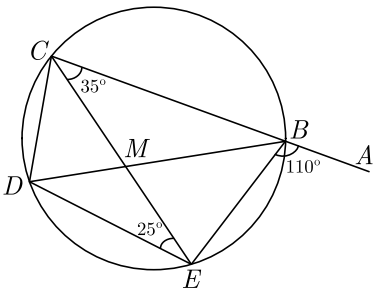
SQP 2020

26. In the given figure side CB of cyclic quadrilateral $BCDE$ is extended to a point A. Given that $\angle ABE = 110^o$, $\angle BCE = 35^o$ and $\angle CED = 25^o$.
(i) Find $\angle DCE$
(ii) Find $\angle CMD$.



Sol :

We have redrawn the given figure as below.



Due to angles in the same segment we have

$$\angle CBD = \angle CED$$
$$= 25^o$$

(i) $\angle DCE$

Sum of all angle on a straight line is always 180^o.

Therefore

$$110^\circ + \angle DBE + 25^\circ = 180^o$$
$$\angle DBE + 135^\circ = 180^o$$
$$\angle DBE = 45^o$$

Due to angles in the same segment we have

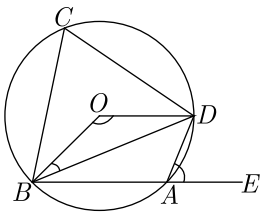
$$\angle DCE = \angle DBE$$
$$= 45^o$$

(ii) $\angle CMD$.

In triangle ΔCBM,

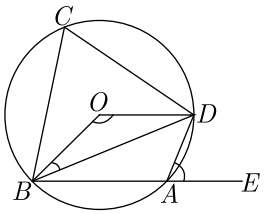
$$\angle CMB + \angle CBM + \angle MCB = 180^o$$
$$\angle CMB + \angle CBD + 35^\circ = 180^o$$
$$\angle CMB + 25^\circ + 35^\circ = 180^o$$
$$\angle CMB + 60^\circ = 180^o$$

NODIA



Sol :

We have redrawn the given figure as below



Here $\angle DAE = 70^\circ$

Due to linear pair axiom we obtain

$$\begin{aligned}\angle DAB &= 180^\circ - \angle DAE \\ &= 180^\circ - 70^\circ \\ &= 110^\circ\end{aligned}$$

(i) $\angle BCD$

Since sum of either pair of opposite angles of a cyclic quadrilateral is 180°. therefore

$$\begin{aligned}\angle BCD + \angle DAB &= 180^\circ \\ \angle BCD &= 180^\circ - \angle DAB \\ &= 180^\circ - 110^\circ \\ &= 70^\circ\end{aligned}$$

(ii) $\angle BOD$

In a circle angle made by an arc at the centre is twice the angle made by it at any point on the remaining part of the circle. Therefore

$$\begin{aligned}\angle BOD &= 2\angle BCD \\ &= 2 \times 70^\circ = 140^\circ\end{aligned}$$

(iii) $\angle OBD$

Here OB and OD are radius of circle.

Angles opposite to equal sides are equal, thus

$$\angle OBD = \angle ODB \qquad ...(i)$$

Using angle sum property in triangle ΔOBD we have

$$\begin{aligned}\angle OBD + \angle BDO + \angle DOB &= 180^\circ \\ \angle OBD + \angle ODB + 140^\circ &= 180^\circ \\ 2\angle OBD &= 180^\circ - 140^\circ \\ &= 40^\circ\end{aligned}$$

$$\angle OBD = \frac{40^\circ}{2} = 20^\circ$$

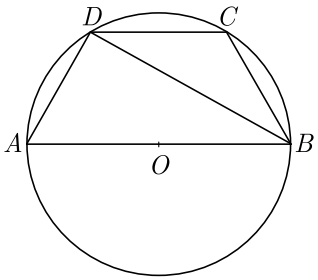
Main 2015

31. In given figure $ABCD$ is a cyclic quadrilateral, O is the centre of the circle. If $\angle DAB = 52^o$, $\angle CBD = 28^o$ find

(i) $\angle ABD$,

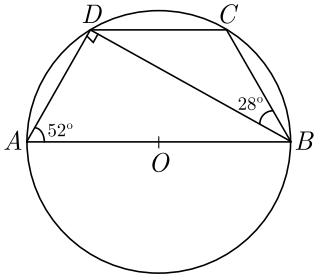
(ii) $\angle BCD$ and

(iii) $\angle CDB$.



Sol :

We have redrawn the given figure as below



The angle in a semi-circle is always a right angle.

$$\angle ADB = 90^o$$

(i) $\angle ABD$

Sum of two acute angles in the right angle is always 90^o.

Thus $$\begin{aligned}\angle ABD + \angle DAB &= 90^o \\ \angle ABD + 52^o &= 90^o \\ \angle ABD &= 38^o\end{aligned}$$

(ii) $\angle BCD$

The opposite angles of a cyclic quadrilateral are always supplementary.

Thus $$\begin{aligned}\angle BCD + \angle DAB &= 180^\circ \\ \angle BCD + 52^o &= 180^\circ \\ \angle BCD &= 180^o - 52^o \\ &= 128^o\end{aligned}$$

(iii) $\angle CDB$.

Sum of all angles of triangle is always 180°.

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always supplementary.

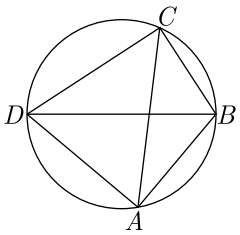
$$\angle ADC + \angle ABC = 180^o$$
$$\angle ADC + \angle ABD + \angle DBC = 180^o$$
$$116^\circ + 26^\circ + \angle DBC = 180^o$$
$$142^\circ + \angle DBC = 180^o$$
$$\angle DBC = 38^o$$

Due to angles in the same segment,

$$\angle DEC = \angle DBC$$
$$= 38^o$$

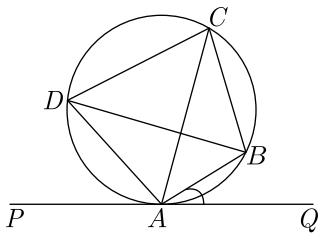
Main 2017

29. In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. If $\angle BAQ = 30^\circ$, prove that
(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle.



Sol :

We have redrawn the given figure as below



(i) BD is a diameter of the circle.

Here $\angle BAQ = 30^\circ$

Since AB is the bisector of $\angle CAQ$,

Thus $\angle CAB = 30^\circ$

and $\angle CAQ = 2\angle BAQ$
$= 2 \times 30^\circ = 60^\circ$

Since PAQ is straight line, we obtain

$$\angle PAC + \angle CAQ = 180^\circ$$
$$\angle PAC = 180^\circ - \angle CAQ$$
$$= 180^\circ - 60^\circ$$
$$= 120^\circ$$

As AD is bisector of $\angle PAC$,

$$\angle DAC = \frac{1}{2}\angle PAC$$
$$= \frac{1}{2} \times 120^\circ = 60^\circ$$

and $\angle DAB = \angle DAC + \angle CAB$
$= 60^\circ + 30^\circ = 90^\circ$

Since angle in a semicircle is always 90° i.e. angle made by diameter to any point made by diameter to any point on the circle is right angle.

Here $\angle DAB$ is angle in semicircle thus BD is a diameter of the circle. Hence proved.

(ii) ABC is an isosceles triangle.

If a chord is drawn through the point of contact of a tangent to a circle, then the angles made by the chord with the given tangent are respectively equal to the angle formed in the corresponding alternate segment.

Thus $\angle CDA = \angle CAQ$
$= 60^\circ$

Here, $ABCD$ is cyclic quadrilateral, therefore

$$\angle CDA + \angle CBA = 180^\circ$$
$$\angle CBA = 180^\circ - \angle CDA$$
$$= 180^\circ - 60^\circ$$
$$= 120^\circ$$

Using angle sum property in triangle ΔABC we have

$$\angle BCA + \angle CAB + \angle CBA = 180^\circ$$
$$\angle BCA + 30^\circ + 120^\circ = 180^\circ$$
$$\angle BCA + 150^\circ = 180^\circ$$
$$\angle BCA = 30^\circ$$

We already proved $\angle CAB = 30^\circ$

Here $\angle BCA = \angle CAB$

Therefore ΔABC is an isosceles triangle.

Hence proved.

Comp 2017

30. In the given figure, O is the centre of the circle. $\angle DAE = 70^\circ$. Find giving suitable reasons, the measure of
(i) $\angle BCD$
(ii) $\angle BOD$
(iii) $\angle OBD$

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(a) 65^o (b) 45^o
(c) 55^o (d) 5^o

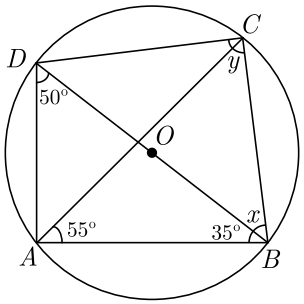
Sol :

Sum of either pair of opposite angles of cyclic quadrilateral is 180^o.

$$(2x+5)^\circ + (x+10)^\circ = 180^\circ$$
$$2x+5+x+10 = 180^\circ$$
$$3x = 180^\circ - 15^\circ$$
$$3x = 165^\circ$$
$$x = 55^\circ$$

The required value of x is 55^o.
Thus (c) is correct option.

18. What are the values of x and y in the following figure where O is the centre of the circle.



(a) 85^o and 40^o (b) 20^o and 70^o
(c) 40^o and 85^o (d) 70^o and 20^o

Sol :

Due to angle in the same segment

$$\angle BDC = \angle BAC$$
$$= 55^o$$
$$\angle ADC = \angle ADB + \angle BDC$$
$$= 50^o + 55^o$$
$$= 105^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

But $$\angle B + \angle D = 180^o$$
$$35^o + x + 105^o = 180^o$$
$$x + 140^o = 180^o$$
$$x = 40^o$$

Due to angle in the same segment

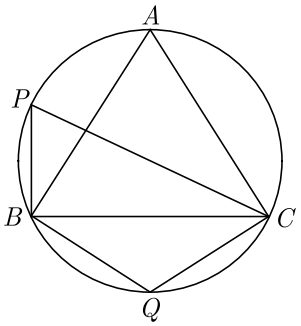
$$\angle DAC = \angle DBC$$
$$= 40^o$$
$$\angle A = 55^o + 40^o = 95^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle A + \angle C = 180^o$$
$$95^o + y = 180^o$$
$$y = 85^o$$

Hence $x = 40^o, y = 85^o$
Thus (c) is correct option.

19. In given figure ABC is an equilateral triangle inscribed in the circle. P and Q are two points on the circumference. What is the value of $\angle BQC$?



(a) 95^o (b) 120^o
(c) 110^o (d) 135^o

Sol :

All angles of equilateral are always 60^o.

$$\angle BAC = \angle ABC = \angle ACB = 60^o$$

Due to angles in the same segment,

$$\angle BPC = \angle BAC$$
$$= 60^o$$

The opposite angles of a cyclic quadrilateral are always supplementary.

$$\angle BQC + \angle BPC = 180^o$$
$$\angle BQC + 60^o = 180^o$$
$$\angle BQC = 120^o$$

Thus (b) is correct option.

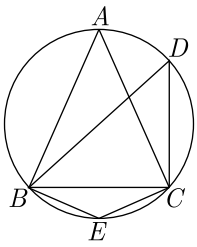
20. In the given figure O is the centre of the circle. What is the value of $y - x$?

NODIA

$$\angle BCD + \angle CDB + \angle DBC = 180^\circ$$
$$128^\circ + \angle CDB + 28^\circ = 180^\circ$$
$$\angle CDB + 156^\circ = 180^\circ$$
$$\angle CDB = 180^o - 156^o$$
$$= 24^o$$

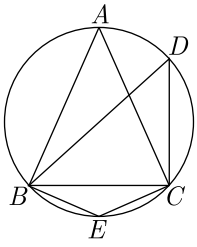
Main 2014

32. In the given figure, $\angle DBC = 58^\circ$. BD is a diameter of the circle. Calculate
(i) $\angle BDC$
(ii) $\angle BEC$
(iii) $\angle BAC$



Sol :

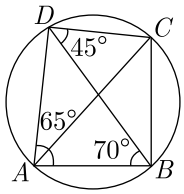
We have redrawn the given figure as below



Here $\angle DBC = 58^\circ$
(i) $\angle BDC$
Since, BD is a diameter of a circle and angle in a semi-circle is always 90°. thus
$$\angle BCD = 90^\circ$$
Using angle sum property in triangle ΔBCD we have
$$\angle DBC + \angle BCD + \angle BDC = 180^\circ$$
$$58^\circ + 90^\circ + \angle BDC = 180^\circ$$
$$\angle BDC = 180^\circ - (90^\circ + 58^\circ)$$
$$= 180^\circ - 148^\circ$$
$$= 32^\circ$$
(ii) $\angle BEC$
Sum of opposite angles in a cyclic quadrilateral is 180°. Here $BECD$ is cyclic quadrilateral, thus
$$\angle BEC + \angle BDC = 180^\circ$$
$$\angle BEC = 180^\circ - 32^\circ$$
$$= 148^\circ$$
$$\angle BEC = 180^\circ - 32^\circ$$
$$= 148^\circ$$
(iii) $\angle BAC$
Since angles is a same segment are equal, thus
$$\angle BAC = \angle BDC$$
$$\angle BAC = 32^\circ \qquad [\angle BDC = 32^\circ]$$

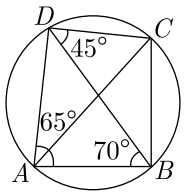
Comp 2013

33. In the given figure, if $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$ and $\angle BDC = 45^\circ$, calculate
(i) $\angle BCD$.
(ii) $\angle ADB$.
Hence, show that AC is a diameter.



Sol :

We have redrawn the given figure as below



Here $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$ and $\angle BDC = 45^\circ$
(i) $\angle BCD$
Sum of opposite angles in a cyclic quadrilateral is 180°. Since, $ABCD$ is a cyclic quadrilateral we have
$$\angle BCD + \angle BAD = 180^\circ$$
$$65^\circ + \angle BCD = 180^\circ$$
$$\angle BCD = 180^\circ - 65^\circ$$
$$= 115^\circ$$
(ii) $\angle ADB$
Using angle sum property in triangle ΔABD we have
$$\angle DAB + \angle ABD + \angle ADB = 180^\circ$$
$$65^\circ + 70^\circ + \angle ADB = 180^\circ$$
$$\angle ADB = 180^\circ - 135^\circ$$

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$= 45^\circ$

In ΔADC,

$\angle BDA + \angle BDC = 45^\circ + 45^\circ$

$= 90^\circ$

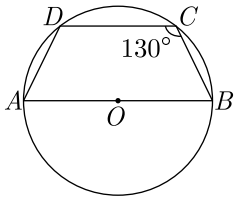
i.e. $\angle ADC$ in semi-circle is 90°, so AC is a diameter.

Mani 2012

34. In the given figure, AB is the diameter of a circle with centre O and $\angle BCD = 130^\circ$. Find

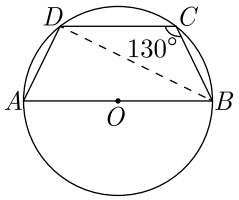
(i) $\angle DAB$.

(ii) $\angle DBA$.



Sol :

We have redrawn the given figure as below



We have drawn diagonal BD.

Here $\angle BCD = 130^\circ$

Since, ΔADB is in the semi-circle and angle in a semi-circle is right angle. Thus

$\angle ADB = 90^\circ$

(i) $\angle DAB$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$\angle BCD + \angle DAB = 180^\circ$

$130^\circ + \angle DAB = 180^\circ$

$\angle DAB = 180^\circ - 130^\circ$

$= 50^\circ$

(ii) $\angle DBA$

Using angle sum property in triangle ΔADB we have

$\angle DAB + \angle ADB + \angle DBA = 180^\circ$

$50^\circ + 90^\circ + \angle DBA = 180^\circ$

$\angle DBA = 180^\circ - 140^\circ$

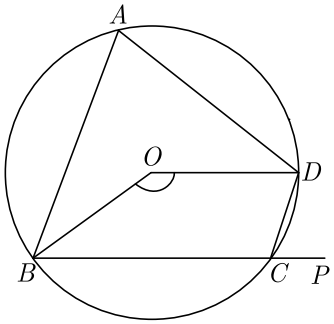
$= 40^\circ$

Comp 2011

35. In given figure O is the centre of the circle. The angle subtended by the arc BCD at the centre is $140°$. BC is produced to P.

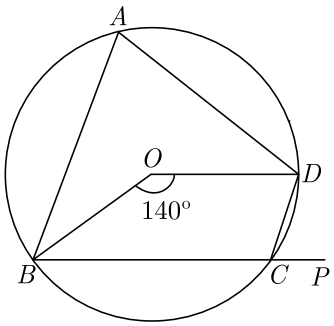
(i) Find $\angle BAD$.

(ii) Find $\angle DCP$.



Sol :

We have redrawn the given figure as below.



(i) $\angle BAD$

Here arc BD makes an angle $\angle BOD = 140^o$ at the centre O of the circle and $\angle BAD$ at a point A in the circle.

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$\angle BOD = 2\angle BAD$

$\angle BAD = \frac{1}{2}\angle BOD$

$= \frac{1}{2} \times 140^o$

$= 70^o$

(ii) Find $\angle DCP$

Since $ABDC$ is a cyclic quadrilateral, the opposite angles of a cyclic quadrilateral are always supplementary.

$\angle BAD + \angle BCD = 180^o$

$70^o + \angle BCD = 180^o$

$\angle BCD = 110^o$

NODIA

Since $\angle BCD$ and $\angle DCP$ are linear pairs, we have

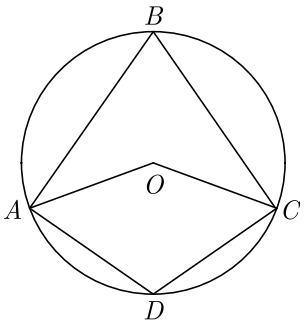
$$\angle BCD + \angle DCP = 180^o$$
$$110^o + \angle DCP = 180^o$$
$$\angle DCP = 70^o$$

The exterior angle of a cyclic quadrilateral is equal to the interior angle at the opposite vertex.

Hence, $\angle BAD = \angle DCP$
$= 70^o$

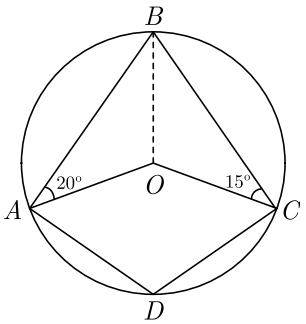
SQP 2011

36. In the given circle, $ABCD$ is a cyclic quadrilateral. $\angle BAO = 20°$ and $\angle BCO = 150^o$.
(i) Find $\angle AOC$.
(ii) Find $\angle ADC$.



Sol :

We have redrawn the given figure as below. Here we have joined OB.



Due to raddi of same circle we have

$$OA = OB = OC$$

Same side make equal angle in any triangle.

$$\angle OAB = \angle OBA$$
$$= 20^o$$

and $\angle OBC = \angle OCB$
$= 15^o$

Now $\angle ABC = \angle OBA + \angle OBC$
$= 20^o + 15^o$
$= 35^o$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

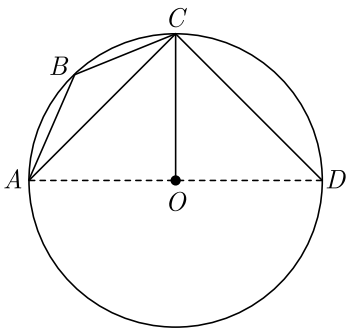
$$\angle AOC = 2\angle ABC$$
$$= 70^o$$

The opposite angles of a cyclic quadrilateral are always supplementary.

Thus $\angle ABC + \angle ADC = 180°$
$35° + \angle ADC = 180°$
$\angle ADC = 180^o - 35^o$
$= 145^o$

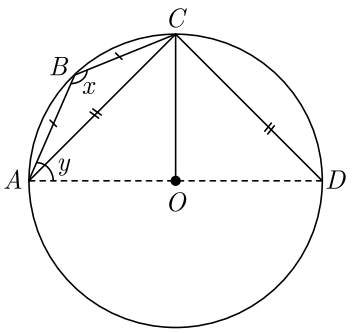
Comp 2008

37. In the given figure $AB = BC$ and $CA = CD$. Find the values of x and y in each of the following where O is the centre of the circle.
(i) Find $\angle ABC$.
(ii) Find $\angle BAD$.



Sol :

We have redrawn the given figure below.



Given $CA = CD$

Thus $\angle CAD = \angle CDA$

(i) $\angle ABC$

Since, $\angle ACD$ is in the semi-circle and angle in a semi-circle is right angle. Thus ΔACD is right angle triangle.

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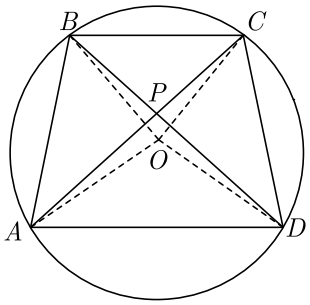
$= 52^\circ$

Comp 2018

45. ABC is a cyclic quadrilateral whose diagonals AC and BD intersect at P. If $AB = DC$, prove that:
(i) $\Delta PAB \cong \Delta PDC$
(ii) $PA = PD$ and $PC = PB$
(iii) $AD \,||\, BC$

Sol :

As per question we have drawn the figure as below.



We have,
(i) $\Delta PAB \cong \Delta PDC$
In ΔPAB and ΔPDC :
Due to angle in same segment

$$\angle ABD = \angle ACD$$

$$\angle ABP = \angle DCP$$

Due to angle in same segment

$$\angle BAC = \angle BDC$$

$$\angle BAP = \angle CDP$$

Given $AB = CD$
So, by AAD congruence criterion, we have

$$\Delta PAB \cong \Delta PDC$$

(ii) $PA = PD$ and $PC = PB$
Since $\Delta PAB \cong \Delta PDC$, Corresponding parts of congruent triangles are equal, thus

$$PA = PD$$

$$PB = PC$$

(iii) $AD \,||\, BC$
Here O is the centre of circle and we have joined O to all point.
Given $AB = CD$
Equal chord make equal angles at the centre,

$$\angle AOB = \angle COD$$

Angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

Thus $\angle AOB = 2\angle ACB$
and $\angle COD = 2\angle CAD$
Submitting above both result we have

$$2\angle ACB = 2\angle CAD$$

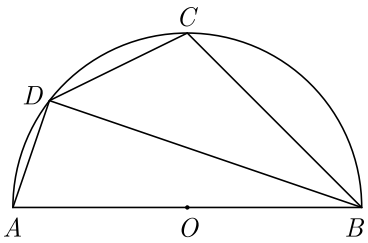
$$\angle ACB = \angle CAD$$

Since alternate angles at equal, we have

$AD \,||\, BC$ Hence Proved

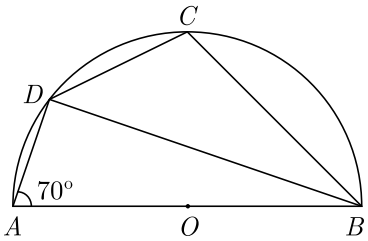
SQP 2017

46. In the given figure O is the centre of the semicircle. Given that $AD = DC$ and $\angle DAB = 70^\circ$
(i) Find $\angle BCD$.
(ii) Find $\angle ABD$.
(iii) Find $\angle ABC$.



Sol :

(i) $\angle BCD$
We have redrawn the given figure as below.



Since $\angle ADB$ is angle in semi-circle

$$\angle ADB = 90^\circ$$

Here $ABCD$ is a cyclic quadrilateral. From angle sum property of a cyclic quadrilateral,

$$\angle DAB + \angle ADC + \angle DCB + \angle CBA = 360^\circ$$

$$70^\circ + 140^\circ + \angle DCB + 40^\circ = 360^\circ$$

$$250^\circ + \angle DCB = 360^\circ$$

$$\angle DCB = 110^\circ$$

(ii) $\angle ABD$
In ΔADB, by angle sum property of triangle,

$$\angle DAB + \angle ABD + \angle ADB = 180^\circ$$

$$70^\circ + \angle ABD + 90^\circ = 180^\circ$$

$$\angle ABD + 160^\circ = 180^\circ$$

NODIA

(i) $\angle BCA$

The opposite angles of a cyclic quadrilateral are always supplementary. Here $OACB$ is a cyclic quadrilateral. Thus

$$\angle BCA + \angle BOA = 180^o$$
$$\angle BCA + 140^o = 180^o$$
$$\angle BCA = 40^o$$

(ii) $\angle ADB$

$$\text{reflex } \angle AOB = 360^o - 140^o$$
$$= 220^o$$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\text{reflex } \angle AOB = 2\angle ADB$$
$$\angle ADB = \tfrac{1}{2} \text{ reflex } \angle AOB$$
$$= \tfrac{1}{2} \times 220^o$$
$$= 110^o$$

(iii) $\angle DBC$

An exterior angle of a triangle is equal to the sum of two opposite interior angles. Thus in $\triangle DBC$,

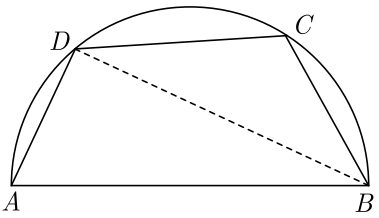
$$\angle DBC + \angle BCD = \angle ADB$$
$$\angle DBC + \angle BCA = 110^o$$
$$\angle DBC + 40^\circ = 110^o$$
$$\angle DBC = 70^o$$

Main 2004

40. In the given figure $ABCD$ is a cyclic quadrilateral inscribed a semicircle. $\angle DAB = 70^\circ$.

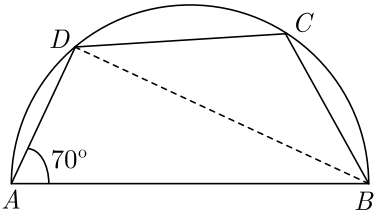
(i) Find $\angle DCB$

(ii) Find $\angle ABD$.



Sol :

We have redrawn the given figure as below.



(i) $\angle DCB$

The opposite angles of a cyclic quadrilateral are always supplementary. Here $ABCD$ is a cyclic quadrilateral. Thus

$$\angle DAB + \angle DCB = 180^o$$
$$70^o + \angle DCB = 180^o$$
$$\angle DCB = 110^o$$

(ii) $\angle ABD$

Since $\angle BDA$ is an angle in a semicircle,

$$\angle BDA = 90^o$$

In $\triangle BAD$, by angle sum property of a triangle

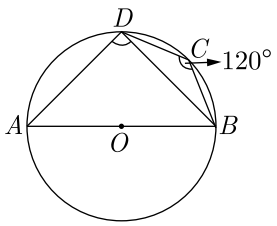
$$\angle BAD + \angle BDA + \angle ABD = 180^o$$
$$70^o + 90^o + \angle ABD = 180^o$$
$$160^o + \angle ABD = 180^o$$
$$\angle ABD = 20^o$$

Main 2002

41. In the given figure, AB is the diameter of a cricle with centre O. $\angle BCD$ is 120°. Find

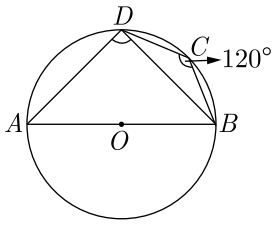
(i) $\angle BAD$.

(ii) $\angle DBA$.



Sol :

We have redrawn the given figure as below



Here AB is a diameter of a circle.

and $\angle BCD = 120^\circ$

(i) $\angle BAD$

Angle in a semi-circle is 90°, Here AB is a diameter of a circle. Thus

$$\angle ADB = 90^\circ$$

Now, A, B, C and D lie on a circle, so $ABCD$ form a cyclic quadrilateral.

Sum of opposite angles in a cyclic quadrilateral is 180°. Thus

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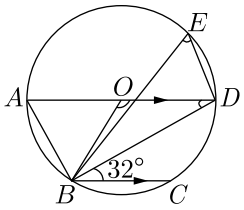


Here AD is a diameter and parallel to BC and

$$\angle CBD = 32^\circ.$$

(i) $\angle OBD$

Since, $AD \,||\, BC$ and due to alternate interior angles we have

$$\angle ADB = \angle CBD = 32^\circ$$

$$\angle ODB = \angle ADB$$

$$= 32^\circ$$

In ΔOBD, OB and OD are radius of circle, thus

$$OB = OD$$

In a triangle, angles opposite to equal sides are equal. Thus

$$\angle OBD = \angle ODB$$

$$= 32^\circ$$

(ii) $\angle AOB$

Since, ΔABD is a right angled triangle because it is in semi-circle.

$$\angle ABD = 90^\circ$$

Now, $$\angle ABO = \angle ABD - \angle OBD$$

$$= 90^\circ - 32^\circ$$

$$= 58^\circ$$

In ΔAOB, OA and OB are radius of circle, thus

$$OA = OB$$

In a triangle, angles opposite to equal sides are equal. Thus

$$\angle OAB = \angle ABO$$

$$= 58^\circ$$

Using angle sum property in triangle ΔOAB we have

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$58^\circ + 58^\circ + \angle AOB = 180^\circ$$

$$116^\circ + \angle AOB = 180^\circ$$

$$\angle AOB = 180^\circ - 116^\circ$$

$$= 64^\circ$$

(iii) $\angle BED$

Since angles in same segment are equal, we have

$$\angle BED = \angle BAD$$

$$= 58^\circ$$

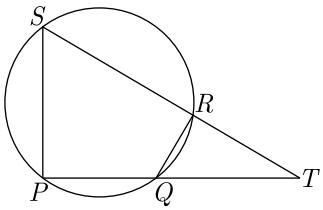
SQP 2016

49. In the given figure, $PQRS$ is a cyclic quadrilateral, PQ and SR produced meet at T.

(i) Prove $\Delta TPS \sim \Delta TRQ$.

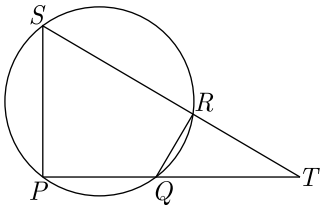
(ii) Find SP, if $TP = 18$ cm, $RQ = 4$ cm and $TR = 6$ cm.

(iii) Find area of quadrilateral $PQRS$, if area of $\Delta PTS = 27$ cm^2.



Sol :

We have redrawn the given figure as below



Here $PQRS$ is a cyclic quadrilateral, PQ and RS are produced such that it intersect at T.

(i) $\Delta TPS \sim \Delta TRQ$.

The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle. Thus in quadrilateral $PQRS$, we have

$$\angle SPQ = \angle QRT \qquad ...(i)$$

Now, in ΔTPS and ΔTRQ,

$$\angle SPQ = \angle QRT \quad \text{[from Eq. (i)]}$$

Due to common angle we obtain

$$\angle PTS = \angle RTQ \quad \text{[common]}$$

By AA similarity criteria we obtain

$$\Delta TPS \sim \Delta TRQ$$

(ii) SP, if $TP = 18$ cm, $RQ = 4$ cm and $TR = 6$ cm.

In similar triangles, corresponding sides are proportional. Since, ΔTPS and ΔTRQ are similar.

$$\frac{TP}{TR} = \frac{PS}{RQ}$$

$$\frac{18}{6} = \frac{PS}{4}$$

NODIA

$$\angle ABC = 90^\circ$$

Using angle sum property in triangle ΔABC we have

$$\begin{aligned}\angle CAB + \angle ABC + \angle BCA &= 180^\circ\\ 50^\circ + 90^\circ + \angle BCA &= 180^\circ\\ 140^\circ + \angle BCA &= 180^\circ\\ \angle BCA &= 180^\circ - 140^\circ\\ &= 40^\circ\end{aligned}$$

(ii) $\angle EDC$

Here $BC || AE$ thus due to alternate angle we have

$$\begin{aligned}\angle EAB + \angle ABC &= 180^\circ\\ \angle EAC + 50^\circ + 90^\circ &= 180^\circ\\ \angle EAC + 140^\circ &= 180^\circ\\ \angle EAC &= 180^\circ - 140^\circ\\ &= 40^\circ\end{aligned}$$

Here, $AEDC$ is a cyclic quadrilateral.

$$\begin{aligned}\angle EAC + \angle EDC &= 180^\circ\\ 40^\circ + \angle EDC &= 180^\circ\\ \angle EDC &= 140^\circ\end{aligned}$$

(iii) $\angle BEC$

Angle on same segment are equal. Therefore

$$\begin{aligned}\angle BEC &= \angle BAC\\ &= 50^\circ\end{aligned}$$

Now $$\begin{aligned}\angle BAE &= \angle BAC + \angle CAE\\ &= 50^\circ + 40^\circ\\ &= 90^\circ\end{aligned}$$

Here AC is a diameter of a circle. Since angle in a semi-circle is always 90°, thus

$$\angle AEC = 90^\circ$$

If an angle of a triangle in circle is 90°, the hypotenuse must be diameter of circle. Hence BE is diameter.

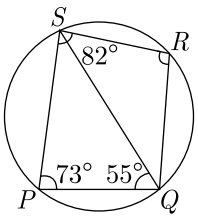
Main 2018

44. $PQRS$ is a cyclic quadrilateral. Given $\angle QPS = 73^\circ$, $\angle PQS = 55^\circ$ and $\angle PSR = 82^\circ$, calculate

(i) $\angle QRS$

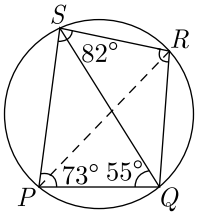
(ii) $\angle RQS$

(iii) $\angle PRQ$.



Sol :

We have redrawn the given figure as below



Here $\angle QPS = 73^\circ$, $\angle PQS = 55^\circ$ and $\angle PSR = 82^\circ$

(i) $\angle QRS$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $PQRS$ is cyclic quadrilateral, thus

$$\begin{aligned}\angle QPS + \angle QRS &= 180^\circ\\ 73^\circ + \angle QRS &= 180^\circ\\ \angle QRS &= 180^\circ - 73^\circ\\ &= 107^\circ\end{aligned}$$

(ii) $\angle RQS$

Using angle sum property in triangle ΔPQS we have

$$\begin{aligned}\angle SPQ + \angle PQS + \angle PSQ &= 180^\circ\\ 73^\circ + 55^\circ + \angle PSQ &= 180^\circ\\ \angle PSQ &= 180^\circ - 128^\circ\\ &= 52^\circ\\ \angle QSR &= \angle PSR - \angle PSQ\\ &= 82^\circ - 52^\circ\\ &= 30^\circ\end{aligned}$$

Using angle sum property in triangle ΔQRS we have

$$\begin{aligned}\angle QSR + \angle SRQ + \angle RQS &= 180^\circ\\ \angle RQS + 107^\circ + 30^\circ &= 180^\circ\\ \angle RQS = + 107^\circ + 30^\circ &= 180^\circ\\ &= 43^\circ\end{aligned}$$

(iii) $\angle PRQ$

Since angles in same segment are equal, we have

$$\angle PRQ = \angle PSQ$$

NODIA

$$\angle BAD + \angle DCB = 180^\circ$$
$$\angle BAD = 180^\circ - \angle DCB$$
$$= 180^\circ - 120^\circ$$
$$= 60^\circ$$

(ii) $\angle DBA$

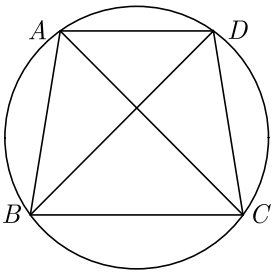
Using angle sum property in triangle ΔABD we have

$$\angle DBA + \angle BDA + \angle DAB = 180^\circ$$
$$\angle DBA + 90^\circ + 60^\circ = 180^\circ$$
$$\angle DBA = 180^\circ - 150^\circ$$
$$= 30^\circ$$

FOUR MARK QUESTIONS

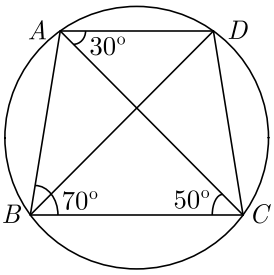
Comp 2023

42. In the given figure, $ABCD$ is a cyclic quadrilateral. $\angle ABC = 70^\circ$, $\angle ACB = 50^\circ$ and $\angle CAD = 30^\circ$.
(i) Find $\angle BAC$.
(i) Find $\angle ADC$.
(iii) Prove that BD is a diameter.



Sol :

We have redrawn the given figure as below



(i) $\angle BAC$

Due to angle sum property of a triangle,

$$\angle ABC + \angle BCA + \angle BAC = 180^\circ$$
$$70^\circ + 50^\circ + \angle BAC = 180^\circ$$
$$120^\circ + \angle BAC = 180^\circ$$
$$\angle BAC = 60^\circ$$

(ii) $\angle ADC$

The opposite angles of a cyclic quadrilateral are always supplementary.

Thus $\angle ABC + \angle ADC = 180^\circ$
$$70^\circ + \angle ADC = 180^\circ$$
$$\angle ADC = 110^\circ$$

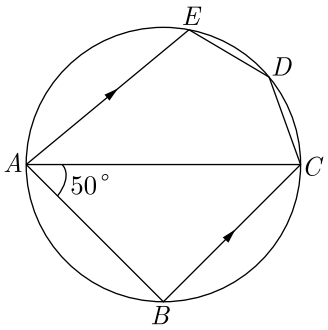
(iii) BD is a diameter.

$$\angle BAD = \angle BAC + 30^\circ$$
$$= 60^\circ + 30^\circ$$
$$= 90^\circ$$

Hence BD is a diameter because BD is sub-tending 90° at the circumference.

Main 2019

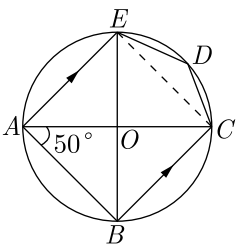
43. In the given figure, $ABCDE$ is a pentagon inscribed in a circle such that AC is a diameter and side $BC \| AE$. If $\angle BAC = 50^\circ$, then find giving reasons



(i) $\angle ACB$
(ii) $\angle EDC$
(iii) $\angle BEC$
Hence prove that BE is also a diameter.

Sol :

We have redrawn the figure as below. Here we have joined EB and EC.



Here we have $\angle BAC = 50^\circ$
(i) $\angle ACB$
Here AC is a diameter of a circle. Since angle in a semi-circle is always 90°, thus

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$$\angle ABD = 20^o$$

(iii) $\angle ABC$

Given $AD = DC$

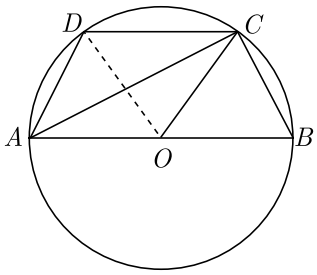
Equal arcs make equal angle. Thus

$$\angle ABD = \angle DBC$$

Now
$$\begin{aligned}\angle ABC &= \angle ABD + \angle DBC \\ &= \angle ABD + \angle ABD \\ &= 2\angle ABD \\ &= 2 \times 20 \\ &= 40^o\end{aligned}$$

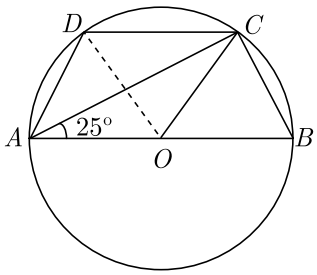
Comp 2016

47. In the given circle $ABCD$ is a cyclic quadrilateral with centre O. Here $AB \,||\, DC$ and $\angle CAB = 25^o$.
(i) Find $\angle COB$
(ii) Find $\angle ADC$
(iii) Find $\angle DAC$.
(iv) Find $\angle DOC$.



Sol :

We have redrawn the given figure as below.



Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

Thus
$$\begin{aligned}\angle COB &= 2\angle CAB \\ &= 2 \times 25^o \\ &= 50^o\end{aligned}$$

Due to radii of same circle we have

$$OC = OB$$

Same side make equal angle in any triangle.

$$\angle OCB = \angle OBC$$

The sum of all angles of a triangle is always equal to 180°. Thus

$$\begin{aligned}\angle OCB + \angle OBC + \angle COB &= 180^o \\ \angle OCB + \angle OCB + 50^\circ &= 180^o \\ 2\angle OCB + 50^\circ &= 180^o \\ 2\angle OCB &= 130^o \\ \angle OCB &= 65^o\end{aligned}$$

The opposite angles of a cyclic quadrilateral are always supplementary.

$$\begin{aligned}\angle ABC + \angle ADC &= 180^o \\ \angle OBC + \angle ADC &= 180^o \\ \angle 65^\circ + \angle ADC &= 180^o \\ \angle ADC &= 115^o\end{aligned}$$

Due to alternate angle we have

$$\begin{aligned}\angle DCA &= \angle CAB \\ &= 25^o\end{aligned}$$

The sum of all angles of a triangle is always equal to 180°. Thus

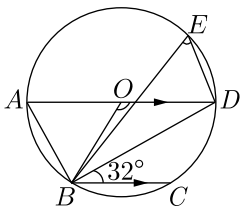
$$\begin{aligned}\angle DAC + \angle DCA + \angle ADC &= 180^o \\ \angle DAC + 25^\circ + 115^\circ &= 180^o \\ \angle DAC + 140^\circ &= 180^o \\ \angle DAC &= 40^o\end{aligned}$$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\begin{aligned}\angle DOC &= 2\angle DAC \\ &= 2 \times 40^o\end{aligned}$$

Main 2016

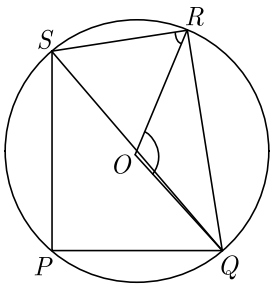
48. In the figure given below, AD is a diameter and O is the centre of the circle. AD is parallel to BC and $\angle CBD = 32^\circ$. Find
(i) $\angle OBD$
(ii) $\angle AOB$
(iii) $\angle BED$



Sol :

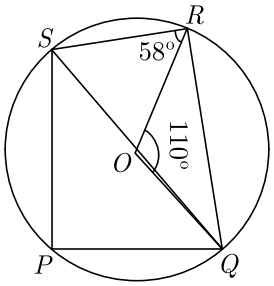
We have redrawn the given figure as below

NODIA



Sol :

We have redrawn the given figure as below.



(i) $\angle QSR$

Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle QOR = 2\angle QSR$$
$$\angle QSR = \tfrac{1}{2}\angle QOR$$
$$= 55^o$$

(ii) $\angle ORQ$

Due to radii of same circle we have

Thus $OQ = OR$

Same side make equal angle in any triangle.

Thus $\angle ORQ = \angle OQR$

Now in $\triangle ORQ$,

$$\angle ORQ + \angle OQR + \angle ROQ = 180^o$$
$$\angle ORQ + \angle ORQ + 110^\circ = 180^o$$
$$2\angle ORQ = 70^o$$
$$\angle ORQ = 35^o$$

(ii) $\angle QPS$

Now $\angle QRS = \angle ORS + \angle OQR$

$$= 58^o + 35^o$$
$$= 93^o$$

The opposite angles of a cyclic quadrilateral are always supplementary. Here $PQRS$ is a cyclic quadrilateral. Thus

$$\angle QPS + \angle QRS = 180^o$$
$$\angle QPS + 93^\circ = 180^o$$
$$\angle QPS = 87^o$$

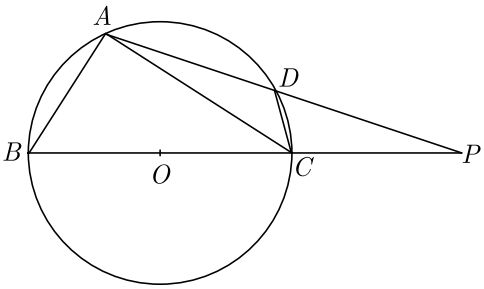
SQP 2014

52. In the given figure $ABCD$ is a cyclic quadrilateral in the circle with centre O. $\angle ABC = 55^o$ and $\angle DAC = 28^o$.

(i) Find $\angle ACB$

(ii) Find $\angle ADC$

(iii) Find $\angle APB$.



Sol :

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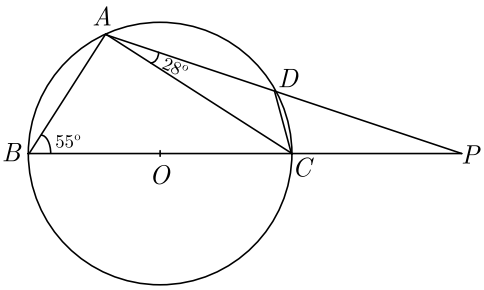

Since $\angle BCA$ is an angle in a semicircle,

$$BAC = 90^o$$

(i) $\angle ACB$

Sum of two acute angles in the right angle is always 90^o.

Thus $\angle ACB + \angle ABC = 90^o$

$$\angle ACB + 55^\circ = 90^o$$
$$\angle ACB = 35^o$$

(ii) $\angle ADC$

The opposite angles of a cyclic quadrilateral are always supplementary. Here $ABCD$ is a cyclic quadrilateral. Thus

$$\angle ADC + \angle ABC = 180^o$$
$$\angle ADC + 55^\circ = 180^o$$
$$\angle ADC = 125^o$$

(ii) $\angle APB$

An exterior angle of a triangle is equal to the sum of two opposite interior angles. Thus in $\triangle APC$,

$$\angle BCA = \angle APC + \angle CAP$$

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$$PS = \frac{18 \times 4}{6} = 12 \text{ cm}$$

(iii) Area of quadrilateral $PQRS$,
The ratio of areas of two similar triangles in equal to the square of the ratio of their corresponding sides. Since, ΔTPS and ΔTRQ are similar, we have

$$\frac{\text{ar}(\Delta TPS)}{\text{ar}(\Delta TRQ)} = \frac{TP^2}{TR^2}$$

Substituting $TP = 18$ cm, $TR = 6$ cm and $ar(\Delta TPS) = 27$ cm^2 we have

$$\frac{27}{ar(\Delta TRQ)} = \frac{18^2}{6^2}$$

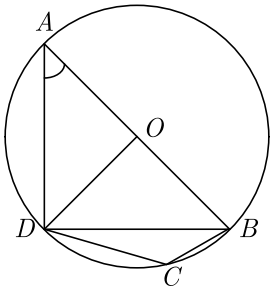
$$ar(\Delta TRQ) = \frac{27 \times (6)^2}{(18)^2}$$

$$ar(\Delta TRQ) = \frac{27 \times 6 \times 6}{18 \times 18} = 3 \text{ cm}^2$$

$$ar(\square PQRS) = ar(\Delta TPS) - ar(\Delta TRQ)$$
$$= 27 - 3 = 24 \text{ cm}^2$$

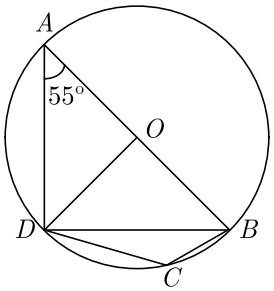
Comp 2015

50. In the given circle $ABCD$ is a cyclic quadrilateral with centre O. Here $OD \,||\, BC$ and $\angle DAB = 55^o$
(i) Find $\angle BCD$.
(ii) Find $\angle BOD$.
(iii) Find $\angle ODB$.
(iv) Find $\angle BDC$.



Sol :

We have redrawn the given figure as below.



(i) $\angle BCD$
The opposite angles of a cyclic quadrilateral are always supplementary.

$$\angle BCD + \angle BAD = 180^o$$
$$\angle BCD + 55^\circ = 180^o$$
$$\angle BCD = 125^o$$

(ii) $\angle BOD$
Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.
Thus

$$\angle BOD = 2\angle BAD$$
$$= 2 \times 55^o$$
$$= 110^o$$

(iii) $\angle OBD$

Due to radii of same circle,

$$OD = OB$$

Same side make equal angle in any triangle.

$$\angle ODB = \angle OBD$$

The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle ODB + \angle OBD + \angle BOD = 180^o$$
$$\angle ODB + \angle ODB + 110^\circ = 180^o$$
$$2\angle ODB = 70^o$$
$$\angle ODB = 35^o$$

(iv) $\angle BDC$

Due to alternate angle we have

$$\angle DBC = \angle ODB$$
$$= 35^o$$

The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle BDC + \angle BCD + \angle DBC = 180^o$$
$$\angle BDC + 125^\circ + 35^\circ = 180^o$$
$$\angle BDC + 160^\circ = 180^o$$
$$\angle BDC = 20^o$$

SQP 2015

51. In the given figure O is the centre of a circle in which $PQRS$ is cyclic quadrilateral. Given that $\angle QOR = 110^o$ and $\angle ORS = 58^o$.
(i) Find $\angle QSR$.
(ii) Find $\angle ORQ$.
(iii) Find $\angle QPS$.

NODIA

Sum of two acute angle in a right angle triangle is always $90°$.

$$\angle CAD + \angle CDA = 90^o$$
$$\angle CAD + \angle CAD = 90^o$$
$$2\angle CAD = 90^o$$
$$\angle CAD = \tfrac{1}{2} \times 90^o$$
$$= 45^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle ABC + \angle CDA = 180^o$$
$$\angle ABC + 45^o = 180^o$$
$$\angle ABC = 135^o$$

(i) $\angle BAD$

In ΔABC $\quad AB = BC$

Thus $\quad \angle BAC = \angle BCA$

But $\quad \angle BAC + \angle BCA + \angle ABC = 180^o$

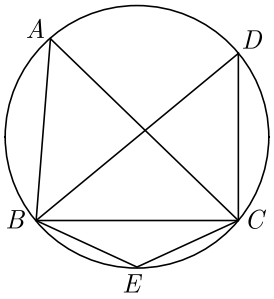
$$\angle BAC + \angle BAC + 135^o = 180^o$$
$$2\angle BAC + 135^o = 180^o$$
$$2\angle BAC = 45^o$$
$$\angle BAC = 22\tfrac{1}{2}^\circ$$

Now $\quad \angle BAD = \angle CAD + \angle BAC$

$$= 45^o + 22\tfrac{1}{2}^o$$
$$= 67\tfrac{1}{2}^o$$

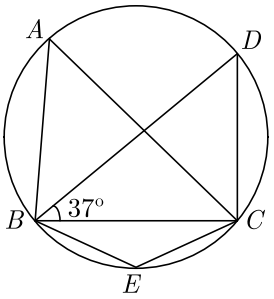
Main 2005

38. In the figure, $\angle DBC = 37^o$. BD is a diameter o the circle, calculate :
(i) Find $\angle BDC$
(ii) Find $\angle BEC$.
(iii) Find $\angle BAC$.



Sol :

We have redrawn the given figure as below.



(i) $\angle BDC$

Since $\angle BCD$ is angle in a semi-circle,

$$\angle BCD = 90^o$$

Thus ΔBCD is right angle triangle. Sum of two acute angles in the right angle is always 90^o. Thus in ΔBCD

$$\angle DBC + \angle BDC = 90^o$$
$$37^\circ + \angle BDC = 90^o$$
$$\angle BDC = 53^o$$

(ii) $\angle BEC$

Here $BDCE$ is a cyclic quadrilateral. The opposite angles of a cyclic quadrilateral are always supplementary.

$$\angle BEC + \angle BDC = 180^o$$
$$\angle BEC + 53^\circ = 180^o$$
$$\angle BEC = 127^o$$

(iii) $\angle BAC$

Equal arcs make equal angle. Thus

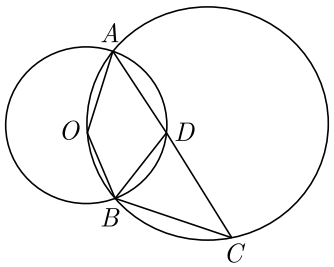
$$\angle BAC = \angle BDC$$
$$= 53^o$$

Comp 2005

39. In the given figure O is the centre of the smaller circle. The two circles are intersecting at A and B. ADC is a straight line and $\angle AOB = 140^o$.
(i) Find $\angle BCA$
(ii) Find $\angle ADB$
(iii) Find $\angle DBC$.

Sol :

We have redrawn the given figure as below.



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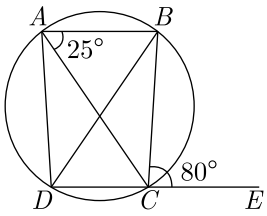
$$\angle ACB = \angle APB + 28^\circ$$
$$35^o = \angle APB + 28^\circ$$
$$7^o = \angle APB$$
$$\angle APB = 7^o$$

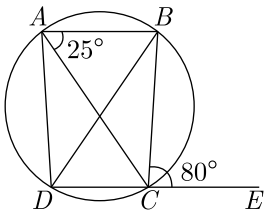
Main 2013

53. In the given figure, AB is parallel to DC, $\angle BCE = 80^\circ$ and $\angle BAC = 25^\circ$. Find
(i) $\angle CAD$.
(ii) $\angle CBD$.
(iii) $\angle ADC$.



Sol :

We have redrawn the given figure as below



Here $\angle BCE = 80^\circ$ and $\angle BAC = 25^\circ$
(i) $\angle CAD$.
Since exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

Thus $\angle BCE = \angle BAD$
$= 80^\circ$

Now, $\angle CAD = \angle BAD - \angle BAC$
$= 80^\circ - 25^\circ$
$= 55^\circ$

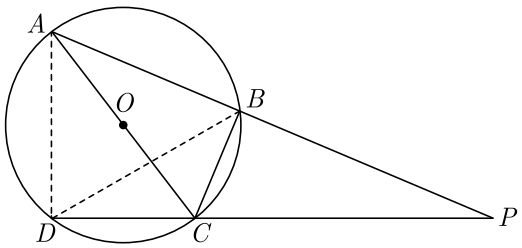
(ii) $\angle CBD$.
Angles in the same segment are equal. Thus
$$\angle CBD = \angle CAD$$
$$= 55^\circ$$

(iii) $\angle ADC$.
Here $AB \,||\, DC$ and $ABCD$ forms a parallelogram. In a parallelogram, sum of two adjacent angles is 180°. Thus
$$\angle BAD + \angle ADC = 180^\circ$$
$$\angle BAC + \angle CAD + \angle ADC = 180^\circ$$
$$25^\circ + 55^\circ + \angle ADC = 180^\circ$$
$$\angle ADC = 180^\circ - 80^\circ$$
$$\angle ADC = 100^\circ$$

NODIA

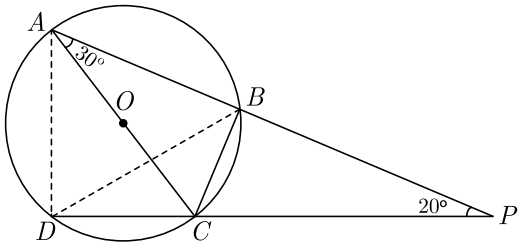
Comp 2012

54. In the given figure two chords AB and CD meet on producing at P. Centre O lies on AC. Given that $\angle OAB = 30^\circ$ and $\angle BPC = 20^\circ$.
(i) Find $\angle DCB$.
(ii) Find $\angle DBC$.
(iii) Find $\angle DAB$



Sol :

We have redrawn the given figure as below.



Due to angle in same segment,
$$\angle BDC = \angle BAC$$
$$= 30^o$$

Since, $\angle ABC$ is in the semi-circle and angle in a semi-circle is right angle. Thus
$$\angle ABC = 90^o$$

Due to linear pair
$$\angle CBP = 180^o - 90^o$$
$$= 90^o$$

(i) $\angle DCB$
In a triangle, the exterior angle is always equal to the sum of the interior opposite angle. In ΔBCP,
$$\angle DCB = \angle CBP + \angle BPC$$
$$= 90^o + 20^o$$
$$= 110^o$$

(ii) $\angle DBC$.
Using angle sum property in triangle ΔBDC we have

Sol :

Tangent at any point of a circle is perpendicular to the radius at the point of contact.

Thus $OT \perp PT$

Now in right-angled triangle PTO

$$OP^2 = OT^2 + PT^2$$
$$= (7)^2 + (24)^2$$
$$= 49 + 576$$
$$= 625$$

Thus $OP = 25$ cm

Since $OR = OT$ because of radii of circle,

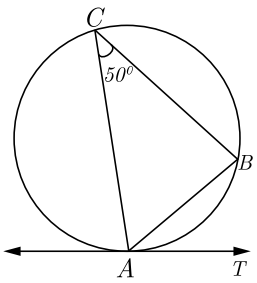
$$PR = OP + OR = 25 + 7 = 32 \text{ cm}$$

Thus (c) is correct option.

12. In the given figure AB is a chord of the circle such that $\angle ACB = 50^\circ$. If AT is tangent to the circle at point A, then $\angle BAT$ is a equal to

(a) 65^o (b) 60^o

(c) 50^o (d) 40^o



Sol :

In the given figure AB is a chord of the circle such that $\angle ACB = 50^\circ$. AT is tangent to the circle at A.

Due to angles in the alternate segments we have

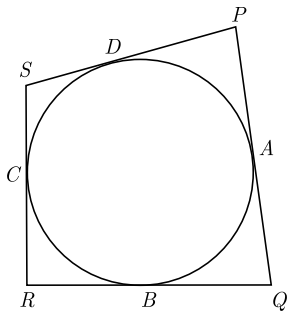
$$\angle ACB = \angle BAT = 50^\circ$$

Thus (c) is correct option.

13. In the given figure if sides PQ, QR, RS and SP of a quadrilateral $PQRS$ touch a circle at point A, B, C and D respectively, then $PD + BQ$ is equal to

(a) PQ (b) QR

(c) PS (d) SR



Sol :

Since PD and PA are the tangents to the circle,

$$PA = PD \quad ...(1)$$

Similarly QA and QB are the tangents

$$QA = QB \quad ...(2)$$

From (1) and (2) we have

Now $PD + BQ = PA + QA = PQ$

Thus (a) is correct option.

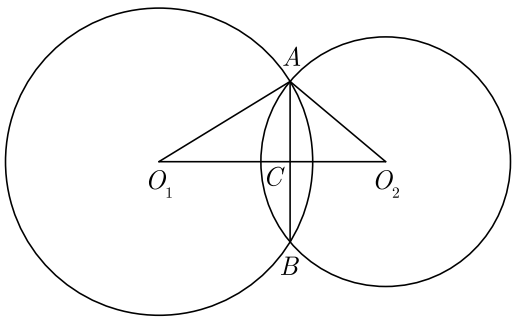
14. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and $AB = 24$ cm, then the distance O_1O_2 is equal to

(a) 44 cm (b) 51 cm

(c) 40.5 cm (d) 45 cm

Sol :

As per given information we have drawn the figure below.



Since C is the mid-point of AB,

$$AC = 12$$
$$AO_1 = 37$$

and $AO_2 = 20$

$$CO_1 = \sqrt{37^2 - 12^2} = 35$$
$$CO_2 = \sqrt{20^2 - 12^2} = 16$$
$$O_1O_2 = 35 + 16 = 51$$

Thus (b) is correct option.

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a semi-circle is right angle. Thus ΔACD is right angle triangle.
Sum of two acute angle in a right angle triangle is always 90°.

$$\angle CAD + \angle ADC = 90^o$$
$$x + 50 = 90^o$$
$$x = 40^o$$

In ΔOCD, due to radii of the same circle,

$$OC = OD$$

Equal side make equal angle.

$$\angle ODC = \angle OCD$$
$$\angle ADC = 50^o$$

Using angle sum property in triangle ΔOCD we

$$\angle OCD + \angle ODC + \angle COD = 180^o$$
$$50^o + 50^o + \angle COD = 180^o$$
$$100^o + \angle COD = 180^o$$
$$\angle COD = 80^o$$

Since $AB \parallel OC$, due to corresponding angles,

$$\angle BAO = \angle COD$$
$$\angle BAD = 80^o$$

Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus
Now in cyclic quad. $ABCD$

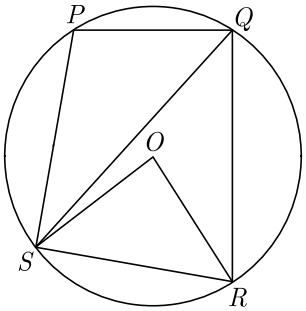
$$\angle BAD + \angle BCD = 180^o$$
$$80^o + y + 50^o = 180^o$$
$$130^o + y = 180^o$$
$$y = 180^o - 130^o$$
$$y = 50^o$$

Now $y - 6x$

Thus $x = 40^o, y = 50^o$

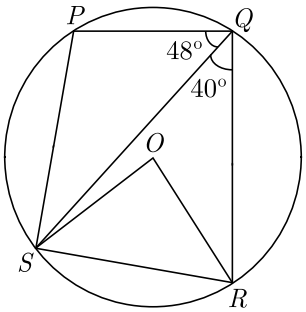
Comp 2010

57. In the given figure $PQRS$ is a cyclic quadrilateral in the circle with centre O. $\angle PQS = 48^o, \angle SQR = 40^o$ and $PS \,||\, QR$.
(i) Find $\angle QRS$
(ii) Find $\angle OSR$.



Sol :

We have redrawn the given figure as below.



(i) $\angle QRS$
Since $PS \,||\, QR$, due to alternate angle

$$\angle PSQ = \angle SQR$$
$$= 40^o$$

The sum of all angles of a triangle is always equal to 180°. Thus

$$40^\circ + 48^\circ + \angle QPS = 180^o$$
$$88^\circ + \angle QPS = 180^o$$
$$\angle QPS = 92^o$$

The opposite angles of a cyclic quadrilateral are always supplementary.

$$\angle QPS + \angle QRS = 180^o$$
$$92^\circ + \angle QRS = 180^o$$
$$\angle QRS = 88^o$$

(ii) $\angle OSR$
Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.

$$\angle SOR = 2\angle SQR$$
$$= 2 \times 40^o$$
$$= 80^o$$

Due to radii of same circle,

$$OS = OR$$

Same side make equal angle in any triangle.

$$\angle OSR = \angle ORS$$

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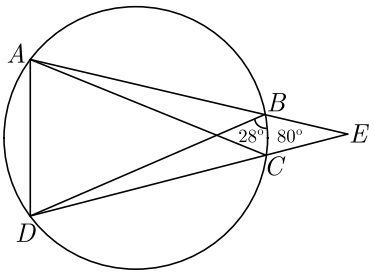
$$\angle ABD = \tfrac{1}{2}\angle AOD$$
$$= \tfrac{1}{2} \times 68^o$$

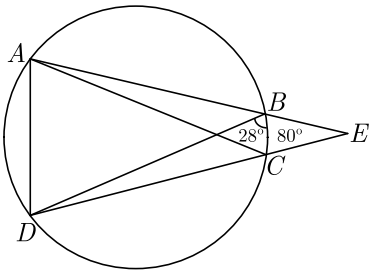
Main 2001

61. In the given figure $ABCD$ is a cyclic quadrilateral. AB and DC are produced to meet at E. Given that $BC = BE$, $\angle CBE = 80^o$ and $\angle DBC = 28°$.
(i) Find $\angle BEC$
(ii) Find $\angle DAC$
(iii) Find $\angle BAC$.



Sol :

We have redrawn the given figure as below.



(i) $\angle BEC$

Given that $BC = BE$

Equal side make equal angle in any triangle.

Thus $\angle BCE = \angle BEC$

Now in $\triangle BCE$,

$$\angle BCE + \angle BEC + \angle CBE = 180^o$$
$$\angle BEC + \angle BEC + 80° = 180^o$$
$$2\angle BEC = 100^o$$
$$\angle BEC = 50^o$$

Now $\angle DBE = \angle DBC + \angle CBE$
$= 28° + 80°$
$= 108°$

(ii) $\angle DAC$

Now in $\triangle BDE$,

$$\angle BDE + \angle BED + \angle DBE = 180^o$$
$$\angle BDC + 50° + 108° = 180^o$$
$$\angle BDC + 158° = 180^o$$
$$\angle BDC = 22^o$$

Due to angles in the same segment,

$$\angle DAC = \angle BDC$$
$$= 28^o$$

(ii) $\angle BAC$

Due to angles in the same segment,

$$\angle BAC = \angle BDC$$

END

NODIA

$$\angle AEC = 180^\circ - 105^\circ$$
$$= 75^\circ$$

By linear axiom we obtain

$$\angle AEC + \angle CEF = 108^\circ$$
$$\angle CEF = 180^\circ - \angle AEC$$
$$= 180^\circ - 75^\circ$$
$$= 105^\circ$$

Since, points A, B, D and E lie on a circle, so $ABDE$ form a cyclic quadrilateral.
Sum of opposite angles in a cyclic quadrilateral is 180°. Thus

$$\angle AEB + \angle EDB = 180^\circ$$
$$62^\circ + \angle EDB = 180^\circ$$
$$\angle EDB = 180^\circ - 62^\circ$$
$$= 118^\circ$$

and $$\angle ABD + \angle AED = 180^\circ$$
$$\angle AED = \angle AEC$$
$$= 75^\circ$$
$$a + 75^\circ = 180^\circ$$
$$a = 180^\circ - 75^\circ$$
$$= 105^\circ$$

Since, BF is a straight, we have

$$\angle BDE + \angle EDF = 180^\circ$$
$$118^\circ + c = 180^\circ$$
$$c = 180^\circ - 118^\circ$$
$$= 62^\circ$$

The exterior angle of a cyclic of quadrilateral is equal to the opposite interior angle i.e.

$$\angle DEF = a = 105^\circ$$

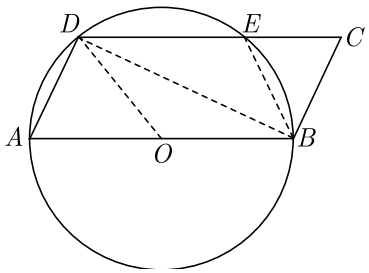
Using angle sum property in triangle ΔDEF we have

$$\angle DEF + \angle EFD + \angle FDE = 180^\circ$$
$$105^\circ + b + 62^\circ = 180^\circ$$
$$167^\circ + b = 180^\circ$$
$$b = 180^\circ - 167^\circ$$
$$= 13^\circ$$

Hence, $a = 105^\circ$, $b = 13^\circ$ and $c = 62^\circ$

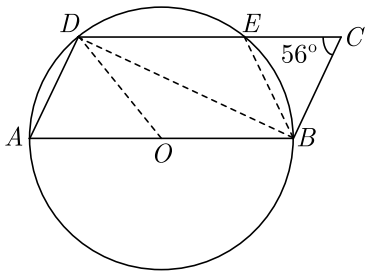
Main 2003

60. In the circle with centre O, $ABCD$ is a parallelogram. $\angle BCD = 56^o$.
(i) Find $\angle BED$.
(ii) Find $\angle AOD$.
(iii) Find $\angle ABD$.



Sol :

We have redrawn the given figure as below.



(i) $\angle BED$
Opposite angles of a parallelogram are equal, Thus

$$\angle DAB = \angle DCB$$
$$= 56^o$$

The opposite angles of a cyclic quadrilateral are always supplementary.

$$\angle BED + \angle DAB = 180^o$$
$$\angle BED + 56^o = 180^o = 124^o$$
$$\angle BED = 124^o$$

(ii) $\angle AOD$
Due to radii of same circle we have

$$OA = OD$$

Same side make equal angle in any triangle.

$$\angle OAD = \angle ODA$$
$$= 56^o$$

In ΔAOD,

$$\angle AOD + \angle OAD + \angle ODA = 180^o$$
$$\angle AOD + 56^\circ + 56^\circ = 180^o$$
$$\angle AOD + 112^\circ = 180^o$$
$$\angle AOD = 68^o$$

(iii) $\angle ABD$.
Angle subtend by an arc at the centre is twice the angle at the remaining part of the circle.
Thus $$\angle AOD = 2\angle ABD$$

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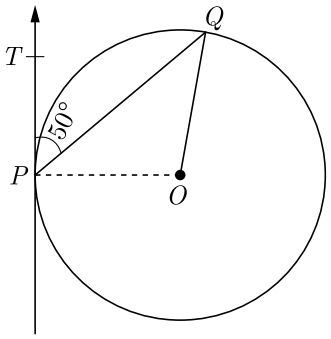


(a) 130° (b) 90°
(c) 100° (d) 75°

Sol :

Due to angle between radius and tangent,

$$\angle OPT = 90^\circ$$

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Also, $OP = OQ$ [Radii of a circle]

Since equal opposite sides have equal opposite angles,

$$\angle OPQ = \angle OQP = 40^\circ$$

$$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$$

$$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

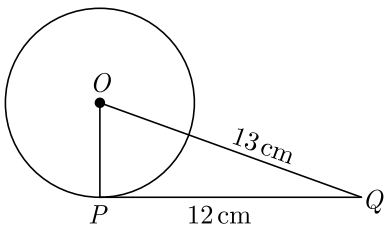
Thus (c) is correct option.

5. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is

(a) 10 (b) 5
(c) 12 (d) 7

Sol :

Let O be the centre of the circle. As per given information we have drawn the figure below.



We have $OQ = 13$ cm

and $PQ = 12$ cm

Radius is perpendicular to the tangent at the point of contact.

Thus $OP \perp PQ$

In ΔOPQ, using Pythagoras theorem,

$$OP^2 + PQ^2 = OQ^2$$

$$OP^2 + 12^2 = 13^2$$

$$OP^2 = 13^2 - 12^2$$

$$= 169 - 144$$

$$= 25$$

Thus $OP = 5$ cm

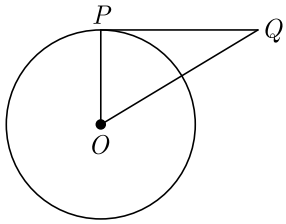
Thus (b) is correct option.

6. QP is a tangent to a circle with centre O at a point P on the circle. If ΔOPQ is isosceles, then $\angle OQR$ equals to

(a) 30° (b) 45°
(c) 60° (d) 90°

Sol :

Let O be the centre of the circle. As per given information we have drawn the figure below.



Radius and tangent are perpendicular at their point of contact.

Now, in isosceles triangle POQ we have

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

Equal sides subtend equal angles in isosceles triangle.

Thus $2\angle OQP + 90^\circ = 180^\circ$

$$\angle OQP = 45^\circ$$

Thus (b) is correct option.

7. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$
(c) $10\sqrt{2}$ (d) $10\sqrt{3}$

Sol :

As per given information we have drawn the figure below.

NODIA

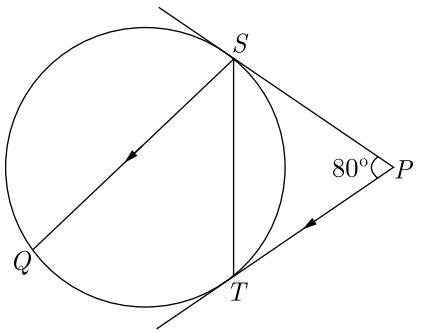
CHAPTER 18

TANGENT OF CIRCLE

MULTIPLE CHOICE QUESTIONS

Main 2024

1. In the given diagram, PS and PT are the tangents to the circle. Given that $SQ \parallel PT$ and $\angle SPT = 80^\circ$ The value of $\angle QST$ is:



(a) 140° (b) 90°
(c) 80° (d) 50°

Sol :

Since PS and PT are tangents from an exterior point to a circle from point P,

$$PS = PT$$

$$\angle PST = \angle PTS$$

Now in $\triangle PST$,

$$\angle PST + \angle PTS + \angle SPT = 180^\circ$$

$$\angle PTS + \angle PTS + 80^\circ = 180^\circ$$

$$2\angle PTS + 80^\circ = 180^\circ$$

$$2\angle PTS = 100^\circ$$

$$\angle PTS = 50^\circ$$

Here $SQ \parallel PT$ and ST is transversal, then due to alternate pair of angle

$$\angle QST = \angle PTS$$

$$= 50^\circ$$

Thus (d) is correct option.

SQP 2024

2. In a circle with radius R, the shortest distance between two parallel tangents is equal to :

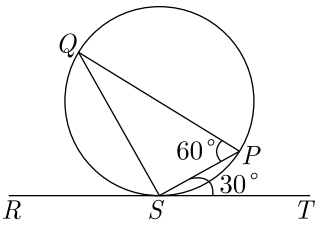
(a) R (b) $2R$
(c) $2\pi R$ (d) πR

Sol :

The tangents drawn to a circle at the end-points of a diameter are parallel. Hence, the distance between the parallel tangents to a circle is equal to the diameter of the circle.
Thus (b) is correct option.

Main 2023

3. In the given diagram RT is a tangent touching the circle at S. If $\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$ then $\angle PSQ$ is equal to:



(a) 40° (b) 30°
(c) 60° (d) 90°

Sol :

In ΔPSQ we have $\angle SPQ = 60^\circ$
Due to alternate segment theorem,
Also, $\angle SQP = \angle PST = 30^\circ$
Using angle sum property in triangle $\angle SPQ$ we have

$$\angle SPQ + \angle SQP + \angle PSQ = 180^\circ$$

$$60^\circ + 30^\circ + \angle PSQ = 180^\circ$$

$$\angle PSQ = 180^\circ - 90^\circ$$

$$= 90^\circ$$

Thus (d) is correct option.

Comp 2023

4. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with $PQ\angle POQ$ is

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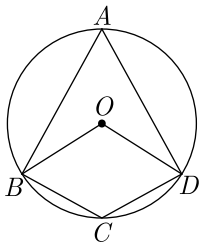
The sum of all angles of a triangle is always equal to 180°. Thus

$$\angle OSR + \angle ORS + \angle SOR = 180^o$$
$$\angle OSR + \angle OSR + 80^\circ = 180^o$$
$$2\angle OSR = 100^o$$
$$\angle OSR = 50^o$$

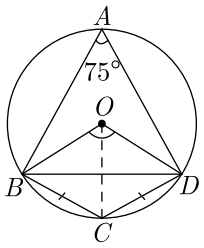
Main 2009

58. In the given figure, O is the centre of the centre, $\angle BAD = 75^\circ$ and chord $BC =$ chord CD. Find
(i) $\angle BOC$.
(ii) $\angle OBD$.
(iii) $\angle BCD$.



Sol :

We have redrawn the given figure as below



Here $\angle BAD = 75^\circ$ and $BC = CD$
(i) $\angle BOC$
An angle subtended by an arc of a circle at the centre is twice of the angle subtended by the same arc at any point on the remaining part of the circle. Thus

$$\angle BOD = 2\angle BAD$$
$$= 2 \times 75^\circ$$
$$= 150^\circ$$
$$\angle BOC = \tfrac{1}{2}\angle BOD$$
$$= \tfrac{1}{2} \times 150^\circ = 75^\circ$$

(ii) $\angle OBD$
In ΔOBD, OB and OD are raddi of same circle, thus

$$OB = OD$$

In a triangle angles opposite to the equal sides are equal, thus

$$\angle ODB = \angle OBD$$

Using angle sum property in triangle ΔOBD we have

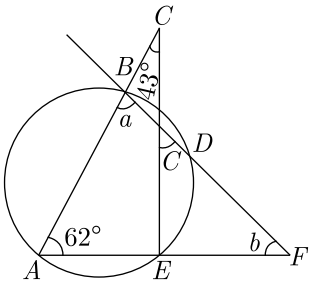
$$\angle OBD + \angle ODB + \angle BOD = 180^\circ$$
$$\angle OBD + \angle OBD + 150^\circ = 180^\circ$$
$$2\angle OBD = 180^\circ - 150^\circ$$
$$= 30^\circ$$
$$\angle OBD = \tfrac{1}{2} \times 30^\circ = 15^\circ$$

(iii) $\angle BCD$
Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle BAD + \angle BCD = 180^\circ$$
$$75^\circ + \angle BCD = 180^\circ$$
$$\angle BCD = 180^\circ - 75^\circ$$
$$= 105^\circ$$

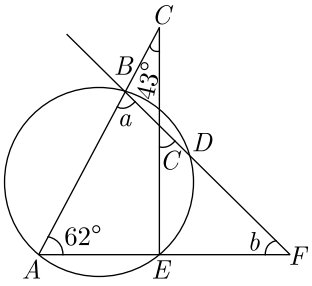
Main 2007

59. In the given figure, if $\angle ACE = 43^\circ$ and $\angle CAF = 62^\circ$, find the values of a, b and c.



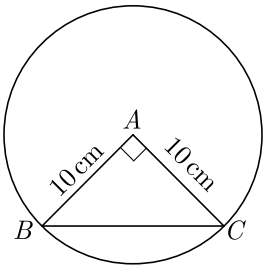
Sol :

We have redrawn the given figure as below



Using angle sum property in triangle ΔACE we have

$$\angle AEC + \angle EAC + \angle ACE = 180^\circ$$
$$\angle AEC + 62^\circ + 43^\circ = 180^\circ$$
$$\angle AEC + 105^\circ = 180^\circ$$



Using Pythagoras theorem in ΔABC, we get

$$BC^2 = AB^2 + AC^2$$
$$= 10^2 + 10^2$$
$$= 100 + 100 = 200$$
$$BC = 10\sqrt{2} \text{ cm}$$

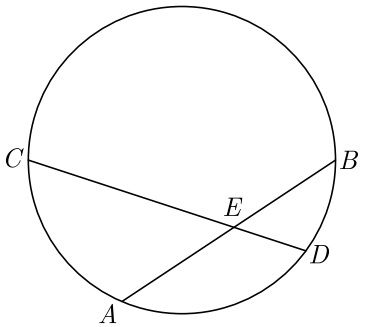
Thus (c) is correct option.

8. Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. The length of DE is

(a) 1.6 cm (b) 3.2 cm
(c) 4.8 cm (d) 6.4 cm

Sol :

As per given information we have drawn the figure below.

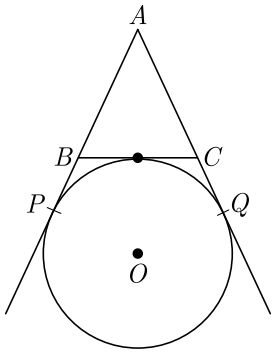


Applying the rule, $AE \times EB = CE \times ED$

$$2.4 \times 3.2 = 1.6 \times ED$$
$$ED = 4.8 \text{ cm}$$

Thus (c) is correct option.

9. In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then the length of AP (in cm) is



(a) 15 (b) 10
(c) 9 (d) 7.5

Sol :

Due to tangents from external points,
$BP = BR, CR = CQ$, and $AP = AQ$
Perimeter of ΔABC,

$$AB + BC + AC = AB + BR + RC + AC$$
$$5 + 4 + 6 = AB + BP + CQ + AC$$
$$15 = AP + AQ$$
$$15 = 2AP$$

Thus $$AP = \frac{15}{2} = 7.5 \text{ cm}$$

Thus (d) is correct option.

10. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is

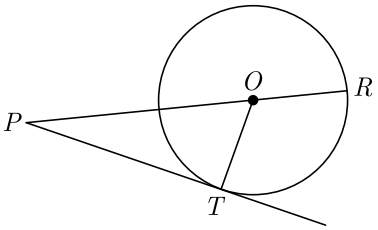
(a) $3r$ (b) $6r$
(c) $9r$ (d) $12r$

Sol :

Side of the regular hexagon inscribed in a circle of radius r is also r, the perimeter is $6r$.

Thus (b) is correct option.

11. In figure, on a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then the length of PR is



(a) 30 cm (b) 28 cm
(c) 32 cm (d) 25 cm

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$$\angle BDC + \angle DBC + \angle DCB = 180^o$$
$$30^o + \angle DBC + 110^o = 80^o$$
$$\angle DBC + 140^o = 180^o$$
$$\angle DBC = 40^o$$

(iii) $\angle DAB$

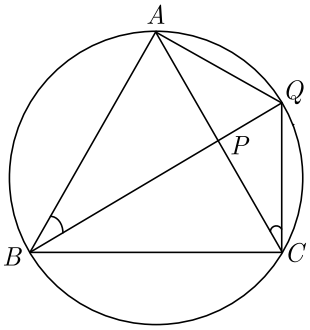
Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle DAB + \angle DCB = 180^o$$
$$\angle DAB + 110^\circ = 180^o$$
$$\angle DAB = 70^\circ$$

Main 2011

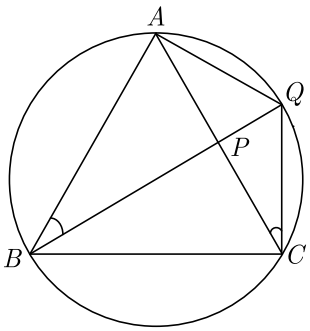
55. In given figure ABC is a triangle in which $AB = AC$ and P is a point on AC Through P a line is drawn to intersect BP produced at Q. Prove that

$$\angle AQC = 90^o + \tfrac{1}{2}\angle BAC$$



Sol :

We have redrawn the given figure as below.



In, ΔABC :

Given $AB = AC$

Equal side make equal angle. Thus

$$\angle ACB = \angle ABC$$

But,

$$\angle BAC + \angle ACB + \angle ABC = 180^o$$
$$\angle BAC + 2\angle ACB = 180^o$$
$$\angle ACB = 90^o - \tfrac{1}{2}\angle BAC \quad ...(i)$$

Angles in the same segment are equal, thus

$$\angle ACB = \angle AQB$$

and $$\angle BAC = \angle BQC$$

Adding above we have

$$\angle BAC + \angle ACB \angle AQB + \angle BQC$$
$$\angle BAC + \angle ACB = \angle AQC$$
$$\angle ACB = \angle AQC - \angle BAC \quad ...(ii)$$

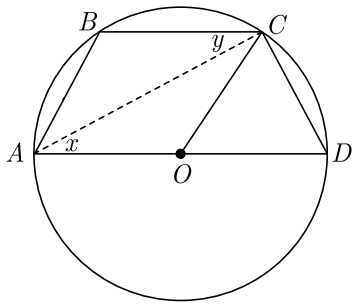
From (i) and (ii), we have

$$\angle AQC - \angle BAC = 90^o - \tfrac{1}{2}\angle BAC$$
$$\angle AQC = 90^o + \tfrac{1}{2}\angle BAC.$$

Main 2010

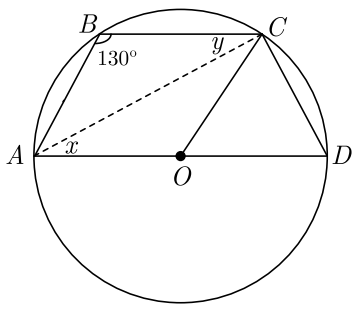
56. In the given figure O is the centre of the circle. Given that $AB \| CD$ and $\angle ABC = 130^o$. What is the value of angle x and y.

NODIA



Sol :

We have redrawn the given figure below.

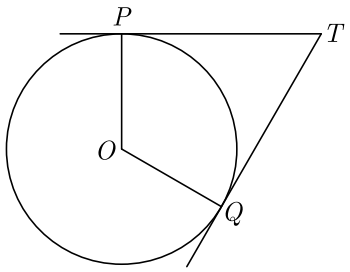


Sum of opposite angles in a cyclic quadrilateral is 180°. Here $ABCD$ is cyclic quadrilateral, thus

$$\angle ABC + \angle ADC = 180^o$$
$$130^o + \angle ADC = 180^o$$
$$\angle ADC = 50^o$$

Since, $\angle ACD$ is in the semi-circle and angle in

15. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110^\circ$, then $\angle PTQ$ is

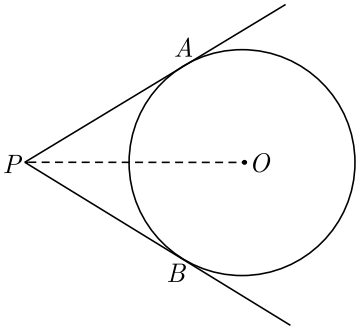


(a) 60° (b) 70°
(c) 80° (d) 90°

Sol :

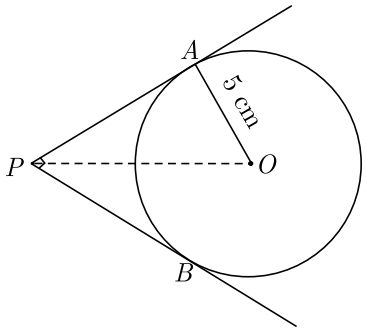
Here $OP \perp PT$ and $OQ \perp QT$,
In quadrilateral $OPTQ$, we have
$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$
$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$
$\angle PTQ = 70^\circ$
Thus (b) is correct option.

16. In the given figure PA and PB are tangents from point P to a circle with centre O. If the radius of the circle is 5 cm and $PA \perp PB$, then the length OP is equal to
(a) 5 cm (b) 10 cm
(c) 7.5 cm (d) $5\sqrt{2}$ cm



Sol :

As per question we have drawn the figure below



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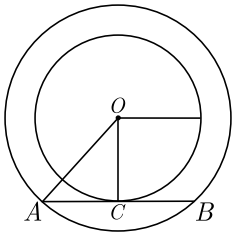
Since OA is radius and PA is tangent to the circle
$\angle OAP = 90^\circ$
Since $PA \perp PB$, we have
$\angle APB = 90^\circ$
$\angle APO = 90^\circ \times \frac{1}{2} = 45^\circ$
$\angle AOP = 90^\circ - 45^\circ$
Thus $AP = OA = 5$ cm
$OP = \sqrt{OA^2 + PA^2}$
$= \sqrt{5^2 + 5^2}$
$= 5\sqrt{2}$ cm
Thus (d) is correct option.

17. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other is
(a) 3 cm (b) 6 cm
(c) 9 cm (d) 1 cm

Sol :

As per question we have drawn the figure below



Here $OC = 4$ cm, $OA = 5$ cm
and $OC \perp ACB$
In right ΔOAC, $OA^2 = OC^2 + AC^2$
$5^2 = 4^2 + AC^2$
$25 = 16 + AC^2$
$AC^2 = 25 - 16 = 9$

$$AC = 3 \text{ cm}$$

Length of chord $AB = 2 \times AC$

$$= 2 \times 3 = 6 \text{ cm}$$

Thus (b) is correct option.

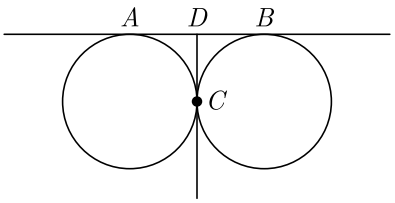
18. AB and CD are two common tangents to circles which touch each other at a point C. If D lies on AB such that $CD = 4$ cm then AB is

(a) 12 cm (b) 8 cm

(c) 4 cm (d) 6 cm

Sol :

As per given information we have drawn the figure below.



$AD = CD$ and $BD = CD$

$AB = AD + BD$

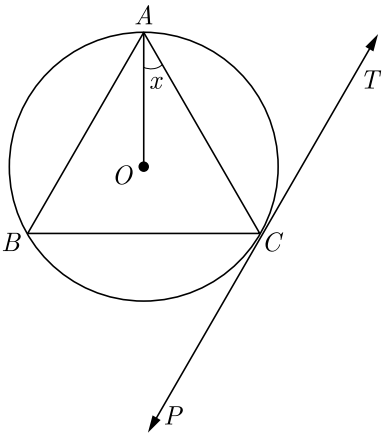
$= CD + CD$

$= 2CD$

$= 2 \times 4 = 8$ cm

Thus (b) is correct option.

19. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118^\circ$, then the measure of $\angle x$ is

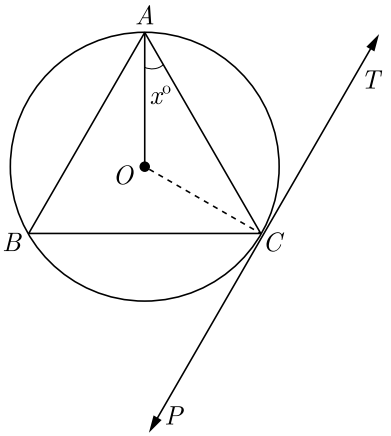


(a) 28° (b) 32°

(c) 42° (d) 38°

Sol :

We join OC as shown in the below figure. Here OC is the radius and PT is the tangent to circle at point C.



Thus $OC \perp PT$

$\angle OCP = 90^\circ$

Given, $\angle ACP = 118^\circ$

$\angle ACO = \angle ACP - \angle OCP$

$= 118^\circ - 90^\circ = 28^\circ$

$\angle ACO = 28^\circ$

Since O is the circumcentre, thus due to radius

$OA = OC$

$\angle OAC = \angle ACO$

$x = 28^\circ$

Thus (a) is correct option.

20. Two concentric circles of radii a and b where $a > b$,The length of a chord of the larger circle which touches the other circle is

(a) $\sqrt{a^2 + b^2}$ (b) $2\sqrt{a^2 + b^2}$

(c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$

Sol :

In ΔOAL, $OA^2 = OL^2 + AL^2$

$a^2 = OL^2 + b^2$

$OL = \sqrt{a^2 - b^2}$

Length of chord,

$2AL = 2\sqrt{a^2 - b^2}$

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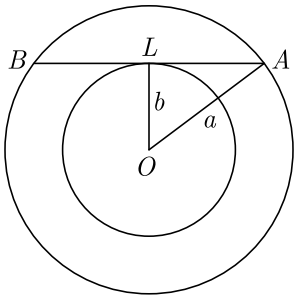
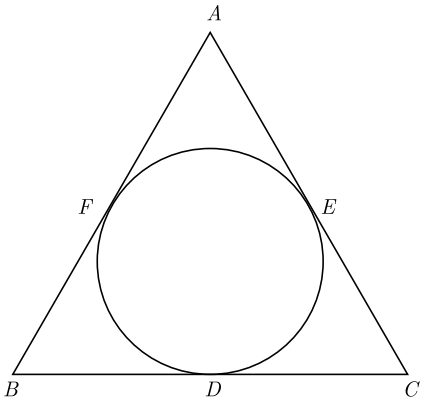


Thus (d) is correct option.

21. In the given figure sides BC, CA and AB of ΔABC touch a circle at point D, E and F respectively, If $BD = 4$ cm, $DC = 3$ cm and $CA = 8$ cm, then the length of side AB is

(a) 12 cm (b) 11 cm
(c) 10 cm (d) 9 cm



Sol :

Here $BD = 4$ cm, $DC = 3$ cm and $CA = 8$ cm
Since BD and BF are tangents to the circle,

$$BF = BD = 4 \text{ cm}$$

Similarly, $CD = CE = 3$ cm

$$AE = AC - CE$$
$$= 8 - 3 = 5 \text{ cm}$$

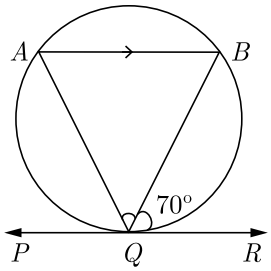
and $AF = AE = 5$ cm

Now $AB = AF + BF$
$= 5 + 4 = 9$ cm

Thus (d) is correct option.

22. In the given figure PQR is a tangent at Q to a circle. If AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal to

(a) 20^o (b) 40^o
(c) 35^o (d) 45^o



Sol :

Due to angles in the alternate segments,

$$\angle BAQ = \angle BQR = 70^\circ$$

Due to alternate angles,

$$\angle QBA = \angle BQR = 70^\circ$$

Using angle sum property in triangle $\angle ABQ$ we have

$$\angle AQB + \angle QAB + \angle QBA = 180^\circ$$
$$\angle AQB + 70^\circ + 70^\circ = 180^\circ$$
$$\angle AQB + 140^\circ = 180^\circ$$
$$\angle AQB = 180^\circ - 140^\circ$$
$$= 40^\circ$$

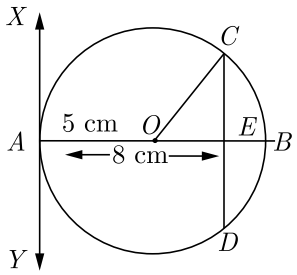
Thus (b) is correct option.

23. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

Sol :

As per given information we have drawn the figure below.



In right ΔOEC,

$$OE = 8 - 5 = 3 \text{ cm}$$
$$OC = 5 \text{ cm}$$
$$CE = \sqrt{OC^2 - OE^2}$$
$$= \sqrt{5^2 - 3^2}$$

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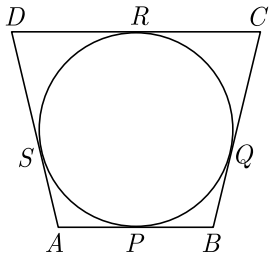
$$= \sqrt{25-9} = \sqrt{16} = 4 \text{ cm}$$

$$CD = 2 \times CE$$

$$= 2 \times 4 = 8 \text{ cm}$$

Thus (d) is correct option.

24. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm, then length of AD is



(a) 3 cm (b) 4 cm
(c) 5 cm (d) 6 cm

Sol :

Four sides of a quadrilateral $ABCD$ are tangent to a circle.

$$AB + CD = BC + AD$$

$$6 + 4 = 7 + AD$$

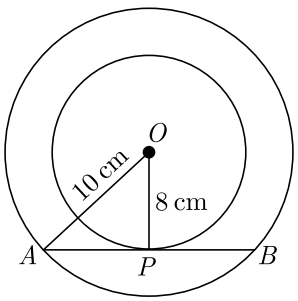
$$AD = 10 - 7 = 3 \text{ cm}$$

Thus (a) is correct option.

25. Two concentric circles are of radii 10 cm and 8 cm, then the length of the chord of the larger circle which touches the smaller circle is
(a) 6 cm (b) 12 cm
(c) 18 cm (d) 9 cm

Sol :

As per given information we have drawn the figure below.



Let O be the centre of the concentric circles of radii 10 cm and 8 cm, respectively. Let AB be a chord of the larger circle touching the smaller circles at P.

Then, $AP = PB$ and $OP \perp AB$

Applying Pythagoras theorem in ΔOPA, we have

$$OA^2 = OP^2 + AP^2$$

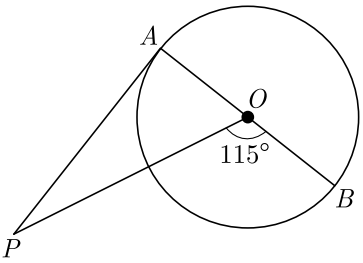
$$100 = 64 + AP^2$$

$$AP^2 = 100 - 64 = 36 \Rightarrow AP = 6 \text{ cm}$$

$$AB = 2AP = 2 \times 6 = 12 \text{ cm}$$

Thus (b) is correct option.

26. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115^\circ$, then perimeter of $\angle APO$ is



(a) 25° (b) 20°
(c) 30° (d) 65°

Sol :

Since tangent at a point to a circle is perpendicular to the radius,

$$\angle OAP = 90^\circ$$

Now, $\angle AOP + \angle BOP = 180^\circ$

$$\angle AOP + 115^\circ = 180^\circ$$

$$\angle AOP = (180^\circ - 115^\circ) = 65^\circ$$

From angle sum property of triangle we have

$$\angle OAP + \angle AOP + \angle APO = 180^\circ$$

$$90^\circ + 65^\circ + \angle APO = 180^\circ$$

$$155^\circ + \angle APO = 180^\circ$$

$$\angle APO = 180^\circ - 155^\circ$$

$$= 25^\circ$$

Thus (a) is correct option.

27. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and $PA = 14$ cm. The perimeter of ΔPCD is
(a) 14 cm (b) 21 cm
(c) 28 cm (d) 35 cm

Sol :

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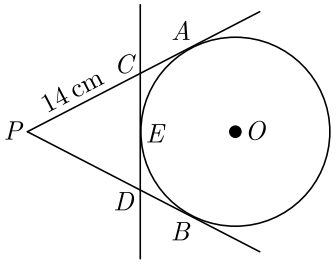
As per information given in question we have drawn figure below.



Here $PA = PB = 14\,\text{cm}$

Also, CD is tangent at point E on the circle.

So, CA and CE are tangent to the circle from point C.

Therefore, $CA = CE$,

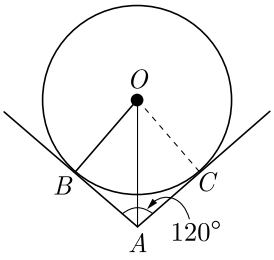
Similarly, $DB = DE$

Now, perimeter of ΔPCD,

$$\begin{aligned} PC + CD + PD &= PC + CE + ED + PD \\ &= PC + CA + PD + DB \\ &= PA + PB \\ &= 14 + 14 \\ &= 28 \text{ cm} \end{aligned}$$

Thus (c) is correct option.

28. In the given figure, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$, then OA is equal to that



(a) $2AB$ (b) $3AB$
(c) $4AB$ (d) $5AB$

Sol :

In ΔOAB and ΔOAC, we have,

$$\angle OBA = \angle OCA = 90^\circ$$

Common side,

$$OA = OA$$

Due to radius of circle,

and $OB = OC$

So, by RHS congruence criterion,

$$\Delta OBA \cong \Delta OCA$$

$$\angle OAB = \angle OAC = \frac{1}{2} \times 120^\circ = 60^\circ$$

In ΔOBA, we have,

$$\cos 60^\circ = \frac{AB}{OA}$$

$$\frac{1}{2} = \frac{AB}{OA}$$

$$OA = 2AB$$

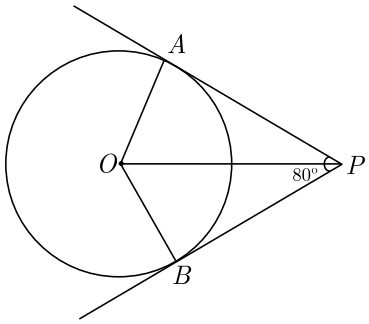
Thus (a) is correct option.

29. If tangents PA and PB from an exterior point P to a circle with centre O are inclined to each other at an angle of 80°, then $\angle POA$ is equal to

(a) 50° (b) 60°
(c) 70° (d) 100°

Sol :

As per question we have drawn the figure below



Here OA is radius and AP is tangent, thus

$$\angle OAP = 90^\circ$$

and $$\angle OPA = \frac{1}{2}\angle APB$$

$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

$$\angle POA = 90^\circ - 40^\circ = 50^\circ$$

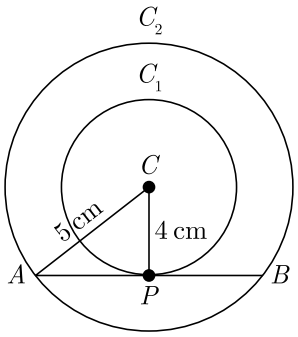
Thus (a) is correct option.

30. In the given figure, three circles with centres P , Q and R are drawn, such that the circles with centres Q and R touch each other externally and they touch the circle with centre P, internally. If $PQ = 10\,\text{cm}$, $PR = 8\,\text{cm}$ and $QR = 12\,\text{cm}$, then the diameter of the largest circle is:

NODIA

Sol :

Let C be the centre of two concentric circles C_1 and C_2, whose radii are $r_1 = 4\,\text{cm}$ and $r_2 = 5\,\text{cm}$. Now, we draw a chord AB of circle C_2, which touches C_1 at P.



AB is tangent at P and CP is radius at P. Tangent at any point of circle is perpendicular to the radius through the point of contact.

Thus $CP \perp AB$

Now, in right triangle PAC

By Pythagoras theorem we have

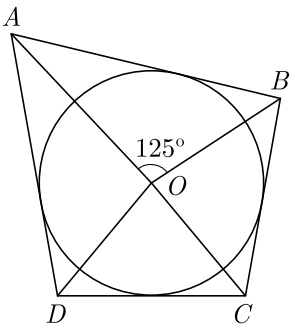
$$AP^2 = AC^2 - PC^2$$
$$= 5^2 - 4^2 = 25 - 16$$
$$= 9$$
$$AP = 3\,\text{cm}$$

So, length of chord,

$$AB = 2AP = 2 \times 3 = 6\,\text{cm}$$

Thus (b) is correct option.

37. In figure, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to



(a) 62.5° (b) 45°
(c) 35° (d) 55°

Sol :

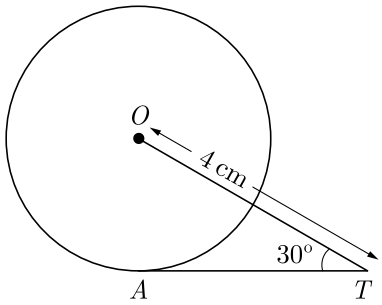
A quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

i.e. $\angle AOB + \angle COD = 180^\circ$

$$125^\circ + \angle COD = 180^\circ$$
$$\angle COD = 180^\circ - 125^\circ$$
$$= 55^\circ$$

Thus (d) is correct option.

38. In figure, AT is a tangent to the circle with centre O such that $OT = 4\,\text{cm}$ and $\angle OTA = 30^\circ$. Then, AT is equal to

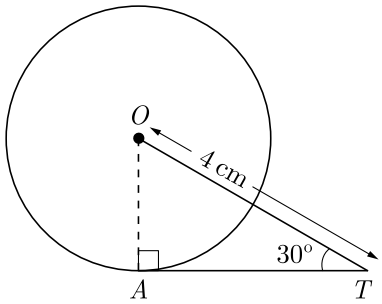


(a) 4 cm (b) 2 cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Sol :

First we joint OA. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OAT = 90^\circ \text{ and } OT = 4\,\text{cm (given)}$$



In ΔOAT, $\cos 30^\circ = \dfrac{AT}{OT}$

$$\frac{AT}{4} = \frac{\sqrt{3}}{2}$$
$$AT = 2\sqrt{3}\,\text{cm}$$

Thus (c) is correct option.

39. In the given figure sides BC, CA and AB of ΔABC touch a circle at the points P,Q and R respectively. if $PC = 5$ cm, $AR = 4$ cm and $RB = 6$ cm, then the perimeter of ΔABC is

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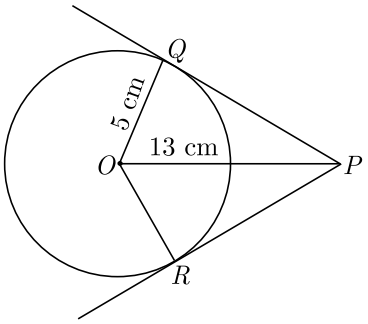


Here point P is 13 cm from O. Here PQ is tangent and OQ is the radius, thus $OP \perp PQ$

Now $PQ^2 = OP^2 - OQ^2$

$= 13^2 - 5^2$

$= 169 - 25$

$= 144 = (12)^2$

$PQ = 12$ cm

Now area of $\Delta OPQ = \frac{1}{2} PQ \times OQ$

$= \frac{1}{2} \times 12 \times 5$

$= 30$ cm^2

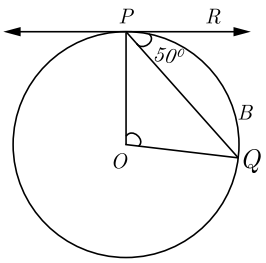
area of quad. $PQOR = 2 \times 30 = 60$ cm^2

Thus (a) is correct option.

34. In the given figure O is the centre of a circle and PQ is a chord. if the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is

(a) 100° (b) 80°

(c) 90° (d) 75°



Sol :

Since OP is radius and PR is tangent to the circle

$OP \perp PR$

But $\angle OPQ + \angle RPQ = 90^\circ$

$\angle OPQ + 50^\circ = 50^\circ$

$= 90^\circ$

$\angle OPQ = 90^\circ - 50^\circ$

$= 40^\circ$

Due to radii of the same circle we have

$OP = OQ$

$\angle OQP = \angle OPQ = 40^\circ$

and $\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$

$= 180^\circ - (40^\circ + 40^\circ)$

$= 180^\circ - 80^\circ$

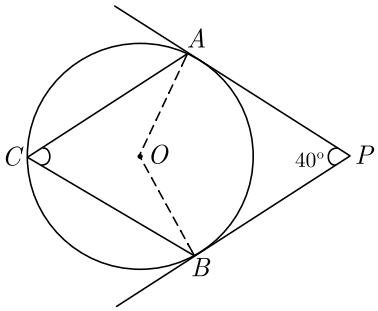
$= 100^\circ$

Thus (a) is correct option.

35. In the given figure PA and PB are tangents at points A and B respectively to a circle with centre O. if C is a point on the circle and $\angle APB = 40^\circ$, then $\angle ACB$ is equal to

(a) 80° (b) 70°

(c) 90° (d) 140°



Sol :

Here $\angle APB = 40^\circ$

But $\angle APB + \angle AOB = 180^\circ$

$40^\circ + \angle AOB = 180^\circ$

$\angle AOB = 180^\circ - 40^\circ$

$= 140^\circ$

Now arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ is on the remaining part of the circle.

Thus $\angle ACB = \frac{1}{2} \angle AOB$

$= \frac{1}{2} \times 140^\circ = 70^\circ$

Thus (b) is correct option.

36. If radii of two concentric circles are 4 cm and 5 cm, then the length of each of one circle which is tangent to the other circle, is

(a) 3 cm (b) 6 cm

(c) 9 cm (d) 1 cm

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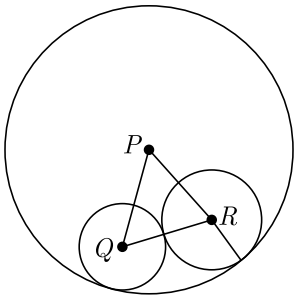


(a) 30 cm (b) 20 cm
(c) 10 cm (d) None of these

Sol :

Let radii of the circles with centres P, Q and R are p, q and r, respectively.

Then, $PQ = p - q = 10$ ...(1)

$PR = p - r = 8$ ...(2)

and $QR = q + r = 12$ ...(3)

Adding equation (2) and (3), we get,

$p + q = 20$ ...(4)

Adding equation (1) and (4), we get,

$2p = 30$

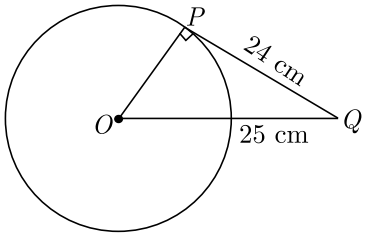
Hence, diameter of the largest circle $2p = 30$.
Thus (a) is correct option.

31. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(a) 7 cm (b) 12 cm
(c) 15 cm (d) 24.5 cm

Sol :

As per question we have drawn the figure below



Lngth of tangent PQ to the circle,

$PQ = 24$ cm

and $QO = 25$ cm

Here PQ is tangent and OP is radius thus $OP \perp PQ$.
Now in right ΔOPQ we have

$$OQ^2 = OP^2 + PQ^2$$

$$25^2 = OP^2 + 24^2$$

$$OP^2 = 25^2 - 24^2$$

$$= 625 - 576$$

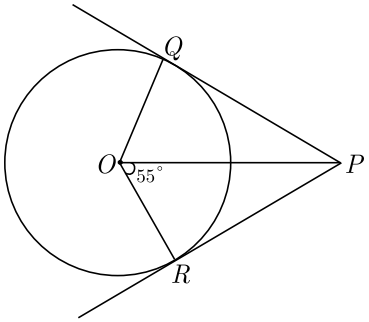
$$OP^2 = 49 = 7^2$$

$$OP = 7 \text{ cm}$$

Thus (c) is correct option.

32. In the given figure PQ and PR are tangents from P to a circle with centre O. If $\angle POR = 55^\circ$, then $\angle QPR$ is

(a) 35^o (b) 55^o
(c) 70^o (d) 80^o



Sol :

Since OR is radius and PR is tangent, $OR \perp PR$

$$\angle ORP = 90^\circ$$

In ΔOPR

$$\angle OPR = 90^\circ - 55^\circ = 35^\circ$$

$$QPR = 2 \times \angle OPR$$

$$= 2 \times 35^\circ = 70^\circ$$

Thus (c) is correct option.

33. From a point which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. The the area of the quadrilateral $PQOR$ is

(a) 60 cm^2 (b) 65 cm^2
(c) 30 cm^2 (d) 32.5 cm^2

Sol :

As per question we have drawn the figure below

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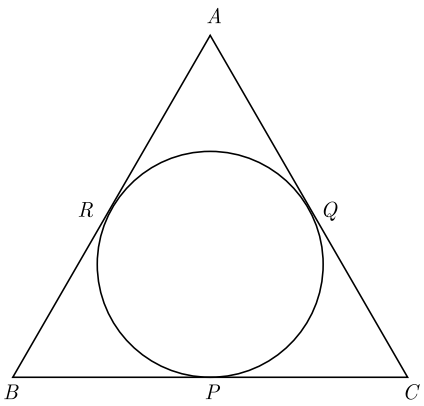
(a) 60 cm (b) 45 cm
(c) 30 cm (d) 15 cm



Sol :

Here $PC = 5$ cm, $AR = 4$ cm, $RB = 6$ cm

Since AR and AQ are tangents to the circles

$$AQ = AR = 4 \text{ cm}$$

Similarly $CQ = CP = 5$ cm

and $BP = BR = 6$ cm

Now $AB = AR + BR = 4 + 6 = 10$ cm

$BC = BP + CP = 6 + 5 = 11$ cm

$AC = AQ + CQ = 4 + 5 = 9$ cm

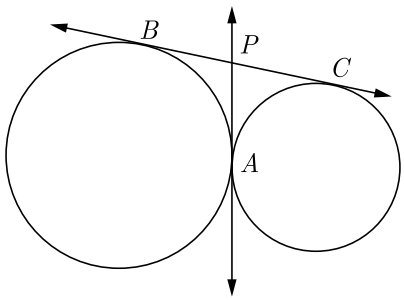
Perimeter of the ΔABC ,

$$AB + BC + CA = 10 + 11 + 9$$
$$= 30 \text{ cm}$$

Thus (c) is correct option.

40. In the given figure two circles touch each other at A. Line BC and AP are common tangents to these circles. If $BP = 3.8$ cm, then the length of BC is equal to

(a) 7.6 cm (b) 1.9 cm
(c) 11.4 cm (d) 5.7 cm



Sol :

Since PB and PA are the tangents to the first circle

$$PB = PA$$
$$= 3.8 \text{ cm}$$

Similarly PC and PA are tangents to the second circle

$$PA = PC$$
$$= 3.8 \text{ cm}$$
$$BC = PB + PC$$
$$= 3.8 + 3.8$$
$$= 7.6 \text{ cm}$$

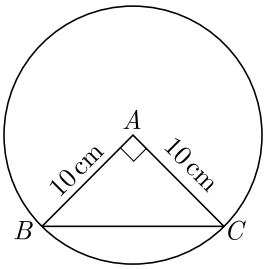
Thus (a) is correct option.

41. A chord of a circle of radius 10 cm, subtends a right angle at its centre. What is the length of the chord?

(a) $10\sqrt{2}$ (b) $2\sqrt{2}$
(c) $5\sqrt{2}$ (d) $3\sqrt{2}$

Sol :

As per given information we have drawn the figure below.



Using Pythagoras theorem in ΔABC, we get

$$BC^2 = AB^2 + AC^2 = 10^2 + 10^2$$
$$= 100 + 100 = 200$$
$$BC = 10\sqrt{2} \text{ cm}$$

Thus (a) is correct option.

42. In the given figure PA and PB are tangents to a circle with centre O. If $\angle APB = 50°$, then $\angle OAB$ is equal to

(a) 25^o (b) 30^o
(c) 40^o (d) 50^o

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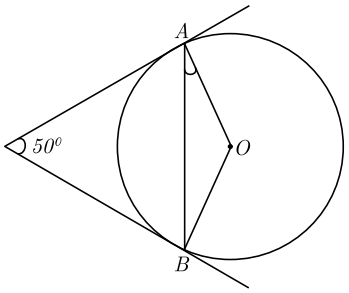


Sol :

In the given figure PA and PB are tangents to the circle with centre O . $\angle APB = 50^\circ$

But $\angle AOB + \angle APB = 180^\circ$

$$\angle AOB + 50^\circ = 180^\circ$$
$$\angle AOB = 180^\circ - 50^\circ$$
$$= 130^\circ$$

Using angle sum property in triangle ΔOAB we have

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$
$$\angle OAB + \angle OBA + 130^\circ = 180^\circ$$
$$\angle OAB + \angle OBA = 180^\circ - 130^\circ$$
$$= 50^\circ$$

In ΔOAB, due to radii of the same circle

$$OA = OB$$
$$\angle OAB = \angle OBA$$

Thus $\angle OAB + \angle OAB = 50^\circ$

$$\angle OAB = \frac{50^\circ}{2} = 25^\circ$$

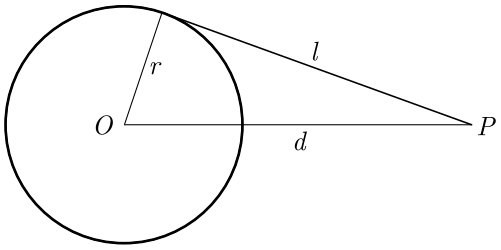
Thus (a) is correct option.

43. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

(a) $10\sqrt{3}$ (b) $2\sqrt{3}$
(c) $5\sqrt{3}$ (d) $3\sqrt{3}$

Sol :

As per the given question we draw the figure as below.



Length of the tangent, $l = \sqrt{d^2 - r^2}$

$$= \sqrt{8^2 - 6^2}$$
$$= \sqrt{64 - 36}$$
$$= \sqrt{28} = 2\sqrt{3} \text{ cm.}$$

Thus (b) is correct option.

44. If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

(a) 62.5° (b) 45°
(c) 35° (d) 50°

Sol :

Sum of the angles between radii and between intersection point of tangent is always 180°.

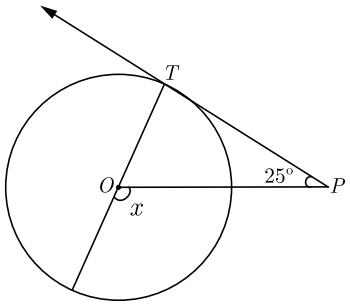
Thus angle at the point of intersection of tangents

$$= 180^\circ - 130^\circ = 50^\circ$$

Thus (d) is correct option.

45. In the given figure PT is a tangent at T to the circle with centre O. if $\angle TPO = 25^\circ$, then the value of x is

(a) 25° (b) 65°
(c) 115° (d) 90°



Sol :

In the given figure PT is the tangent at T to the circle with centre O. $\angle TPO = 25^\circ$

Since OT is radius and TP is the tangent, thus $OT \perp TP$.

Now in ΔOPT we have

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$$\angle TOP + \angle OPT = 90^\circ$$
$$\angle TOP + 25^\circ = 90^\circ$$
$$\angle TOP = 90^\circ - 25^\circ$$
$$= 65^\circ$$

Due to linear pair we have

$$\angle TOP + x = 180^\circ$$
$$65^\circ + x = 180^\circ$$
$$x = 180^\circ - 65^\circ$$
$$= 115^\circ$$

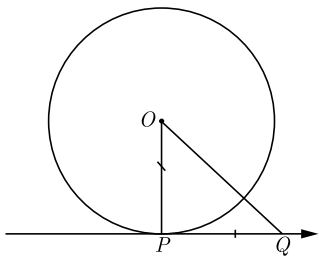
Thus (c) is correct option.

46. PQ is a tangent to a circle at point P. Centre of circle is O. If ΔOPQ is an isoceles triangle, then $\angle QOP$ is equal to

(a) 30^o (b) 60^o
(c) 45^o (d) 90^o

Sol :

As per given information we have drawn the figure below.



Here OP is radius and PQ is tangent to the circle Thus $OP \perp PQ$ i.e.,

$$\angle OPQ = 90^\circ$$

Thus $\angle QOP + \angle PQO = 90^\circ$

Since ΔOPQ is a isosceles triangle,

$$OP = PQ$$
$$\angle QOP = \angle PQO$$
$$= \frac{90^\circ}{2} = 45^\circ$$

Thus (c) is correct option.

47. **Assertion (A):** The tangents drawn at the end points of a diameter of a circle, are parallel.
Reason (R): Diameter of a circle is the longest chord.

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A)
(b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A)
(c) Assertion (A) is true but Reason (R) is false
(d) Assertion (A) is false but Reason (R) is true

Sol :

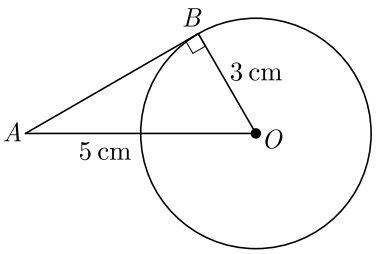
Both assertion (A) and reason (R) are true statements, but they are not related to each other. Therefore, the correct answer is (A) and (R) are true, but (R) is not the correct explanation of (A).

Thus (b) is correct option.

48. **Assertion :** If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.
Reason : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :



$$OA^2 = AB^2 + OB^2$$
$$5^2 = AB^2 + 3^2$$
$$AB = \sqrt{25 - 9} = 4\,\text{cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

49. **Assertion :** PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110^\circ$, then $\angle APB = 90^\circ$.
Reason : The length of two tangents drawn from an external point are equal.

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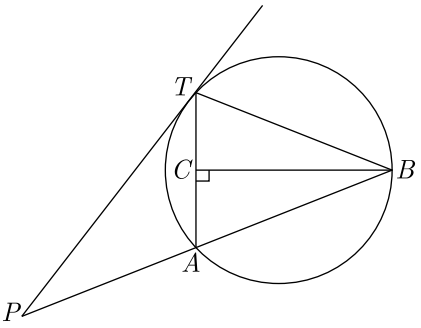


Sol :

We have redrawn the figure below.

(i) AB

In ΔABC and ΔTBC :

Since BC is perpendicular bisector of TA,

$$TC = CA$$

$$\angle TCB = \angle ACB = 90^{\circ}$$

Due to common angle,

$$BC = BC$$

By SAS similarity criterion we get

$$\Delta ABC \cong \Delta TBC$$

By CPCT we get

$$TB = AB$$

Then, $AB = TB$

$$= 7 \text{ cm}$$

(ii) PT

Using properly $PA \times PB = PT^2$

$$9 \times (9 + 7) = PT^2$$

$$PT^2 = 9 \times 16$$

$$PT = 3 \times 4$$

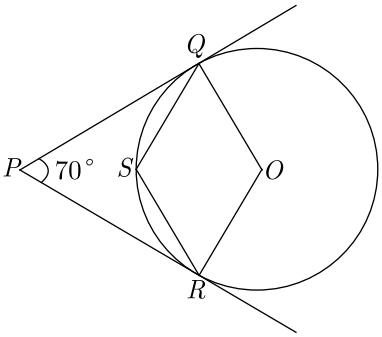
$$= 12 \text{ cm}$$

Main 2022 Sem II

55. In the given figure O is the centre of the circle. PQ and PR are tangents and $\angle QPR = 70^{\circ}$ Calculate.

(i) $\angle QOR$

(ii) $\angle QSR$



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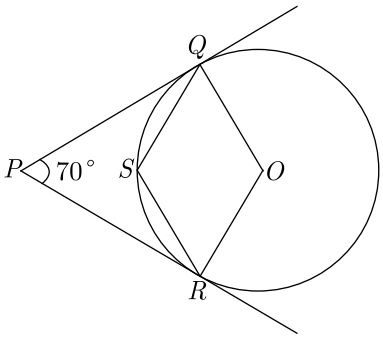
Sol :

We have redrawn the figure as below.



(i) $\angle QOR$

Tangent is always perpendicular to radius at the point of contact. Thus

$$\angle PQO = \angle PRO$$

$$= 90^{\circ}$$

In quadrilateral $PQOR$ we have

$$\angle QPR + \angle PQO + \angle QOR + \angle PRO = 360^{\circ}$$

$$70^{\circ} + 90^{\circ} + \angle QOR + 90^{\circ} = 360^{\circ}$$

$$\angle QOR = 110^{\circ}$$

(ii) $\angle QSR$

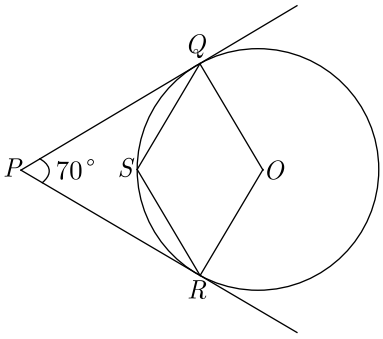
$$\angle QOR = 360^{\circ} - 110^{\circ}$$

$$= 250^{\circ}$$

$$\angle QSR = \tfrac{1}{2} \angle QOR$$

$$= \tfrac{1}{2} \times 250^{\circ}$$

$$= 125^{\circ}$$



Main 2020

56. In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C, respectively. If $\angle BCQ = 55^{\circ}$ and $\angle BAP = 60^{\circ}$, find

(i) $\angle OBA$ and $\angle OBC$

(ii) $\angle AOC$

(iii) $\angle ATC$

$$= (16+9)\times 16$$
$$= 25\times 16$$
$$PT = \sqrt{25\times 16}$$
$$= 5\times 4 = 20$$

Hence, the length of PT is 20 cm.

(ii) $\angle BAT$

Due to alternate segment theorem,

$$\angle BAT = \angle PTB = 50^\circ$$

(iii) $\angle BOT$

Tangent is always perpendicular to radius at the point of contact. Thus

$$\angle OTP = 90^\circ$$
$$\angle OTB + \angle PTB = 90^\circ$$
$$\angle OTB + 50^\circ = 90^\circ$$
$$\angle OTB = 40^\circ$$

Due to radii of circle $OB = OT$ and angles opposite to equal sides are equal. Thus

$$\angle OTB = \angle OBT$$
$$= 40^\circ$$

Using angle sum property in triangle ΔBOT we have

$$\angle OTB + \angle BOT + \angle OBT = 180^\circ$$
$$40^\circ + \angle BOT + 40^\circ = 180^\circ$$
$$\angle BOT + 80^\circ = 180^\circ$$
$$\angle BOT = 100^\circ$$

(iv) $\angle ABT$

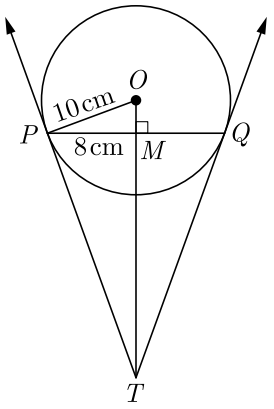
$$\angle ABT = \angle OBA + \angle OBT$$

Substituting $\angle OBA = 45^\circ$ and $\angle OBT = 40^\circ$ we have

$$\angle ABT = 45^\circ + 40^\circ$$
$$= 85^\circ$$

Comp 2023

53. In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP.



Sol :

Here PQ is chord of circle and OM will be perpendicular on it and it bisect PQ. Thus $\Delta\, OMP$ is a right angled triangle.

We have $OP = 10$ cm (Radius)

$PM = 8$ cm $(PQ = 16$ cm$)$

Now in $\Delta\, OMP$, $OM = \sqrt{10^2 - 8^2}$

$$= \sqrt{100-64} = \sqrt{36}$$
$$= 6 \text{ cm}$$

Now $\angle TPM + \angle MPO = 90^\circ$

Also, $\angle TPM + \angle PTM = 90^\circ$

$$\angle MPO = \angle PTM$$
$$\angle TMP = \angle OMP = 90^\circ$$

Thus due to AA symmetry we have

$$\Delta\, TMP \sim \Delta\, PMO$$

Now
$$\frac{TP}{PO} = \frac{MP}{MO}$$
$$\frac{TP}{10} = \frac{8}{6}$$
$$TP = \frac{80}{6} = \frac{40}{3}$$

Hence length of TP is $\frac{40}{3}$ cm.

Main 2022 Sem II

54. In the given figure, PT is a tangent to the circle at T, chord BA is produced to meet the tangent at P. Perpendicular BC bisects the chord TA at C. If $PA = 9$ cm and $TB = 7$ cm, find the lengths of :

(i) AB

(ii) PT

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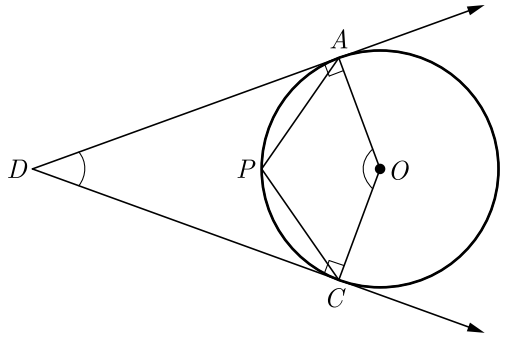


Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90^\circ$$

and

$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360^\circ$$

$$50^\circ + 90^\circ + 90^\circ + \angle AOC = 360^\circ$$

$$230^\circ + \angle AOC = 360^\circ$$

$$\angle AOC = 360^\circ - 230^\circ = 130^\circ$$

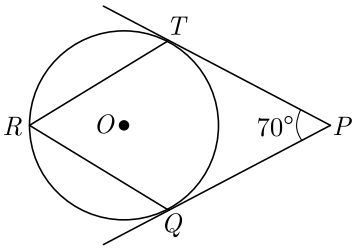
Now Reflex $\angle AOC = 360^\circ - 130^\circ = 230^\circ$

$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$

$$= \frac{1}{2} \times 230^\circ = 115^\circ$$

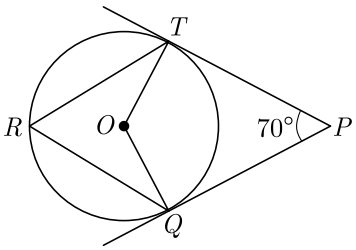
Comp 2017

63. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



Sol :

We redraw the given figure by joining O to T and Q as shown below.



Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $\angle TOQ = 180^\circ - \angle TPQ$

$$= 180^\circ - 70^\circ = 110^\circ$$

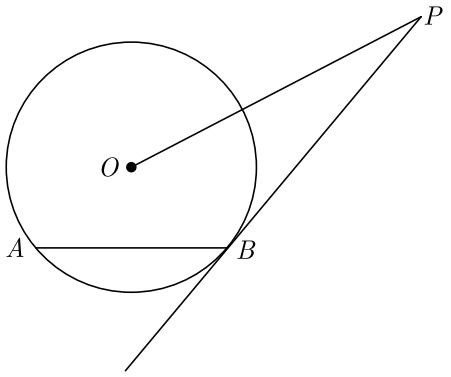
Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle TRQ = \frac{1}{2} \angle TOQ$$

$$= \frac{1}{2} \times 110^\circ = 55^\circ$$

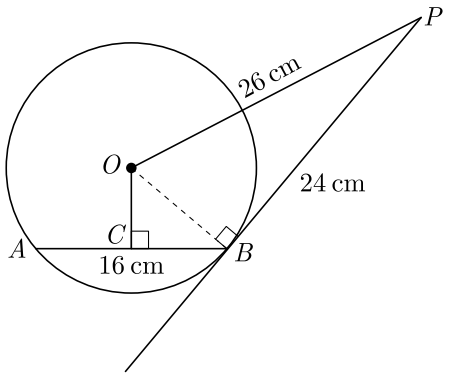
Main 2016

64. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Sol :

We redraw the given figure by joining O to B as shown below.



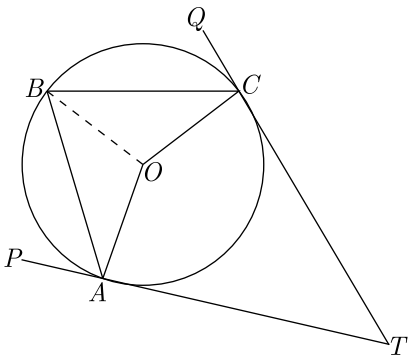
Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle,

We have $PB = 24$ cm, $OP = 26$ cm.

Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90^\circ$.

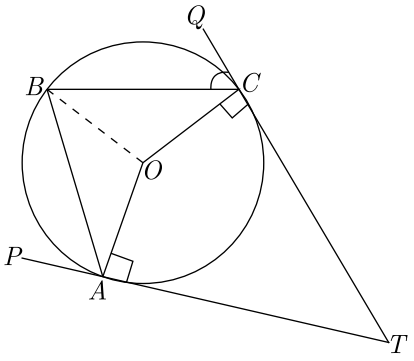
In right angled ΔOPB, we have

NODIA



Sol :

We have redrawn the figure as below.



Here we have $\angle BCQ = 55^\circ$ and $\angle BAP = 60^\circ$

(i) $\angle OBA$ and $\angle OBC$

Here TP and TQ are two tangents to the circle with centre O touching at A and C, respectively.

Since radius is always perpendicular to tangent,

$$\angle OCQ = 90^\circ$$

$$\angle BCO = \angle OCQ - \angle BCQ$$

$$= 90^\circ - 55^\circ$$

$$= 35^\circ$$

Due to radii of circle $OC = OB$ and angles opposite to equal sides are equal. Thus

$$\angle OBC = \angle BCO$$

$$= 35^\circ$$

Similarly $\angle OAP = 90^\circ$

$$\angle BAO = \angle OAP - \angle BAP$$

$$= 90^\circ - 60^\circ$$

$$= 30^\circ$$

Due to radii of circle $OA = OB$ and angles opposite to equal sides are equal. Thus

$$\angle OBA = \angle BAC$$

$$= 30^\circ$$

(ii) $\angle AOC$

We have $\angle ABC = \angle ABO + \angle CBO$

$$= 30^\circ + 35^\circ = 65^\circ$$

$$\angle AOC = 2 \times ABC$$

$$= 2 \times 65^\circ = 130^\circ$$

(iii) $\angle ATC$

Sum of all the angles of a quadrilateral is 360°. thus In quadrilateral $AOCT$,

$$\angle AOC + \angle OAT + \angle OCT + \angle ATC = 360^\circ$$

$$130^\circ + 90^\circ + 90^\circ + \angle ATC = 360^\circ$$

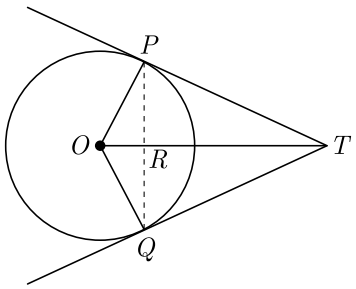
$$310^\circ + \angle ATC = 360^\circ$$

$$\angle ATC = 360^\circ - 310^\circ$$

$$= 50^\circ$$

Comp 2020

57. In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T. Find the length of TP.



Sol :

Since length of tangents from an external point to a circle are equal,

$$PT = QT$$

Thus ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$.

Thus $OT \perp PQ$ and OT also bisects PQ.

Thus $PR = PQ = \frac{8}{2} = 4$ cm

Since ΔOPR is right angled isosceles triangle,

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$

$$= 3 \text{ cm}$$

Now, Let $TP = x$ and $TR = y$ then we have

$$x^2 = y^2 + 16 \quad (1)$$

Also in ΔOPT,

$$x^2 + (5)^2 = (y+3)^2 \quad (2)$$

Solving (1) and (2) we get

$$y = \frac{16}{3} \text{ and } x = \frac{20}{3}$$

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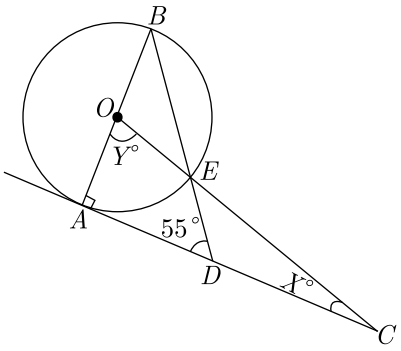
Hence, $TP = \frac{20}{3}$

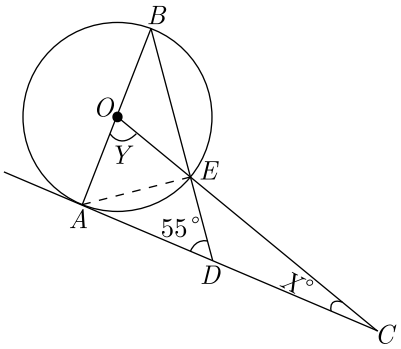
Main 2019

58. In the given figure AC is a tangent to the circle with centre O. If $\angle ADB = 55^\circ$, find x and y. Give reasons for your answers.



Sol :

We have redrawn the figure as below. Here we have joined AE



Here $\angle ADB = 55^\circ$

Here, AC is a tangent. Tangent is always perpendicular to radius at the point of contact. Thus

$$\angle OAC = 90^\circ$$

Using angle sum property in triangle ΔABD we have

$$\begin{aligned}\angle ABD + \angle BAD + \angle ADB &= 180^\circ \\ \angle ABD + 90^\circ + 55^\circ &= 180^\circ \\ \angle ABD + 145^\circ &= 180^\circ \\ \angle ABD &= 180^\circ - 145^\circ \\ &= 35^\circ\end{aligned}$$

Angle made by a chord at centre is double the angle made by it any remaining part.

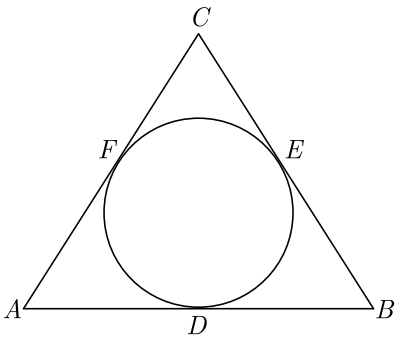
$$\begin{aligned}y &= 2\angle ABD \\ &= 2 \times 35^\circ \\ &= 70^\circ\end{aligned}$$

Using angle sum property in triangle ΔOAC we have

$$\begin{aligned}COA + \angle OAC + \angle ACO &= 180^\circ \\ 70^\circ + 90^\circ + x^\circ &= 180^\circ \\ 160^\circ + x^\circ &= 180^\circ \\ x^\circ &= 180^\circ - 160^\circ \\ &= 20^\circ\end{aligned}$$

Comp 2019

59. In the given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



NODIA

Sol :

Since AF and AD are tangent of the circle, $AF = AD$

Let $AF = AD = x$

Now $DB = AB - AD = 12 - x$

Since BD and BE are tangent of the circle, $BD = BE$

Thus $BE = BD = 12 - x$

Now $CE = CB - BE = 8 - (12 - x)$

Since CF and CE are tangent of the circle, $CF = CE$

Thus $CF = CE = 8 - (12 - x)$ cm

But $AC = CF + FA$

Substituting values we have

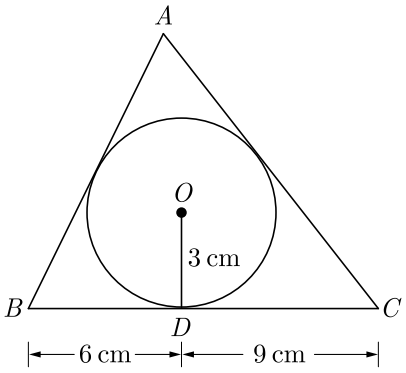
$$\begin{aligned}10 &= 8 - (12 - x) + x \\ 10 &= 2x - 4 \\ 2x &= 10 + 4 = 14\end{aligned}$$

$$x = 7$$

Thus $AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm

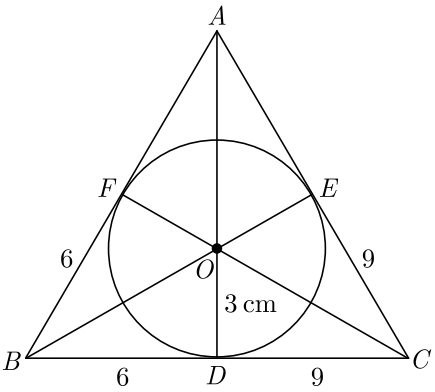
Main 2018

60. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.



Sol :

We redraw the given circle as shown below.



Since tangents from an external point to a circle are equal,

$$AF = AE$$
$$BF = BD = 6 \text{ cm}$$
$$CE = CD = 9 \text{ cm}$$

Let $AF = AE = x$

Now $AB = AF + FB = 6 + x$

$AC = AE + EC = x + 9$

$BC = 6 + 9 = 15$ cm

Perimeter of ΔABC,

$$p = 15 + 6 + x + 9 + x = 30 + 2x$$

Now area, $\Delta ABC = \frac{1}{2} rp$

Here $r = 3$ is the radius of circle. Substituting all values we have

$$54 = \tfrac{1}{2} \times 3 \times (30 + 2x)$$
$$54 = 45 + 3x$$

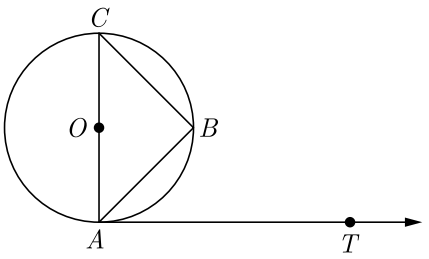
or $x = 3$

Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

Comp 2018

61. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, find $\angle BAT$.

NODIA



Sol :

We have $\angle ACB = 50^\circ$

Since $\angle CBA$ is angle in semi-circle,

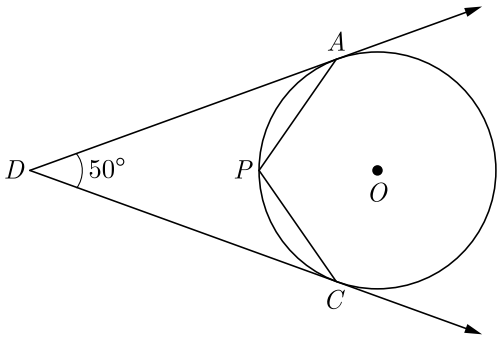
$$\angle CBA = 90^\circ$$

Now $\angle OAB = 180^\circ - 90^\circ - 50^\circ = 40^\circ$

$$\angle BAT = 90^\circ - \angle OAB$$
$$= 90^\circ - 40^\circ = 50^\circ$$

Main 2017

62. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50^\circ$.



Sol :

We redraw the given figure by joining A and C to O as shown below.

$$\angle TPR = \angle TQR$$

Thus $\Delta PTR \cong \Delta QTR$

and $PR = QR$

and $\angle PRT = \angle QRT$

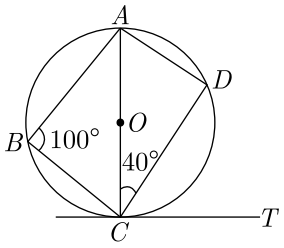
But $\angle PRT + QRT = 180^\circ$ as PQ is line segment,

$$\angle PRT = \angle QRT = 90^\circ$$

Therefore TR or OT is the right bisector of line segment PQ. Hence proved.

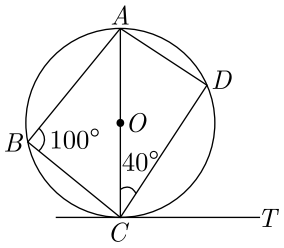
Main 2013

67. In the given circle with centre O, $\angle ABC = 100^\circ$, $\angle ACD = 40^\circ$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$



Sol :

We have redrawn the given figure as below



Here $\angle ABC = 100^\circ$, $\angle ACD = 40^\circ$

Sum of opposite angles in a cyclic quadrilateral is 180°. Since, $ABCD$ is a cyclic quadrilateral we have

$$\angle ABC + \angle ADC = 180^\circ$$
$$100^\circ + \angle ADC = 180^\circ$$
$$\angle ADC = 180^\circ - 100^\circ$$
$$\angle ADC = 80^\circ$$

Using angle sum property in triangle ΔADC we have

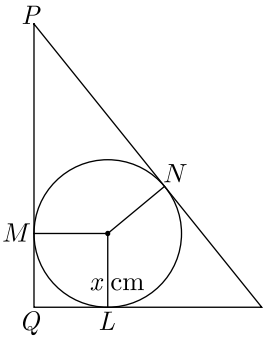
$$\angle ACD + \angle ADC + \angle DAC = 180^\circ$$
$$40^\circ + 80^\circ + \angle DAC = 180^\circ$$
$$\angle DAC = 180^\circ - 120^\circ$$
$$= 60^\circ$$

Using alternate segment property we have

$$\angle DCT = \angle DAC = 60^\circ$$

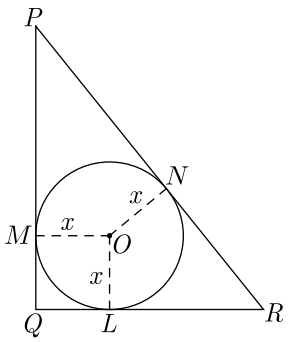
Main 2012

68. In ΔPQR, $PQ = 24$ cm, $QR = 7$ cm and $\angle PQR = 90^\circ$. Find the radius of the inscribed circle



Sol :

We have redrawn the given figure as below



Here $PQ = 24$ cm, $QR = 7$ cm and $\angle PQR = 90^\circ$

In right ΔPQR, using Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$
$$= 24^2 + 7^2$$
$$= 25^2$$
$$PR = 25 \text{ cm}$$

Here, PQ, QR and PR are the tangent of inscribed circle.

The tangent at any point of a circle and the radius through the point are perpendicular to each other.

Thus $\angle OMQ = \angle OLQ = \angle ONR = 90^\circ$

Since tangent drawn from an external point to the circle are equal to length thus

$$QL = QM$$

Since, all angles of a quadrilateral $OLQM$ are 90° and $QL = QM$ we get that $OLQM$ is a square.

Let x be side of square $OLQM$

$$QL = QM = x$$

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$$OB = \sqrt{OP^2 - BP^2} = \sqrt{26^2 - 24^2}$$
$$= \sqrt{676 - 576} = \sqrt{100} = 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2}AB = \frac{16}{2} = 8 \text{ cm}$$

Now in ΔOBC, $OC^2 = OB^2 - BC^2$

$$= 10^2 - 8^2$$
$$= 36$$
$$OC = 6 \text{ cm}$$

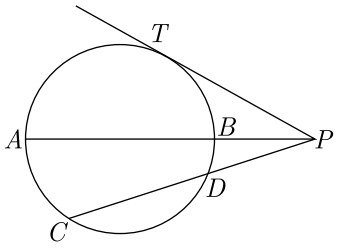
Thus distance of the chord from the centre is 6 cm.

Main 2014

65. In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T and $CD = 7.8$ cm, $PD = 5$ cm, $PB = 4$ cm. Find

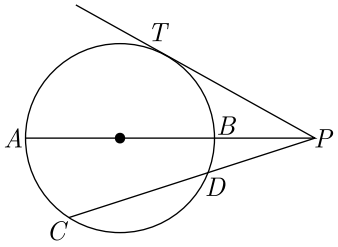
(i) AB,

(ii) the length of tangent PT.



Sol :

We have redrawn the given figure as below



Here $CD = 7.8$ cm, $PB = 5$ cm, $PB = 4$ cm

(i) AB

Since, chord CD and tangent at point T intersect each other at P.

$$PC \times PD = PT^2 \quad ...(1)$$

Since, chord AB and tangent at point T intersect each other at P.

$$PA \times PB = PT^2 \quad ...(2)$$

From Eqs. (1) and (2), we get

$$PC \times PD = PA \times PB \quad ...(3)$$

Now, $PA = PB + BA$

$$= 4 + AB$$

and $PC = PD + DC$

$$= 5 + 7.8 = 12.8 \text{ cm}$$

From Eq. (3), we get

$$12.8 \times 5 = (4 + AB) \times 4$$
$$4 + AB = \frac{12.8 \times 5}{4}$$
$$AB = 16 - 4$$
$$AB = 12 \text{ cm}$$

(ii)Length of tangent PT.

From Eq. (2), we get

$$PT^2 = PA \times PB$$
$$= (PB + BA) \times PB$$
$$= (4 + 12) \times 4$$
$$= 16 \times 4$$
$$PT^2 = 64$$

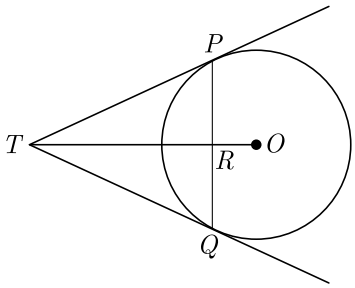
Comp 2014

66. From a point T outside a circle of centre O , tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

NODIA

Sol :

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

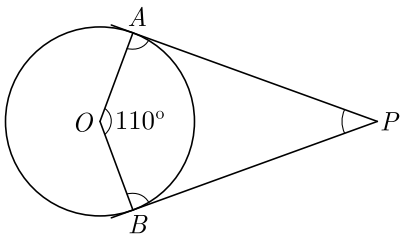
Now in ΔPTR and ΔQTR

$$TP = TQ$$
$$TR = TR \quad \text{(Common)}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

As per information given in question we have drawn figure below.



Radius is perpendicular to the tangent at point of contact.

Thus, $OA \perp AP$ and $OB \perp PB$.

In quadrilateral, $OAPB$, we have

$$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$$

$$90^\circ + \angle APB + 90^\circ + 110^\circ = 360^\circ$$

$$\angle APB = 70^\circ$$

Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

50. **Assertion :** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.
Reason : A parallelogram circumscribing a circle is a rhombus.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

From an external point the two tangents drawn subtend equal angles at the centre.
So assertion is true.
Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

Thus (b) is correct option.

51. **Assertion :** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.
Reason : A parallelogram circumscribing a circle is a rhombus.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

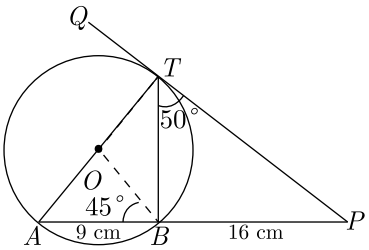
From an external point the two tangents drawn subtend equal angles at the centre.
So assertion is true.
Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

Thus (b) is correct option.

THREE MARKS QUESTIONS

Main 2023

52. In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. Chord AB produced meets the tangent at P.
$AB = 9$ cm, $BP = 16$ cm, $\angle PTB = 50^\circ$, $\angle OBA = 45^\circ$



Find:
(i) length of PT (ii) $\angle BAT$
(iii) $\angle BOT$ (iv) $\angle ABT$

Sol :

(i) Length of PT

Here, $PT^2 = PA \times PB$

$= (PB + AB) \times PB$

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Since tangent drawn from an external point to the circle are equal to length thus

$$RN = RL$$
$$= QR - QL$$
$$= 7 - x$$

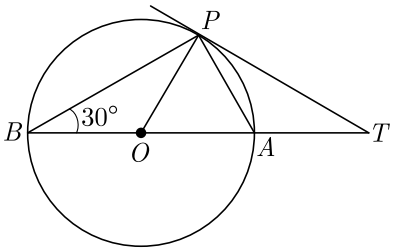
and
$$PN = PM$$
$$= PQ - QM$$
$$= 24 - x$$

Now
$$PR = PN + NR$$
$$25 = 24 - x + 7 - x$$
$$25 = 31 - 2x$$
$$2x = 31 - 25$$
$$2x = 6$$
$$x = \frac{6}{3} = 3 \text{ cm}$$

Hence, the radius of the inscribed circle is 3 cm.

Comp 2012

69. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^o$, what is the measure of $\angle PTA$?



Sol :

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^o$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^o$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^\circ + 30^\circ = 60^\circ$$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^o$$

Now in right angle ΔOPT,

$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

Substituting $\angle OPT = 90^o$ and $\angle POT = 60^\circ$ we have

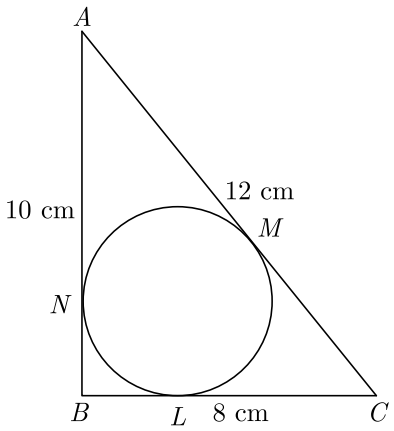
$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$
$$= 180^\circ - 150^\circ$$
$$= 30^o$$

Thus $\angle PTA = \angle PTO = 30^o$

Comp 2011

70. In figure, a circle is inscribed in a ΔABC having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the length BL, CM and AN.

NODIA



Sol :

Tangents from external a point on a circle are always equal in length.

Let x be length of BL, then we have

$$BL = x = BN$$

So,
$$LC = MC = (8 - x)$$

and
$$AN = AM = (10 - x)$$

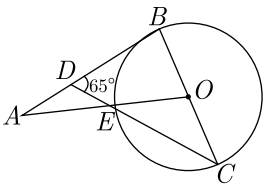
Since,
$$AC = 12$$
$$AM + MC = 12$$
$$(10 - x) + (8 - x) = 12$$
$$18 - 2x = 12 \Rightarrow x = 3$$

Hence,
$$BL = 3 \text{ cm}$$
$$CM = 8 - 3 = 5 \text{ cm}$$

and
$$AN = 10 - 3 = 7 \text{ cm}$$

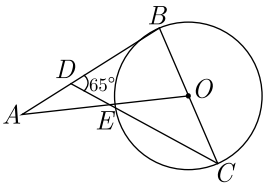
Main 2010

71. In the following figure, O is the centre of the circle and AB is a tangent to it at point B, if $\angle BDC = 65^\circ$. Find $\angle BAO$.



Sol :

We have redrawn the given figure as below



Here $\angle BDC = 65^\circ$

Tangent at any point of a circle is perpendicular to the radius drawn through the point of contact.

Since, AB is a tangent to the circle at point B.

$$\angle ABO = 90^\circ$$
$$= \angle DBC$$

Using angle sum property in triangle ΔBCD we have

$$\angle DBC + \angle BCD + \angle CDB = 180^\circ$$
$$90^\circ + \angle BCD + 65^\circ = 180^\circ$$
$$\angle BCD = 180^\circ - 155^\circ$$
$$= 25^\circ$$
$$\angle BCE = \angle BCD$$
$$= 25^\circ$$

Angle that an arc of a circle subtends at the centre is double of the angle which it subtends at any point on the remaining part of the circle.

Thus $\angle BOE = 2\angle BCE$
$$= 2\angle BCD$$
$$= 2 \times 25^\circ$$
$$= 50^\circ$$
$$\angle AOB = \angle BOE = 50^\circ$$

Using angle sum property in triangle ΔABO we have

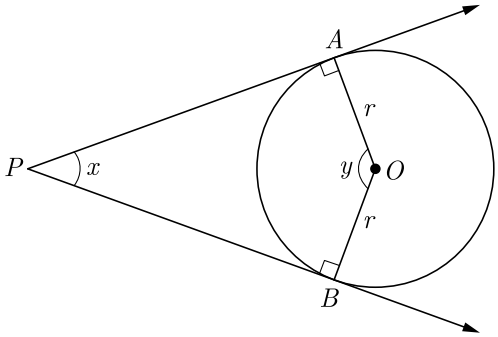
$$\angle BAO = 180^\circ - \angle ABO - \angle AOB$$
$$= 180^\circ - 90^\circ - 50^\circ$$
$$= 40^\circ$$

Comp 2010

72. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

Sol :

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle A = \angle B = 90^\circ$

Since, $AOBP$ is a quadrilateral,

$$\angle A + \angle B + x + y = 360^\circ$$
$$90^\circ + 90^\circ + x + y = 360^\circ$$
$$180 + x + y = 360^\circ$$
$$x + y = 180^\circ$$

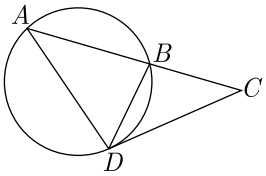
Therefore opposite angle are supplementary.

Main 2009

73. In the following figure, $AB = 7$ cm and $BC = 9$.

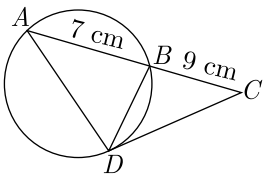
(i) Prove that $\Delta ACD \sim \Delta DCB$.

(ii) Find the length of CD.



Sol :

We have redrawn the given figure as below



Here $AB = 7$ cm and $BC = 9$ cm

(i) $\Delta ACD \sim \Delta DCB$.

In ΔACD and ΔDCB,

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Due to common angles we obtain

$$\angle C = \angle C$$

Due to angles in alternate segment,

$$\angle CDB = \angle BAD$$

By AA axiom of similarity]

$$\Delta ACD \sim \Delta DCB \quad \text{Hence proved.}$$

(ii) Length of CD.

Since, chord AB and tangent at D intersect each other at point C, we have

$$AC \times BC = CD^2$$

$$16 \times 9 = CD^2$$

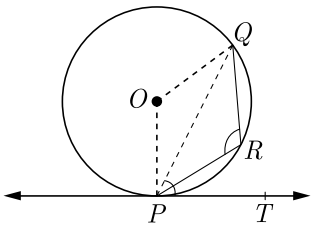
$$CD^2 = 144 = 12^2$$

$$CD = 12 \text{ cm}$$

Comp 2009

74. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

Sol :



We have $\angle QPT = 60^\circ$

Here $\angle OPT = 90^\circ$ because of tangent at radius.

Now $\angle OPQ = \angle OQP$

$$= \angle OPT - \angle QTP$$

$$= 90^\circ - 60^\circ = 30^\circ$$

$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$

$$= 180^\circ - (30^\circ + 30^\circ)$$

$$= 180^\circ - 60^\circ = 120^\circ$$

Now Reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$$\angle PRQ = \frac{1}{2} \text{ Reflex } \angle POQ$$

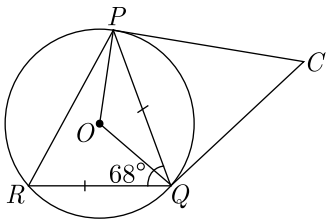
$$= \frac{1}{2} \times 240^\circ = 120^\circ$$

Main 2008

75. In the given figure, $PQ = RQ$, $\angle RQP = 68^\circ$, PC and QC are tangents to the circle with centre O. Calculate the values of

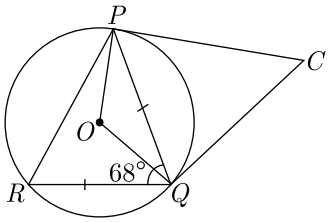
(i) $\angle QOP$

(ii) $\angle QCP$.



Sol :

We have redrawn the given figure as below



Here $PQ = QR$, $\angle RQP = 68^\circ$

(i) In ΔPQR,

In ΔPQR $PQ = QR$

Since angles opposite to equal sides are equal, thus

$$\angle PRQ = \angle QPR \quad \text{...(i)}$$

Using angle sum property in triangle ΔPRQ we have

$$\angle PRQ + \angle QPR + \angle RQP = 180^\circ$$

$$\angle PRQ + \angle PRQ + 68^\circ = 180^\circ$$

$$2\angle PRQ = 180^\circ - 68^\circ$$

$$= 112^\circ$$

$$\angle PRQ = \frac{112^\circ}{2} = 56^\circ$$

Angle that an arc of a circle subtends at the centre is twice the angle subtends by it any point on the remaining circumference.

$$\angle QOP = 2 \times \angle PRQ$$

$$= 2 \times 56^\circ$$

$$= 112^\circ$$

(ii) $\angle QCP$.

Due to angles in alternate segments we have

$$\angle PQC = \angle PRQ$$

$$\angle PQC = 56^\circ$$

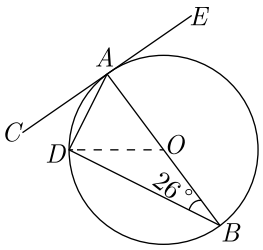
Using angle sum property in triangle ΔPQC we have

$$\angle PQC + \angle QPC + \angle QCP = 180^\circ$$

$$56^\circ + 56^\circ + \angle QCP = 180^\circ$$

$$112^\circ + \angle QCP = 180^\circ$$

NODIA



Here $\angle ABD = 26^\circ$

(i) $\angle BDA$

Since angle in a semi-circle is always 90°, thus

$$\angle BDA = 90^\circ$$

(ii) $\angle BAD$

In ΔABD, using angle sum property in triangle ΔABD we have

$$\angle ABD + \angle BDA + \angle BAD = 180^\circ$$
$$26^\circ + 90^\circ + \angle BAD = 180^\circ$$
$$\angle BAD = 180^\circ - 116^\circ$$
$$= 64^\circ$$

(iii) $\angle CAD$

Due to alternate segment theorem,

$$\angle CAD = \angle ABD$$
$$= 26^\circ$$

(iv) $\angle ODB$

Due to radii of circle $OB = OD$ and angles opposite to equal sides are equal. Thus

$$\angle ODB = \angle OBD$$
$$= \angle ABD$$
$$= 26^\circ$$

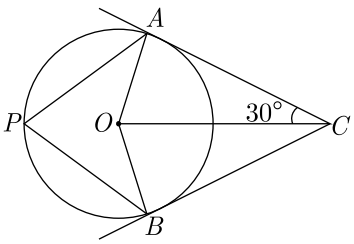
Main 2011

91. In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If $\angle ACO = 30^\circ$, find

(i) $\angle BCO$.

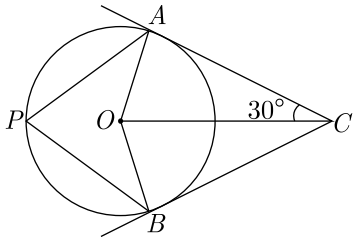
(ii) $\angle AOB$.

(iii) $\angle APB$.



Sol :

We have redrawn the given figure as below.



(i) $\angle BCO$

Here, ΔACO and ΔBCO are congruent by RHS congruence rule.

Thus by CPCT we have

$$\angle BCO = \angle ACO = 30^\circ$$

(ii) $\angle AOB$

Using angle sum property in triangle ΔOAC we have

$$\angle OAC + \angle ACO + \angle COA = 180^\circ$$
$$90^\circ + 30^\circ + \angle COA = 180^\circ$$
$$\angle COA = 180^\circ - 120^\circ$$
$$= 60^\circ$$
$$\angle AOB = \angle AOC + \angle BOC$$
$$= \angle AOC + \angle AOC$$
$$= 2\angle AOC$$
$$= 2\angle COA$$
$$= 2 \times 60^\circ = 120^\circ$$

(iii) $\angle APB$

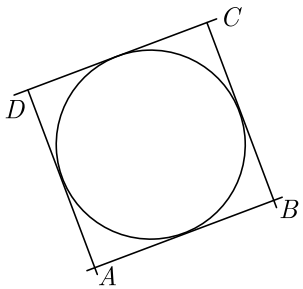
Angle that an arc of a circle subtends as the centre is twice of the angle subtends by it at any point of remaining part of the circle.

Thus $2\angle APB = \angle AOB$

$$\angle APB = \frac{1}{2} \times 120^\circ = 60^\circ$$

Main 2014

92. In figure, a circle touches all the four sides of a quadrilateral $ABCD$. If $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm, then find the length of AD.



Sol :

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$$OE = OF$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle OEA = \angle OFA = 90^\circ$

Now in ΔAEO and ΔAFO,

$$OE = OF$$
$$\angle OEA = \angle OFA = 90^\circ$$
$$OA = OA \qquad \text{(Common side)}$$

Thus $\Delta AEO \cong \Delta AFO$ (SAS congruency)

$$\angle 7 = \angle 8$$

Similarly, $\angle 1 = \angle 2$

$$\angle 3 = \angle 4$$
$$\angle 5 = \angle 6$$

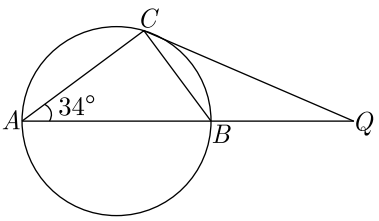
Since angle around a point is 360°,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$
$$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$$
$$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$$
$$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$$
$$\angle AOB + \angle COD = 180^\circ$$

Hence Proved.

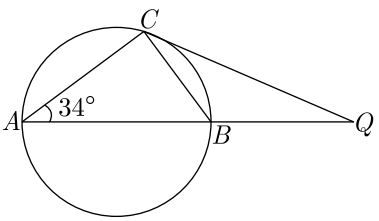
Main 2006

79. In the given figure, AB is a diameter. The tangent at C meets AB produced at Q, $\angle CAB = 34^\circ$. Find (i) $\angle CBA$. (ii) $\angle CQA$.



Sol :

We have redrawn the given figure as below



(i) $\angle CBA$.

Since AB is a diameter of circle, triangle in a semi-circle is always 90°, thus

$$\angle ACB = 90^\circ$$

Using angle sum property in triangle ΔABC we have

$$\angle ACB + \angle CBA + \angle BAC = 180^\circ$$
$$90^\circ + \angle CBA + 34^\circ = 180^\circ$$
$$\angle CBA = 180^\circ - 124^\circ = 56^\circ$$

(ii) $\angle CQA$

Since, CQ is a tangent to the circle, due to alternate segment property

$$\angle BCQ = \angle BAC = 34^\circ$$

Since, AQ is a straight line, we have

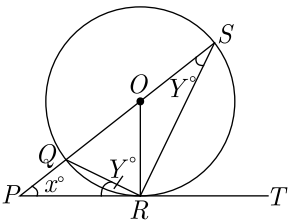
$$\angle ABC + \angle CBQ = 180^\circ$$
$$56^\circ + \angle CBQ = 180^\circ$$
$$\angle CBQ = 180^\circ - 56^\circ = 124^\circ$$

Using angle sum property in triangle ΔCBQ we have

$$\angle CBQ + \angle CQB + \angle BCQ = 180^\circ$$
$$124^\circ + \angle CQB + 34^\circ = 180^\circ$$
$$\angle CQB = \angle CQA = 22^\circ$$

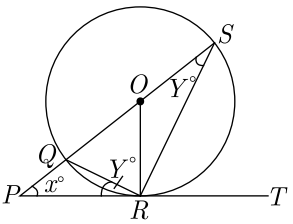
SQP 2006

80. In the given figure, PT touches the circle, whose centre is O, at R. Diameter SQ, when produced meets PT at P. If $\angle SPR = x^\circ$ and $\angle QRP = y^\circ$. Show that $x^\circ + 2y^\circ = 90^\circ$.



Sol :

We have redrawn the given figure as below



Since, PRT is a tangent at R and QR is a chord, using alternate segment property, we have

$$\angle QRP = \angle RSQ = y^\circ$$

Since, SQ is a diameter of a circle and $\angle QRS$

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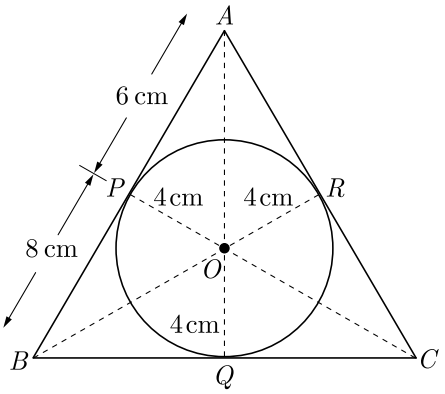
$$\angle QCP = 180^\circ - 112^\circ$$
$$= 68^\circ$$

Comp 2008

76. In Figure the radius of incircle of ΔABC of area $84\,\text{cm}^2$ and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Sol :

Since length of tangents from an external point to a circle are equal,

At A, $\quad AP = AR = 6$ cm $\quad (1)$

At B, $\quad ,BP = BQ = 8$ cm $\quad (2)$

At C, $\quad CR = CQ = x \quad (3)$

Perimeter of ΔABC,

$$p = AP + PB + BQ + QC + CR + RA$$
$$= 6 + 8 + 8 + x + x + 6 = 28 + 2x$$

Now area $\quad \Delta ABC = \frac{1}{2}rp$

Here $r = 4$ is the radius of circle. Substituting all values we have

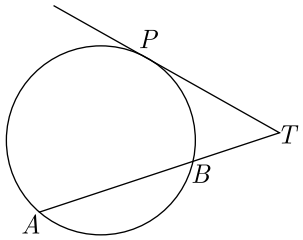
$$84 = \tfrac{1}{2} \times 4 \times (28 + 2x)$$
$$84 = 56 + 4x$$
$$21 = 14 + x \Rightarrow x = 7$$

Thus $\quad AC = AR + RC = 6 + 7 = 13$ cm

$$BC = BQ + QC = 8 + 7 = 15 \text{ cm}$$

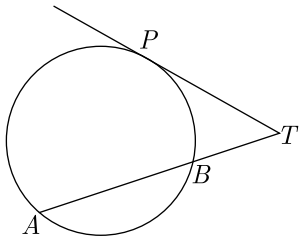
Main 2007, 2001

77. In the given figure, PT is a tangent to the circle. Find PT, if $AT = 16$ cm and $AB = 12$ cm.



Sol :

We have redrawn the given figure as below



Here $AT = 16$ cm and $AB = 12$ cm

If chord AB and tangent at P meet at point T , then

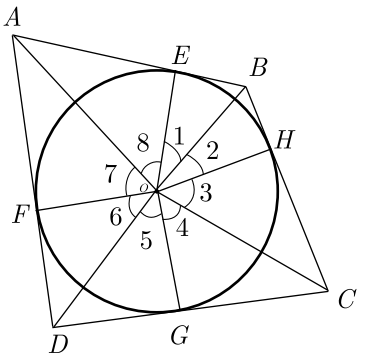
$$TA \times TB = PT^2$$
$$16 \times (AT - AB) = PT^2$$
$$16 \times (16 - 12) = PT^2$$
$$PT^2 = 16 \times 4$$
$$PT^2 = 64$$
$$PT = 8 \text{ cm}$$

Comp 2007

78. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol :

A circle centre O is inscribed in a quadrilateral $ABCD$ as shown in figure given below.



Since OE and OF are radius of circle,

NODIA

$$= PQ - QM$$
$$= 3 - x$$

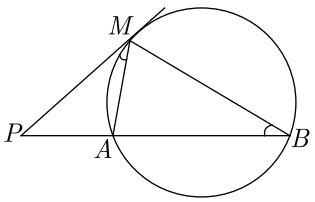
Now $PR = PN + NR$

$$5 = 3 - x + 4 - x$$
$$5 = 7 - 2x$$
$$2x = 7 - 5 = 2$$
$$x = 1 \text{ cm}$$

Hence, the radius of the inscribed circle is 1 cm.

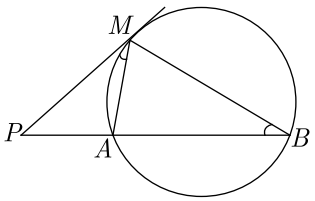
Main 2005

83. In the given figure, PM is a tangent to the circle and $PA = AM$. Prove that
(i) ΔPMB is an isosceles.
(ii) $PA \cdot PB = MB^2$.



Sol :

We have redrawn the given figure as below



(i) ΔPMB is an isosceles.

Since given $PA = AM$ [given]

Angles opposite to equal sides are equal, thus

$$\angle AMP = \angle APM \quad ...(1)$$

Since, angle between the tangent and chord is equal to the angle subtended by same chord in alternate segment.

Thus $\angle AMP = \angle ABM = \angle PBM$

$$\angle APM = \angle PBM$$

or $\angle BPM = \angle PBM$ [from Eq. (i)]

$$PM = MB \quad ...(ii)$$

Hence, ΔPMB is an isosoceles triangle.
(ii) $PA \cdot PB = MB^2$.
Since, PM is tangent to the circle and PAB is a secant.

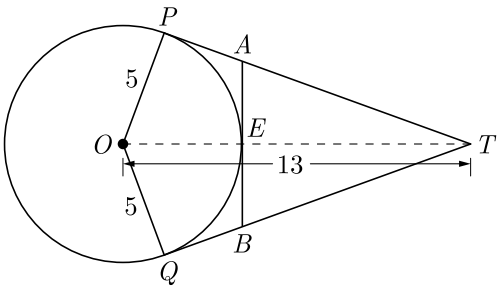
$$PA \cdot PB = PM^2$$

Since $PM = MB$ we have

$$PA \cdot PB = MB^2$$ Hence proved.

Main 2003

84. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Sol :

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $PT = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE = 13 - 5 = 8$ cm

Since length of tangents from an external point to a circle are equal,

Let $PA = AE = x$

Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $TA^2 = TE^2 + EA^2$

$$(TP - PA)^2 = 8^2 + x^2$$
$$(12 - x)^2 = 64 + x^2$$
$$144 - 24x + x^2 = 64 + x^2$$
$$24x = 144 - 64 = 80$$

or, $x = 3.3$ cm.

Thus $AB = 2 \times x = 2 \times 3.3$

$$= 6.6 \text{ cm}.$$

Comn 2003

85. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

Sol :

As per the given question we draw the figure as below.

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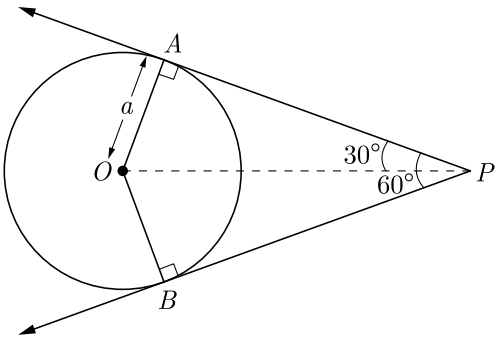


Tangents are always equally inclined to line joining the external point P to centre O.

$$\angle APO = \angle BPO = \frac{60^o}{2} = 30^o$$

Also radius is also perpendicular to tangent at point of contact.

In right $\Delta\, OAP$ we have,

$$\angle APO = 30^o$$

Now, $\sin 30^o = \dfrac{OA}{OP}$

Here OA is radius whose length is a, thus

$$\frac{1}{2} = \frac{a}{OP}$$

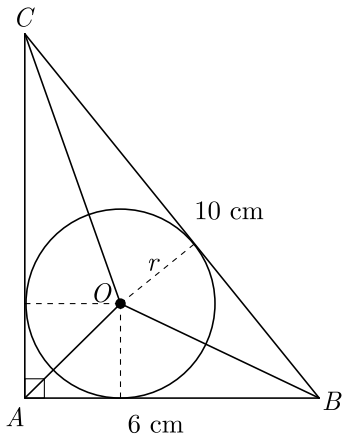
or $OP = 2a$

Main 2002

86. A right triangle ABC, right angled at A is circumscribing a circle. If $AB = 6$ cm and $BC = 10$ cm, find the radius r of the circle.

Sol :

As per question we draw figure shown below.



In triangle $\Delta\, ABC$,

$$AC = \sqrt{10^2 - 36^2} = 8 \text{ cm}$$

Area of triangle $\Delta\, ABC$,

$$\Delta\, ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

Here we have joined AO, BO and CO.

For area of triangle we have

$$\Delta\, ABC = \Delta\, OBC + \Delta\, OCA + \Delta\, OAB$$

$$24 = \tfrac{1}{2} rBC + \tfrac{1}{2} rAC + \tfrac{1}{2} rAB$$

$$= \tfrac{1}{2} r(BC + AC + AB)$$

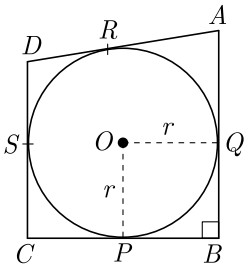
$$= \tfrac{1}{2} r(6 + 10 + 8) = 12r$$

or $12r = 24$

Thus $r = 2$ cm.

Comp 2002

87. In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



Sol :

Since length of tangents from an external point to a circle are equal,

$$DR = DS = 5 \text{ cm}$$

$$AR = AQ$$

$$BQ = BP$$

Now $AR = AD - DR$

$$= 23 - 5 = 18 \text{ cm}$$

$$AQ = AR = 18 \text{ cm}$$

$$QB = AB - AQ = 29 - 18 = 11 \text{ cm}$$

$$PB = QB = 11$$

Now $\angle OQB = \angle OPB = 90^\circ$ because radius is always perpendicular to tangent.

Thus $OP = OQ = PB = BQ$

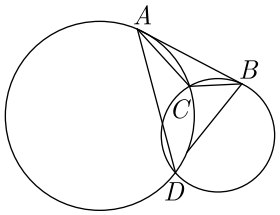
So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

Main 2000

88. In the given figure, AB is a common tangent to

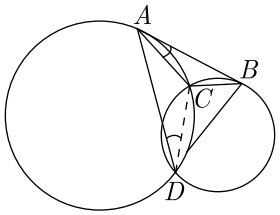
NODIA

two circles intersecting at C and D. Write down the measure of $(\angle ACB + \angle ADB)$. Justify your answer.



Sol :

We have redrawn the given figure as below.



Here AB is a common tangent and we have joined CD.
From the figure,
Due to angles in the alternate segments,

$$\angle CAB = \angle ADC \quad ...(1)$$

and $$\angle ACB = \angle BDC \quad ...(2)$$

Using angle sum property in triangle ΔABC we have

$$\angle ACB + \angle CBA + \angle BAC = 180^\circ$$

$$\angle ACB = 180^\circ - (\angle CAB + \angle CBA)$$

From eq (1) and (2) we have

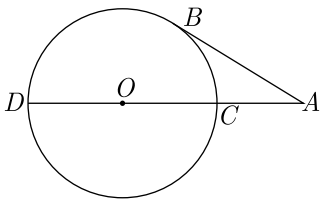
$$\angle ACB = 180^\circ - (\angle ADC + \angle BDC)$$

$$= 180^\circ - \angle ADB$$

$$\angle ACB + \angle ADB = 180^\circ$$

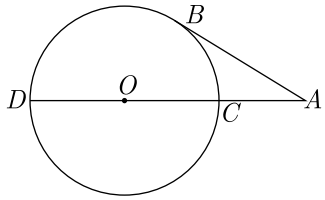
Main 2000

89. In the given figure, O is the centre of the circle and AB is a tangent at B. If $AB = 15$ cm and $AC = 7.5$ cm. Calculate the radius of the circle.



Sol :

We have redrawn the given figure as below.



Here $AB = 15$ cm and $AC = 7.5$ cm
If chord AB and tangent at P meet at point T , then

$$TA \times TB = PT^2$$

Here diameter DC and tangent at B meet at point A, thus

$$AC \times AD = AB^2$$

$$AC \times (AC + CD) = AB^2$$

$$7.5 \times (7.5 + CD) = 15^2$$

$$7.5 \times (7.5 + CD) = 15 \times 15$$

$$(7.5 + CD) = 2 \times 15$$

$$7.5 + CD = 30$$

$$CD = 30 - 7.5 = 22.5$$

$$OC = \tfrac{1}{2} CD = \tfrac{1}{2} \times 22.5$$

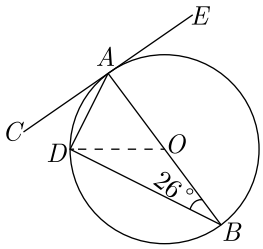
$$= 11.25 \text{ cm}$$

Thus radius is 11.25 cm.

FOUR MARK QUESTIONS

Main 2023

90. In the given figure O, is the centre of the circle. CE is a tangent to the circle at A. If $\angle ABD = 26^\circ$, then find



(i) $\angle BDA$ (ii) $\angle BAD$
(iii) $\angle CAD$ (iv) $\angle ODB$

Sol :

We have redrawn the figure as below.

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angle in semi-circle, we can write

$$\angle QRS = 90^\circ$$

Using angle sum property in triangle ΔPRS we have

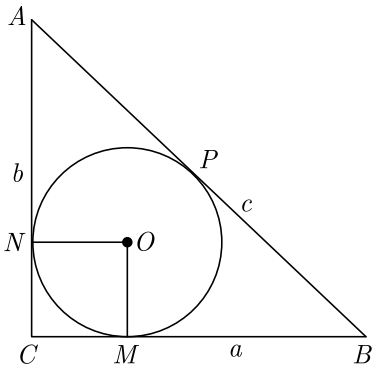
$$\angle SPR + \angle PRS + \angle RSP = 180^\circ$$
$$x^\circ + (y^\circ + 90^\circ) + y^\circ = 180^\circ$$
$$x^\circ + 2y^\circ = 180^\circ - 90^\circ$$
$$x^\circ + 2y^\circ = 90^\circ \text{ Hence proved.}$$

Comp 2006

81. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle, of radius r, touches the sides of the triangle. Prove that $r = \frac{a+b-c}{2}$.

Sol :

As per question we have drawn figure shown below.



Let the circle touches CB at M, CA at N and AB at P.

Now $OM \perp CB$ and $ON \perp AC$ because radius is always perpendicular to tangent

OM and ON are radius of circle, thus

$$OM = ON$$

CM and CN are tangent from C, thus

$$CM = CN$$

Therefore $OMCN$ is a square. Let

Let $OM = r = CM = CN = ON$

Since length of tangents from an external point to a circle are equal,

$$AN = AP, CN = CM \text{ and } BM = BP$$

Now taking $AN = AP$

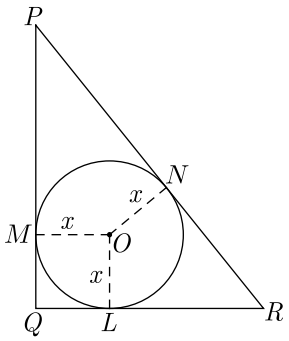
$$AC - CN = AB - BP$$
$$b - r = c - BM$$
$$b - r = c - (a - r)$$
$$b - r = c - a + r$$
$$2r = a + b - c$$
$$r = \frac{a+b-c}{2} \quad \text{Hence Proved.}$$

Main 2005

82. PQR is a right angled triangle with $PQ = 3$ cm and $QR = 4$ cm. A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle.

Sol :

As per question we have drawn figure shown below.



Here $PQ = 3$ cm, $QR = 4$ cm and $\angle PQR = 90^\circ$

In right ΔPQR, using Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$
$$= 3^2 + 4^2$$
$$= 5^2$$
$$PR = 5 \text{ cm}$$

Here, PQ, QR and PR are the tangent of inscribed circle.

The tangent at any point of a circle and the radius through the point are perpendicular to each other.

Thus $\angle OMQ = \angle OLQ = \angle ONR = 90^\circ$

Since tangent drawn from an external point to the circle are equal to length thus

$$QL = QM$$

Since, all angles of a quadrilateral $OLQM$ are 90° and $QL = QM$ we get that $OLQM$ is a square.

Let x be side of square $OLQM$

$$QL = QM = x$$

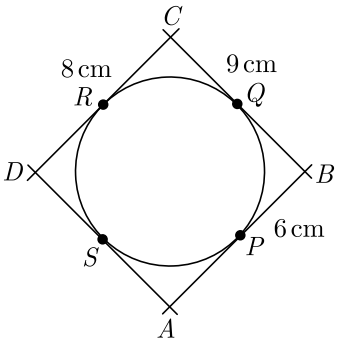
Since tangent drawn from an external point to the circle are equal to length thus

$$RN = RL$$
$$= QR - QL$$
$$= 4 - x$$

and $PN = PM$

NODIA

As per given information we have redrawn the figure below.



Tangents drawn from an external point to a circle are equal in length.

Thus $AP = AS$ and let it be x.

Similarly, $BP = BQ$, $CQ = CR$ and $RD = DS$

Now $BP = AB - AP = 6 - x$

$BP = BQ = 6 - x$

$CQ = BC - BQ$

$= 9 - (6 - x) = 3 + x$

Now, $CQ = CR = 3 + x$

$RD = CD - CR$

$= 8 - (3 + x) = 5 - x$

Now, $RD = DS = 5 - x$

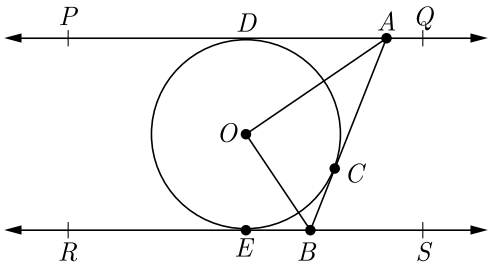
$AD = AS + SD$

$= x + 5 - x = 5$

Thus AD is 5 cm.

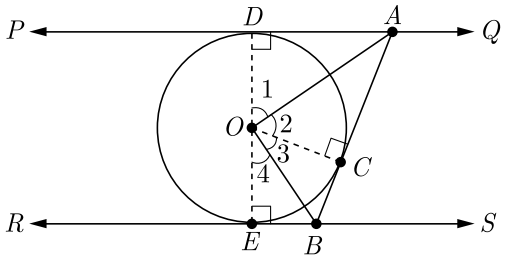
Comp 2013

93. In Figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90°$.



Sol :

We redraw the given figure as shown below.



In ΔDOA and ΔCOA, DA and AC are tangents drawn from common point,

Thus $DA = AC$

Due to angle between tangent and radius,

$\angle ODA = \angle OCA = 90°$

Due to radius of circle,

$OD = OC$

By SAS symmetry we have

$\Delta DOA \cong \Delta COA$

Hence, by CPCT,

$\angle 1 = \angle 2$

i.e., $\angle DOA = \angle COA$ ...(1)

Similarly, by SAS

$\Delta BOC = \Delta BOE$

and by CPCT $\angle 3 = \angle 4$

i.e., $\angle COB = \angle BOE$ ...(2)

Now, angles on a straight line,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

From equation (1) and (2) we have

$2\angle 2 + 2\angle 3 = 180°$

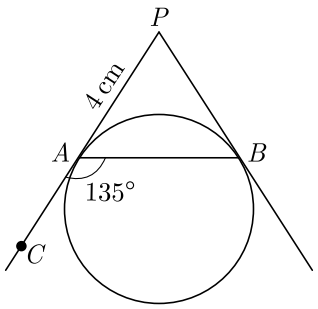
$\angle 2 + \angle 3 = 90°$

i.e., $\angle AOC + \angle BOC = 90°$

or $\angle AOB = 90°$ Hence Proved

Comp 2005

94. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135^0$. Find the length of chord AB.



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Sol :

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

In triangle ΔAPB we have

$$\angle APB = 180^\circ - \angle ABP - \angle BAP$$
$$= 180^\circ - 45^\circ - 45^\circ$$
$$= 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

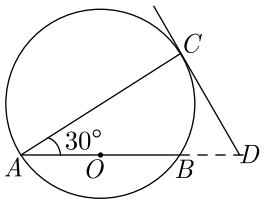
Now $AB^2 = AP^2 + BP^2 = 2AP^2$

$$= 2 \times 4^2 = 32$$

Hence $AB = \sqrt{32} = 4\sqrt{2}$ cm

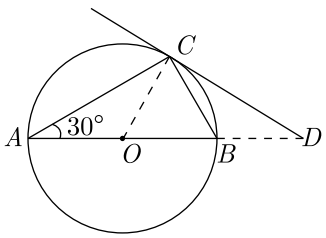
Main 2004

95. In the given figure, AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30^\circ$. The tangent at C intersects AB produced at D. Prove that $BC = BD$.



Sol :

We have redrawn the given figure as below.



Here we have $\angle BAC = 30^\circ$. We have joined OC. Here AB is a diameter of a circle and $\angle ACB$ is in a semi-circle, thus

$$\angle ACB = 90^\circ$$

Using angle sum property in triangle ΔACB we have

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$
$$\angle ABC + 90^\circ + 30^\circ = 180^\circ$$
$$\angle ABC = 180^\circ - 120^\circ$$
$$= 60^\circ$$

Due to radii of same circle, $OC = OB$ and angles opposite to equal sides are equal, thus

$$\angle OBC = \angle OCB$$

Now, $\angle OCD = \angle OCB + \angle BCD$

$$\angle BCD = \angle OCD - \angle OCB$$
$$= 90^\circ - 60^\circ$$
$$\angle BCD = 30^\circ \quad (1)$$

Since, tangent at any point of a circle and the radius through the point are perpendicular,

$$\angle OCD = 90^\circ$$

Due to linear pair we have

$$\angle ABC + \angle DBC = 180^\circ$$
$$60^\circ + \angle DBC = 180^\circ$$
$$\angle DBC = 180^\circ - 60^\circ$$
$$= 120^\circ$$

Using angle sum property in triangle ΔBCD we have

$$\angle BCD + \angle CDB + \angle DBC = 180^\circ$$
$$30^\circ + \angle BDC + 120^\circ = 180^\circ$$
$$\angle BDC + 150^\circ = 180^\circ$$
$$\angle BDC = 180^\circ - 150^\circ$$
$$\angle BDC = 30^\circ \quad (2)$$

From eq. (1) and (2) we have

$$\angle BDC = \angle BCD = 30^\circ$$

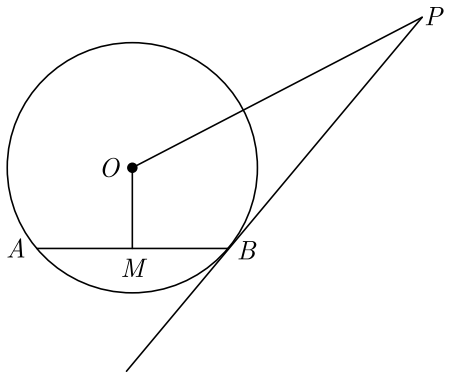
Since sides opposite to equal angles are equal angles are equal,

$$BC = BD$$ Hence proved.

NODIA

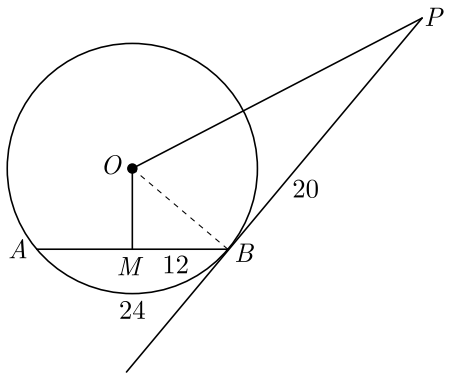
Comp 2004

96. PB is a tangent to the circle with centre O to B.AB is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is length 20 cm, find the length of PO.



Sol :

We redraw the given figure by joining O to B as shown below.



Here ΔOMB right angled triangle because AB is chord and OM is perpendicular on it.
In right angled triangle ΔOMB we have,

$$OB^2 = OM^2 + MB^2$$
$$= 5^2 + 12^2 = 13^2$$

Thus $OB = 13$

Here ΔOBP right angled triangle because PB is tangent on radius OB.

This in right angled triangle ΔOBP we have,

$$OP^2 = OB^2 + BP^2 = 13^2 + 20^2 = 569$$

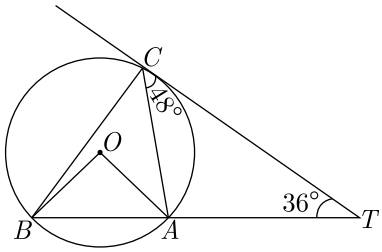
Thus $OP = \sqrt{569} = 23.85$ cm

Main 2001

97. A, B and C are three points on a circle. The tangent at C meets BA produced at T. Given that $\angle ATC = 36^\circ$ and $\angle ACT = 48^\circ$. Calculate the angle subtended by AB at the centre of the circle.

Sol :

We have redrawn the given figure as below.



Using angle sum property in triangle ΔATC we have

$$\angle ACT + \angle CTA + \angle TAC = 180^\circ$$
$$48^\circ + 36^\circ + \angle TAC = 180^\circ$$
$$84^\circ + \angle TAC = 180^\circ$$
$$\angle TAC = 180^\circ - 84^\circ$$
$$= 96^\circ$$

Since, BT is a straight line, we have

$$\angle BAC + \angle CAT = 180^\circ$$
$$\angle BAC + 96^\circ = 180^\circ$$
$$\angle BAC = 180^\circ - 96^\circ$$
$$= 84^\circ$$

Due to angle in alternate segment we have

$$\angle ABC = \angle ACT = 48^\circ$$

Using angle sum property in triangle ΔABC we have

$$\angle BCA + \angle CAB + \angle ABC = 180^\circ$$
$$\angle BCA + 84^\circ + 48^\circ = 180^\circ$$
$$\angle BCA + 132^\circ = 180^\circ$$
$$\angle BCA = 180^\circ - 132^\circ$$
$$= 48^\circ$$

Angle subtended by an arc at the centre is twice the angle subtended by it at remaining part of the circle.

Thus $\angle BOA = 2 \times \angle BCA$
$= 2 \times 48^\circ$
$= 96^\circ$

Comp 2001

98. In Figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.

NODIA

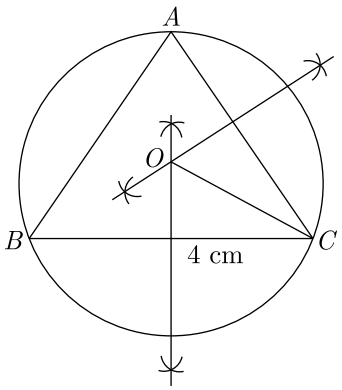
draw two arcs intersecting eachother at A.

Step 3 : Join AB and AC. ΔABC is the required equilateral triangle.

Step 4 : Draw the perpendicular bisectors of AC and BC intersecting each other at O.

Step 5 : With centre O and radius OC draw a circle which will pass through A, B and C.

This is the required circle.



SQP 2015

11. Construct $\Delta ABC, BC = 6$ cm, $\angle ABC = 45°$ and perpendicular from A to BC is 3.5 cm. Construct a circumcircle of the triangle.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 6$cm.

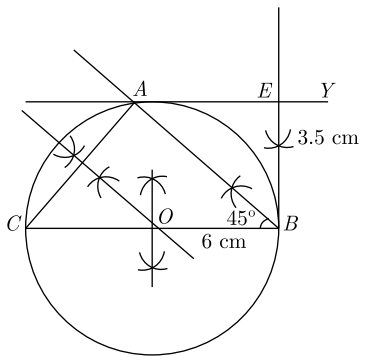
Step 2 : At 13, draw a perpendicular and cut off $BX = 3.5$ cm. From X, draw a line parallel to BC which intersect BX at A.

Step 3 : Join AC.

Step 4 : Draw perpendicular bisectors of BC and CA intersecting each other at O.

Step 5 : With centre O and radius OA, OB or OC draw a circle which passes through A, B and C.

This is the required circumcircle of ΔABC.



Comp 2015

12. Only ruler and compass may be used in this question.

(i) Construct ΔABC such that $AB = AC = 7$ cm and $BC = 5$ cm.

(ii) Draw AX, the perpendicular bisector of side BC.

(iii) Draw a circle with centre A and radius 3 cm cutting AX at Y.

(iv) Construct another circle to touch the circle with centre A externally at Y and passing through B and C.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 5$ cm.

Step 2 : With centres A and B and radius 7 cm draw arcs intersecting other at C. Join CA and CB.

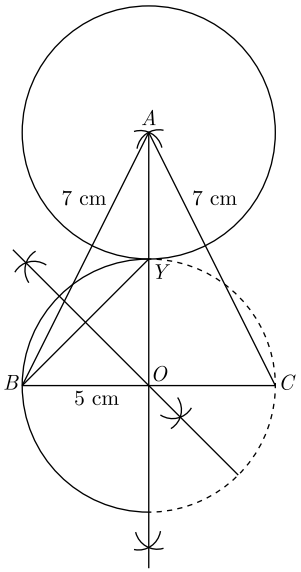
Step 3 : Draw perpendicular bisector of BC from A to meet it at D.

Step 4 : Draw a circle with centre A and radius 3 cm cutting BX at Y.

Step 5 : Draw the perpendicular bisector of AY which intersects YD at O.

Step 6 : With centre O and radius OB, draw a circle which will touch the just circle as Y and pass through B and C.

This is the required circle.



Comp 2014

13. In $\Delta ABC, AB = BC = 4.5$ cm and $AC = 2.8$ cm. Draw the circle passing through A, B and C.

Sol :

Steps of Construction :

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CHAPTER 19

CONSTRUCTIONS

FOUR MARK QUESTIONS

Main 2023

1. Using ruler and compass construct a triangle ABC in which $AB = 6\,\text{cm}$, $\angle BAC = 120^\circ$ and $AC = 5\,\text{cm}$. Construct a circle passing through A, B, and C. Measure and write down the radius of the circle.

Sol :

Steps of Construction :

Step 1. Draw a line segment $AB = 6$ cm.

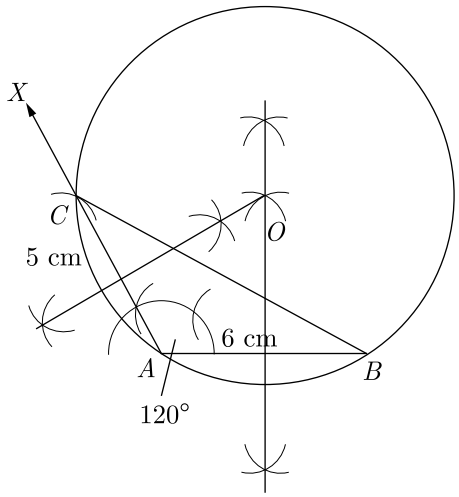
Step 2. From the point A, draw $\angle XAB = 120^\circ$.

Step 3. Taking radius 5 cm, with A as centre cut $CA = 5$ cm.

Step 4. Join C to B, which is the required ΔABC.

Step 5. Draw perpendicular bisector of AB and AC. They both intersect at point O.

Step 6. Draw a circle, with O as centre and OA or OC or OB as radius. Then, we get the required circumcircle. Radius of circle = 5.3 cm (approx)



Comp 2023

2. Draw a circle of radius 4 cm. Draw two tangents to this circle so that the angle between the tangents is 60°.

Sol :

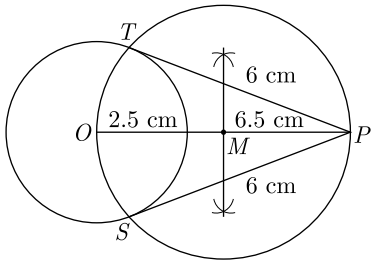
Steps of Construction :

Step 1 : Draw a circle of radius 4cm with centre O .

Step 2 : Draw two arcs OT and PT with $180^\circ - 60^\circ = 120^\circ$ between them.

Step 3 : At S and T draw rays making 90° each which meet each other at P.

Now $\angle TPS = 60^\circ$ and PT and PS are required tangents to the circle.



Main 2020

3. Using ruler and compass construct a ΔABC where $AB = 3$ cm, $BC = 4$ cm and $\angle ABC = 90^\circ$. Hence construct a circle circumscribing the ΔABC. Measure and write down the radius of the circle.

Sol :

Steps of Construction :

Step 1. Draw a line segment $BC = 4$ cm

Step 2. At point B, draw a ray BX making an $\angle B = 90^\circ$ and cut $AB = 3$ cm from BX.

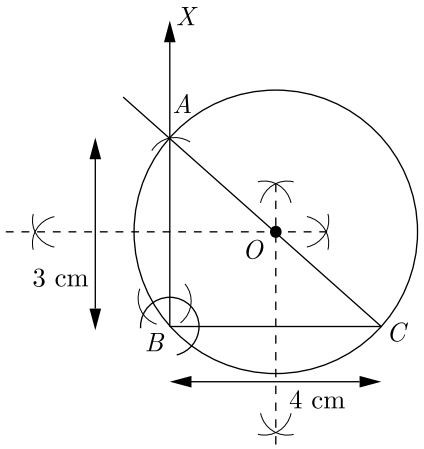
Step 3. Join AC, which is the required ΔABC.

Step 4. Draw the perpendicular bisectors of BC and AB, which meets at point O.

Step 5. Draw a circle, taking O as centre and OA as radius. This circle passes through the points A, B and C, which is the required circle.

Step 6. On measuring the length of radius with the help of ruler, we get $OB = 2.5$ cm.

NODIA



SQP 2020

4. Construct a triangle ABC, given $AB = 4$ cm, $BC = 6$ cm and $\angle ABC = 90°$. Construct a circle which passes through the points A, B and C. Mark its centre as O.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 6$ cm.

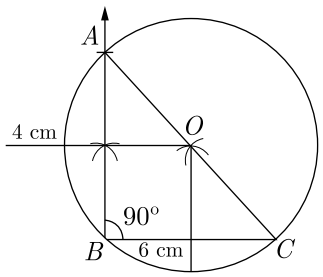
Step 2 : AB, draw a ray making an angle of $90°$ and cut off $BA = 4$ cm.

Step 3 : Join AC.

Step 4 : Draw the perpendicular bisectors of AB and BC intersecting each other at O.

Step 5 : With centre O and radius equal to OA or OB draw a circle which will pass through A, B and C.

This is the required circumcircle of ΔABC.



Comp 2019

5. Construct a triangle DEF, with $DE = 6$ cm, $\angle FDE = 60°$ and $\angle FED = 45°$. Circumscribe a circle about the triangle.

Sol :

Steps of Construction.

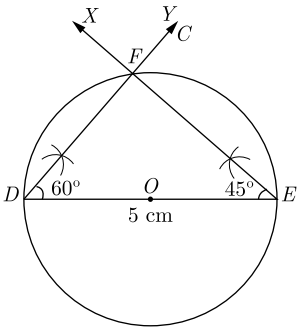
Step 1 : Draw a line segment $DE = 6$ cm.

Step 2 : At D draw a ray of $60°$ and at E, draw a ray making an angle of $45°$ meeting each other at F.

Step 3 : Draw the perpendicular bisectors of DE and EF intersecting each other at O.

Step 4 : With centre O and radius equal to OD or OE or OF, draw a circle which will pass through D, E and F.

This is the required circum circle of ΔDEF.



SQP 2019

6. Construct a ΔPQR, given $PQ = 5$ cm, $QR = 7$ cm and $\angle PQR = 60°$. Inscribe a circle in the triangle.

Sol :

Steps of Construction.

Step 1 : Draw a line segment $QR = 7$ cm

Step 2 : AQ, draw a ray making an angle of $60°$ and cut off $QP = 5$ cm.

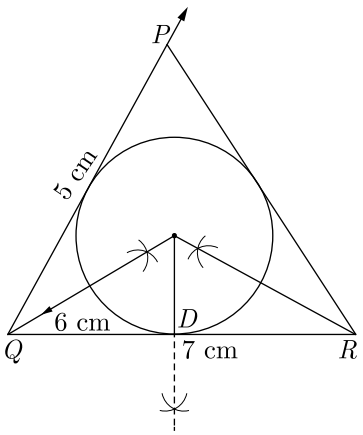
Step 3 : Join PR.

Step 4 : Draw the angle bisectors of LQ and Zit intersecting each other at I.

Step 5 : From I, draw $ID \perp QR$.

Step 6 : With cente I and radius equal to ID, draw a circle which touches the sides of ΔPQR.

This is the required in circle of ΔPQR.



Main 2017

7. Construct a regular hexagon of side 3.5 cm. Construct a circumcircle of the hexagon.

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Sol :

Steps of Construction :

Step 1. Draw a line $AB = 3.5$ cm.

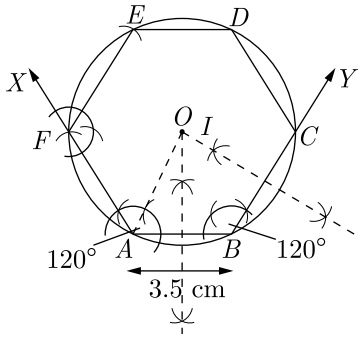
Step 2. From point A and B draw two rays AX and BY making an angle of 120°.

Step 3. Cut $AF = 4$ cm from AX and $BC = 3.5$ cm from BY.

Step 4. Now on point F and C repeat steps 2 and 3. We get a regular hexagon $ABCDEF$ with each side equal to 4 cm and each angle is 120°.

Step 5. Draw the perpendicular bisector of sides AB and BC, which intersect each other at point O.

Step 6. Draw a circle, with O as centre and OA as radius, which passes through all the vectices of the regular hexagon $ABCDEF$



Comp 2017

8. Construct a regular hexagon of side 5 cm. Inscribe a circle in it.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $AB - 5$ cm.

Step 2 : At A and B, draw rays making angles of 120° each and cut off $AF = BC = 5$ cm.

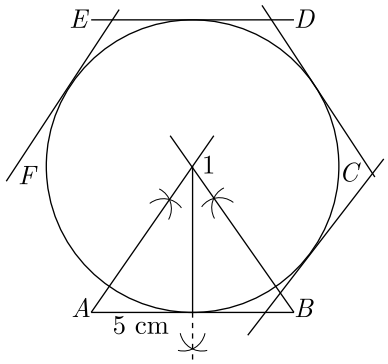
Step 3 : At C draw a ray making an angle of 120° and cut off $CD = 5$ cm.

Step 4 : With centre D and F and radius 5cm draw arcs intersecting each other at E and join EF and ED, $ABCDEF$ is the required hexagon.

Step 5 : Draw the angle bisector of $\angle A$ and $\angle B$ intersecting each other at I and from I, draw $IL \perp AB$.

Step 6 : With centre I and radius equal to IL draw a circle which touches the sides of hexagon.

This is the required in circle of hexagon $ABCDEF$.



Comp 2016

9. Construct $\Delta PQR, QR = 5$ cm, $\angle PQR = 60^\circ$ and perpendicular from P to QR is 3 cm. Draw an incircle of ΔPQR.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $QR = 5$ cm

Step 2 : At Q, draw a ray making an angle of 60°.

Step 3 : Draw a perpendicular at Q and cut off $QS = 3$ cm and draw a line parallel to QR from S which intersects QX at P.

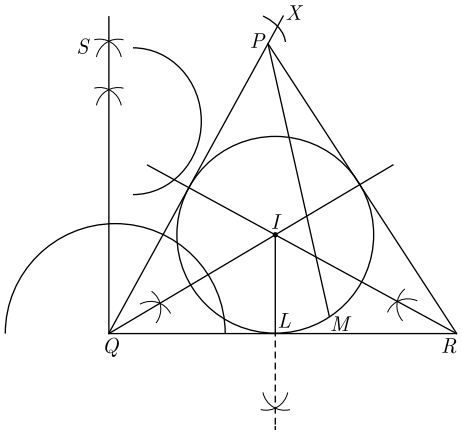
Step 4 : Join PQ.

Step 5 : Draw the bisectors of $\angle Q$ and $\angle R$ intersecting each other at I. From I draw $IL \perp QR$.

Step 6 : With centre I and radius IL, draw a circle which touches the sides of ΔPQR.

This is the required in circle of ΔPQR.

NODIA



Main 2015

10. Draw an equilateral triangle of side 4 cm. Draw its circumcircle.

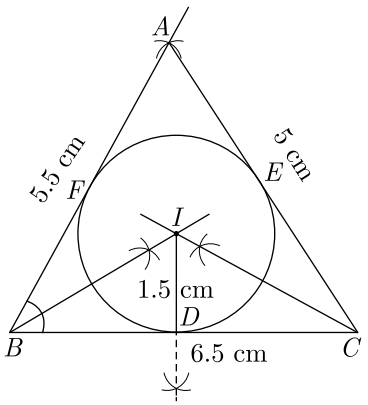
Sol :

Steps of construction :

Step 1 : Draw a line segment $BC = 4$ cm.

Step 2 : With centre B and C, and radius 4 cm,

This is the required incircle.



Comp 2013

17. Using ruler and compasses only, draw an equilateral triangle of height 6 cm and draw its inscribed circle. Measure the radius of the circle.

Sol :

Steps of Construction:

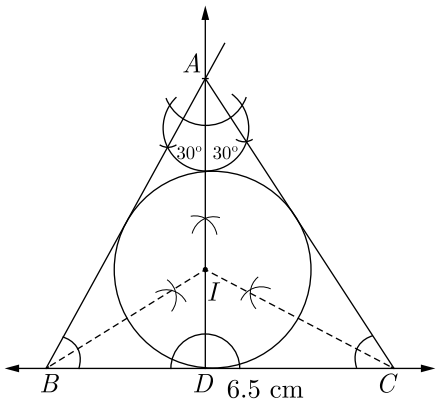
Step 1 : Draw a line XY and take a point D on it.

Step 2 : At D, draw a perpendicular and cut off $DA = 6$ cm.

Step 3 : At A, draw rays making an angle of 30° with AD both sides meeting the line XY at B and C respectively.

Step 4 : Draw the angle bisectors of B and C intersecting euch other at I and from I, draw a perpendicular $I\,D$ on BC.

Step 5 : With I as centre and $I\,D$ as radius, draw a circle which to makes the sides of the triangle internally. This is the required incircle and its radius, on measuring it is 2 cm.



Comp 2012

18. Using a ruler and a pair of compasses only, construct :

(a) a triangle ABC, given $AB = 4$ cm, $BC = 6$ cm and $\angle ABC = 90°$.

(b) a circle which passes through the points A, B and C and nark its centre as O.

Sol :

Steps of Construction:

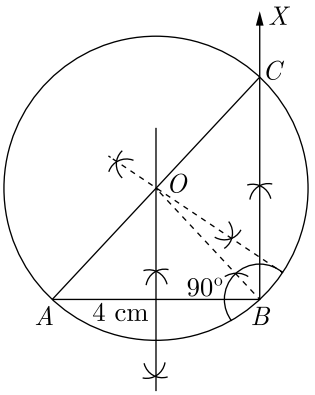
Step 1 : Draw a line segment $AB = 4$ cm

Step 2 : At B, draw a ray $B\,x$ making an angle of 90° and cut off $BC = 6$ cm.

Step 3 : Join AC.

Step 4 : Draw the perpendicular bisectors of sides AB and AC which intersect each other at O.

Step 5 : With centre O, and radius equal to OB or OA or OC draw a circle which passes through A and C. , This is the required circle.



SQP 2012

19. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Sol :

Steps of Construction:

Step 1. Draw a circle with centre O and radius 6 cm.

Step 2. Draw two radii OA and OB inclined to each other at an angle of 120°.

Step 3. Draw $AP \perp OA$ at A and $BP \perp OB$ at B, which meet at P.

Step 4. PA and PB are the required tangents inclined to each other an angle of 60°.

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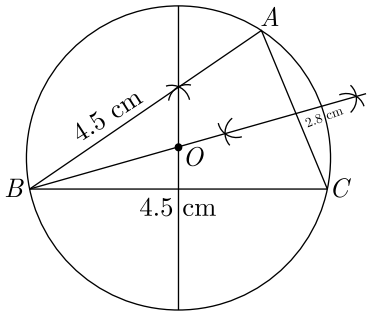
Step 1 : Draw a line segment $BC = 4.5$ cm.
Step 2 : With centre B and radius 4.5 cm and with centre C, and radius 2.8 cm draw arcs intersecting each other at A.
Step 3 : Join AB and AC.
Step 4 : Draw the perpendicular bisectors of BC and AC respectively which intersect each other at O.
Step 5 : With centre O and radius OB or OC, draw a circle which will pass through A, B and C.
This is the required circle.

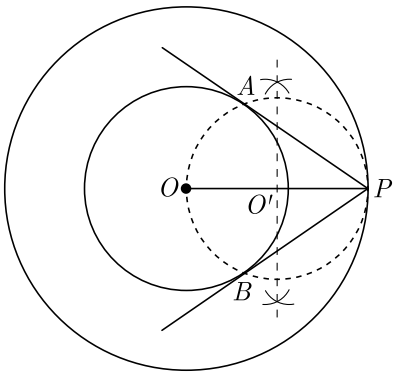


SQP 2014

14. Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

Sol :

Steps of Construction:
Step 1. Draw a circle with radius 3 cm and centre O.
Step 2. Draw another circle with centre O and radius 5 cm.
Step 3. Take a point P on the circumference of outer circle and join O to P.
Step 4. Taking OP as diameter draw another circle which intersect the smallest circle at A and B.
Step 5. Join A to P and B to P. AP and BP are the required tangents.

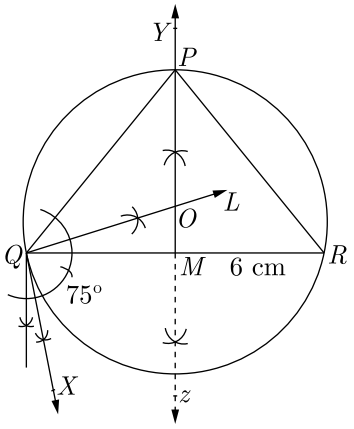


SQP 2013

15. Without calculating base angles, construct an isosceles ΔPQR whose base $QR = 6$ cm and $LP = 75°$. Draw the circumcircle of ΔPQR.

Sol :

Steps of construction :
Step 1 : Draw a line segment $QR = 6$ cm.
Step 2 : Draw a perpendicular bisector YZ of QR.
Step 3 : Draw a ray QX making an angle of $75°$ with QR.
Step 6 : Though Q, draw perpendicular on QN intersecting YZ at O.
Step 5 : With centre O and radius OQ. draw a segment on QR. Which intersects the perpendicular bisector YZ at P.
Step 6 : Join PQ and PR. Now is an isosceles triangle whose $LP = 75°$
Step 7 : With centre O, and radius OQ complete the circle.
This is the required circumcircle of ΔPQR whose $LP = 75°$.



Main 2013

16. Construct a ΔABC with $BC = 6.5$ cm, $AB = 5.5$ cm and $AC = 5$ cm. Construct the incircle of the triangle. Measure the radius of the incircle.

Sol :

Steps of construction :
Step 1 : Draw a line segment $BC = 6.5$ cm.
Step 2 : With centres B and C, and radii 5.5 cm and 5 cm, draw arcs intersecting each other at A.
Step 3 : Join AB and AC.
Step 4 : Draw the angle bisectors of $\angle B$ and $\angle C$ intersecting at I.
Step 5 : Through I, draw $ID \perp BC$.
Step 6 : With centre I and radius ID draw a circle which touches the sides of the ΔABC.

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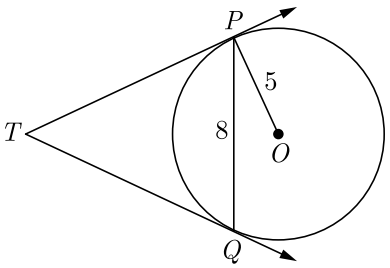
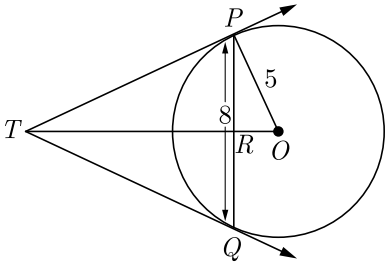


END

Sol :

We redraw the given figure as shown below. Here OT is perpendicular bisector of PQ,



Since, OT is perpendicular bisector of PQ,

, $PR = QR = 4$ cm

In right angle triangle ΔOTP and ΔPTR, we have

$$TP^2 = TR^2 + PR^2 \qquad ...(1)$$

Also, $OT^2 = TP^2 + OP^2$

Substituting TP^2 from equation (1) we have

$$OT^2 = (TR^2 + PR^2) + OP^2$$

$$(TR + OR)^2 = TR^2 + PR^2 + OR^2$$

Now $OR^2 = OP^2 - PR^2$

$$= 5^2 - 4^2 = 3^2$$

Thus $OR = 3$ cm

Thus substituting $OR = 3$ cm we have

$$(TR + 3)^2 = TR^2 + 4^2 + 5^2$$

$$TR^2 + 9 + 6TR = TR^2 + 16 + 25$$

$$6TR = 32$$

$$TR = \frac{16}{3}$$

Now, from (1), $TP^2 = TR^2 + PR^2$

$$= \left(\frac{16}{3}\right)^2 + 4^2$$

$$= \frac{400}{9}$$

$$TP = \frac{20}{3} \text{ cm}$$

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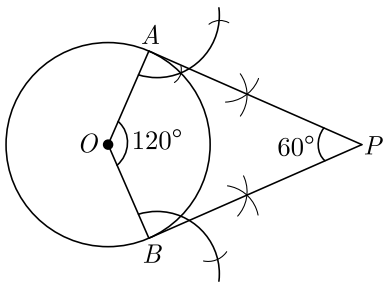


Comp 2011

20. In $\Delta ABC, AB = 5$ cm, $BC = 6.5$ cm and $CA = 4.8$ cm. Draw the circumcircle of ΔABC.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 6.5$ cm.

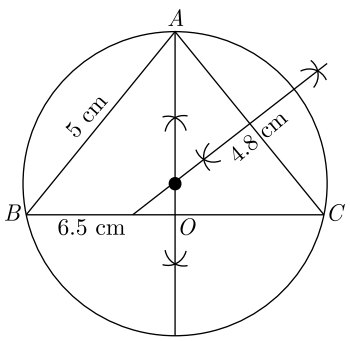
Step 2 : With centre B and radius 5 cm and with centre C and radius 4.8 cm, draw intersecting each other at A.

Step 3 : Join AB and AC.

Step 4 : Draw perpendicular bisector of sides AC and BC intersecting each other at O.

Step 5 : With centre O and radius OA or OC, draw a circle which will pass through A, B and C.

This is the required circle.



Main 2010, Main 2002

21. Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon.

Sol :

Steps of Construction :

Step 1. Draw a line $AB = 4$ cm.

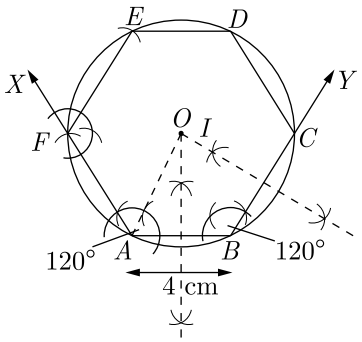
Step 2. From point A and B draw two rays AX and BY making an angle of 120°.

Step 3. Cut $AF = 4$ cm from AX and $BC = 4$ cm from BY.

Step 4. Now on point F and C repeat steps 2 and 3. We get a regular hexagon $ABCDEF$ with each side equal to 4 cm and each angle is 120°.

Step 5. Draw the perpendicular bisector of sides AB and BC, which intersect each other at point O.

Step 6. Draw a circle, with O as centre and OA as radius, which passes through all the vectices of the regular hexagon $ABCDEF$



Comp 2010

22. Draw an isosecles ΔABC in which base $BC = 6$ cm and the altitude from vertex to the base is 4 cm. Draw its inscribed circle.

Sol :

Steps of Construction :

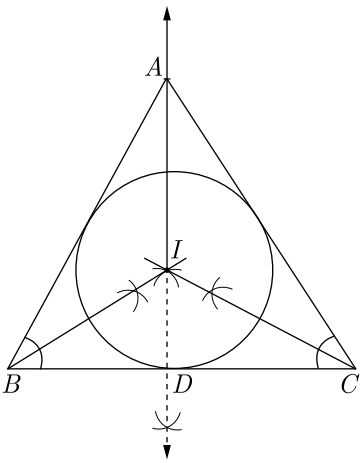
Step 1 : Draw a line segment $BC = 6$ cm.

Step 2 : Draw the perpendicular bisector and cut off $DA = 4$ cm.

Step 3 : Join AB and AC.

Step 4 : Draw the bisector of $\angle B$ and $\angle C$ intersecting each other at I.

Step 5 : With I as centre and ID as radius draw a circle which will touch its sides.



SQP 2010

23. Draw a circle of radius 3.5 cm. From a point P, 6 cm from its centre, draw two tangents to the circle.

Sol :

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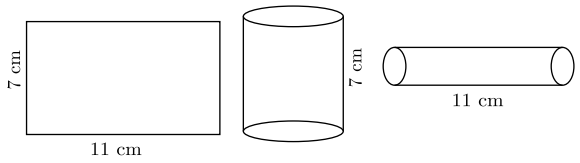
CHAPTER 20

SURFACE AREA AND VOLUME

MULTIPLE CHOICE QUESTIONS

Main 2024

1. A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side 11 cm and then about the side 7 cm to form a cylinder, as shown in the diagram.
The ratio of their curved surface areas is :



(a) 1 : 1 (b) 7 : 11
(c) 11 : 7 (d) $\frac{11\pi}{7} : \frac{7\pi}{11}$

Sol :

In both case are will be equal to the area of sheet.
Thus ratio is 1 : 1.
Thus (a) is correct option.

Main 2023

2. Volume of a cylinder of height 3 cm is 48π. Radius of the cylinder is:
(a) 48 cm (b) 16 cm
(c) 4 cm (d) 24 cm

Sol :

Volume of cylinder $V = \pi r^2 h$

$$48\pi = \pi \times r^2 \times 3 \qquad (h = 3 \text{ cm})$$

$$r^2 = 16$$

$$r = \sqrt{16} = 4$$

Thus (c) is correct option.

Comp 2023

3. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(a) $4\pi r^2$ (b) $6\pi r^2$
(c) $3\pi r^2$ (d) $8\pi r^2$

Sol :

Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.
Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$
Thus (a) is correct option.

Main 2022 Sem II

4. The volume of a conical tent is 462 m^3 and the area of base is 154 m^2. The height of the cone is :
(a) 15 m (b) 12 m
(c) 9 m (d) 24 m

Sol :

Volume of conical tent,

$$\frac{1}{3}\pi r^2 h = 462 \text{ m}^2$$

Area of base $\pi r^2 = 154 \text{ m}^2$

Now, $\frac{1}{3}(154) \times h = 462$

$$h = \frac{462 \times 3}{154}$$

$$h = 9 \text{ m}$$

Thus (c) is correct option.

Main 2022 Sem II

5. The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is : $\left(\text{Take } \pi = \frac{22}{7}\right)$
(a) 13,200 cm^2 (b) 15,400 cm^2
(c) 4,400 cm^2 (d) 8,800 cm^2

Sol :

We have $r = 14$ cm

$h = 100$ cm

Curved surface area,

$$\text{CSA} = 2\pi rh$$

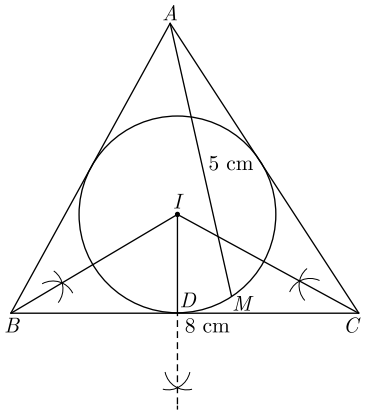
$$= 2 \times \frac{22}{7} \times 14 \times 100$$

$$= 8{,}800 \text{ cm}^2$$

Thus (d) is correct option.

6. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is

NODIA



Main 2006

27. Draw a circle of radius 3 cm. Draw the tangents to the circle so that the angle between them is 45°.

Sol :

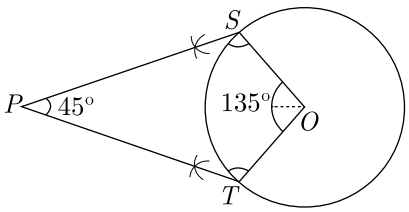
Steps of Construction :

Step 1 : Draw a circle with radius 3 cm and centre O.

Step 2 : Draw two radii OS and OT in such a way that between angle will be $180° - 45° = 135°$.

Step 3 : Now draw rays From S and T making an angle 90° each which meet each other at P.

The angle SPT will be 45° and PS and PT are required tangents to the circle.



Main 2005

28. Construct a triangle ABC, given that $AB = 4$ cm, $BC = 6$ cm and median $AM = 3$ cm. Circumscribe a circle about the triangle.

Sol :

Steps of Construction.

Step 1 : Draw a line segment $BC = 6$ cm and draw its mid point M.

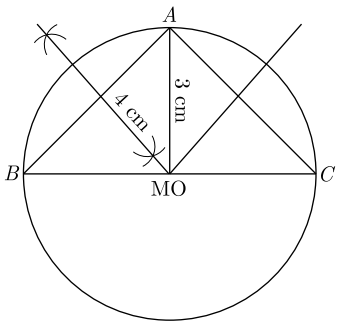
Step 2 : With centre B and radius 4cm and with centre M and radius 3cm, draw arcs intersecting each other at A.

Step 3 : Join AB, AM and AC.

Step 4 : Draw the perpendicular bisectors of AB and AC intersecting each other at O.

Step 5 : With centre O and radius equal to OA and OB or OC, draw a circle which passes through A, B and C.

This is the required circumcircle of ΔABC.



Comp 2005

29. Construct a triangle PQR, given $PQ = 5$ cm, $QR = 6.5$ cm and $\Delta PQR = 120°$.
Construct a circumcircle of ΔPQR.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $QR = 6.5$ cm.

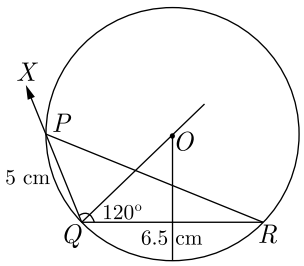
Step 2 : At Q, draw a ray making an angle of 120° and cut of $QP = 5$ cm.

Step 3 : Join PR.

Step 4 : Draw the perpendicular bisectors of PQ and QR intersecting each other at O.

Step 5 : With centre O and radius equal to $OP - OQ - OR$. Draw a circle which will pass through P, Q and R.

This is the required circumcircle of ΔPQR.



Main 2003

30. Construct a ΔPMN, given that $MN = 5$ cm, $PN = 7$ cm and $\angle PMN = 90°$. Construct an incircle of the triangle.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $MN = 5$ cm.

Step 2 : At M, draw a ray making an angle of 90°.

Step 3 : With centre N and radius 7 cm draw an arc which intersects MX at P.

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Steps of Construction :

Step 1. Draw a line segment OP of length 6 cm.

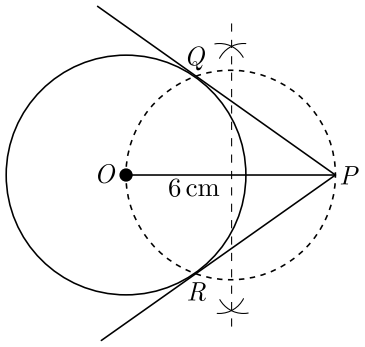
Step 2. From the point O, draw a circle of radius $= 3.5$ cm.

Step 3. Draw a perpendicular bisector of OP. Let M be the mid point of OP.

Step 4. Taking M as centre and OM as radius draw a circle.

Step 5. This circle intersects the given circle at Q and R.

Step 6. Join PQ and PR, which are tangents to the circles.



Main 2009

24. Draw a circle of radius 2.5 cm. Mark a point P at a distance of 6.5 cm from the centre of the circle. Draw two tangents to the circle from P and measure the length of each.

Sol :

Steps of Construction :

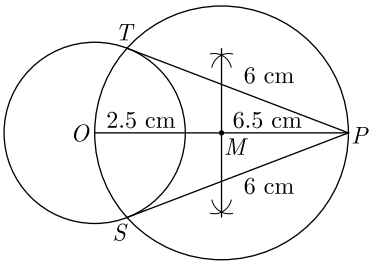
Step 1 : Draw a circle of radius 2.5 cm with centre O.

Step 2 : Take a point P at a distance of 6.5 cm from the centre O.

Step 3 : With diameter OP, draw a circle which intersects the given circle at T and S.

Step 4 : Join PT and PS are required tangents to the circle.

On measuring $PT = PS = 6$ cm.



Main 2008

25. Using a ruler and a pair of compass only, construct

(i) a ΔABC, given $AB = 4$ cm, $BC = 6$ cm and $\angle ABC = 90^\circ$.

(ii) a circle that passes through the point A, B and C and mark its centre as O.

Sol :

Steps of Construction :

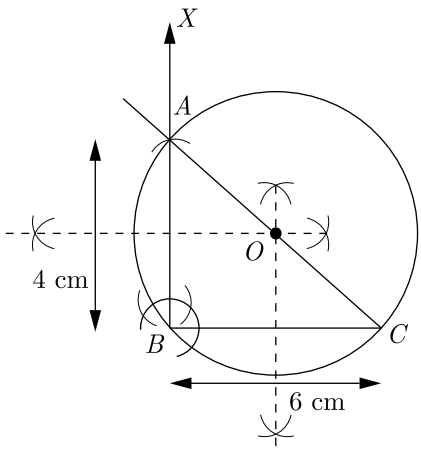
Step 1. Draw a line segment $BC = 6$ cm

Step 2. At point B, draw a ray BX making an $\angle B = 90^\circ$ and cut $AB = 4$ cm from BX.

Step 3. Join AC, which is the required ΔABC.

Step 4. Draw a bisector of BC and AB, which meets at point O.

Step 5. Draw a circle, taking O as centre and OA as radius. This circle passes through the points A, B and C, which is the required circle.



Comp 2007

26. Construct a triangle ABC, given that $AB = 6$ cm, $BC = 8$ cm and medium $AM = 5$ cm. Construct an incircle of triangle ABC and measure its radius.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 8$ cm and locate its mid point M.

Step 2 : With centre B and radius 6cm and with centre M and radius 5cm, draw arcs intersecting each other at A.

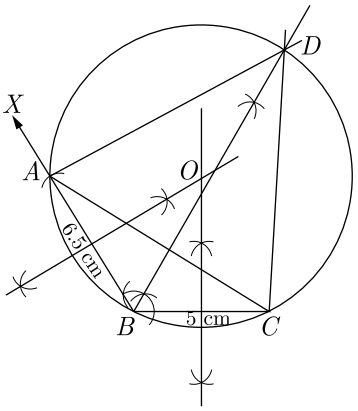
Step 3 : Join AM and AC.

Step 4 : Draw the bisectors of $\angle B$ and $\angle C$ intersecting each other at I and from I, draw $ID \perp BC$.

Step 6 : With centre I and radius equal to ID, draw a circle which touches the sides of' the ΔABC.

This is the required in circle of ΔABC.

NODIA



SQP 2018

33. Construct a triangle with sides. 5 cm, 4 cm and 3 cm. Draw its circumcircle and measure its radius.

Sol :

Steps of Construction :

Step 1 : Draw a line segment $BC = 5$ cm.

Step 2 : With centre B and radius 4 cm, and with centre C and radius 3 cm, draw arcs intersecting each other at A.

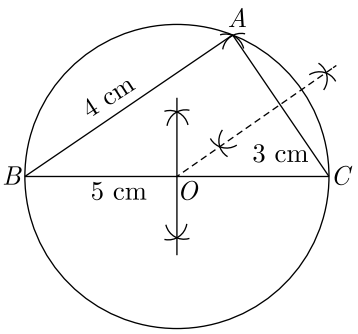
Step 3 : Join AB and AC.

Step 4 : Draw the perpendicular bisectors of sides AC and BC intersecting each other at O.

Step 5 : We see that O lies on the side BC.

Step 6 : With centre O and radius OB, draw a circle which will pass through A, B and C respectively. On measuring the radius of the circle, it is 2.5 cm.

Step 7 : In ΔABC, $AB = 4$ cm, $BC = 6$ cm $\angle ABC = 90°$



Main 2016

34. Draw a line $AB = 5$ cm. Mark a point C on AB such that $AC = 3$ cm. Using a ruler and a compasses only, construct

(i) a cirlce of radius 2.5 cm, passing through A and C.

(ii) two tangents to the circle from the external point B. Measure and record the length of the tangents.

Sol :

Steps of Construction :

Step 1. Draw $AB = 5$ cm and mark a point C on AB such that $AC = 3$ cm.

Step 2. Draw perpendicular bisector of AC, which intersect AC at D.

Step 3. With radius equals to 2.5 cm and A as centre make an arc which cuts the perpendicular bisector of AC.

Step 4. Again, make an other arc with same radius and C as centre which cuts the previous arc at point O.

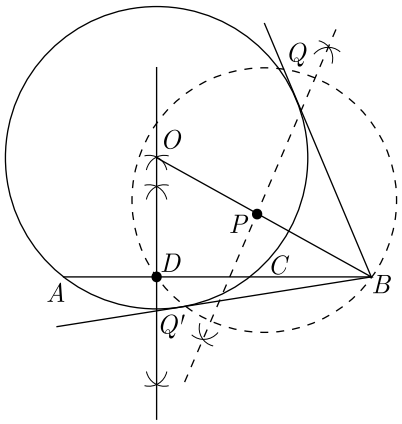
Step 5. O as centre and radius equals to OA or OC. Draw a circle which passes through the points A and C.

Step 6. Join O to B.

Step 7. Draw perpendicular bisector of OB, which intersect OB at P.

Step 8. With P as centre and radius equal to OP or PB. Draw a circle (dotted) which intersect the previous circle at Q and Q'.

Step 9. Join BQ and BQ', which are required tangents to the circle.



Main 2014

35. Construct a ΔABC with $BC = 6.5$ cm, $AB = 5.5$ cm and $AC = 5$ cm. Construct the incircle of the triangle. Measure and record the radius of the incircle.

Sol :

Steps of Construction :

Step 1. Draw a line $BC = 6.5$ cm of triangle.

Step 2. Taking B and C as centre, draw two arcs of radius 5.5 cm and 5 cm, respectively, which intersect each other at A.

Step 3. Join AB and AC. Thus, ΔABC is the required given triangle.

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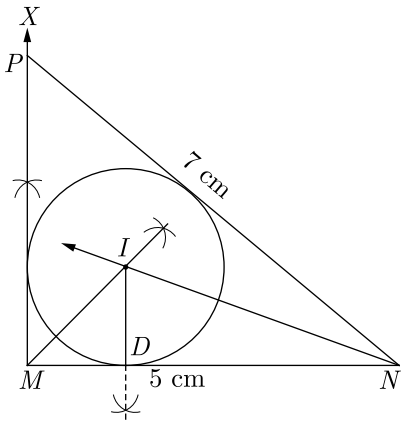
Step 4 : Join PN.
Step 5 : Draw the bisectors of $\angle M$ and $\angle N$ which intersect each other at I.
Step 6 : From I, draw $ID \perp MN$.
Step 7 : With centre I and radius equal to ID, draw a circle which touches the sides of ΔPMN.
This is the required in circle of ΔPMN.



FIVE MARK QUESTIONS

Main 2019

31. Use ruler and compass only for answering this question. Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre. Construct two tangents to the circle from the external point P. Measure and write down the length of any one tangent.

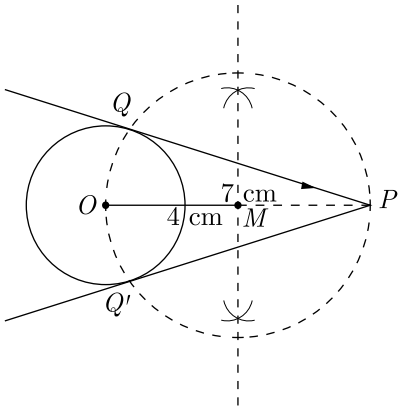
Sol :

Steps of Construction :

Step 1. Draw a circle with centre O and radius 4 cm
Step 2. Take a point P outside the circle at a distance of 7 cm from the centre. Join OP.
Step 3. Draw a perpendicular bisector of OP, which intersects OP at (the mid-point) M.
Step 4. Taking M as centre and OM as radius draw a circle, which intersect the given circle at Q and Q'.
Step 5. Join PQ and PQ', which are the required tangents.
Step 6. On measuring the length of tangent with the help of ruler, we get

$$PQ = PQ' = 5.7 \text{ cm}$$

NODIA



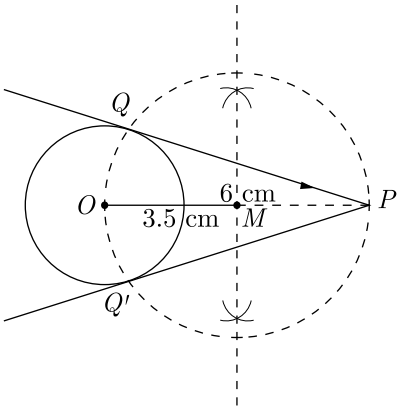
Main 2018

32. Using ruler and compass only, construct a ΔABC such that $BC = 5$ cm and $AB = 6.5$ cm and $\angle ABC = 120^\circ$
(i) Construct a circumcircle of ΔABC.
(ii) Construct a cyclic quadrilateral $ABCD$, such that D is equidistant from AB and BC.

Sol :

Steps of Construction :

Step 1. Draw a line segment $BC = 5$ cm.
Step 2. At B, draw a ray BX making an angle of 120° with BC.
Step 3. With B as centre and radius 6.5 cm, draw an arc intersecting BX at A.
Step 4. Join AC, thus ABC is the required triangle.
Step 5. Draw perpendicular bisector of AB and BC which intersect each other at O.
Step 6. With O as centre and radius OA or OB or OC, draw a circle which is passing through A, B and C. The circle, so obtained is required circum-circle of ΔABC.
Step 7. Draw angle bisector of $\angle ABC$ which meets the circle at D. Then, point $D$is equidistant from AB and BC.
Step 8. Join AD and CD to get required cyclic quadrilateral $ABCD$.



Main 2007

38. Using a ruler, construct a ΔABC with $BC = 6.4$ cm, $CA = 5.8$ cm and $\angle ABC = 60^\circ$. Draw its incircle. Measure and record the radius of the circle.

Sol :

Steps of Construction :

Step 1. Draw a line segment $BC = 6.4$ cm.

Step 2. Taking B as centre, draw a ray BX making an $\angle ABC = 60^\circ$.

Step 3. Taking C as centre, draw an arc of radius 5.8 cm, which intersects ray BX at A.

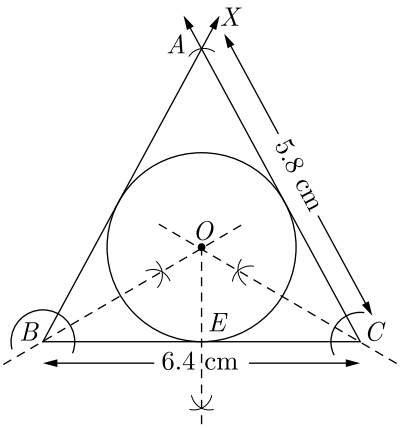
Step 4. Join AC, then ΔABC is the required triangle.

Step 5. Draw angle bisectors of $\angle ACB$ and $\angle ABC$, which intersects at O.

Step 6. Draw a perpendicular line from O to BC , which meets at a point E.

Step 7. Taking Oas centre and OE as radius, draw a circle and it is required incircle of given triangle.

Step 8. On measuring the radius of incircle is $OE = 1.5$ cm.



Main 2004

39. Using a ruler and compass only, construct a ΔABC such that $AB = 5$ cm, $\angle ABC = 75^\circ$ and the radius of the circumcircle of ΔABC is 3.5 cm. On the same diagram, construct a circle, touching AB at its middle point and also touching the side AC.

Sol :

Steps of Construction :

Step 1. Draw a line segment $AB = 5$ cm.

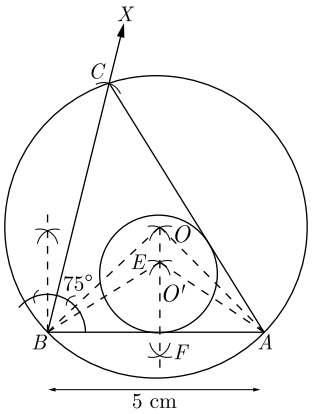
Step 2. From point B, draw a ray BX making an $\angle ABX = 75^\circ$.

Step 3. Taking A and B as centres one-by-one marks two arcs of radius 3.5 cm, which intersect at the point O.

Step 4. Taking O as the centre and OB as radius, draw a circumcircle, which passes through the points A and B. It also intersects BX at point C. Join AC, hence ΔABC is the required triagle.

Step 5. Draw perpendicular bisector EF of AB , which intersect AB at point M an draw an angle bisector of $\angle CAB$, which intesect EF of O'.

Step 6. Taking O' as centre and $O'M$ as radius, draw a circle, which touches AB and AC .



END

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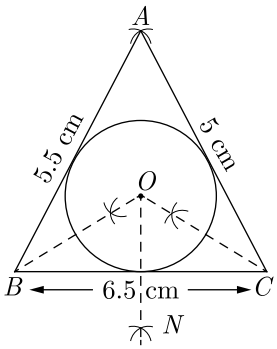
Step 4. Now, draw the internal bisectors of $\angle B$ and $\angle C$. Let these bisectors meet at point O.

Step 5. From point O, draw a perpendicular on side BC, which cuts BC at N.

Step 6. Taking O as centre and radius equal to ON, draw a circle which touches all the sides of the ΔABC and it is the required incircle.

Step 7. Measure ON, which is the required radius of the incircle. $ON = 1.5$ cm



Main 2012

36. Construct a ΔABC in which base $BC = 6$ cm, $AB = 5.5$ cm and $\angle ABC = 120^\circ$.

(i) Construct a circle circumscribing the ΔABC.

(ii) Draw a cyclic quadrilateral $ABCD$ so that D is equidistance from B and C.

Sol :

Steps of Construction :

Step 1. Draw a line segment $BC = 6$ cm.

Step 2. At B, draw a ray BX making an angle of 120° with BC.

Step 3. Taking radius 5.5 cm, with B as centre cut $BA = 5.5$ cm.

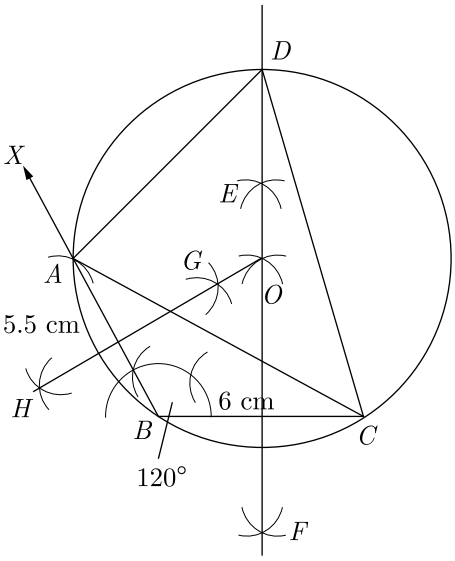
Step 4. Join A to C, which is the required ΔABC.

Step 5. Draw perpendicular bisector EF of BC and GH of AB. They both intersect at point O.

Step 6. Draw a circle, with O as centre and OA or OC or OB as radius. Then, we get the required circumcircle.

Step 7. Extend FE line in the diretion of E, which meet the circle at point D, which is equidistant from B and C.

Step 8. Join AD and CD. Hence, $ABCD$ is the required cyclic quadrilateral.



Main 2011

37. Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.

Sol :

Steps of Construction :

Step 1. Draw a circle with centre O and radius 3.5 cm

Step 2. Take a point P outside the circle at a distance of 6 cm from the centre. Join OP.

Step 3. Draw a perpendicular bisector of OP, which intersects OP at (the mid-point) M.

Step 4. Taking M as centre and OM as radius draw a circle, which intersect the given circle at Q and Q'.

Step 5. Join PQ and PQ', which are the required tangent is 4.8 cm.

Step 6. On measuring the length of tangent with the help of ruler, we get

$$PQ = PQ' = 4.7 \text{ cm}$$

NODIA

increased by
(a) 200% (b) 500%
(c) 700% (d) 800%

Sol :

Let r be the original radius of sphere. If we increased radius by 100 %. it will be $2r$.

$$V_r = \frac{4}{3}\pi r^3$$

Now $$V_{2r} = \frac{4}{3}\pi \times (2r)^3$$

$$= \frac{4}{3}\pi \times 8r^2$$

Thus new volume is 8 times of original volume.
Hence when the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.
Thus (c) is correct option.

7. A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is
(a) $\left(\frac{4}{3}\right)^{1/3}$ (b) $\left(\frac{8}{3}\right)^{1/3}$
(c) $(3)^{1/3}$ (d) 2

Sol :

As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$\frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

Thus (b) is correct option.

8. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is
(a) 2 : 1 (b) 1 : 2
(c) 1 : 3 (d) 3 : 1

Sol :

$$\pi r l = 2\pi r h$$

$$\frac{l}{h} = \frac{2}{1}$$

Thus (a) is correct option.

9. If the perimeter of one face of a cube is 20 cm, then its surface area is
(a) 120 cm^2 (b) 150 cm^2
(c) 125 cm^2 (d) 400 cm^2

Sol :

Edge of cube, $a = \frac{20}{4}\text{ cm} = 5\text{ cm}$

Surface area $6a^2 = 6 \times 5^2\text{ cm}^2 = 150\text{ cm}^2$
Thus (b) is correct option.

10. Ratio of lateral surface areas of two cylinders with equal height is
(a) 1 : 2 (b) $H : h$
(c) $R : r$ (d) None of these

Sol :

$$2\pi Rh : 2\pi rh = R : r$$

Thus (c) is correct option.

11. Volume of a spherical shell is given by
(a) $4\pi(R^2 - r^2)$ (b) $\pi(R^3 - r^3)$
(c) $4\pi(R^3 - r^3)$ (d) $\frac{4}{3}\pi(R^3 - r^3)$

Sol :

$$\text{Volume of spherical shell} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(R^3 - r^3)$$

Thus (d) is correct option.

12. Ratio of volumes of two cylinders with equal height is
(a) $H : h$ (b) $R : r$
(c) $R^2 : r^2$ (d) None of these

Sol :

$$\pi R^2 h : \pi r^2 h = R^2 : r^2$$

Thus (c) is correct option.

13. Ratio of volumes of two cones with same radii is
(a) $h_1 : h_2$ (b) $s_1 : s_2$
(c) $r_1 : r_2$ (d) None of these

Sol :

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

Since $$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2 \qquad (r_1 = r_2)$$

$$h_1 : h_2$$

Thus (a) is correct option.

14. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is
(a) 12 m (b) 18 m
(c) 36 m (d) 66 m

NODIA

Sol :

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l,
Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$

$$\pi \times \left(\frac{2}{2}\times\frac{1}{10}\right)^2 \times l = \frac{4}{3}\times\pi\times\left(\frac{6}{2}\right)^3$$

$$\pi \times \frac{1}{100}\times h = \frac{4}{3}\times\pi\times 3^3$$

$$\frac{l}{100} = 4\times 3^2 = 36$$

$$l = 3600\,\text{cm} = 36\,\text{m}$$

Thus (c) is correct option.

15. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then the volume of remaining solid is

(a) $280\,\pi\text{cm}^3$ (b) $330\,\pi\text{cm}^3$
(c) $240\,\pi\text{cm}^3$ (d) $440\,\pi\,\text{cm}^3$

Sol :

Volume of the remaining solid

=Volume of the cylinder − Volume of the cone

$$= \pi\times 6^2\times 10 - \frac{1}{3}\times\pi\times 6^2\times 10$$

$$= (360\pi - 120\pi)$$

$$= 240\pi\ \text{cm}^3$$

Thus (c) is correct option.

16. A right circular cylinder of radius r and height h (where, $h > 2r$) just encloses a sphere of diameter

(a) r (b) $2r$
(c) h (d) $2h$

Sol :

Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is $2r$.
Thus (b) is correct option.

17. During conversion of a solid from one shape to another, the volume of the new shape will

(a) increase (b) decrease
(c) remain unaltered (d) be doubled

Sol :

During conversion of a solid from one shape to another, the volume of the new shape will remain unaltered.

Thus (c) is correct option.

18. A solid piece of iron in the form of a cuboid of dimensions $49\,\text{cm}\times 33\,\text{cm}\times 24\,\text{cm}$, is moulded to form a solid sphere. The radius of the sphere is

(a) 21 cm (b) 23 cm
(c) 25 cm (d) 19 cm

Sol :

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49\times 33\times 24$$

$$= 38808\,\text{cm}^3$$

$$4\times\frac{22}{7}r^3 = 38808\times 3$$

$$r^3 = \frac{38808\times 3\times 7}{4\times 22} = 441\times 21$$

$$r^3 = 21\times 21\times 21$$

$$r = 21\,\text{cm}$$

Thus (a) is correct option.

19. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

Volume of the twelve solid sphere is equal to the volume of cylinder.

$$V_{12\,\text{sphere}} = V_{\text{cylinder}}$$

$$12\times\frac{4}{3}\pi r^3 = \pi\left(\frac{2}{1}\right)^2\times 16$$

$$16\pi r^3 = 16\pi$$

$$r^3 = 1 \Rightarrow r = 1\,\text{cm}$$

Diameter of each sphere, $d = 2r$

$$= 2\times 1 = 2\,\text{cm}$$

Thus (c) is correct option.

20. Volumes of two spheres are in the ratio 64 : 27. The the ratio of their surface areas is

(a) 3 : 4 (b) 4 : 3
(c) 9 : 16 (d) 16 : 9

Sol :

Let the radii of the two spheres are r_1 and r_2, respectively.
Given, ratio of their volumes,

$$V_1 : V_2 = 64 : 27$$

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$$\frac{V_1}{V_2} = \frac{64}{27}$$
$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$
$$\frac{r_1^3}{r_2^3} = \frac{64}{27}$$
$$\frac{r_1}{r_2} = \frac{4}{3}$$

Now, ratio of their surface area,

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the required ratio of their surface area is 16 :9.

Thus (d) is correct option.

21. The radii of two cylinders are in the ratio 2 ∶ 3 and their heights are in the ratio 5 ∶ 3, what is the ratio of their volumes ?

(a) 10 : 9 (b) 27 : 20
(c) 9 : 10 (d) 20 : 27

Sol :

$$\frac{\text{Volume of } 1^{\text{st}} \text{cylinder}}{\text{Volume of } 2^{\text{nd}} \text{cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$
$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$
$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$
$$= 20 \ : \ 27$$

Thus (d) is correct option.

22. Three cubes of iron whose edges are 3 cm, 4 cm and 5 cm respectively are melted and formed into a single cube, what will be the edge of the new cube formed ?

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

Let the edge of single cube be x.

Volume of single cube= Volume of three cubes

$$x^3 = 3^3 + 4^3 + 5^3$$
$$= 27 + 64 + 125 = 216$$
$$x = 6 \text{ cm}$$

Thus (d) is correct option.

NODIA

23. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. What is the diameter of each sphere?

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

Volume of the twelve solid sphere is equal to the volume of cylinder.

$$V_{12\,\text{sphere}} = V_{\text{cylinder}}$$
$$12 \times \frac{4}{3}\pi r^3 = \pi\left(\frac{2}{2}\right)^2 \times 16$$
$$16\pi r^3 = 16\pi$$
$$r^3 = 1 \ \Rightarrow \ r = 1\,\text{cm}$$

Diameter of each sphere,

$$d = 2r = 2 \times 1 = 2\,\text{cm}$$

Thus (c) is correct option.

24. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

(a) 424.4 m^2 (b) 418.4 m^2
(c) 671.4 m^2 (d) 471.4 m^2

Sol :

Radius of conical heap $r = 12\,\text{m}$

Height of heap, $h = 3.5$ m

Volume of rice,

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5\,\text{m}^3$$
$$= 528\,\text{m}^3$$

Slanted height,

$$l = \sqrt{12^2 + (3.5)^2} = 12.5\,\text{m}$$

Area of canvas cloth required,

$$\pi r l = \frac{22}{7} \times 12 \times 12.5$$
$$= 471.4\,\text{m}^2$$

Thus (d) is correct option.

25. A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. What is the ratio of the side of the cube to the radius of the new small sphere ?

(a) $\left(\frac{2}{9}\right)^{1/3}$ (b) $\left(\frac{8}{3}\right)^{1/3}$

Thus (c) is correct option.

38. A solid sphere of radius r melted and recast into the shape of a solid cone of height r. What is the radius of the base of a cone?

(a) $\frac{1}{2}r$ (b) $2r$
(c) $9r$ (d) $3r$

Sol :

Let the radius of cone be R cm.

$$\text{Volume of sphere} = \text{Volume of cone}$$
$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$$
$$4r^3 = R^2 r$$
$$R^2 = 4r^2$$
$$R = 2r$$

Thus (b) is correct option.

39. A glass cylinder with radius 10 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. Use $\pi = \frac{22}{7}$

(a) 3.25 cm (b) 2.41 cm
(c) 4.56 cm (d) 1.63 cm

Sol :

Let h be the height of water raised measured.

Volume of water displaced in cylinder $= \pi(10)^2 h$

Volume of cube,

$$\pi(10)^2 h = 8 \times 8 \times 8$$
$$h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$$
$$= 1.629 \text{ cm}.$$

Thus (d) is correct option.

40. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder ?

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

$$\text{Volume of cylinder} = \text{Volume of sphere},$$
$$\pi r^2 h = \pi$$

where r and h are radius of base and height of cylinder

$$(0.5)^2 h = 1$$
$$0.25h = 1 \Rightarrow h = 4 \text{ cm}.$$

Thus (a) is correct option.

41. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. (Use $\pi = \frac{22}{7}$)

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

Let r be the radius of base of cylinder and h be height.

Volume of a right circular cylinder $= 25\frac{1}{7}$ cm

$$\pi r^2 h = \frac{176}{7}$$
$$\frac{22}{7} \times h^2 \times h = \frac{176}{7}$$
$$h^3 = \frac{176}{22} = 8 = 2^3.$$

Hence, height of the cylinder $= 2$ cm.

Thus (c) is correct option.

42. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then what is the ratio of the slant height of the cone to the height of the cylinder?

(a) 2 (b) 3
(c) 4 (d) $\frac{1}{2}$

Sol :

We have $\pi r l = 2\pi r h$

$$\frac{l}{h} = \frac{2}{1}$$

Thus (a) is correct option.

43. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. What is the length of the wire?

(a) 36 m (b) 24 m
(c) 12 cm (d) 16 m

Sol :

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l,

Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$
$$\pi \times \left(\frac{2}{2} \times \frac{1}{10}\right)^2 \times l = \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$$
$$\pi \times \frac{1}{100} \times h = \frac{4}{3} \times \pi \times 3^3$$

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of bowl,

$$2\pi r = 132$$
$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Capacity i.e volume of the bowl,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$
$$= 19404 \text{ cm}^3$$

Thus (c) is correct option.

32. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

(a) 5 units (b) 9 units
(c) 2 units (d) 6 units

Sol :

Let radius of sphere be r.

Now Volume of sphere= S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$
$$r = \frac{9}{2} \text{ units}$$

Diameter $d = \frac{9}{2} \times 2 = 9$ units

Thus (b) is correct option.

33. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, its radius will be

(a) 5 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

Let the radius of spherical ball be r.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi[3^3 + 4^3 + 5^3]$$
$$r^3 = 27 + 64 + 125 = 216$$
, $$r = 6 \text{ cm}$$

Thus (d) is correct option.

34. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

(a) 9 : 1 (b) 3 : 1
(c) 1 : 9 (d) 1 : 3

Sol :

Let h_1 and h_2 be height and r_1 and r_2 be radii of two cones.

Now $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

Ratio of their volumes,

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right) = \left(\frac{3}{1}\right)^2\left(\frac{1}{3}\right) = \frac{3}{1}$$

Hence, ratio of their volumes is 3 :1.

Thus (b) is correct option.

NODIA

35. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. What is the radius of sphere ?

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 6 cm

Sol :

Let R be the radius of sphere.

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$
$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (2)^2 \times 8$$
$$4R^3 = 4 \times 8$$
$$R^3 = 8 \Rightarrow R = 2 \text{ cm}$$

Thus (c) is correct option.

36. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? Use $\pi = \frac{22}{7}$

(a) 1540 cm^3 (b) 1520 cm^3
(c) 2520 cm^3 (d) 2430 cm^3

Sol :

We have $r = 7$ cm, $h = 10$ cm,

Volume of cylinder,

$$\pi r^2 h = \frac{22}{7} \times (7)^2 \times 10$$
$$= 1540 \text{ cm}^3$$

Thus (a) is correct option.

37. A cylinder, a cone and a hemisphere have same base and same height. What is the ratio of their volumes?

(a) 3 : 2 : 1 (b) 2 : 3 : 1
(c) 3 : 1 : 2 (d) 2 : 1 : 3

Sol :

$$V_{\text{cylinder}} : V_{\text{cone}} : V_{\text{hemisphere}} = \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$$
$$= \pi r^2 r : \frac{1}{3}\pi r^2 r : \frac{2}{3}\pi r^3 \quad (h = r)$$
$$= 1 : \frac{1}{3} : \frac{2}{3}$$
$$= 3 : 1 : 2$$

THREE MARKS QUESTIONS

Comp 2023

48. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Sol :

Height and radius of cylinder are equal to the height and radius of cone.

$$\text{Height of cylinder} = \text{height of cone} = 8 \text{ cm}$$

$$\text{radius of cylinder} = \text{radius of cone} = 6 \text{ cm}$$

$$\text{Slant height of cone} = \sqrt{r^2 + h^2}$$

$$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$= 10 \text{ cm}$$

Total surface area of remaining solid,

$$= \text{CSA of cylinder} + \text{CSA of cone} + \text{area of top}$$

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 6(2 \times 8 + 10 + 6)$$

$$= \frac{22}{7} \times 6 \times 32$$

$$= 603.43$$

Hence total surface area is 603.43 cm^2.

Main 2022 Sem II

49. A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed.

Sol :

Let n be the number of cones which are melted.

Radius of solid cone $R = 5$ cm

Height of solid cone $H = 9$ m

Radius of small cylinder $r = 0.5$ cm

Height of small cylinder $h = 1.5$ m

Let n cylinders are formed from one solid cone. Volume of n cylinders is equal to the volume of single cone.

$$V_{\text{cone}} = n V_{\text{cylinder}}$$

$$\frac{1}{3}\pi R^2 H = n\pi r^2 h$$

$$\frac{1}{3} R^2 H = n r^2 h$$

$$\frac{1}{3}(5)^2 \times 9 = n(0.5)^2 \times 1.5$$

$$25 \times 3 = n \times 0.25 \times 1.5$$

$$4 \times 25 \times 3 = n \times 1.5$$

$$4 \times 25 \times 2 = n$$

$$200 = n$$

Hence, number of cylinder formed are 200.

Comp 2022

50. Small 126 cones, each of diameter 7 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. Use $\pi = \frac{22}{7}$

Sol :

Volume of single cone,

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{\pi}{3} \times \left(\frac{7}{2}\right)^2 \times 3$$

Volume of recast sphere,

$$V_{\text{sphere}} = \frac{4}{3}\pi R^3$$

Volume of sphere is equal to the volume of 126 cones.

Thus $V_{\text{sphere}} = 126\, V_{\text{cone}}$

$$\frac{4\pi}{3} \times R^3 = 126 \times \frac{\pi}{3} \times \left(\frac{7}{2}\right)^2 \times 3$$

$$4R^3 = 126 \times \frac{7}{2} \times \frac{7}{2} \times 3$$

$$4R^3 = 7 \times 9 \times 7 \times \frac{7}{2} \times 3$$

$$8R^3 = 7 \times 9 \times 7 \times 7 \times 3$$

$$2R = 7 \times 3$$

$$R = \frac{7 \times 3}{2} = 10.5 \text{ cm}$$

Thus diameter is 21 cm.

Surface area $4\pi R^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$

$$= 1386 \text{ cm}^2$$

Main 2020

51. From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameter of the cones are also of 6 cm and height 10.5 cm. Taking $\pi = \frac{22}{7}$ find the volume of the remaining solid.

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$$\frac{l}{100} = 4 \times 3^2 = 36$$
$$l = 3600 \text{ cm}$$
$$= 36 \text{ m}$$

Thus (a) is correct option.

44. What is the volume of the largest solid right circular cone that can be cut out off a solid cube of side 42 cm?

(a) 9720 cm^3 (b) 9702 cm^3
(c) 19440 cm^3 (d) 19404 cm^3

Sol :

The base of cone is the largest circle that can be inscribed in the face of the cube and the height will be equal to edge of the cube.

Radius of cone, $r = \frac{42}{2} = 21$ cm

Height of cone, $h = 42$ cm

Volume of cone, $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 42$$
$$= 22 \times 21 \times 42$$
$$= 19404 \text{ cm}^3$$

Thus (d) is correct option.

45. **Assertion :** The slant height of the frustum of a cone is 5 cm and the difference between the radii of its two circular ends is 4 cm. Then the height of the frustum is 3 cm.

Reason : Slant height of the frustum of the cone is given by $l = \sqrt{(R-r)^2 + h^2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have, $l = 5 \text{ cm}, \ R - r = 4 \text{ cm}$

$$5 = \sqrt{(4)^2 + h^2}$$
$$16 + h^2 = 25$$
$$h^2 = 25 - 16 = 9$$
$$h = 3 \text{ cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

NODIA

46. **Assertion :** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2

Reason : If r be the radius and h be the height of the cylinder, then total surface area $= (2\pi rh + 2\pi r^2)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Total surface area,

$$2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$
$$= 2 \times \frac{22}{7} \times 14(30 + 14) = 88(44)$$
$$= 3872 \text{ cm}^2$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

47. **Assertion :** If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason : If r be the radius and h be the slant height of the cone, then slant height $= \sqrt{h^2 + r^2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Slant height $l = \sqrt{(\frac{14}{2})^2 + (24)^2}$

$$= \sqrt{49 + 576}$$
$$= \sqrt{625}$$
$$= 25$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Hence, the number of cones recast from solid metallic sphere is 270.

Comp 2014

62. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.

Sol :

Volume of water in cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (5)^3 \times 8 = \frac{200}{3}\pi \text{ cm}^3$$

Volume of water flows out

$$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi \text{ cm}^3$$

Let r be the radius of one spherical ball.

Volume of 100 spherical ball,

$$\frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$$

$$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$$

$$r = \frac{1}{2} = 0.5 \text{ cm}$$

Main 2012

63. A hollow sphere of internal and external radii 6 cm and 8 cm respectively, is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones.

Sol :

Internal radius of hollow sphere,

$$R_1 = 6 \text{ cm}$$

and external radius of hollow sphere,

$$R_2 = 8 \text{ cm}$$

Radius of base cone,

$$r = 2 \text{ cm}$$

Height of base cone,

$$h = 8 \text{ cm}$$

Let hollow sphere be melted into form of n cone. Since volume of hollow sphere is equal to volume of cone formed, we have

$$V_{\text{sphere}} = nV_{\text{cone}}$$

$$\frac{4}{3}\pi(R_2^3 - R_1^3) = n\frac{1}{3}\pi r^2 h$$

$$4(R_2^3 - R_1^3) = nr^2 h$$

$$4 \times (8^3 - 6^3) = n(2)^2 \times 8$$

$$(512 - 216) = n \times 4 \times 2$$

$$296 = n \times 4 \times 2$$

$$n = 37 \text{ cm}$$

Hence, the number of cones is 37.

Main 2011

64. A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of sphere formed.

Sol :

Radius of cone $R = 5$ cm

Height of cone $H = 8$ cm

Radius of sphere $r = 0.5$ cm

Let n sphere are formed from the melted cone. Thus volume of n sphere is equal to the volume of single cone

$$V_{\text{cone}} = nV_{\text{sphere}}$$

$$\frac{1}{3}\pi R^2 H = n\frac{4}{3}\pi r^3$$

$$R^2 H = 4r^3$$

$$(5)^2 \times 8 = n \times 4 \times (0.5)^3$$

$$5 \times 5 \times 2 = n \times 0.5 \times 0.5 \times 0.5$$

$$10 \times 10 \times 2 = n \times 0.5$$

$$200 = 0.5n$$

$$n = 400$$

Hence, the number of spheres formed on melting a cone is 400.

Main 2010

65. A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone.

Sol :

Given, diameter of hemispherical bowl

$$D = 7.2 \text{ cm}$$

Radius of hemispherical bowl

$$R = \frac{d}{2} = \frac{7.2}{2} = 3.6 \text{ cm}$$

Let $r = 4.8$ cm be radius of base of cone and h be height of cone.

Since, the chocolate sauce from hemispherical bowl is completely poured into an inverted cone. Volume of material in hemispherical bowl is equal to the volume of cone.

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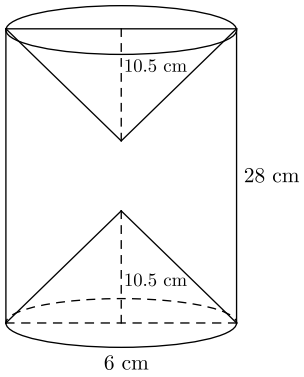

Sol :

Height of cylinder, $H = 28$ cm

Radius of cylinder, $R = \frac{6}{2} = 3$ cm

Height of cavity cone $h = 10.5$ cm

Radius of cavity cone $r = \frac{6}{2} = 3$ cm

Volume of the remaining solid,

$$\begin{aligned} V_{\text{Remaining}} &= V_{\text{cylinder}} - 2V_{\text{cone}} \\ &= \pi R^2 H - 2 \times \frac{1}{3}\pi r^2 h \\ &= \pi \times 3^2 \times 28 - 2 \times \frac{1}{3}\pi \times 3^2 \times (10.5) \\ &= \pi \times 3^2\left[28 - \frac{2}{3} \times 10.5\right] \\ &= \frac{22}{7} \times 9[28 - 7] \\ &= \frac{22 \times 9 \times 21}{7} \\ &= 594 \text{ cm}^3 \end{aligned}$$

Comp 2020

52. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

Sol :

Radius of given sphere $R = \frac{16}{2} = 8$ cm

Volume of sphere,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(8)^3 \text{ cm}^3$$

Radius of one recasted cone,

$$r = 4 \text{ cm}$$

Height $h = 8$ cm

Volume $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2 \times 8$

Let the number of recasted cones be n. Volume of sphere is equal to the n recasted cone.

$$\begin{aligned} n \times \frac{1}{3}\pi(4)^2 \times 8 &= \frac{4}{3}\pi(8)^3 \\ n = \frac{4 \times (8)^3}{(4)^2 \times 8} &= \frac{8^2}{4} \\ = \frac{64}{4} = 16 & \end{aligned}$$

Hence, number of recasted cones is 16.

Main 2019

53. A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble.

Sol :

Let R be the radius of big spherical ball and r be the radius of recasted small spherical marble.
Let n spherical marbles are formed from single big spherical ball.
Volume of n spherical marbles is equal to the volume of single big spherical ball.

$$\begin{aligned} V_{\text{sphere}} &= nv_{\text{sphere}} \\ \frac{4}{3}\pi R^3 &= n\frac{4}{3}\pi r^3 \\ R^3 &= nr^3 \\ 6^3 &= 64r^3 \\ 6^3 &= (4)^3 r^3 \\ 6^3 &= (4r)^3 \\ 6 &= (4r) \\ r &= \frac{6}{4} = 1.5 \text{ cm} \end{aligned}$$

Comp 2019

54. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Sol :

Radius of conical vessel $= 5$ cm

Height of conical vessel $= 24$ cm

Volume of this vessel, $= \frac{\pi}{3} \times (5)^2 \times 24$

$= 200\pi$ cm^3

Internal radius of cylindrical vessel $= 10$

Let the h be the height of emptied water.
Volume of water in cylinder,

$$\pi r^2 h = \frac{3}{4} \times \text{Volume of cone}$$

$$\pi \times 10 \times 10 \times h = \frac{3}{4} \times 200\pi$$

$$100h = 150 \Rightarrow h = 1.5 \text{ cm}$$

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Hence the height of water is 1.5 cm.

Main 2018

55. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the
(i) radius of the cylinder.
(ii) Volume of cylinder. [use $\pi = \frac{22}{7}$]

Sol :

Let r be the radius of base of circle and h be the height of cylinder.
Given, circumference of the base of a cylindrical vessel $= 132$ cm and height $= 25$ cm
(i) Radius of the cylinder
Circumference of base of a cylinder

$$2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$2 \times 22 \times r = 132 \times 7$$

$$r = 3 \times 7 = 21 \text{ cm}$$

(ii) Volume of cylinder
Volume of cylinder, $V = \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 66 \times 21 \times 25$$

$$= 34650 \text{ cm}^3$$

Comp 2018

56. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Sol :

Height and radius of cylinder are equal to the height and radius of cone.

$$\text{Height of cylinder} = \text{height of cone} = 8 \text{ cm}$$

$$\text{radius of cylinder} = \text{radius of cone} = 6 \text{ cm}$$

$$\text{Slant height of cone} = \sqrt{r^2 + h^2}$$

$$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$= 10 \text{ cm}$$

Total surface area of remaining solid,

$$= \text{CSA of cylinder} + \text{CSA of cone} + \text{area of top}$$

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 6(2 \times 8 + 10 + 6)$$

$$= \frac{22}{7} \times 6 \times 32$$

$$= 603.43$$

Hence total surface area is 603.43 cm^2.

Main 2017

57. A conical tent is to accommodate 77 persons. Each person must have 16 m^3 of air to breathe. Given the radius of the tent as 7 m. Find the height of the tent and also its curved surface area.

Sol :

Given, in the conical tent, each person requires 16 cm^3 of air to breath.
Required volume of the conical tent $= 77 \times 16$

$$= 1232 \text{ m}^3$$

Let r be the radius of base of tent and h be the height of tent.
From volume of tent we have

$$\frac{1}{2}\pi r^2 h = 1232$$

$$\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times h = 1232 \text{ m}^3$$

$$\frac{1}{3} \times 22 \times 7 \times h = 1232$$

$$h = \frac{1232 \times 3}{22 \times 7}$$

$$= 24 \text{ m}$$

Thus, height of the conical tent is 24 m.
Slant height l of cone is given by

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(24)^2 + (7)^2}$$

$$= \sqrt{576 + 49}$$

$$= \sqrt{625} \text{ m}$$

$$= 25 \text{ m}$$

Curved surface area of the conical tent,

$$\pi rl = \frac{22}{7} \times 7 \times 25$$

$$= 22 \times 25$$

$$= 550 \text{ m}^2$$

Main 2016

58. A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted and recast into a solid sphere of radius 6 cm. Find the number of cones.

Sol :

Let n be the number of cones which are melted.

NODIA

Radius of small cone $r = 2$ cm

Height of small cone $h = 3$ m

Radius of sphere $R = 6$ cm

Volume of n melted cone is equal to the volume of single sphere.

$$V_{\text{sphere}} = nV_{\text{cone}}$$
$$\frac{4}{3}\pi R^3 = n\frac{1}{3}\pi r^2 h$$
$$4R^3 = nr^2 h$$
$$4 \times (6)^3 = n(2)^2 \times 3$$
$$4 \times 6 \times 6 \times 6 = n \times 2 \times 2 \times 3$$
$$2 \times 6 \times 6 = n$$
$$n = 72$$

Hence, the number of cones is 72.

Comp 2016

59. A girl empties a cylindrical bucket, full of sand, of radius 12 cm and height 16 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 12 cm, then find its slant height correct upto one place of decimal.

Sol :

Here volume of cone is equal to the volume of cylinder.

Let r_1 and r_2 be the radii of the cylinder and cone respectively.

$$\text{Volume of cone} = \text{Volume of Cylinder}$$
$$\frac{1}{3}\pi r_2^2 h_2 = \pi r_1^2 h_1$$
$$\frac{1}{3} r_2^2 h_2 = \pi r_1^2 h_1$$
$$\frac{1}{3} \times r_2^2 \times 12 = 12 \times 12 \times 16$$
$$r_2^2 = \frac{3 \times 12 \times 12 \times 16}{12}$$
$$= 3 \times 12 \times 16$$
$$= 36 \times 16$$
$$r_2 = 24 \text{ cm}$$

Radius of cone $r_2 = 24$ cm

Now, slant height of cone

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{12^2 + 24^2}$$
$$= \sqrt{144 + 576}$$
$$= 26.83 \text{ cm}.$$

Main 2015

60. Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed.

Sol :

Radius of smaller sphere $r_1 = 2$ cm

Radius of bigger sphere $r_2 = 4$ cm

Let r be radius of base of recasted cone and $h = 8$ cm be height of cone.

Volume of recasted cone is equal to the volume of both sphere. Thus

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$
$$r^2 h = 4r_1^3 + 4r_2^3$$
$$r^2 \times 8 = 4 \times (2)^3 + 4 \times (4)^3$$
$$r^2 \times 2 = (2)^3 + 4 \times (4)^2$$
$$r^2 = (2)^2 + 2 \times (4)^2$$
$$= 4 + 2 \times 16$$
$$= 36$$
$$r = \sqrt{36} = 6 \text{ cm}$$

Hence, the radius of cone is 6 cm.

Main 2013

61. A solid sphere of radius 15 cm is melted and recast into a solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.

Sol :

Radius of sphere $R = 15$ cm

Let r be the radius of base of cone and h be the height of cone.

We have $r = 2.5$ cm and $h = 8$ cm.

Let n cone are formed from the melted sphere. Thus volume of n cone is equal to the volume of single sphere.

$$V_{\text{sphere}} = nV_{\text{cone}}$$
$$\frac{4}{3}\pi R^3 = n\frac{1}{3}\pi r^2 h$$
$$4R^3 = nr^2 h$$
$$4 \times (15)^3 = n(2.5)^2 \times 8$$
$$(15)^3 = n(2.5)^2 \times 2$$
$$15 \times 15 \times 15 = n \times 2.5 \times 2.5 \times 2$$
$$6 \times 6 \times 15 = n \times 2$$
$$3 \times 6 \times 15 = n$$
$$n = 270$$

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(c) $\left(\frac{9}{2}\right)^{1/3}$ (d) $\left(\frac{3}{8}\right)^{1/3}$

Sol :

As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$\frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

Thus (b) is correct option.

26. A solid piece of iron in the form of a cuboid of dimensions $49\text{ cm} \times 33\text{ cm} \times 24\text{ cm}$, is moulded to form a solid sphere. What is the radius of the sphere ?

(a) 21 cm (b) 32 cm
(c) 28 cm (d) 36 cm

Sol :

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24 = 38808\text{ cm}^3$$

$$4 \times \frac{22}{7} r^3 = 38808 \times 3$$

$$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22}$$

$$= 441 \times 21$$

$$r^3 = 21 \times 21 \times 21$$

$$r = 21\text{ cm}$$

Thus (a) is correct option.

27. A solid metallic cuboid 24 cm × 11 cm × 7 cm is melted and recast into n solid cones of base radius 3.5 cm and height 6 cm. What is the value of n?

(a) 42 (b) 12
(c) 24 (d) 18

Sol :

Let n be the number of cones formed.
Now, according to question,

Volume of n cones = Volume of cuboid

$$n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 = 24 \times 11 \times 7$$

$$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$$

Thus $n = 24$.
Thus (c) is correct option.

28. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder.

(a) 1 : 4 (b) 4 : 1
(c) 3 : 2 (d) 2 : 3

Sol :

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h} = \frac{1}{4}$$

$$= 1 : 4$$

Thus (a) is correct option.

29. Volume of two spheres are in the ratio 64 : 27, what is the ratio of their surface areas ?

(a) 64 : 27 (b) 16 : 9
(c) 27 : 64 (d) 9 : 16

Sol :

$$\frac{\text{Volume of I}^{\text{st}}\text{sphere}}{\text{Volume of II}^{\text{nd}}} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Ratio of their surface areas,

$$\frac{2\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Thus (b) is correct option.

30. What is the volume of the largest right circular cone that can be cut off from a cube of edge 4.2 cm?

(a) 32.4 cm³ (b) 25.4 cm³
(c) 12.4 cm³ (d) 19.4 cm³

Sol :

Edge of the cube = 4.2 cm.

Height of the cone = 4.2 cm.

Radius of the cone = $\frac{4.2}{2} = 2.1$ cm.

Volume of the cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$$

$$= 19.4\text{ cm}^3$$

Thus (d) is correct option.

31. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm³.

(a) 12580 cm³ (b) 22540 cm³
(c) 19404 cm³ (d) 16550 cm³

Sol :

Let r be the radius of bowl, then circumference

NODIA

$$V_{\text{bowl}} = V_{\text{cone}}$$
$$\frac{2\pi R^3}{3} = \frac{1}{3}\pi r^2 h$$
$$2(R)^3 = r^2 h$$
$$2 \times (3.6)^3 = (4.8)^2 \times h$$
$$2 \times 3.6 \times 3.6 \times 3.6 = 4.8 \times 4.8 \times h$$
$$2 \times 3 \times 3 \times 3.6 = 4 \times 4 \times h$$
$$3 \times 3 \times 0.9 = 2 \times h$$
$$8.1 = 2h$$
$$h = \frac{8.1}{2} = 4.05 \text{ cm}$$

Hence, height of the cone is 4.05 cm.

Comp 2006

66. Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is Rs. 40 per meter, find the amount by which Rampal helped the money.

Sol :

Radius of tent $r = \frac{14}{2} = 7$ m

Height $h = 24$ m

Slant height $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576} = 25$ m

Surface area of the tent,

$$\pi rl = \pi rl$$
$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Surface area of 10 tents,

$$= 550 \times 10 = 5500$$
$$\text{Total cost} = 5500 \times \frac{40}{2} = 110000$$

Hence, Rampal helped the centre of 110000 Rs.

Comp 2005

67. Water is flowing at the rate of 0.7 m/sec through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the level of water in half hour.

Sol :

Length of water that flows in 1 sec is 0.7 m or 70 cm.

Radius of pipe is $\frac{2}{2} = 1$ cm.

Volume of water in 1 second,

$$= \pi \times (1)^2 \times 70 = 70\pi \text{cm}^3$$

Volume of water in 30 minutes,

$$= 70\pi \times 60 \times 30 \text{ cm}^3$$

Let h be height of water in tank. Radius of tank is 40 cm.

Volume of water in the tank,

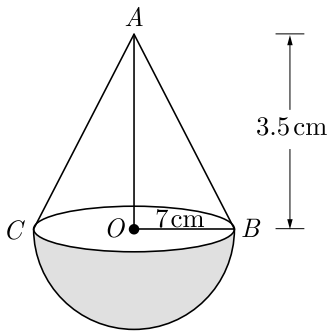
$$\pi 40^2 \times h = 70\pi \times 60 \times 30$$
$$h = \frac{70 \times 60 \times 30}{40 \times 40} = 78.75 \text{ cm}$$

Comp 2003

68. A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of cone is 7 cm and height of cone is 3.5 cm, find the volume of the solid. (Take $\pi = \frac{22}{7}$)

Sol :

As per given information in question we have drawn the figure below,



Here, radius $r = 7$ cm

and height of a cone $= 3.5$ cm

Volume of the solid,

= Volume of hemisphere + Volume of a cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$
$$= \frac{2}{3} \times \frac{22}{7} \times 7^3 + \frac{1}{3} \times \frac{22}{7} \times 7^3 \times 3.5$$
$$= \frac{1}{3}(2156 + 539)$$
$$= \frac{1}{3} \times 2695$$
$$= 898.33 \text{ cm}^3.$$

Main 2002

69. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively, is melted into a cone of base diameter 8 cm. Find the height of the cone.

Sol :

Given, internal and external diameters of hollow

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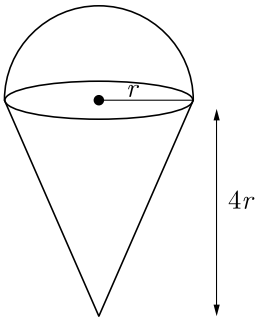


Volume of each cone with hemispherical top

$$\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3=\frac{1}{3}\pi r^2(h+2r)$$
$$=\frac{1}{3}\pi r^2(4r+2r)$$
$$=2\pi r^3$$

Now, volume of ice-cream in container is equal to 10 cone of ice-cream.

$$36\times 15\pi=10\times 2\pi x^3$$
$$r^3=\frac{36\times 15}{20}=27$$
$$r=3\,\text{cm}$$

Comp 2012

81. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Sol :

Radius of sphere $=\frac{12}{2}=6$ cm

Volume $=\frac{4}{3}\pi\times 6^3$ cm^3

It is submerged into water, in cylindrical vessel, then water level rise by $3\frac{5}{9}=\frac{32}{9}$ cm. Volume of submerged sphere is equal to the volume of water rise in cylinder.

Volume submerged = Volume rise

Let r be radius of cylinder. Therefore

$$\pi\times r^2\times\frac{32}{9}=\frac{4}{3}\pi\times 6^3\text{ cm}$$
$$r^2=\frac{216\times 3\times 4}{32}$$
$$=\frac{27\times 3\times 4}{4}$$
$$r^2=27\times 3=81$$
$$r=9\text{ cm}$$

Diameter $2r=2\times 9=18$ cm.

Comp 2011

82. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m $\times$ 44 m. Find the time in which the level of water in the tank will rise by 7 cm.

Sol :

Radius of pipe, $r=\frac{14}{2}=7$ cm

Cross section area of pipe,

$$\pi r^2=\frac{22}{7}\times\left(\frac{7}{100}\right)^2$$

Speed of water flowing through the pipe

$$=5\text{ km/hr}=15000\text{ m/hr}$$

In an hour length of water $=5000$ m

Volume of water flowing from pipe in 1 hr,

$$\pi r^2 h=\frac{22}{7}\times\left(\frac{7}{100}\right)^2\times 5000\text{ m}^3$$

Let t be time taken to fill the tank. Now total volume of water flowing in time t,

$$\pi r^2 ht=\frac{22}{7}\times\left(\frac{7}{100}\right)^2\times 5000t$$

Volume of water flown= Volume of water in tank

$$\pi r^2 ht=l\times b\times y$$
$$\frac{22}{7}\times\left(\frac{7}{100}\right)^2\times 500t=50\times 44\times\frac{7}{100}$$
$$\frac{22}{7}\times\frac{7}{100}\times\frac{7}{100}\times 5000t=50\times 44\times\frac{7}{100}$$
$$22\times 50t=50\times 44$$
$$t=\frac{50\times 44}{22\times 50}=2$$

Hence, Time taken to fill the tank is 2 hours.

Comp 2010

83. A toy is in the form of a cone radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi=\frac{22}{7}$

Sol :

As per question the figure is shown below. Here total surface area of the toy is equal to the sum of surface area of hemisphere and curved surface area of cone.

NODIA

Comp 2000

73. A well diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment.

Sol :

The volume of soil taken out from the well,

$$\pi^2 rh = \pi \times \left(\frac{3}{2}\right)^2 \times 14 \text{ m}^3$$

The radius of embankment with well

$$= \frac{3}{5} + 5 = \frac{13}{2} \text{ m}$$

Let the y be height of embankment. Then the volume of soil used in embankment,

$$\pi(R^2 - r^2)y = \pi r^2 h$$

$$\pi\left[\left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]y = \pi \times \left(\frac{3}{2}\right)^2 \times 14$$

$$\frac{160}{4}y = \frac{3}{2} \times \frac{3}{2} \times 14$$

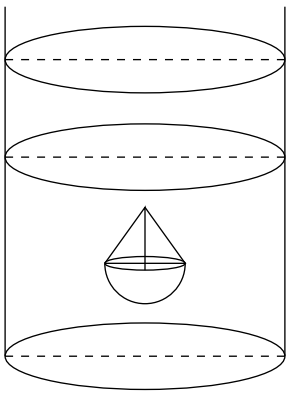
$$y = \frac{3 \times 3 \times 14}{160} = 0.7875 \text{ m}$$

Hence the height of embankment is 78.75 cm.

FOUR MARK QUESTIONS

Main 2023

74. A solid is in the shape of a hemisphere of radius 7 cm, surmounted by a cone of height 4 cm. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is 14 cm, find the rise in the water level.



Sol :

For the solid, radius of hemisphere is same as the radius of cone.

Radius $r = 7$ cm

Height of cone, $h = 4$ cm

Volume of Solid,

$$V_{\text{solid}} = V_{\text{hemisphere}} + V_{\text{cone}}$$

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2(2r + h)$$

Radius of the cylinder,

$$R = 14 \text{ cm}$$

Due to volume V_{solid} of solid water rises in cylinder. Let the rise in water level be H. The amount of water rise in cylinder is equal to the volume of solid.

Thus $V_{\text{rise}} = V_{\text{solid}}$

$$\pi R^2 H = \frac{1}{3}\pi r^2(2r + h)$$

$$R^2 H = \frac{1}{3} r^2(2r + h)$$

$$14 \times 14 \times H = \frac{1}{3} \times 7 \times 7(2 \times 7 + 4)$$

$$2 \times 2 \times H = \frac{1}{3}(14 + 4)$$

$$4H = \frac{18}{3}$$

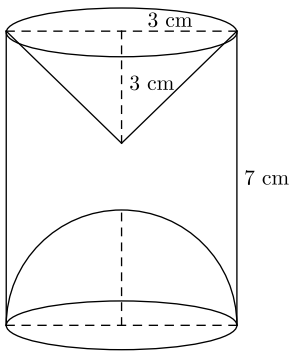
$$H = \frac{18}{12} = \frac{3}{2} = 1.5$$

Hence, the rise in water level in the cylinder is 1.5 cm.

Main 2019

75. A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows

The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm. Give your answer correct to the nearest whole number. [Take $\pi = \frac{22}{7}$].



Sol :

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$$l = \sqrt{r^2 + h^2} = \sqrt{0.7^2 + 2.4^2} = 2.5$$

m

Total surface area of remaining solid,

= CSA of cylinder + CSA of cone + Area of top.

$$= 2\pi rh + \pi rl + \pi r^2$$
$$= \pi r(2h + l + r)$$
$$= \frac{22}{7} \times 0.7(2 \times 2.4 + 2.5 + 0.7)$$
$$= \frac{22}{7} \times 0.7 \times 8 = \frac{176}{10}$$

Hence total surface area is 17.6 cm^2

Main 2008

86. The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the

(i) radius of the floor.

(ii) height of the tent.

(iii) length of the canvas required area of cover this conical tent, if its width is 2 m.

Sol :

Let r be the radius of base of tent and h be the height of tent.

(i) Radius of the floor

Since area of the base floor is 154 m, thus

$$\pi r^2 = 154$$
$$r^2 = \frac{154}{22} \times 7$$
$$r^2 = 7^2$$
$$r = 7 \text{ m}$$

(ii) Height of the tent

Since volume of a conical tent is 1232 m^3, we have

$$\frac{1}{3}\pi r^2 h = 1232$$
$$\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times h = 1232$$
$$22 \times 7 \times h = 1232 \times 3$$
$$7 \times h = 56 \times 3$$
$$h = 8 \times 3 = 24 \text{ m}$$

(iii) Required length of the canvas

Slant height l of cone,

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{24^2 + 7^2}$$
$$= \sqrt{576 + 49}$$
$$= \sqrt{625} \text{ m}$$
$$= 25 \text{ m}$$

Curved surface area of the conical tent,

$$\pi rl = \frac{22}{7} \times 7 \times 25$$
$$= 22 \times 25$$
$$= 550 \text{ m}^2$$

Thus we require 550 m^2 canvas. Since width of canvas is 2 m, required length of canvas is $\frac{550}{2} = 275$ m.

Comp 2008

87. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

Sol :

Radius of given sphere $R = \frac{16}{2} = 8$ cm

Volume of sphere,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(8)^3 \text{ cm}^3$$

Radius of one recasted cone,

$$r = 4 \text{ cm}$$

Height $h = 8$ cm

Volume $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2 \times 8$

Let the number of recasted cones be n. Volume of sphere is equal to the n recasted cone.

$$n \times \frac{1}{3}\pi(4)^2 \times 8 = \frac{4}{3}\pi(8)^3$$
$$n = \frac{4 \times (8)^3}{(4)^2 \times 8}$$
$$= \frac{8^2}{4} = \frac{64}{4} = 16$$

Hence, number of recasted cones is 16.

Main 2007

88. The surface area of a solid metallic, sphere is 616 cm^2. It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained?

Sol :

Let R cm be the radius of bigger sphere.

Since surface area of a bigger sphere is 616 cm^2 we have

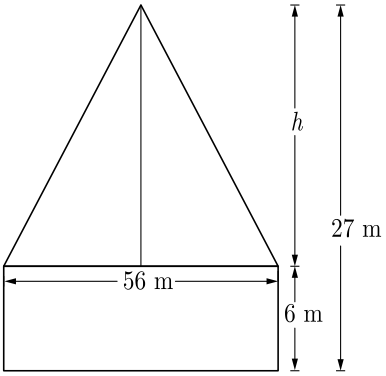
$$4\pi R^2 = 616$$
$$4 \times \frac{22}{7} \times R^2 = 616$$
$$4 \times \frac{1}{7} \times R^2 = 28$$
$$4 \times R^2 = 28 \times 7$$

NODIA

canvas used in the tent.

Sol :

As per question the figure is shown below.



Total height of tent $H_{\text{Total}} = 27$ m

Height of cylindrical part $h = 6$ m

Height of conical part $H = 27 - 6 = 21$ m

Radius of cone $R = \frac{56}{2} = 28$ m

Radius of cylinder $R = \frac{56}{2} = 28$ m

Slant height of cone

$$L = \sqrt{R^2 + H^2}$$
$$= \sqrt{28^2 + 21^2}$$
$$= \sqrt{784 + 441}$$
$$= \sqrt{1225}$$
$$= 35 \text{ m}$$

Area of canvas used,

$$2\pi rh + \pi rl = \pi r(2h + l)$$
$$= \frac{22}{7} \times 28(2 \times 6 + 35)$$
$$= 22 \times 4 \times 47$$
$$= 4136 \text{ m}^2$$

Main 2014

79. The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate

(i) the radius of the sphere.

(ii) the number of cones recast. (take $\pi = \frac{22}{7}$)

Sol :

(i) Radius of the sphere

Let R cm be the radius of sphere.

Since surface area of a sphere is 2462 cm^2 we have

$$4\pi R^2 = 2464$$
$$4 \times \frac{22}{7} \times R^2 = 2464$$
$$4 \times \frac{1}{7} \times R^2 = 112$$
$$\frac{1}{7} \times R^2 = 28$$
$$R^2 = 28 \times 7$$
$$= 4 \times 7 \times 7$$
$$R = 2 \times 7 = 14$$

NODIA

(ii) Number of cones recast

Let r be the radius of base of recast cone and h be the height of cone. From melted sphere such n cone are recast.

We have $r = 3.5$ cm and $h = 7$ cm.

Thus volume of sphere must be equal to volume of n cone. Thus

$$V_{\text{sphere}} = nV_{\text{cone}}$$
$$\frac{4}{3}\pi R^3 = n\frac{1}{3}\pi r^2 h$$
$$4R^3 = nr^2 h$$
$$4(14)^3 = n(3.5)^2 \times 7$$
$$n = \frac{4(14)^3}{(3.5)^2 \times 7}$$
$$= \frac{4 \times 14 \times 14 \times 14}{3.5 \times 3.5 \times 7}$$
$$= 4 \times 4 \times 4 \times 2$$
$$= 128$$

Hence, the number of cones recast from solid metallic sphere is 128.

Comp 2013

80. A right cylindrical container of radius 6 cm and height 15 cm if full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

Sol :

For cylindrical container $R = 6$ cm and $H = 15$ cm

Volume of ice cream in the cylindrical container

$$\pi R^2 H = \pi(6)^2 \times 15 = 36 \times 15\pi$$

As per given information in question we have drawn the figure of cone as given below. Here r is the common radius of cone and hemisphere.

sphere are $D_1 = 4$ cm and $D_2 = 8$ cm.
Internal radius of hollow sphere

$$R_1 = \frac{D_1}{2} = \frac{4}{2} = 2 \text{ cm}$$

and external radius of hollow sphere

$$R_2 = \frac{D_2}{2} = \frac{8}{2} = 4 \text{ cm}$$

Also, given diameter of base cone

$$d = 8 \text{ cm}$$

Radius of base cone,

$$r = \frac{d}{2} = \frac{8}{2} = 4 \text{ cm}$$

Now hollow sphere is melted into form of a cone. Let h be the height of cone formed. Since volume of hollow sphere is equal to volume of cone formed, we have

$$V_{\text{sphere}} = V_{\text{cone}}$$

$$\frac{4}{3}\pi(R_2^3 - R_1^3) = \frac{1}{3}\pi r^2 h$$

$$4(4^3 - 2^3) = (4)^2 \times h$$

$$4(64 - 8) = 16 \times h$$

$$h = \frac{4 \times 56}{16} = 14 \text{ cm}$$

Hence, height of the cone formed is 14 cm.

Comp 2002

70. Water is flowing at 7 m/s through a circular pipe of internal diameter of 4 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

Sol :

Length of water that flows in 1 sec is 7 m or 700 cm.
Radius of pipe is $\frac{4}{2} = 2$ cm.
Thus volume of water in 1 second,

$$= \pi \times (2)^2 \times 700 \text{ cm}^3$$

Volume of water in 30 minutes,

$$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$$

Let h be height of water in tank. Radius of tank is 40 cm.
Volume of water in the tank,

$$\pi 40^2 \times h = \pi \times 4 \times 700 \times 60 \times 30$$

$$h = \frac{700 \times 60 \times 30 \times 4}{40 \times 40} = 3150 \text{ cm}$$

Hence, water level increased is 3150 cm or 31.5 m.

Main 2001

71. The volume of a right circular cone is 1232 cm^3. If the radius of its base is 14 cm, find its curved surface area.

Sol :

Let $r = 14$ cm be the radius, h be the height and l be the slant height of the cone.
Since volume of right circular cone is 1232 cm^3, we have

$$\frac{1}{3}\pi r^2 h = 1232 \text{ cm}^3$$

$$\frac{1}{3} \times \frac{22}{7} \times (14)^2 h = 1232$$

$$\frac{1}{3} \times \frac{1}{7} \times 14 \times 14 \times h = 56$$

$$\frac{1}{3} \times 2 \times h = 4$$

$$h = 2 \times 3 = 6 \text{ cm}$$

Now, slant height of the cone,

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{(14)^2 + (6)^2}$$
$$= \sqrt{196 + 36}$$
$$= \sqrt{232} = 2\sqrt{58} \text{ cm}$$

Curved surface area of the curve $= \pi r l$

$$\pi r l = \frac{22}{7} \times 14 \times 2\sqrt{58}$$

$$= 88\sqrt{58} \text{ cm}^2$$

Main 2000

72. Find the volume and surface area of hemisphere of diameter 7 cm. (Take $\pi = \frac{22}{7}$)

Sol :

Diameter of hemisphere $d = 7$ cm
Radius of hemisphere $r = \frac{d}{2} = \frac{7}{2}$ cm
Volume of hemisphere, $V = \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$$
$$= \frac{22}{3} \times \left(\frac{7}{2}\right)^2$$
$$= \frac{22 \times 7 \times 7}{3 \times 2 \times 2}$$
$$= 89.83 \text{ cm}^3$$

Surface area of hemisphere,

$$S = 3\pi r^2$$
$$= 3 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$
$$= \frac{3 \times 22 \times 7}{2 \times 2}$$
$$= 115.5 \text{ cm}^2$$

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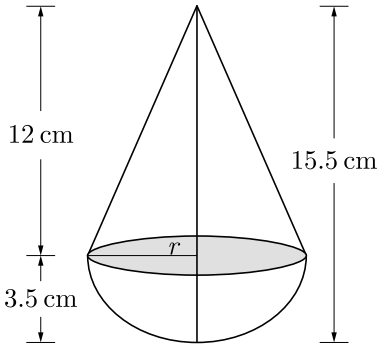


Radius $r = \frac{7}{2} = 3.5$ cm

and height $h = 12$ cm

Slant height of cone,

$$l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2} = 12.5$$

Total surface area of the toy

= Surface area of hemisphere +

+ Curved surface area of cone

$= 2\pi r^2 + \pi rl$

$= \pi r(2r + l)$

$= \frac{22}{7} \times 3.5 \times (2 \times 3.5 + 12.5)$

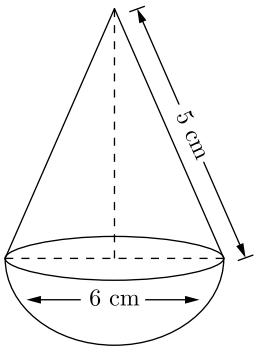
$= 11 \times 19.5 = 214.5$ cm^2

Main 2009

84. The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate

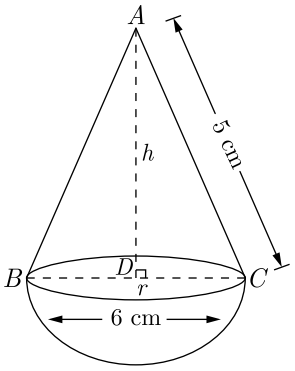
(i) the height of the cone.

(ii) the volume of the solid.



Sol :

Let r be common radius of hemisphere and cone cm and h be height of the cone.



Slant height of cone $l = 5$ cm

Common diameter, $d = 6$ cm

Common radius $r = \frac{6}{2} = 3$ cm

(i) Height of the cone

Height of Cone $h = \sqrt{l^2 - r^2}$

$= \sqrt{5^2 - 3^2}$

$= 4$ cm

(ii) Volume of the solid

Volume of solid is the total volume of cone and hemisphere. Thus volume of solid,

$$V = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2(2r + h)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3^2 \times (2 \times 3 + 4)$$

$$= \frac{22}{7} \times 3 \times (6 + 4)$$

$$= \frac{22}{7} \times 3 \times 10$$

$$= \frac{660}{7}$$

$$= 94.28 \text{ cm}^3$$

Hence, the volume of the solid figure is 94.28 cm^3.

Comp 2009

85. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.

Sol :

Radius of cylinder and cone,

$r = 0.7$ cm

Height of cylinder and cone,

$h = 2.4$ cm

Slant height of cone,

NODIA

$$n = \frac{4(10.5)^3}{(3.5)^2 \times 3}$$
$$= \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3}$$
$$= 4 \times 3 \times 3 \times 3.5$$
$$= 126$$

Hence, the number of cones recast from solid metallic sphere is 126.

Comp 2004

92. From a solid cylinder of height 24 cm and diameter 14 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

Sol :

Height and radius of cylinder are equal to the height and radius of cone.

Height of cylinder= height of the cone $= 24$ cm

Radius of cylinder= radius of cone $= \frac{14}{2} = 7$ cm

Slant height of cone $= \sqrt{r^2 + h^2}$
$= \sqrt{7^2 + 24^2}$
$= \sqrt{49 + 576} = 25$ cm

Total surface area of remaining part
= Surface area of cylinder+
+ Surface area of cone + area of top
$= 2\pi rh + \pi rl + \pi r^2$
$= \pi r(2h + l + r)$
$= \frac{22}{7} \times 7(2 \times 24 + 25 + 7)$
$= 22 \times 80$
$= 1760$ cm^2

Main 2003

93. A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top which is open, is 2.5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.25 cm are dropped into the vessel, $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel.

Sol :

Height of vessel $H = 11$ cm

Radius of vessel $R = 2.5$ cm

Let $r = 0.25$ cm be radius of each lead shot and such n lead shot are dropped in vessel.
Since $\frac{2}{5}$ of the water in the vessel overflows, due to n lead shot, volume of n lead shot is equal to the volume of $\frac{2}{5}$ of the water in vessel.

Thus $n V_{\text{Lead shot}} = \frac{2}{5} V_{\text{vessel}}$

$$n\frac{4}{3}\pi r^3 = \frac{2}{5} \times \frac{1}{3}\pi R^2 H$$
$$4nr^3 = \frac{2}{5}R^2 H$$
$$2nr^3 = \frac{1}{5}R^2 H$$
$$5 \times 2nr^3 = R^2 H$$
$$10 \times n \times (0.25)^3 = (2.5)^2 \times 11$$
$$n = \frac{(2.5)^2 \times 11}{10 \times (0.25)^3}$$
$$= \frac{2.5 \times 2.5 \times 11}{10 \times 0.25 \times 0.25 \times 0.25}$$
$$= \frac{10 \times 10 \times 11}{10 \times 0.25}$$
$$= \frac{10 \times 11}{0.25}$$
$$= 40 \times 11 = 440$$

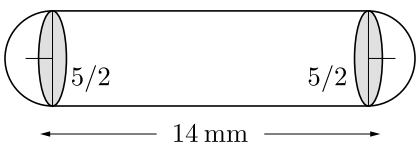
Hence, number of lead shots is 440.

Comp 2001

94. A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 15 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule.

Sol :

As per question the figure is shown below.



Total height $= 14$ mm

Height of cylinder $= 14 - 2 \times 2.5 = 9$ mm

Radius of cylinder $= 2.5$mm

Radius of hemisphere $= 2.5$ mm

Volume of capsule= Volume of two hemispheres
+ Volume of cylinder
$= 2 \times \frac{2\pi r^3}{3} + \pi r^2 h$
$= \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 + \pi\left(\frac{5}{2}\right)^2 \times 9$
$= \pi\left(\frac{5}{2}\right)^2\left(\frac{4}{3} \times \frac{5}{2} + 9\right)$
$= \frac{25\pi}{4}\left(\frac{10}{3} + 9\right)$

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$$R^2 = 7 \times 7$$

$$R = 7 \text{ cm}$$

Let $r = \frac{3.5}{2}$ be the radius of smaller sphere and such n sphere are formed from melted bigger sphere.

Volume of single bigger sphere must be equal to volume of n smaller sphere. Thus

$$V_{\text{big}} = nV_{\text{small}}$$

$$\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$$

$$R^3 = nr^3$$

$$7^3 = n\left(\frac{3.5}{2}\right)^3$$

$$7^3 = n\left(\frac{7}{4}\right)^3$$

$$1 = n\left(\frac{1}{4}\right)^3$$

$$n = 4^3 = 64$$

Comp 2007

89. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

Sol :

If a cone of maximum size is curved out from a cube edge a, diameter and height of cone will be a

Side of cube $a = 14$ cm.

If cone of maximum size is curved out,

Radius of cone $r = \frac{14}{2} = 7$ cm

Height of cone $h = 7$ cm

Slant height $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 14^2}$

$= \sqrt{49 + 196} = \sqrt{245}$

$= 15.65$ cm.

Total surface area,

= Surface area cube + curved Surface area of cone − Circular area of base of cone

$= 6a^2 + \pi rl - \pi r^2$

$= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 15.65 - \frac{22}{7} \times 7 \times 7$

$= 1176 + [22(15.65 - 7)]$

$= 1176 \times 22 \times 8.65$

$= 223792.8 \text{ cm}^2$

Main 2006

90. A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16.8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one-third of the water in the original cone overflows. What is the volume of each of the solid cones submerged?

Sol :

Height of vessel $h = 20$ cm

Radius of vessel $r = \frac{16.8}{2} = 8.4$ cm

Let v be volume of each solid cone. Since $\frac{1}{3}$ of the water in the original cone overflows, volume of 2 solid cone is equal to the volume of $\frac{1}{3}$ of the water in vessel.

Thus $2v = \frac{1}{3} \times V_{\text{vessel}}$

$= \frac{1}{3} \times \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{22}{7} \times (8.4)^2 \times 20$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{22}{7} \times 8.4 \times 8.4 \times 20$

$= \frac{22}{7} \times 2.8 \times 2.8 \times 20$

$= 22 \times 0.4 \times 2.8 \times 20$

$2v = 22 \times 8 \times 2.8$

$v = 11 \times 8 \times 2.8$

$= 246.4 \text{ cm}^3$

Hence, volume of each of the solid cones is 246.4 cm^3.

NODIA

Main 2005

91. A metallic sphere of radius 10.5 cm is melted and then recast into small cones, of radius 3.5 cm and height 3 cm. Find the number of cones thus obtained.

Sol :

Radius of sphere $R = 10.5$ cm

Let r be the radius of base of recast cone and h be the height of cone. From melted sphere such n cone are recast.

We have $r = 3.5$ cm and $h = 3$ cm.

Thus volume of sphere must be equal to volume of n cone. Thus

$$V_{\text{sphere}} = nV_{\text{cone}}$$

$$\frac{4}{3}\pi R^3 = n\frac{1}{3}\pi r^2 h$$

$$4R^3 = nr^2 h$$

$$4(10.5)^3 = n(3.5)^2 \times 3$$

Height of cylinder, $H = 7$ cm

Radius of cylinder, $R = 3$ cm

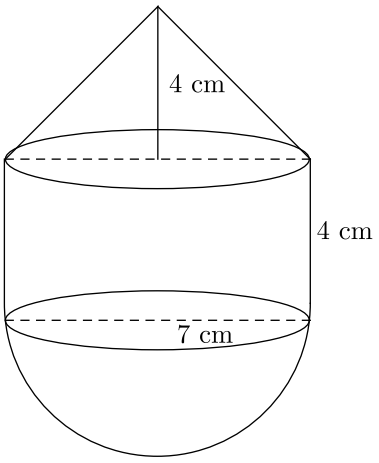
Height of cavity cone $h = 3$ cm

Radius of cone and hemisphere will be equal to the radius of cylinder i.e $R = 3$ cm

$$\begin{aligned} V_{\text{Remaining}} &= V_{\text{cylinder}} - V_{\text{cone}} - V_{\text{hemisphere}} \\ &= \pi R^2 H - \frac{1}{3}\pi R^2 h - \frac{2}{3}\pi R^3 \\ &= \frac{1}{3}\pi R^2(3H - h - 2R) \\ &= \frac{1}{3} \times \frac{22}{7} \times 3^2(3 \times 7 - 3 - 2 \times 3) \\ &= \frac{22}{7} \times 3(21 - 3 - 6) \\ &= \frac{22}{7} \times 3 \times 12 \\ &= \frac{792}{7} \\ &= 113\,\text{cm}^3 \end{aligned}$$

Main 2018

76. The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid.



Sol :

Given, common radius, $r = 7$ cm and height of cone = height of cylinder, $h = 4$ cm
Volume of solid,

$$\begin{aligned} V_{\text{solid}} &= V_{\text{cone}} + V_{\text{cylinder}} + V_{\text{hemisphere}} \\ &= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3 \\ &= \frac{1}{3}\pi r^2(h + 3h + 2r) \\ &= \frac{1}{3}\pi r^2(4h + 2r) \\ &= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times (4 \times 4 + 2 \times 7) \\ &= \frac{1}{3} \times \frac{22}{1} \times 7 \times (16 + 14) \\ &= \frac{1}{3} \times 22 \times 7 \times 30 \\ &= \frac{1}{3} \times 22 \times 7 \times 30 \\ &= 22 \times 7 \times 10 \\ &= 1540\ \text{cm}^3 \end{aligned}$$

Comp 2017

77. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

Sol :

If a cone of maximum size is curved out from a cube edge a, diameter and height of cone will be a

Side of cube $a = 14$ cm.

If cone of maximum size is curved out,

Radius of cone $r = \frac{14}{2} = 7$ cm

Height of cone $h = 7$ cm

Slant height $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 14^2}$

$= \sqrt{49 + 196} = \sqrt{245}$

$= 15.65$ cm.

Total surface area,

= Surface area cube + curved Surface area of cone − Circular area of base of cone

$$\begin{aligned} &= 6a^2 + \pi r l - \pi r^2 \\ &= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 15.65 - \frac{22}{7} \times 7 \times 7 \\ &= 1176 + [22(15.65 - 7)] \\ &= 1176 \times 22 \times 8.65 \\ &= 223792.8\ \text{cm}^2 \end{aligned}$$

Comp 2015

78. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter. If their common diameter is 56 m, the height of cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area of

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$$= \frac{25}{4}\pi\left(\frac{10+27}{3}\right)$$
$$= \frac{25}{4}\pi\left[\frac{37}{3}\right]$$
$$= \frac{25}{4} \times \frac{22}{7} \times \frac{37}{3}$$
$$= \frac{10175}{42} \text{ mm}^3$$
$$= 242.26 \text{ mm}^3.$$

FIVE MARK QUESTIONS

Main 2000

95. The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cone of radius 2.5 cm and height of 8 cm. Calculate

(i) the radius of the solid sphere.

(ii) the number of cones recast. (take $\pi = 3.14$)

Sol :

(i) Radius of the sphere

Let R cm be the radius of sphere.

Since surface area of a sphere is 2462 cm^2 we have

$$4\pi R^2 = 1256$$
$$4 \times 3.14 \times R^2 = 1256$$
$$R^2 = 100$$
$$R = 10 \text{ cm}$$

(ii) Number of cones recast

Let r be the radius of base of cone and h be the height of cone. From melted sphere such n cone are recast.

We have $r = 2.5$ cm and $h = 8$ cm.

Thus volume of sphere must be equal to volume of n cone. Thus

$$V_{\text{sphere}} = nV_{\text{cone}}$$
$$\frac{4}{3}\pi R^3 = n\frac{1}{3}\pi r^2 h$$
$$4R^3 = nr^2 h$$
$$4(10)^3 = n(2.5)^2 \times 8$$
$$(10)^3 = n(2.5)^2 \times 2$$
$$10 \times 10 \times 10 = n \times 2.5 \times 2.5 \times 2$$
$$4 \times 4 \times 5 = n$$
$$n = 80$$

Hence, the number of cones recast from solid metallic sphere is 80.

END

NODIA

$(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$

Sol :

$$\begin{aligned}
\text{LHS} &= (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 \\
&= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2 \\
&= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \\
&\qquad + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A} \\
&= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + \\
&\qquad + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right) \\
&= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right) \\
&= 1 + \frac{1}{\sin^2 A \cos^2 A} + \frac{2}{\sin A \cos A} \\
&= \left(1 + \frac{1}{\sin A \cos A}\right)^2 \\
&= (1 + \sec A \operatorname{cosec} A)^2 \qquad \text{Hence Proved}
\end{aligned}$$

Main 2016

37. Prove that $\dfrac{\cos A}{1 + \sin A} + \tan A = \sec A$.

Sol :

$$\begin{aligned}
\frac{\cos A}{1 + \sin A} + \tan A &= \frac{\cos A}{1 + \sin A} + \frac{\sin A}{\cos A} \\
&= \frac{\cos^2 A + \sin A (1 + \sin A)}{\cos A (1 + \sin A)} \\
&= \frac{\cos^2 A + \sin A + \sin^2 A}{\cos A (1 + \sin A)} \\
&= \frac{\cos^2 A + \sin^2 A + \sin A}{\cos A (1 + \sin A)} \\
&= \frac{1 + \sin A}{\cos A (1 + \sin A)} \\
&= \frac{1}{\cos A} \\
&= \sec A \qquad \text{Hence proved}
\end{aligned}$$

Comp 2014

38. Prove that : $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$.

Sol :

We have $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$.

$$\begin{aligned}
\text{LHS} &= \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} \\
&= \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}} \\
&= \frac{2\sec\theta}{\sqrt{\sec^2\theta - 1}} \\
&= \frac{2\sec\theta}{\tan\theta} \\
&= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} \\
&= 2 \times \frac{1}{\sin\theta} \\
&= 2\operatorname{cosec}\theta \qquad \text{Hence Proved}
\end{aligned}$$

Main 2013, Main 2000

39. Show that $\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \dfrac{\sin A}{1 + \cos A}$.

Sol :

$$\begin{aligned}
\sqrt{\frac{1 - \cos A}{1 + \cos A}} &= \sqrt{\frac{1 - \cos A}{1 + \cos A}} \times \sqrt{\frac{1 + \cos A}{1 + \cos A}} \\
&= \sqrt{\frac{(1 - \cos A)(1 + \cos A)}{(1 + \cos A)^2}} \\
&= \frac{\sqrt{1 - \cos^2 A}}{1 + \cos A} \\
&= \frac{\sqrt{\sin^2 A}}{1 + \cos A} \\
&= \frac{\sin A}{1 + \cos A} \qquad \text{Hence proved}
\end{aligned}$$

Comp 2013

40. Prove that :
$\dfrac{\cos^2\theta}{1 - \tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta$.

Sol :

$$\begin{aligned}
\text{LHS} &= \frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} \\
&= \frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} \\
&= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta} \\
&= \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} \\
&= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)} \\
&= 1 + \sin\theta\cos\theta \qquad \text{Hence Proved}
\end{aligned}$$

Main 2012

41. Prove that $\dfrac{\tan^2\theta}{(\sec\theta - 1)^2} = \dfrac{1 + \cos\theta}{1 - \cos\theta}$.

Sol :

$$\begin{aligned}
\frac{\tan^2\theta}{(\sec\theta - 1)^2} &= \frac{\sec^2\theta - 1}{(\sec\theta - 1)^2} \\
&= \frac{(\sec\theta - 1)(\sec\theta + 1)}{(\sec\theta - 1)^2}
\end{aligned}$$

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(c) $4\sqrt{3}$ cm (d) 3 cm

Sol :

In ΔABC, $\sin 30^\circ = \frac{BC}{AC}$

$\frac{1}{2} = \frac{BC}{6}$

$BC = 3$ cm

Thus (d) is correct option.

7. If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

Sol :

We have $\sin A = \frac{1}{2}$

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$

Now, Base $= \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \frac{\text{Base}}{\text{Perpendicular}}$

$= \frac{\sqrt{3}}{1} = \sqrt{3}$

Hence, the required value of $\cot A$ is $\sqrt{3}$.
Thus (a) is correct option.

8. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is

(a) 0° (b) 90°

(c) 60° (d) 30°

Sol :

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ$ ...(1)

and $\cos\beta = 0$

$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ$ ...(2)

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

9. If ΔABC is right angled at C, then the value of $\sec(A + B)$ is

(a) 0 (b) 1

(c) $\frac{2}{\sqrt{3}}$ (d) not defined

Sol :

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle B = 180^\circ - \angle C$

$= 180^\circ - 90^\circ = 90^\circ$

Now, $\sec(A + B) = \sec 90^\circ$ not defined

Thus (d) is correct option.

10. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90^\circ)$ then the value of $\tan\theta$ is

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

Sol :

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$

$\tan\theta + 1 = \sqrt{2}$

$\tan\theta = \sqrt{2} - 1$

Thus (a) is correct option.

11. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

Sol :

We have $\cos A = \frac{4}{5}$

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

Perpendicular $= \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16} = 3$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

Thus (b) is correct option.

12. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

Sol :

We have $\sin\theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Base $= \sqrt{b^2 - a^2}$

So, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

Thus (c) is correct option.

13. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90^\circ$, then the value of

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$\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) 0

Sol :

We have $\cos 9\alpha = \sin\alpha \quad$ where $9\alpha < 90^\circ$

$$\sin(90^\circ - 9\alpha) = \sin\alpha$$
$$90^\circ - 9\alpha = \alpha$$
$$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$$
$$\tan 5\alpha = \tan(5 \times 9^\circ)$$
$$= \tan 45^\circ = 1 \left[\tan 45^\circ = 1\right]$$

Thus (c) is correct option.

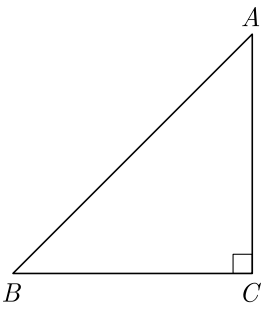
14. If ΔABC is right angled at C, then the value of $\cos(A+B)$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Sol :

We know that in ΔABC,



$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$
$$A + B = 90^\circ$$
$$\cos(A+B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

15. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha+\beta)$ is

(a) 0° (b) 30°

(c) 60° (d) 90°

Sol :

Given, $\sin\alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos\beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$$\alpha + \beta = 30^\circ + 60^\circ = 90^\circ$$

Thus (d) is correct option.

16. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Sol :

Given, $4\tan\theta = 3$

$$\tan\theta = \frac{3}{4} \quad \ldots(i)$$
$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$$
$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1}$$
$$= \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

17. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$

(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

Sol :

We know, $\sec^2\theta - \tan^2\theta = 1$

Substituting $\sec\theta = \frac{x}{p}$ and $\tan\theta = \frac{y}{q}$ in above equation we have

$$\left(\frac{x}{p}\right)^2 - \left(\frac{y}{q}\right)^2 = 1$$
$$x^2q^2 - y^2p^2 = p^2q^2$$

Thus (d) is correct option.

18. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Sol :

We have $\tan\theta = \frac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} - b}{a\frac{\sin\theta}{\cos\theta} + b} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2-b^2}{a^2+b^2}$$

Thus (d) is correct option.

19. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$

(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

NODIA

Sol :

$$\begin{aligned}\cos^4 A - \sin^4 A &= (\cos^2 A)^2 - (\sin^2 A)^2 \\ &= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A) \\ &= (\cos^2 A - \sin^2 A)(1) \\ &= \cos^2 A - (1 - \cos^2 A) \\ &= 2\cos^2 A - 1\end{aligned}$$

Thus (d) is correct option.

20. If $\sec 5A = \operatorname{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

(a) 15° (b) 5°
(c) 20° (d) 10°

Sol :

We have, $\sec 5A = \operatorname{cosec}(A + 30^\circ)$

$$\begin{aligned}\sec 5A &= \sec[90^\circ - (A - 30^\circ)] \\ \sec 5A &= \sec(60^\circ - A) \\ 5A &= 60^\circ - A \\ 6A &= 60^\circ \Rightarrow A = 10^\circ\end{aligned}$$

Thus (d) is correct option.

21. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0 (b) 1/2
(c) 1 (d) 3/2

Sol :

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$\begin{aligned}(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta &= \sin\theta\cos\theta \\ x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta &= \sin\theta\cos\theta \\ x\sin\theta(\sin^2\theta + \cos^2\theta) &= \sin\theta\cos\theta \\ x\sin\theta &= \sin\theta\cos\theta \\ x &= \cos\theta\end{aligned}$$

Now, $x\sin\theta = y\cos\theta$

$$\begin{aligned}\cos\theta\sin\theta &= y\cos\theta \\ y &= \sin\theta\end{aligned}$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Thus (c) is correct option.

22. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2$ is equal to

(a) $\sqrt{mn}$ (b) $\sqrt{\frac{m}{n}}$
(c) $4\sqrt{mn}$ (d) None of these

Sol :

Given, $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$$\begin{aligned}m^2 - n^2 &= (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2 \\ &= 4\tan\theta\sin\theta \\ &= 4\sqrt{\tan^2\theta\sin^2\theta} \\ &= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}} \\ &= 4\sqrt{\sin^2\theta\frac{(1 - \cos^2\theta)}{\cos^2\theta}} \\ &= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta} \\ &= 4\sqrt{\tan^2\theta - \sin^2\theta} \\ &= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)} \\ &= 4\sqrt{mn}\end{aligned}$$

Thus (c) is correct option.

23. If $f(x) = \cos^2 x + \sec^2 x$, then $f(x)$

(a) ≥ 1 (b) ≤ 1
(c) ≥ 2 (d) ≤ 2

Sol : ≥ 2

$$\begin{aligned}f(x) &= \cos^2 x + \sec^2 x \\ &= \cos^2 x + \sec^2 x - 2 + 2 \\ &= \cos^2 x + \sec^2 x - 2\cos x \cdot \sec x + 2 \\ &= (\cos x - \sec x)^2 + 2\end{aligned}$$

We know that, square of any expression is always greater than equal to zero.

$f(x) \geq 2$ Hence proved.

Thus (c) is correct option.

24. **Assertion (A):** For $0 < \theta \leq 90^\circ$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.
Reason (R): $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Sol :

According to trigonometry identity

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$$

$$(\operatorname{cosec}\theta - \cot\theta) = \frac{1}{(\operatorname{cosec}\theta + \cot\theta)}$$

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$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta \qquad \text{Hence Proved}$$

Main 2008

47. Prove the identity $\dfrac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A$.

Sol :

We have $\dfrac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A$

$$\text{LHS} = \frac{\sin A}{1+\cos A}$$

$$= \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{\sin A(1-\cos A)}{1^2 - \cos^2 A}$$

$$= \frac{\sin A(1-\cos A)}{\sin^2 A}$$

$$= \frac{1-\cos A}{\sin A}$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A - \cot A$$

$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2007

48. Prove that : $\dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Sol :

We have $\dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

$$\text{LHS} = \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1}$$

$$= \operatorname{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$$

$$= \operatorname{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}\right]$$

$$= \frac{1}{\sin^2\theta}\sin\theta\left[\frac{(1+\sin\theta) - (1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta \qquad \text{Hence Proved}$$

Main 2006

49. Prove that $\dfrac{\sin\theta\tan\theta}{1-\cos\theta} = 1 + \sec\theta$.

Sol :

We have $\dfrac{\sin\theta\tan\theta}{1-\cos\theta} = 1 + \sec\theta$

$$\text{LHS} = \frac{\sin\theta\tan\theta}{1-\cos\theta}$$

$$= \frac{\sin\theta \cdot \frac{\sin\theta}{\cos\theta}}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\cos\theta(1-\cos\theta)}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta(1-\cos\theta)}$$

$$= \frac{1+\cos\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$$

$$= \sec\theta + 1 \qquad \text{Hence proved}$$

Comp 2006

50. Prove that :
$\dfrac{1}{\operatorname{cosec} A - \cot A} - \dfrac{1}{\sin A} = \dfrac{1}{\sin A} - \dfrac{1}{\operatorname{cosec} A + \cot A}$.

Sol :

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

$$\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\frac{\operatorname{cosec} A + \cot A + \operatorname{cosec} A - \cot A}{(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot a)} = \frac{2}{\sin A}$$

$$\frac{2\operatorname{cosec} A}{\operatorname{cosec}^2 A - \cot^2 A} = \frac{2}{\sin A}$$

$$\frac{2\frac{1}{\sin A}}{1} = \frac{2}{\sin A}$$

$$\frac{2}{\sin A} = \frac{2}{\sin A} \qquad \text{Hence Proved.}$$

Main 2005

51. Prove that $(1+\tan A)^2 + (1-\tan A)^2 = 2\sec^2 A$.

Sol :

$$\text{LHS} = (1+\tan A)^2 + (1-\tan A)^2$$

$$= 1^2 + \tan^2 A + 2\tan A + 1^2 + \tan^2 A - 2\tan A$$

$$= 2 + 2\tan^2 A = 2(1+\tan^2 A)$$

$$= 2\sec^2 A \qquad \text{Hence proved}$$

Comp 2005

52. Prove that: $\dfrac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

Sol :

We have $\dfrac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

$$\text{LHS} = \frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta}$$

$$= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta + 1}$$

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Sol :

$$\text{LHS} = \frac{\sin}{1+\cot A} - \frac{\cos A}{1+\tan A}$$
$$= \frac{\sin A}{1+\frac{\cos A}{\sin A}} - \frac{\cos A}{1+\frac{\sin A}{\cos A}}$$
$$= \frac{\sin^2 A}{\sin A+\cos A} - \frac{\cos^2 A}{\cos A+\sin A}$$
$$= \frac{\sin^2 A}{\sin A+\cos A} - \frac{\cos^2 A}{\sin A+\cos A}$$
$$= \frac{\sin^2 A - \cos^2 A}{\sin A+\cos A}$$
$$= \frac{(\sin A+\cos A)(\sin A-\cos A)}{\sin A+\cos A}$$
$$= \sin A - \cos A$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2020

31. Prove that : $\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1} = \dfrac{1+\cot\theta}{\sin\theta}$

Sol :

$$\text{LHS} = \frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$$
$$= \frac{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}-1}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}+1}$$
$$= \frac{\sin\theta(\cos\theta+1-\sin\theta)}{\sin\theta(\cos\theta-1+\sin\theta)}$$
$$= \frac{\sin\theta\cos\theta+\sin\theta-\sin^2\theta}{\sin\theta(\cos\theta+\sin\theta-1)}$$
$$= \frac{\sin\theta\cos\theta+\sin\theta-(1-\cos^2\theta)}{\sin\theta(\cos\theta+\sin\theta-1)}$$
$$= \frac{\sin\theta(\cos\theta+1)-(1-\cos^2\theta)}{\sin\theta(\cos\theta+\sin\theta-1)}$$
$$= \frac{(1+\cos\theta)(\sin\theta-1+\cos\theta)}{\sin\theta(\cos\theta+\sin\theta-1)}$$
$$= \frac{1+\cos\theta}{\sin\theta} = \text{RHS}$$

Main 2019

32. Prove that
$(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$

Sol :

LHS
$$= (\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)$$
$$= \left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$
$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$$
$$= \frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos\theta}\left(\frac{1}{\sin\theta\cos\theta}\right)$$
$$= 1$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2019

33. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$.

Sol :

Given, $\sin\theta+\cos\theta=\sqrt{3}$

Squaring above equation, we have

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=3$$
$$1+2\sin\theta\cos\theta=3$$
$$2\sin\theta\cos\theta=3-1=2$$
$$\sin\theta\cos\theta=1$$

Now, $\tan\theta+\cot\theta = \dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$
$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

Substituting value of $\sin\theta\cos\theta$ we have

$$\tan\theta+\cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

Main 2018

34. Prove that $\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta} = \tan\theta+\cot\theta$.

Sol :

$$\sec^2\theta+\operatorname{cosec}^2\theta = 1+\tan^2\theta+1+\cot^2\theta$$
$$= \tan^2\theta+\cot^2\theta+2$$
$$= \tan^2\theta+\cot^2\theta+2\tan\theta\cot\theta$$
$$= (\tan\theta+\cot\theta)^2$$
$$\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta} = \tan\theta+\cot\theta \qquad \text{Hence proved}$$

Comp 2018

35. Prove that
$(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2 = 7+\tan^2\theta+\cot^2\theta$

Sol :

$$\text{LHS} = (\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2$$
$$= (\sin^2\theta+\operatorname{cosec}^2\theta+2\sin\theta\operatorname{cosec}\theta)+$$
$$+(\cos^2\theta+\sec^2\theta+2\cos\theta\sec\theta)$$
$$= (\sin^2\theta+\cos^2\theta)+(\operatorname{cosec}^2\theta+\sec^2\theta)$$
$$+2\sin\theta\times\frac{1}{\sin\theta}+2\cos\theta\times\frac{1}{\cos\theta}$$
$$= 1+(1+\cot^2\theta)+(1+\tan^2\theta)+2+2$$
$$= 7+\tan^2\theta+\cot^2\theta$$
$$= \text{RHS}$$

Comp 2017

36. Prove that :

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CHAPTER 21

TRIGONOMETRIC IDENTITIES

MULTIPLE CHOICE QUESTIONS

NODIA

Main 2024

1. Statement 1: $\sin^2\theta + \cos^2\theta = 1$
Statement 2: $\text{cosec}^2\theta + \cot^2\theta = 1$
Which of the following is valid?
(a) only 1 (b) only 2
(c) both 1 and 2 (d) neither 1 nor 2

Sol :

Correct equations are

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

Thus only 1 is valid.

Main 2023

2. $(1+\sin A)(1-\sin A)$ is equal to:
(a) $\text{cosec}^2 A$ (b) $\sin^2 A$
(c) $\sec^2 A$ (d) $\cos^2 A$

Sol :

$$\begin{aligned}(1+\sin A)(1-\sin A) &= 1-\sin^2 A \\ &= 1-\sin^2 A \\ &= \cos^2 A \quad [\sin^2\theta + \cos^2\theta = 1]\end{aligned}$$

Thus (d) is correct option.

Main 2022 Sem II

3. Prove that : $1 + \dfrac{\tan^2\theta}{1+\sec\theta} = \sec\theta$

Sol :

$$\begin{aligned}\text{L.H.S.} &= 1 + \frac{\tan^2}{1+\sec\theta} \\ &= 1 + \frac{\sec^2\theta - 1}{1+\sec\theta} \\ &= 1 + \frac{(\sec\theta+1)(\sec\theta-1)}{(1+\sec\theta)} \\ \text{z} \qquad &= 1 + \sec\theta - 1 \\ &= \sec\theta \\ &= \text{RHS}\end{aligned}$$

Main 2022 Sem II

4. Prove that : $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol :

$$\begin{aligned}\text{L.H.S.} &= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} \\ &= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} \\ &= \frac{2}{1-\sin^2\theta} \\ &= \frac{2}{\cos^2\theta} \\ &= 2\sec^2\theta \\ &= \text{RHS} \qquad \text{Hence, proved}\end{aligned}$$

5. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is
(a) 1 (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Sol :

We have $\sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

$$\sin\theta = \sin(90^\circ - \theta)$$

$$\theta = 90^\circ - \theta \;\Rightarrow\; \theta = 45^\circ$$

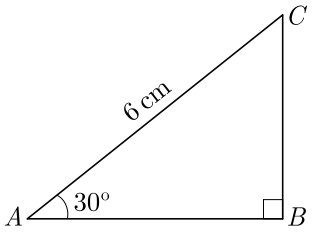
Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Thus (c) is correct option.

6. In the adjoining figure, the length of BC is



(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

$$= \frac{\sec\theta + 1}{\sec\theta - 1}$$
$$= \frac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1}$$
$$= \frac{1 + \cos\theta}{1 - \cos\theta}$$ Hence proved.

Comp 2012

42. Prove that :
$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta.$$

Sol :

We have $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta.$

$$\text{LHS} = \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta}$$
$$= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1 - \tan\theta)\tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$
$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$
$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$
$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$
$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

Comp 2011

43. Prove that: $\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} + \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = 2\sec\theta.$

Sol :

We have $\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} + \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = 2\sec\theta$

$$\text{LHS} = \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} + \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}$$
$$= \sqrt{\frac{(1 + \sin\theta)}{(1 - \sin\theta)} \times \frac{(1 + \sin\theta)}{(1 + \sin\theta)}} + \sqrt{\frac{(1 - \sin\theta)}{(1 + \sin\theta)} \times \frac{(1 - \sin\theta)}{(1 - \sin\theta)}}$$
$$= \sqrt{\frac{(1 + \sin\theta)^2}{(1 - \sin^2\theta)}} + \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}}$$
$$= \sqrt{\frac{(1 + \sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1 + \sin\theta}{\cos\theta} + \frac{1 - \sin\theta}{\cos\theta}$$
$$= \frac{1 + \sin\theta + 1 - \sin\theta}{\cos\theta}$$
$$= \frac{2}{\cos\theta}$$
$$= 2\sec\theta$$ Hence Proved

Comp 2010

44. Prove that
$(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta).$

Sol :

$$\text{LHS} = (1 - \sin\theta + \cos\theta)^2$$
$$= 1 + \sin^2\theta + \cos^2\theta - 2\sin\theta - 2\sin\theta\cos\theta + 2\cos\theta$$
$$= 1 + 1 - 2\sin\theta - 2\sin\theta\cos\theta + 2\cos\theta$$
$$= 2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta$$
$$= 2(1 + \cos\theta) - 2\sin\theta(1 + \cos\theta)$$
$$= (1 + \cos\theta)(2 - 2\sin\theta)$$
$$= 2(1 + \cos\theta)(1 - \sin\theta)$$ Hence Proved

Main 2009, Main 2004

45. Prove the following identity
$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2\,\text{cosec}\,A.$

Sol :

We have $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2\,\text{cosec}\,A$

$$\text{LHS} = \frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$$
$$= \frac{\sin^2 A + (1 + \cos A)^2}{\sin A\,(1 + \cos A)}$$
$$= \frac{\sin^2 A + 1^2 + \cos^2 A + 2\cos A}{\sin A\,(1 + \cos A)}$$
$$= \frac{(\sin^2 A + \cos^2 A) + 1 + 2\cos A}{\sin A\,(1 + \cos A)}$$
$$= \frac{1 + 1 + 2\cos A}{\sin A\,(1 + \cos A)}$$
$$= \frac{2\,(1 + \cos A)}{\sin A\,(1 + \cos A)}$$
$$= \frac{2}{\sin A}$$
$$= 2\,\text{cosec}\,A$$ Hence proved

Comp 2009

46. Prove that : $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta - 1} = \sec\theta + \tan\theta$

Sol :

We have $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta - 1} = \sec\theta + \tan\theta$

$$\text{LHS} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$
$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$
$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

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Thus assertion is True.
Reason : It is a Trigonometric Identity which is used in assertion. Thus reason is also true and correct explanation of assertion.
Thus (a) is correct option.

25. **Assertion :** $\sin^2 67^\circ + \cos^2 67^\circ = 1$
Reason : For any value of $\theta, \sin^2\theta + \cos^2\theta = 1$
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $\sin^2\theta + \cos^2\theta = 1$

$\sin^2 67^\circ + \cos^2 67^\circ = 1$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
Thus (a) is correct option.

26. **Assertion :** The value of $\sin\theta = \frac{4}{3}$ is not possible.
Reason : Hypotenuse is the largest side in any right angled triangle.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

$$\sin\theta = \frac{P}{H} = \frac{4}{3}$$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.
Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
Thus (a) is correct option.

27. **Assertion :** The value of $\sec^2 10^\circ - \cot^2 80^\circ$ is 1.
Reason : The value of $\sin 30^\circ = \frac{1}{2}$.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

We have,

$$\sec^2 10^\circ - \cot^2 80^\circ = \sec^2 10^\circ - \cot^2(90^\circ - 10^\circ)$$
$$= \sec^2 10^\circ - \tan^2 10^\circ = 1$$

Also, $\sin 30^\circ = \frac{1}{2}$

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A) Thus (b) is correct option.

THREE MARKS QUESTIONS

Comp 2023

28. Prove that
$(1 + \tan A - \sec A) \times (1 + \tan A + \sec A) = 2\tan A$

Sol :

$$\text{LHS} = (1 + \tan A - \sec A) \times (1 + \tan A + \sec A)$$
$$= (1 + \tan A)^2 - \sec^2 A$$
$$= 1 + \tan^2 A + 2\tan A - \sec^2 A$$
$$= \sec^2 A + 2\tan A - \sec^2 A$$
$$= 2\tan A$$
$$= \text{RHS}$$

Comp 2022

29. Prove that : $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Sol :

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$
$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$
$$= \tan A \frac{[1 - 2(1 - \cos^2 A)]}{(2\cos^2 A - 1)}$$
$$= \tan A \frac{[1 - 2 + 2\cos^2 A)]}{(2\cos^2 A - 1)}$$
$$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$
$$= \tan A \qquad \text{Hence Proved}$$

Main 2020

30. Prove that $\frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A} = \sin A - \cos A$

NODIA

$$= \frac{1-\cos^2\theta}{\cos\theta+1}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta+1}$$

$$= 1-\cos\theta \qquad ...(1)$$

Now, $\text{RHS} = 2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

$$= 2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}}$$

$$= 2+\frac{\sin^2\theta}{\cos\theta-1}$$

$$= 2+\frac{1-\cos^2\theta}{\cos\theta-1}$$

$$= 2-\frac{(\cos^2\theta-1)}{(\cos\theta-1)}$$

$$= 2-\frac{(\cos\theta-1)(\cos\theta+1)}{\cos\theta-1}$$

$$= 2-(\cos\theta+1)$$

$$= 1-\cos\theta$$

$$= \text{LHS} \qquad \text{Hence Proved}$$

Main 2004

53. If $\operatorname{cosec}\theta-\cot\theta=\sqrt{2}\cot\theta$, then prove that $\operatorname{cosec}\theta+\cot\theta=\sqrt{2}\operatorname{cosec}\theta$.

Sol :

We have $\operatorname{cosec}\theta-\cot\theta=\sqrt{2}\cot\theta$

Squaring both sides we have

$$\operatorname{cosec}^2\theta+\cot^2\theta-2\operatorname{cosec}\theta\cot\theta = 2\cot^2\theta$$

$$\operatorname{cosec}^2\theta-\cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta) = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta+\cot\theta)\sqrt{2}\cot\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$\operatorname{cosec}\theta+\cot\theta = \sqrt{2}\operatorname{cosec}\theta$$

Hence Proved.

Comp 2004

54. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$.

Sol :

We have $\sin\theta+\cos\theta=\sqrt{3}$

Squaring both the sides, we get

$$(\sin\theta+\cos\theta)^2=(\sqrt{3})^2$$

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=3$$

$$1+2\sin\theta\cos\theta=3$$

$$2\sin\theta\cos\theta=3-1=2$$

$$\sin\theta\cos\theta=1 \qquad ...(1)$$

Now $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$$=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

or $\tan\theta+\cot\theta=\frac{1}{\sin\theta\cos\theta}$

Substituting the value of $\sin\theta\cos\theta$ from equation (1) we have

$$\tan\theta+\cot\theta=\frac{1}{1}=1$$

Hence, $\tan\theta+\cot\theta=1$

Comp 2003

55. Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

Sol :

We have $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

$$\text{LHS}=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$

$$=\frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$=\frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}$$

$$=\frac{\tan^2\theta+1+\tan\theta}{\tan\theta}$$

$$=\tan\theta+\cot\theta+1$$

$$=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$$

$$=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1$$

$$=\frac{1}{\sin\theta\cos\theta}+1$$

$$=\operatorname{cosec}\theta\sec\theta+1$$

$$=1+\sec\theta\operatorname{cosec}\theta \qquad \text{Hence Proved}$$

Main 2002

56. If $\tan A+\sin A=m$ and $\tan A-\sin A=n$, show that $m^2-n^2=4\sqrt{mn}$.

Sol :

We have $\tan A+\sin A=m$

and $\tan A-\sin A=n$

NODIA

$$BD = 10\text{ m}$$

In ΔCDB, $\tan 30^\circ = \frac{CB}{DB} = \frac{CB}{10}$

$$\frac{1}{\sqrt{3}} = \frac{CB}{10}$$

$$CB = \frac{10}{\sqrt{3}}$$

Also, $\cos 30^\circ = \frac{DB}{DC} = \frac{10}{DC}$

$$DC = \frac{20}{\sqrt{3}} = AC$$

Height of tree,

$$AC + CB = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3}\text{ m}$$

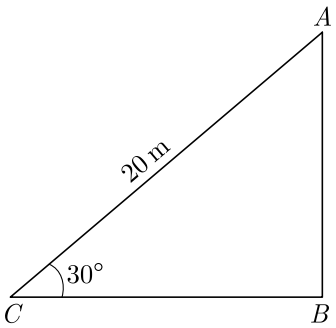
Thus (a) is correct option.

8. A circle artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the angle made by the rope with the ground level is 30°, is

(a) 5 m (b) 10 m
(c) 15 m (d) 20 m

Sol :

Let AB be the vertical pole and CA be the 20 m long rope such that its one end A is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.



In ΔABC, we have

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20}$$

$$AB = 10\text{ m}$$

Hence, the height of the pole is 10 m.
Thus (b) is correct option.

9. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, then the height of kite is

(a) 75 m (b) 78.05 m
(c) 226 m (d) None of these

Sol :

Length of the string of the kite,

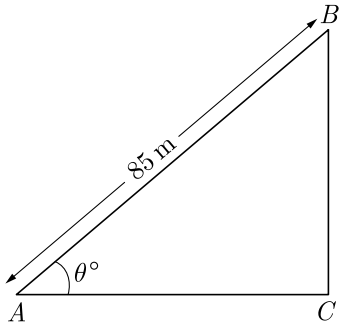
$$AB = 85\text{ m}$$

and $\tan\theta = \frac{15}{8}$

$$\cot\theta = \frac{8}{15}$$

$$\text{cosec}^2\theta - 1 = \frac{64}{225}$$

$$\text{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$$



$$\text{cosec}\,\theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$$

$$\sin\theta = \frac{15}{17}$$

In ΔABC, $\sin\theta = \frac{BC}{AB}$

$$\frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75\text{ m}$$

Thus height of kite is 75 m.
Thus (a) is correct option.

10. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

(a) 12 m (b) 10 m
(c) 8 m (d) 6 m

Sol :

Height of big pole $CD = 20\text{ m}$

Height of small pole $AB = 14\text{ m}$

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$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

When $\sec\theta = x + \frac{1}{4x}$ and $\tan\theta = x - \frac{1}{4x}$ we have

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right)$$
$$= 2x$$

When $\sec\theta = x + \frac{1}{4x}$ and $\tan\theta = -\left(x - \frac{1}{4x}\right)$ we have

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left\{-\left(x - \frac{1}{4x}\right)\right\}$$
$$= x + \frac{1}{4x} - x + \frac{1}{4x}$$
$$= \frac{2}{4x} = \frac{1}{2x}$$

FOUR MARK QUESTIONS

Main 2023

61. Prove the following identity
$(\sin^2\theta - 1)(\tan^2\theta + 1) + 1 = 0$

Sol :

$$\text{LHS} = (\sin^2\theta - 1)(\tan^2\theta + 1) + 1$$
$$= (-\cos^2\theta)(\sec^2\theta) + 1$$
$$= (-\cos^2\theta)\left(\frac{1}{\cos^2\theta}\right) + 1$$
$$= \frac{-\cos^2\theta \times 1}{\cos^2\theta} + 1$$
$$= -1 + 1$$
$$= 0 = \text{RHS}$$ Hence, proved.

Main 2020

62. Prove the identity $\left(\frac{1-\tan\theta}{1-\cot\theta}\right) = \tan^2\theta$

Sol :

$$\text{LHS} = \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$$
$$= \left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2$$
$$= \left(\frac{1-\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}\right)^2$$
$$= \left(\frac{\tan\theta - \tan^2\theta}{\tan\theta - 1}\right)^2$$
$$= \tan^2\theta\left(\frac{1-\tan\theta}{\tan\theta-1}\right)^2$$
$$= \tan^2\theta(-1)^2$$
$$= \tan^2\theta$$
$$= \text{RHS}$$ Hence proved.

Main 2018

63. Prove that
$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2.$

Sol :

$$\text{LHS} = (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$$
$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$
$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$
$$= \frac{[(\sin\theta + \cos\theta) - 1][(\sin\theta + \cos\theta) + 1]}{\sin\theta\cos\theta}$$
$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$
$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$
$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$
$$= 2 = \text{RHS}$$ Hence proved.

Main 2017

64. Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.$

Sol :

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^3\theta - 1)}$$
$$= \frac{\sin\theta[1 - 2(1 - \cos^2\theta)]}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta(1 - 2 + 2\cos^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta(2\cos^2\theta - 1)}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta$$ Hence proved.

Main 2016

65. Prove that $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \cos\theta + \sin\theta.$

Sol :

We have $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \cos\theta + \sin\theta$

$$\text{LHS} = \frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$$
$$= \frac{\sin\theta}{\left[1 - \frac{\cos\theta}{\sin\theta}\right]} + \frac{\cos\theta}{\left[1 - \frac{\cos\theta}{\sin\theta}\right]}$$
$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta}$$

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$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$$
$$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)}$$
$$= \sin\theta + \cos\theta$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2016

66. Prove that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$.

Sol :

We have $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

$$\text{LHS} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$$
$$= \operatorname{cosec}\theta\left[\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}\right]$$
$$= \operatorname{cosec}\theta\left[\frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}\right]$$
$$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}\right)$$
$$= \frac{2\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$$
$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$
$$= 2\sec^2\theta = \text{RHS} \qquad \text{Hence Proved}$$

Main 2015

67. Prove that : $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

Sol :

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$$
$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}$$
$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$
$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$
$$= \operatorname{cosec} A - \cot A \quad \text{Hence Proved.}$$

Comp 2015

68. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2 - 1) = 2p$

Sol :

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

$$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)[(\cos\theta + \sin\theta)^2 - 1]$$
$$= (\sec\theta + \operatorname{cosec}\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$$
$$= (\sec\theta + \operatorname{cosec}\theta)[1 + 2\sin\theta\cos\theta - 1]$$
$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$
$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right)2\sin\theta\cos\theta$$
$$= 2(\sin\theta + \cos\theta) = 2p \quad \text{Hence Proved.}$$

Main 2014

69. Prove the identity
$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$.

Sol :

We have
$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$.

$$\text{LHS} = (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$
$$= (\sin\theta + \cos\theta)\frac{1}{\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$
$$= \sec\theta + \operatorname{cosec}\theta$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2013

70. Prove that :
$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta + \sin\theta)^2} = \frac{\sec\theta\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$

Sol :

$$\text{LHS} = \frac{\cot^3\theta\sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{\frac{\cos^3\theta}{\sin^3\theta} \times \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta} \times \cos^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{\cos^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{1 - \sin\theta\cos\theta}{\cos\theta + \sin\theta}$$
$$= \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$
$$= \frac{\operatorname{cosec}\theta\sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta} \qquad \text{Hence Proved}$$

Main 2011

71. Prove that
$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)\sec^2 A = \tan A$.

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Sol :

We have

$(\text{cosec}\,A - \sin A)(\sec A - \cos A)\sec^2 A = \tan A$

$$\text{LHS} = (\text{cosec}\,A - \sin A)(\sec A - \cos A)\sec^2 A$$
$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\sec^2 A$$
$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\frac{1}{\cos^2 A}$$
$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A}$$
$$= \frac{\sin A}{\cos A}$$
$$= \tan A$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

Comp 2008

72. If $\text{cosec}\,\theta + \cot\theta = p$, then prove that $\cos\theta = \dfrac{p^2-1}{p^2+1}$.

Sol :

$$\frac{p^2-1}{p^2+1} = \frac{(\text{cosec}\,\theta + \cot\theta)^2 - 1}{(\text{cosec}\,\theta + \cot\theta)^2 + 1}$$
$$= \frac{\text{cosec}^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta - 1}{\text{cosec}^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta + 1}$$
$$= \frac{1 + \cot^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta - 1}{\text{cosec}^2\theta + \text{cosec}^2\theta - 1 + 2\,\text{cosec}\,\theta\cot\theta + 1}$$
$$= \frac{2\cot\theta(\cot\theta + \text{cosec}\,\theta)}{2\,\text{cosec}\,\theta(\text{cosec}\,\theta + \cot\theta)}$$
$$= \frac{\cos\theta}{\sin\theta} \times \sin\theta = \cos\theta$$

Main 2007

73. Prove that : $(\cot\theta - \text{cosec}\,\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Sol :

We have $\cot\theta - \text{cosec}\,\theta = \dfrac{1-\cos\theta}{1+\cos\theta}$

Now $\cot\theta - \text{cosec}\,\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}$

$$(\cot\theta - \text{cosec}\,\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$
$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$
$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$
$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$
$$= \frac{1-\cos\theta}{1+\cos\theta} \qquad \text{Hence Proved.}$$

Comp 2006

74. If $7\,\text{cosec}\,\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\,\text{cosec}\,\phi = 3$.

Sol :

We have $7\,\text{cosec}\,\phi - 3\cot\phi = 7$

$$7\,\text{cosec}\,\phi - 7 = 3\cot\phi$$
$$7(\text{cosec}\,\phi - 1) = 3\cot\phi$$
$$7(\text{cosec}\,\phi - 1)(\text{cosec}\,\phi + 1) = 3\cot\phi(\text{cosec}\,\phi + 1)$$
$$7(\text{cosec}^2\phi - 1) = 3\cot\phi(\text{cosec}\,\phi + 1)$$
$$7\cot^2\phi = 3\cot\phi(\text{cosec}\,\phi + 1)$$
$$7\cot\phi = 3(\text{cosec}\,\phi + 1)$$
$$7\cot\phi - 3\,\text{cosec}\,\phi = 3 \qquad \text{Hence Proved}$$

Main 2005

75. If $4\sin\theta = 3$, find the value of x if $\sqrt{\dfrac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \dfrac{\sqrt{7}}{x} + \cos\theta$

Sol :

We have $\sin\theta = \dfrac{3}{4}$

or, $\sin^2\theta = \dfrac{9}{16}$

Since $\sin^2\theta + \cos^2 = 1$, we have

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{9}{16} = \frac{7}{16}$$
$$\cos\theta = \frac{\sqrt{7}}{4}$$

and $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \dfrac{3}{\sqrt{7}}$

Thus $\sqrt{\dfrac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \dfrac{\sqrt{7}}{x} + \cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$
$$\frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$
$$\frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$
$$\frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$
$$\frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

Thus $x = \dfrac{4}{3}$

Comp 2004

76. Prove that $\dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

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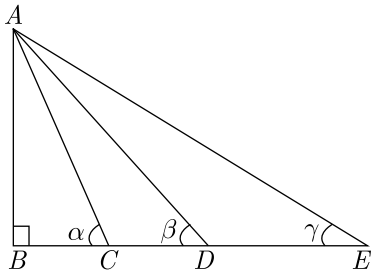
CHAPTER 22

HEIGHT AND DISTANCE

MULTIPLE CHOICE QUESTIONS

SQP 2024

1. An observer at point E, which is at a certain distance from the lamp post AB, finds the angle of elevation of top of lamp post from positions C, D and E as α, β and γ. It is given that B, C, D and E are along a straight line. Which of the following condition is satisfied?



(a) $\tan\alpha > \tan\beta$
(b) $\tan\beta < \tan\gamma$
(c) $\tan\gamma > \tan\alpha$
(d) $\tan\alpha < \tan\beta$

Sol :

It may be easily seen that

$$\alpha > \beta$$

Thus $\tan\alpha > \tan\beta$

Thus (a) is correct option.

2. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

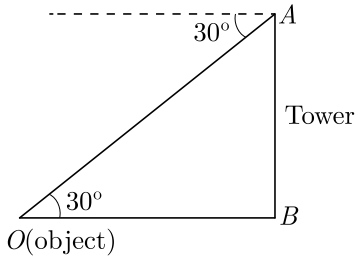
(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m
(c) $75\sqrt{3}$ m (d) 150 m

Sol :

We have $\tan 30^\circ = \dfrac{AB}{OB}$

$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$



Thus (c) is correct option.

3. If the height and length of the shadow of a man are equal, then the angle of elevation of the sun is,

(a) 45° (b) 60°
(c) 90° (d) 120°

Sol :

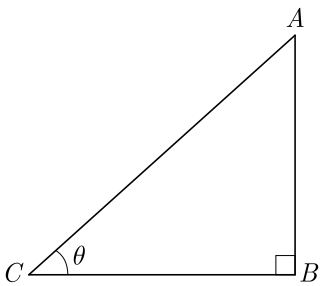
Let AB be the height of a man and BC be the shadow of a man.

$$AB = BC$$

In ΔABC, $\tan\theta = \dfrac{AB}{BC}$

$$\frac{AB}{AB} = \tan\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$



Thus (a) is correct option.

4. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45°. The height of a tree is

(a) 10 m (b) 14 m
(c) 8 m (d) 15 m

Sol :

Let BC be the tree of height h meter. Let AB be

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Sol :

We have $(\operatorname{cosec}\theta+\cot\theta)^2=\dfrac{\sec\theta+1}{\sec\theta-1}$

$$\begin{aligned}
\text{LHS} &= (\operatorname{cosec}\theta+\cot\theta)^2\\
&= \operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta\\
&= \left(\frac{1}{\sin\theta}\right)^2+\left(\frac{\cos\theta}{\sin\theta}\right)^2+2\times\frac{1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}\\
&= \frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}+\frac{2\cos\theta}{\sin^2\theta}\\
&= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta}\\
&= \frac{(1+\cos\theta)^2}{1-\cos^2\theta}\\
&= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}\\
&= \frac{1+\cos\theta}{1-\cos\theta}\\
&= \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}\\
&= \frac{\sec\theta+1}{\sec\theta-1} \qquad \text{Hence Prove.}
\end{aligned}$$

Comp 2000

81. Prove that $\dfrac{\sin A-\cos A-1}{\sin A+\cos A-1}=\dfrac{1}{\sec A-\tan A}$

Sol :

$$\begin{aligned}
\text{LHS} &= \frac{\sin A-\cos A+1}{\sin A+\cos A-1}\\
&= \frac{\sin A-\cos A+1}{\sin A+\cos A-1}\times\frac{1+\sin A}{1+\sin A}\\
&= \frac{(\sin A-\cos A+1)(1+\sin A)}{\sin A+\cos A-1+\sin^2 A+\cos A\sin A-\sin A}\\
&= \frac{(\sin A-\cos A+1)(1+\sin A)}{-1+\cos A+(1-\cos^2 A)+\sin A\cos A}\\
&= \frac{(\sin A-\cos A+1)(1+\sin A)}{\cos A(1-\cos A+\sin A)}\\
&= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A}+\frac{\sin A}{\cos A}\\
&= \sec A+\tan A\\
&= \frac{(\sec A+\tan A)}{(\sec A-\tan A)}\times(\sec A-\tan A)\\
&= \frac{\sec^2 A-\tan^2 A}{\sec A-\tan A}\\
&= \frac{1}{\sec A-\tan A} = \text{RHS}
\end{aligned}$$

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END

Sol :

$$X = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta}$$
$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$
$$= (1 - \sin\theta\cos\theta)$$
$$Y = \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$
$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$
$$= (1 + \sin\theta\cos\theta)$$

Now given expression becomes

$$X + Y = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$
$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$
$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$
$$= 2$$
$$= \text{RHS} \qquad \text{Hence Proved.}$$

Comp 2002

77. In an acute angled triangle ABC if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Sol :

We have $\sin(A+B-C) = \frac{1}{2} = \sin 30^\circ$

$$A + B - C = 30^\circ \qquad \ldots(1)$$

and $\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$$B + C - A = 45^\circ \qquad \ldots(2)$$

Adding equation (1) and (2), we get

$$2B = 75^\circ \Rightarrow B = 37.5^\circ$$

Subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$
$$A - C = -7.5^\circ \qquad \ldots(3)$$

Now $A + B + C = 180^\circ$

$$A + C = 180^\circ - 37.5^\circ$$
$$= 142.5^\circ \qquad \ldots(4)$$

Adding equation (3) and (4), we have

$$2A = 135^\circ \Rightarrow A = 67.5^\circ$$

and, $C = 75^\circ$

Hence, $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

Main 2002

78. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove that $x^2 + y^2 = 1$.

Sol :

We have $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \qquad (1)$

and $x\sin\theta = y\cos\theta$

or, $x = \frac{y\cos\theta}{\sin\theta} \qquad (2)$

Eliminating x from equation (1) and (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$
$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$
$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$
$$y(\sin^2\theta + \cos^2\theta) = \sin\theta$$
$$y = \sin\theta \qquad \ldots(3)$$

Substituting this value of y in equation (2) we have,

$$x = \cos\theta \qquad (4)$$

Squaring and adding equation (3) and (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1 \qquad \text{Hence Proved.}$$

Main 2001

79. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Sol :

We have $\sec\theta + \tan\theta = p \qquad (1)$

Now $\frac{1}{p} = \frac{1}{\sec\theta + \tan\theta} \times \frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$

$$= \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta - \tan\theta$$

or $\frac{1}{p} = \sec\theta - \tan\theta \qquad (2)$

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \frac{1}{p}$,

$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$$

Thus $\cos\theta = \frac{2p}{p^2+1}$

and $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right) = \frac{p^2-1}{2p}$

and $\sin\theta = \tan\theta\cos\theta = \frac{p^2-1}{p^2+1}$

Main 2000

80. Prove that : $(\text{cosec}\,\theta + \cot\theta)^2 = \frac{\sec\theta + 1}{\sec\theta - 1}$

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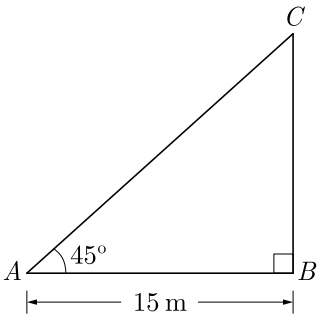
the shadow of tree.



In ΔABC, $CB = 90^\circ$

$$\frac{BC}{BA} = \tan 45^\circ$$

$$BC = AB = 15\text{ m}$$

Thus (d) is correct option.

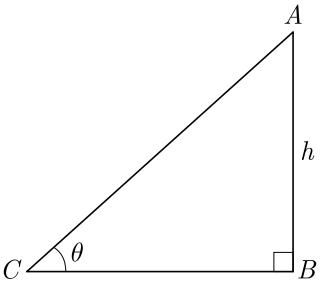
5. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then the angle of elevation of the sun is

(a) 90° (b) 45°

(c) 30° (d) 75°

Sol :

Let AB be the rod of length h, BC be its shadow of length $\sqrt{3}\,h$, θ be the angle of elevation of the sun.

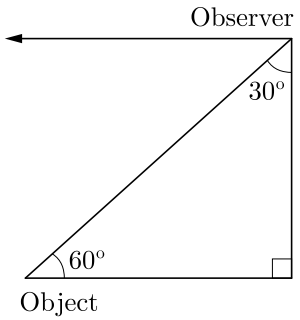


In ΔABC, $\dfrac{h}{\sqrt{3}\,h} = \tan\theta$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Thus (c) is correct option.

6. In the given figure, the positions of the observer and the object are mentioned, the angle of depression is



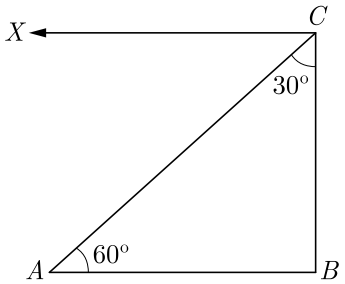
(a) 30° (b) 90°

(c) 60° (d) 45°

Sol :

$$\angle XCA = \angle CAB = 60^\circ$$

Hence, angle of depression $= 60^\circ$



Thus (c) is correct option.

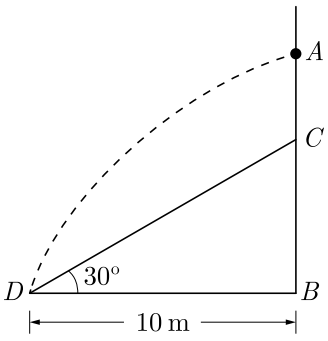
7. A tree is broken by the wind. The top struck the ground at an angle of 30° and at distance of 10 m from its root. The whole height of the tree is $(\sqrt{3} = 1.732)$

(a) $10\sqrt{3}$ m (b) $3\sqrt{10}$ m

(c) $20\sqrt{3}$ m (d) $3\sqrt{20}$ m

Sol :

Let AB be the tree of height x, and AC be the broken part of tree.



Now $AC = CD$

$$\angle CDB = 30^\circ$$

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$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$
$$= (\tan^2 A + \sin^2 A + 2\sin A \tan A) - (\tan^2 A + \sin^2 A - 2\sin A \tan A)$$
$$= \tan^2 A + \sin^2 A + 2\sin A \tan A - \tan^2 A - \sin^2 A + 2\sin A \tan A$$
$$= 4\sin A \tan A$$
$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$
$$= 4\sqrt{\tan^2 A - \sin^2 A}$$
$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$
$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}}$$
$$= 4\sqrt{\frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}}$$
$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$
$$= 4\frac{\sin A \times \sin A}{\cos A}$$
$$= 4\sin A \times \frac{\sin A}{\cos A}$$
$$= 4\sin A \tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

Comp 2002

57. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

Sol :

We have $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$
$$(m^2 + n^2)\cos^2\beta = \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$
$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]\cos^2\beta$$
$$= \cos^2\alpha\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta$$
$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$
$$= \frac{\cos^2\alpha}{\sin^2\beta}$$
$$= n^2 \qquad \text{Hence Proved.}$$

Main 2001

58. Prove that $1 - \frac{\cos^2\theta}{1 + \sin\theta} = \sin\theta$.

Sol :

$$1 - \frac{\cos^2\theta}{1 + \sin\theta} = \frac{1 + \sin\theta - \cos^2\theta}{1 + \sin\theta}$$
$$= \frac{\sin\theta + 1 - \cos^2\theta}{1 + \sin\theta}$$
$$= \frac{\sin\theta + \sin^2\theta}{1 + \sin\theta}$$
$$= \frac{\sin\theta(1 + \sin\theta)}{(1 + \sin\theta)}$$
$$= \sin\theta \qquad \text{Hence proved.}$$

Comp 2001

59. Prove that : $\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Sol :

We have $\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

$$\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$$
$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{\sin\theta\cos\theta + \sin\theta - (1 - \cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{\sin\theta(\cos\theta + 1) - [(1 - \cos\theta)(1 + \cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{(1 + \cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{(1 + \cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{1 + \cos\theta}{\sin\theta}$$
$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \operatorname{cosec}\theta + \cot\theta \qquad \text{Hence Proved}$$

Main 2000

60. If $\sec\theta = x + \frac{1}{4x}, x \neq 0$ find $(\sec\theta + \tan\theta)$.

Sol :

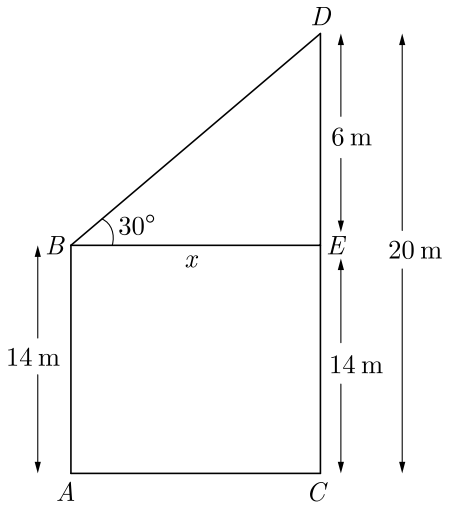
We have $\sec\theta = x + \frac{1}{4x}$ ...(1)

Since, $\tan^2\theta = \sec^2\theta - 1$

Substituting value of $\sec\theta$ we have

$$\tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1$$
$$= x^2 + \frac{2x}{4x} + \frac{1}{16x^2} - 1$$
$$= x^2 + \frac{1}{16x^2} - \frac{1}{2}$$
$$= \left(x - \frac{1}{4x}\right)^2$$

NODIA



$$DE = CD - CE$$
$$= CD - AB \quad [AB = CE]$$
$$= 20 - 14 = 6\text{ m}$$

In $\triangle BDE$, $\sin 30^\circ = \frac{DE}{BD}$

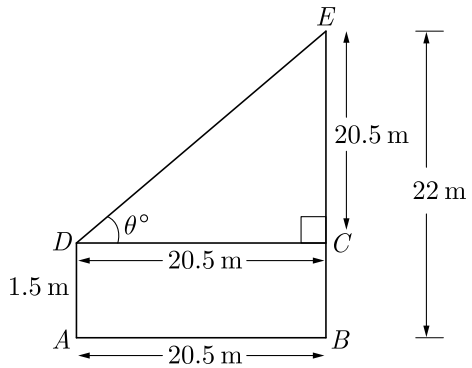
$$\frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12\text{ m}$$

Thus length of wire is 12 m.
Thus (a) is correct option.

11. An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of observer is
(a) 30° (b) 45°
(c) 60° (d) 90°

Sol :

Let $BE = 22\text{ m}$ be the height of the tower and $AD = 1.5\text{ m}$ be the height of the observer. The point D be the observer's eye. We draw $DC \parallel AB$ as shown below.



Then, $AB = 20.5\text{ m} = DC$

and $EC = BE - BC = BE - AD$

$$= 22 - 1.5 = 20.5\text{ m} \quad [BC = AD]$$

Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle DCE$,

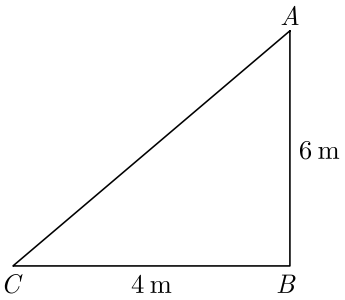
$$\tan\theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$
$$\tan\theta = 1$$
$$\tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

Thus (b) is correct option.

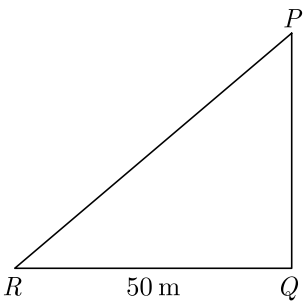
12. A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?
(a) 75 m (b) 100 m
(c) 150 m (d) 50 m

Sol :

Let AB be height of tree and BC its shadow.



Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal i.e $\triangle ABC \sim PQR$



Thus $\frac{AB}{BC} = \frac{PQ}{QR}$

$$\frac{6}{4} = \frac{PQ}{50}$$
$$PQ = \frac{50 \times 6}{4} = 75\text{ m}$$

Thus (a) is correct option.

13. From the top of a 7 m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°, then the height of the tower is

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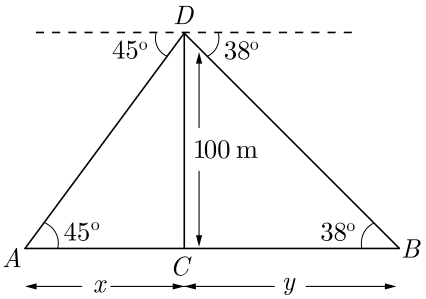
two ships the opposite side of tower. As per given in question we have drawn figure below.



In right ΔACD,

$$\tan 45^\circ = \frac{CD}{AC}$$

$$1 = \frac{100}{x}$$

$$x = 100 \text{ m} \qquad ...(1)$$

In right ΔBCD,

$$\tan 38^\circ = \frac{CD}{CB}$$

$$0.78 = \frac{100}{y}$$

$$y = \frac{100}{0.78} = 128.2 \text{ m}$$

Distance between two ships,

$$AB = AD + DB$$

$$= x + y$$

$$= 100 + 128.2$$

$$= 228.2 \text{ m}$$

$$= 228 \text{ m}$$

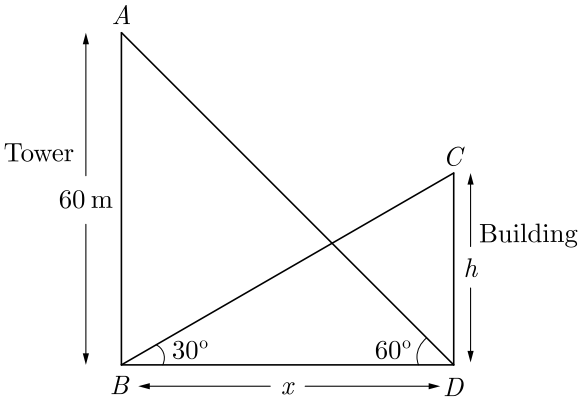
Hence, the distance between two ships A and B is 228 m.

Comp 2023

29. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Sol :

Let AB be the tower of 60 m height and CD be the building of h height. As per given in question we have drawn figure below.



In right ΔABD we have

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Now, in right ΔBCD we have

$$\tan 30^\circ = \frac{CD}{BD} = \frac{h}{x}$$

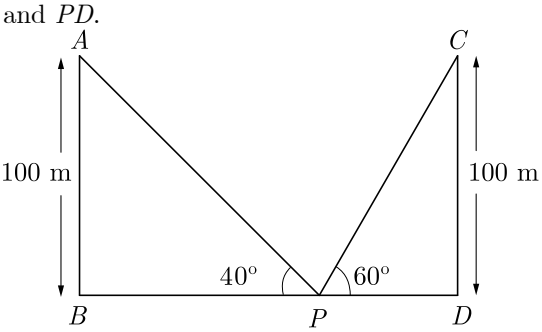
$$\frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

$$h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Hence height of the building is 20 m.

Main 2022 Sem II

30. Two lamp posts AB and CD each of heights 100 m are on either side of the road. P is a point on the road between the two lamp pasts. The angles of elevation of the top of the lamp posts from the point P are 40° and 60°. Find the distance PB and PD.



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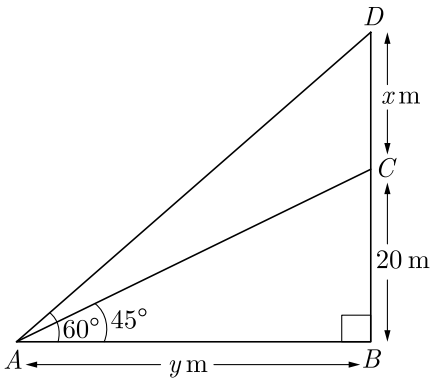
Sol :

Let the distance between pole AB and point P be x and distance between pole CD and point P be y. As per given in question we have drawn figure below.



Now, in $\triangle ABC$, $\dfrac{BC}{AB} = \tan 45^\circ = 1$

$$\frac{20}{y} = 1 \Rightarrow y = 20 \text{ m}$$

i.e. $AB = 20$ m

Now, in $\triangle ABC$, $\dfrac{BD}{AB} = \tan 60^\circ = \sqrt{3}$

$$\frac{BD}{AB} = \sqrt{3}$$

$$\frac{20+x}{20} = \sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

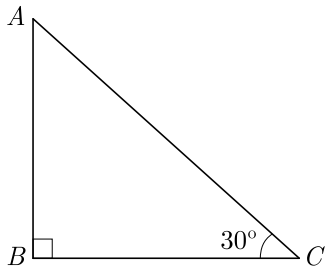
$$x = 20\sqrt{3} - 20$$

$$= 20 \times 0.732$$

$$= 14.64 \text{ m}$$

Thus (a) is correct option.

16. **Assertion :** In the figure, if $BC = 20$ m, then height AB is 11.56 m.



Reason : $\tan\theta = \dfrac{AB}{BC} = \dfrac{\text{perpendicular}}{\text{base}}$ where θ is the angle $\angle ACB$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $\tan 30^\circ = \dfrac{AB}{BC} = \dfrac{AB}{20}$

$$AB = \frac{1}{\sqrt{3}} \times 20$$

$$= \frac{20}{1.73} = 11.56 \text{ m}$$

Both the assertion and reason are correct, reason is the correct explanation of the assertion.
Thus (a) is correct option.

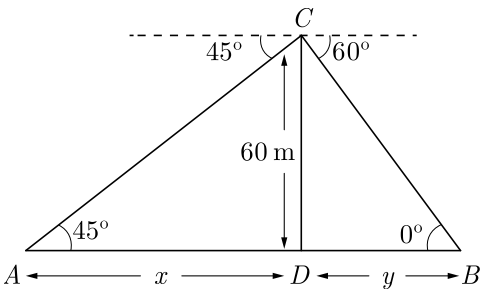
THREE MARKS QUESTIONS

Main 2017

17. The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60° and 45°, respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships. Give your answer correct to the nearest whole number.

Sol :

Let DC be tower of height 100 m. A and B be two ships the opposite side of light house. As per given in question we have drawn figure below.



In right $\triangle ADC$,

$$\tan 45^\circ = \frac{CD}{AD}$$

$$1 = \frac{60}{x}$$

$$x = 60 \text{ m} \quad \ldots(1)$$

In right $\triangle BDC$,

$$\tan 60^\circ = \frac{CD}{DB}$$

$$\sqrt{3} = \frac{60}{y}$$

$$y = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$$

Distance between two ships,

$$AB = AD + DB$$

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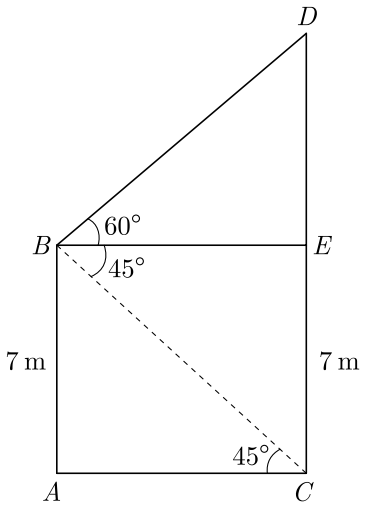
(a) 14.124 m (b) 17.124 m
(c) 19.124 m (d) 15.124 m

Sol :

Let AB be the building and CD be the tower. We draw $BE \perp CD$ as shown below.



Here $CE = AB = 7\text{ m}$

$\angle EBD = 60^\circ$

and $\angle ACB = \angle CBE = 45^\circ$

From $\triangle ACB$, we have

$$\cot 45^\circ = \frac{AC}{AB}$$

$$\frac{AC}{7} = 1 \Rightarrow AC = 7\text{ m}$$

$$BE = AC = 7\text{ m}$$

From $\triangle EBD$, we have

$$\tan 60^\circ = \frac{DE}{BE}$$

$$\frac{DE}{7} = \sqrt{3} \Rightarrow DE = 7\sqrt{3}\text{ m}$$

$$\text{Height of the tower} = (7 + 7\sqrt{3})$$

$$= 7(\sqrt{3} + 1)$$

$$= 7(1.732 + 1)$$

$$= 7 \times 2.732$$

$$= 19.124\text{ m}$$

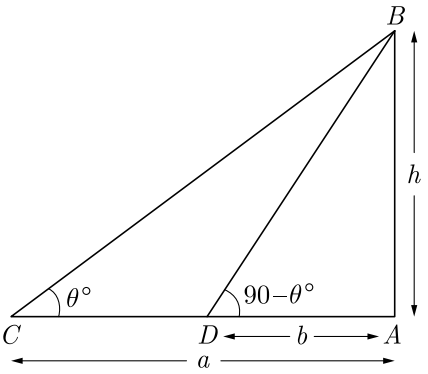
Thus (c) is correct option.

14. The angles of elevation of the top of a tower from the points P and Q at distance of a and b respectively from the base and in the same straight line with it, are complementary. The height of the tower is

(a) ab (b) $\sqrt{ab}$
(c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

Sol :

Let AB be the tower. Let C and D be two points at distance a and b respectively from the base of the tower.



In $\triangle ABC$, $\tan\theta = \frac{AB}{AC}$

$$\tan\theta = \frac{h}{a} \quad \ldots(1)$$

In $\triangle ABD$, $\tan(90^\circ - \theta) = \frac{AB}{AD}$

$$\cot\theta = \frac{h}{b} \quad \ldots(2)$$

From equation (1) and (2), we have

$$\tan\theta \times \cot\theta = \frac{h}{a} \times \frac{h}{b}$$

$$1 = \frac{h^2}{ab} \Rightarrow h = \sqrt{ab}$$

Thus (b) is correct option.

15. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively, then the height of the tower is

(a) 14.64 m (b) 28.64 m
(c) 38.64 m (d) 19.64 m

Sol :

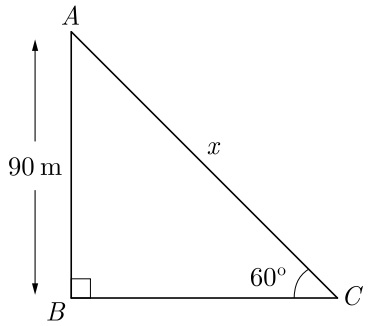
Let the height of the building be BC, $BC = 20\text{ m}$ and height of the tower be CD. Let the point A be at a distance y from the foot of the building.

NODIA

is no slack in the string.

Sol :

As per given in question we have drawn figure below.



In right ΔABC, we have

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{90}{x}$$

$$x = \frac{90 \times 2}{\sqrt{3}} = \frac{180}{\sqrt{3}} = \frac{3 \times 60}{\sqrt{3}}$$

$$= 60\sqrt{3} \ = 60 \times 1.732$$

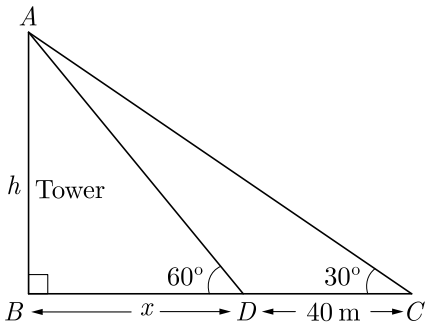
Hence length of string is 103.92 m.

Comp 2010

21. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30°, then when it is 60°. Find the height of the tower.

Sol :

Let AB be the tower of height h. Let BC be the shadow at 60° and BD be shadow at 30°.
As per given in question we have drawn figure below.



In right ΔABC we get,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$$

In right ΔABD we have,

$$\tan 30^\circ = \frac{AB}{BC + 40}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + 40}$$

$$x + 40 = \sqrt{3}\,h$$

$$= \sqrt{3} \times \sqrt{3}\,x$$

$$x + 40 = 3x$$

$$40 = 2x \Rightarrow x = 20 \text{ m}$$

$$h = \sqrt{3} \times 20 = 20\sqrt{3} \text{ m}$$

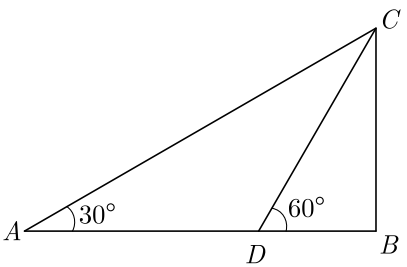
Thus height of tower is $20\sqrt{3}$ m.

Main 2011

22. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

Sol :

As per given information in question we have drawn the figure below.



Here D is first position and A is position after 2 minutes.

Height of the light house,

$$BC = 100 \text{ m}$$

From ΔDBC, $\angle B = 90^\circ$

So, $$\tan 60^\circ = \frac{BC}{BD}$$

$$\sqrt{3} = \frac{100}{BD}$$

$$BD = \frac{100}{\sqrt{3}} \text{m}$$

Now, after time 2 minute boat is at A. New distance from light house is AB and angle is 30°.

From ΔABC, $\angle B = 90^\circ$

So, $$\tan 30^\circ = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

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$$= x + y$$
$$= (60 + 20\sqrt{3})$$
$$= 20(3 + \sqrt{3})$$
$$= 20(3 + 1.73)$$
$$= 20 \times 4.73$$
$$= 94.8 \text{ m}$$

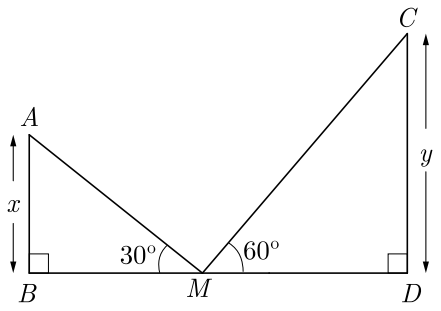
Hence, distance between two ships is 94.8 m.

Comp 2017

18. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

Sol :

Let AB be the tower of height x and CD be the tower of height y. Angle of depressions of both tower at centre point M are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here M is the centre of the line joining their feet. Let $BM = MD = z$

In right ΔABM we have,

$$\frac{x}{z} = \tan 30^\circ$$
$$x = z \times \frac{1}{\sqrt{3}}$$

In right ΔCDM we have,

$$\frac{y}{z} = \tan 60^\circ$$
$$y = z \times \sqrt{3}$$

From (1) and (2), we get

$$\frac{x}{y} = \frac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}}$$
$$\frac{x}{y} = \frac{1}{3}$$

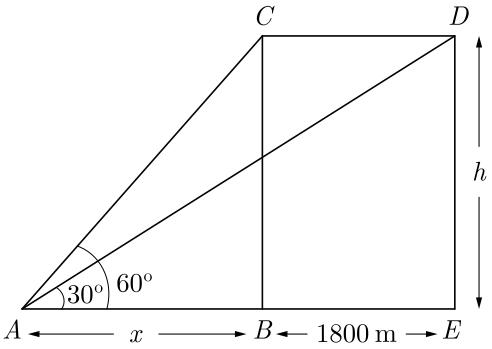
Thus $x : y = 1 : 3$

Comp 2015

19. The angle of elevation of a jet fighter point A on ground is 60°. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying.

Sol :

Let C and D are the point of location of jet at height h. Point B and E are foot print on ground of get at thee location. As per given in question we have drawn figure below.



In 3600 sec distance traveled by plane = 648000 m

In 10 sec distance traveled by plane $= \frac{648000}{3600} \times 10$

$$= 1800 \text{ m}$$

In right ΔABC, we have

$$\frac{h}{x} = \tan 60^\circ = \sqrt{3}$$
$$h = x\sqrt{3} \quad \ldots(1)$$

In right ΔADE we have

$$\frac{h}{x + 1800} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$
$$h = \frac{x + 1800}{\sqrt{3}} \quad \ldots(2)$$

From equations (1) and (2), we get

$$x\sqrt{3} = \frac{x + 1800}{\sqrt{3}}$$
$$3x = x + 1800$$
$$2x = 1800$$
$$x = 900 \text{ m}$$
$$h = x\sqrt{3}$$
$$= 900 \times 1.732 = 1558.5 \text{ m}$$

Thus height of jet is 1558.8 m.

Main 2011

20. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there

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$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

In ΔABD, $\quad \frac{AB}{BD} = \tan 60^\circ$

$$\frac{10\sqrt{3}}{BD} = \tan 60^\circ = \sqrt{3}$$

$$BD = 10 \text{ m}$$

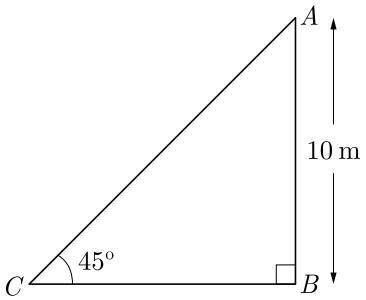
Hence the length of shadow is 10 m.

Comp 2003

26. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$]

Sol :

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.
As per given in question we have drawn figure below.



In ΔABC we have,

$$\sin 45^\circ = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{AC}$$

$$AC = 10\sqrt{2} = 10 \times 1.414$$

$$= 14.14 \text{ m}$$

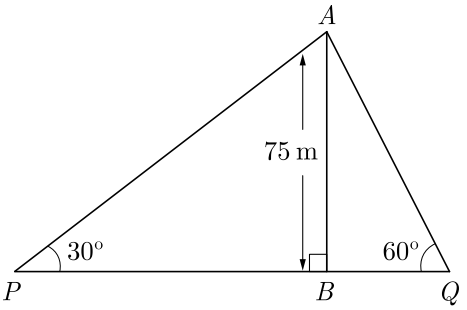
Hence, the length of wire is 14.14 m

Main 2002

27. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Sol :

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



In ΔABP, $\quad \tan 30^\circ = \frac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

In ΔABQ, $\quad \tan 60^\circ = \frac{AB}{BQ}$

$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

$$PQ = BP + BQ$$

$$= 75\sqrt{3} + 25\sqrt{3}$$

$$= 100\sqrt{3}$$

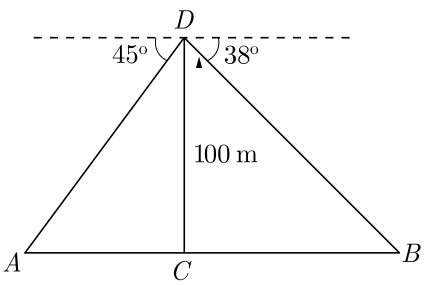
$$= 100 \times 1.73 = 173$$

FOUR MARK QUESTIONS

Main 2023

28. From the top of a tower 100 m high a man observes the angles of depression of two ships A and B, on opposite sides of the tower as 45° and 38° respectively. If the foot of the tower and the ships are in the same horizontal line find the distance between the two ships A and B to the nearest metre. [4]
(Use Mathematical Tables for this question.)

NODIA



Sol :

Let DC be tower of height 100 m. A and B be

$$AB = 100\sqrt{3}$$

Therefore, distance d travelled in 2 min,

$$AD = AB - DB = 100\sqrt{3} - \frac{100}{3}$$
$$= 173.2 - \frac{100}{3}\sqrt{3}$$
$$= 173.2 - 57.73$$
$$= 115.47 \text{ m}$$

Speed $s = \frac{d}{t} = \frac{115.47 \text{ m}}{2 \text{ min}}$

$$= 57.74 \text{ m/ min}$$

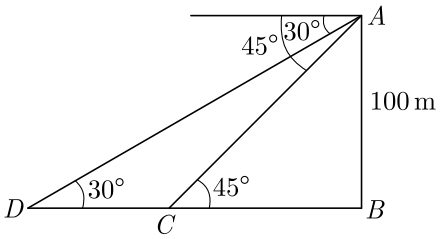
Hence, going away from the light house with a speed of 57.74 m/ min.

Main 2008

23. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Sol :

Let AB be the tower and ships are at points C and D. As per question statement we have shown digram below.



Now in ΔABC we have

$$\tan 45^\circ = \frac{AB}{AC}$$
$$\frac{AB}{AC} = 1 \Rightarrow AB = BC$$

Now in ΔABD we have

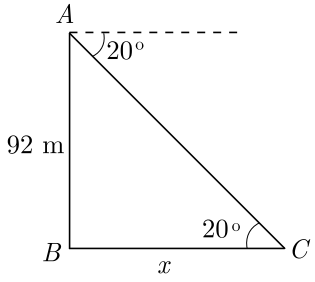
$$\tan 30^\circ = \frac{AB}{BD}$$
$$\frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$
$$\frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$
$$AB + CD = \sqrt{3}\, AB$$
$$CD = AB(\sqrt{3} - 1)$$
$$= 100 \times (1.732 - 1)$$
$$= 73.2 \text{ m}$$

Main 2005

24. From the top of a cliff 92 m high, the angle of depression of a buoy is 20°. Calculate to the nearest meter, the distance of the buoy from the foot of the cliff.

Sol :

Let x be the distance of the buoy from cliff. As per given in question we have drawn figure below. Here AB is cliff of height 92 m. Distance between buoy C and cliff is x.



In right ΔABC, we have

$$\tan 20^\circ = \frac{AB}{BC}$$
$$0.36 = \frac{92}{x}$$
$$x = \frac{92}{0.36}$$
$$= 255.56 \text{ m}$$

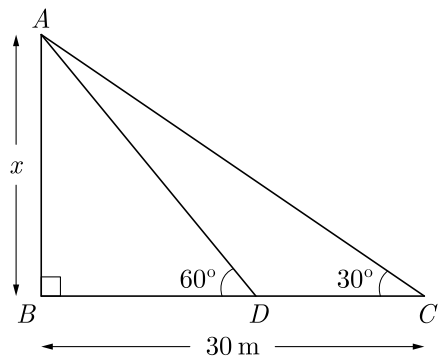
Hence, the distance of the buoy from the foot of the cliff is 255.56 m.

Main 2003

25. If the shadow of a tower is 30 m long, when the Sun's elevation is 30°. What is the length of the shadow, when Sun's elevation is 60° ?

Sol :

As per given in question we have drawn figure below. Here AB is tower and BD is shadow at 60° and BC is shadow at 30° elevation.



In ΔABC, $\frac{AB}{BC} = \tan 30^\circ$

$$\frac{AB}{30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

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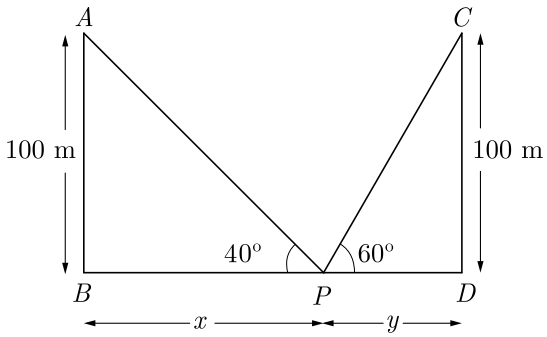


In right angle triangle ΔABP,

$$\tan 40^\circ = \frac{100}{x}$$

$$0.84 = \frac{100}{x}$$

$$x = \frac{100}{0.84} = 119$$

In angle triangle ΔCDP,

$$\tan 60^\circ = \frac{100}{y}$$

$$\sqrt{3} = \frac{100}{y}$$

$$y = \frac{100}{\sqrt{3}}$$

$$= \frac{100\sqrt{3}}{3}$$

$$= \frac{173.2}{3}$$

$$= 57.73 \text{ m}$$

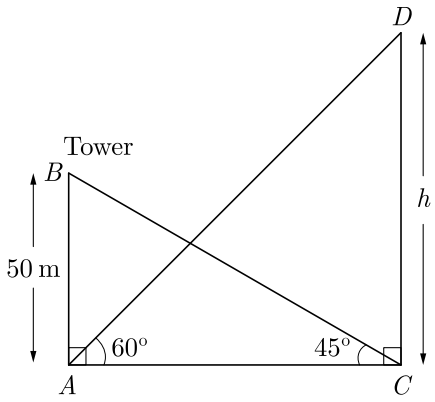
Hence PB= 119 m and PD = 57.73 m

Comp 2022

31. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.

Sol :

Let AB be tower of height of 50 m and DC be hill of height h. As per given in question we have drawn figure below.



In right ΔBAC,

$$\cos 30^\circ = \frac{AC}{50}$$

$$\sqrt{3} = \frac{AC}{50}$$

$$AC = 50\sqrt{3}$$

In right ΔACD,

$$\tan 60^\circ = \frac{CD}{50\sqrt{3}}$$

$$\sqrt{3} = \frac{CD}{50\sqrt{3}}$$

$$CD = 50\sqrt{3} \times \sqrt{3} = 150 \text{ m}$$

Thus height of the hill $CD = 150$ m

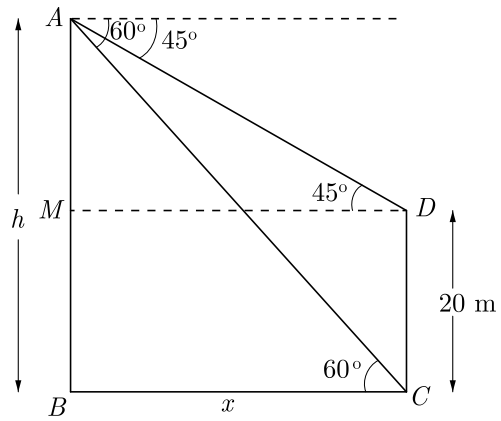
Main 2020

32. From the top of a cliff, the angle of depression of the top and bottom of a tower are observed to be 45° and 60°, respectively. If the height of the tower is 20 m.
Find
(i) the height of the cliff.
(ii) the distance between the cliff and the tower.

Sol :

Let AB be the cliff of height h and CD be the tower of height 20 m. From the top of a cliff the angles of depression of the top and bottom of tower are 45° and 60° respectively. Let x be the distance between cliff and tower. As per given in question we have drawn figure below.

NODIA



In right ΔAMD we have

$$\tan 45^\circ = \frac{AM}{MD}$$

$$1 = \frac{h-20}{x}$$

$$x = h - 20 \qquad (1)$$

In right ΔABC we have,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x}$$

$$\sqrt{3}\,x = h \qquad (2)$$

Substituting the value of x from eq. (1) in eq. (2), we get

$$\sqrt{3}\,(h-20) = h$$

$$\sqrt{3}\,h - 20\sqrt{3} = h$$

$$\sqrt{3}\,h - h = 20\sqrt{3}$$

$$h(\sqrt{3}-1) = 20\sqrt{3}$$

$$h = \frac{20\sqrt{3}}{(\sqrt{3}-1)}$$

$$= \frac{20\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{3-1}$$

$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{2}$$

$$= 10\,(3+\sqrt{3})$$

$$= 10\,(3+1.732)$$

$$= 10 \times 4.732$$

$$= 47.32\text{ m}$$

Substituting above value of h i eq (1) we have

$$x = 47.32 - 20$$

$$= 27.32\text{ m}$$

(i) The height of the cliff is $h = 47.32\,\text{m}$

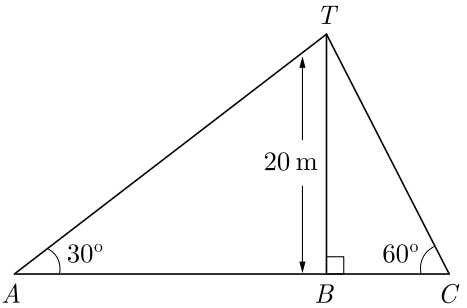
(ii) The distance between the cliff and the tower is $x = 27.32\,\text{m}$.

Comp 2020

33. Two men standing on opposite sides of a tower measure the angles of elevation of he top of the tower as 30° and 60° respectively. If the height of the tower in 20 m, then find the distance between the two men.

Sol :

Let two men are standing at A and C and BT is the tower. As per given in question we have drawn figure below.



In right angle triangle ΔABT,

$$\tan 30^\circ = \frac{BT}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{20}{AB}$$

$$AB = \sqrt{3}\,20$$

In right angle triangle ΔTBC,

$$\tan 60^\circ = \frac{BT}{BC}$$

$$\sqrt{3} = \frac{20}{BC}$$

$$BC = \frac{20}{\sqrt{3}}$$

Thus distance between two men,

$$AB + BC = 20\sqrt{3} + \frac{20}{\sqrt{3}}$$

$$= \frac{60+20}{\sqrt{3}} = \frac{80\sqrt{3}}{3}\text{ m}.$$

Hence, distance between the men is $\frac{80\sqrt{3}}{3}$ m.

Main 2019

34. A man observes the angle of elevation of the top of the tower to be $45°$. He walks towards it in a horizontal line through its base. On covering 20 m

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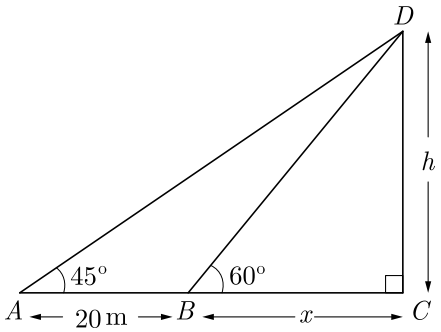
the angle of elevation changes to 60°. Find the height of the tower correct to 2 significant figures.

Sol :

As per given in question we have drawn figure below. Let height of building CD be h and distance BC be x.



In right ΔDBC, $\quad \frac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x+60} = \tan 45^\circ = 1$$

$$h = x + 20 \qquad ...(2)$$

$$x = h - 20 \qquad ...(2)$$

Substituting the value of x from eq. (2) in eq. (1), we get

$$h = \sqrt{3}\,(h-20)$$
$$= \sqrt{3}\,h - 20\sqrt{3}$$
$$\sqrt{3}\,h - h = 20\sqrt{3}$$
$$h(\sqrt{3}-1) = 20\sqrt{3}$$
$$h = \frac{20\sqrt{3}}{(\sqrt{3}-1)}$$
$$= \frac{20\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$
$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2}$$
$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{3-1}$$
$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{2}$$
$$= 10\,(3+\sqrt{3})$$
$$= 10\,(3+1.732)$$
$$= 10 \times 4.732$$
$$= 47.32\ \text{m}$$
$$= 47\ \text{m}$$

Hence, height of building is 51.96 m.

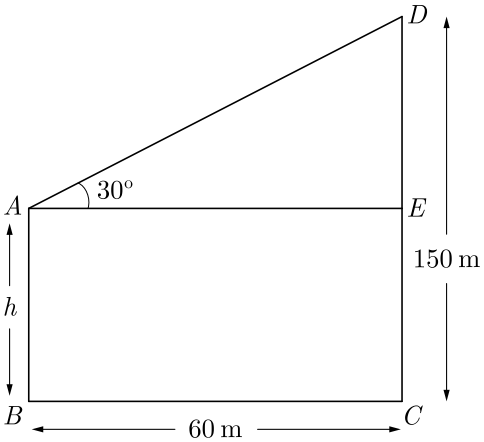
Comp 2019

35. The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30°. If the height of the taller tower is 150 m, then find the height of the shorter tower.

Sol :

Let AB and CD be two towers. Let the height of the shorter tower $AB = h$. As per given in question we have drawn figure below.

NODIA



Here $\quad BC = AE = 60\ \text{m},$

$$DE = DC - EC = (150 - h)$$

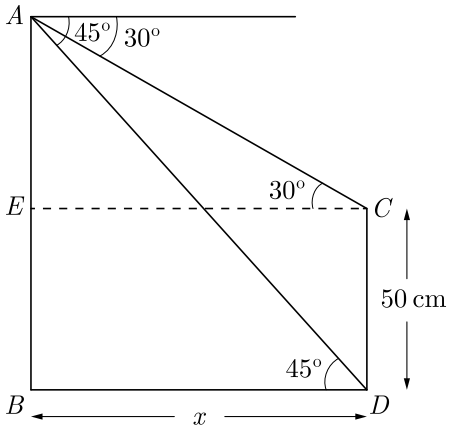
In ΔAED, $\quad \frac{DE}{AE} = \tan 30^\circ$

$$\frac{150-h}{60} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$
$$150\sqrt{3} - h\sqrt{3} = 60$$
$$\sqrt{3}\,h = 150\sqrt{3} - 60$$
$$\sqrt{3}\,h = 150\sqrt{3} - 20\sqrt{3} \times \sqrt{3}$$

or $\quad h = (150 - 20\sqrt{3})\ \text{m}$

Main 2018

36. The angle of elevation from a point P of the top tower QR, 50 m heigh is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre.



Let distance between BD be x.

Now, in right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{h}{x} = 1 \Rightarrow h = x \qquad ...(1)$$

In right ΔAEC we have

,
$$\frac{AE}{EC} = \tan 30^\circ$$

$$\frac{h-50}{x} = \frac{1}{\sqrt{3}}$$

$$x = h\sqrt{3} - 50\sqrt{3} \qquad ...(2)$$

From (1) and (2) we get

$$h = h\sqrt{3} - 50\sqrt{3}$$

$$h\sqrt{3} - h = 50\sqrt{3}$$

$$h(\sqrt{3}-1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1}$$

$$= \frac{50\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{50(3+\sqrt{3})}{3-1}$$

$$h = 25(3+\sqrt{3})$$

$$= 25 \times 4.732 = 118.3 \text{ m}$$

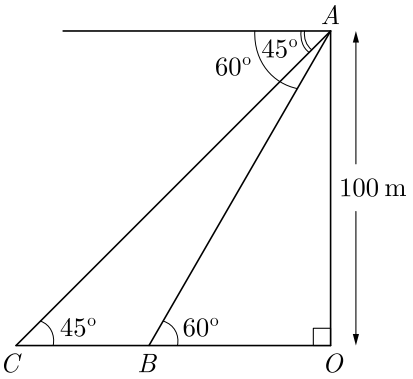
Hence, the height of tower = distance between building and tower = 118.3 m

Comp 2015

41. From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45° and 60°. Find the distance between the objects.

Sol :

Let A be a point on top of building and B, C be two objects. As per given in question we have drawn figure below.



Here $\angle ACO = \angle CAX = 45^\circ$

and $\angle ABO = \angle XAB = 60^\circ$

In right ΔAOC, $\frac{AO}{CO} = \tan 45^\circ$

$$\frac{100}{CO} = 1$$

$$CO = 100 \text{ m}$$

Also in right ΔAOB, we have

$$\frac{AO}{OB} = \tan 60^\circ$$

$$\frac{100}{OB} = \sqrt{3}$$

$$OB = \frac{100}{\sqrt{3}}$$

Thus $BC = CO - OB$

$$= 100 - \frac{100}{\sqrt{3}}$$

$$= 100\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$= 100\frac{(\sqrt{3}-1)}{\sqrt{3}}$$

$$= 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{100(3-\sqrt{3})}{3} \text{ m}$$

Main 2014

42. An aeroplane at an altitude of 250 m observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60°, respectively. Find width of the river. Write the answer correct to the nearest whole number.

Sol :

Let A be helicopter flying at a height of 250 m above the point O on ground. Let B and C be the bank of river. As per given in question we have drawn figure below.

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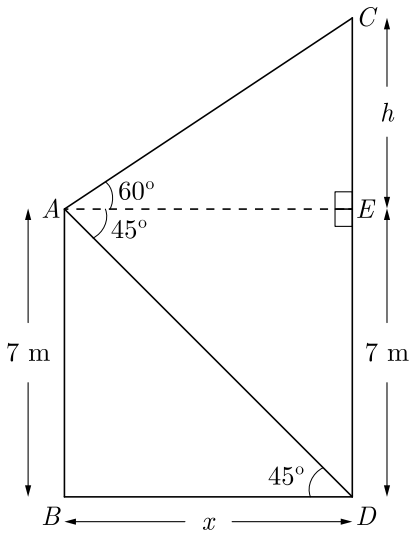


Here $\angle CBD = \angle ECB = 45^\circ$ due to alternate angles.

In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$
$$\frac{7}{x} = 1 \Rightarrow x = 7$$

In right ΔAEC we have

$$\frac{CE}{AE} = \tan 60^\circ$$
$$\frac{h-7}{x} = \sqrt{3}$$
$$h - 7 = x\sqrt{3} = 7\sqrt{3}$$
$$h = 7\sqrt{3} + 7$$
$$= 7(\sqrt{3}+1)$$
$$= 7(1.732+1)$$

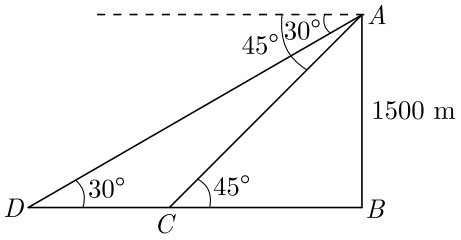
Hence, height of tower $= 19.124$ m

Main 2016

39. An aeroplane at an altitude of 1500 m finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45° and 30°, respectively. Find the distance between the two ships.

Sol :

Let aeroplane be at point A at any instant 1500 m above the earth surface point B. Let ships are at points C and D at same instant. As per question statement we have shown digram below.



Now in ΔABC we have

$$\tan 45^\circ = \frac{AB}{AC}$$
$$\frac{AB}{AC} = 1$$
$$AB = BC$$

Now in ΔABD we have

$$\tan 30^\circ = \frac{AB}{BD}$$
$$\frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$
$$\frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$
$$AB + CD = \sqrt{3}\,AB$$
$$CD = AB(\sqrt{3}-1)$$
$$= 1500 \times (1.732 - 1)$$
$$= 1098 \text{ m}$$

Distance between two ships is 1098 m.

Comp 2016

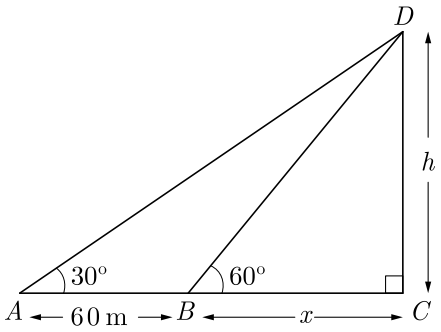
40. The angle of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 45° respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

Sol :

Let CD be the building of height 50 m and AB be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.

NODIA

below.



In right ΔDBC, $\quad \frac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad \ldots(1)$$

In right ΔADC,

$$\frac{h}{x+60} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 60 \qquad \ldots(2)$$

Substituting the value of x from eq. (1) in eq. (2), we get

$$\sqrt{3}\,h = \frac{h}{\sqrt{3}} + 60$$

$$3h = h + 60\sqrt{3}$$

$$2h = 60\sqrt{3}$$

$$h = 30\sqrt{3}$$

$$= 30 \times 1.732$$

$$= 51.96 \text{ m}$$

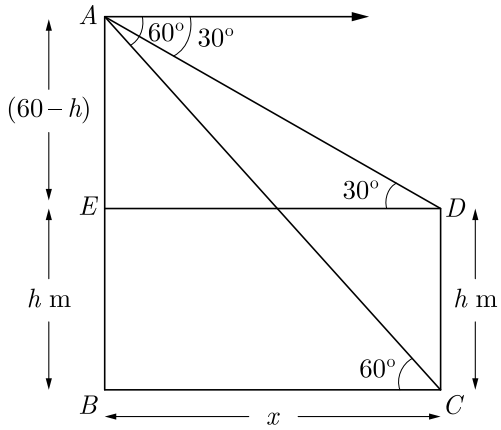
Hence, height of building is 51.96 m.

Comp 2011

49. From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

Sol :

Let AB be the building of height 60 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $\quad DC = EB = h$ and let $BC = x$

$$AE = (60 - h) \text{ m}$$

In right angled ΔAED we have

$$\frac{60-h}{ED} = \tan 30^\circ$$

$$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}(60-h) = x \qquad \ldots(1)$$

In right ΔABC we have

$$\frac{60}{x} = \tan 60^\circ$$

$$60 = \sqrt{3}\,x \qquad \ldots(2)$$

Substituting the value of x from equation (1) in equation (2), we have

$$60 = \sqrt{3} \times \sqrt{3}(60-h)$$

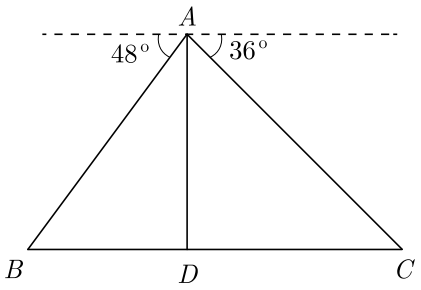
$$60 = 3 \times (60-h)$$

$$20 = 60 - h \quad \Rightarrow \quad h = 40 \text{ m}$$

Hence, height of tower is 40 m.

Main 2010

50. From the top of a light house 100 m high, the angles of depression of two ships on opposite sides of it, are 48° and 36°, respectively. Find the distance between the two ships to the nearest metre.



Sol :

As per question we have shown the digram below.

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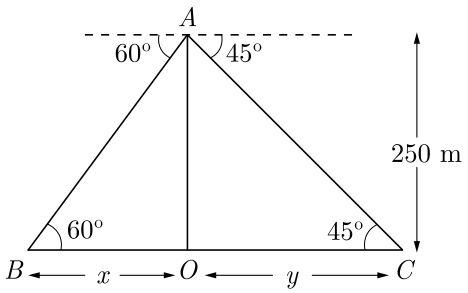


Let BO be x and OC be y.

In right ΔAOC we have

$$\frac{AO}{OC} = \tan 45^\circ$$

$$\frac{250}{y} = 1$$

$$y = 250$$

In right ΔAOB we have

$$\frac{AO}{BO} = \tan 60^\circ$$

$$\frac{250}{x} = \sqrt{3}$$

$$x = \frac{250}{\sqrt{3}}$$

$$BC = y + x$$

$$= 250 + \frac{250}{\sqrt{3}}$$

$$= 250\left(1 + \frac{1}{\sqrt{3}}\right)$$

$$= \frac{250}{3}(3 + \sqrt{3})$$

$$= \frac{250}{3}(3 + 1.732)$$

$$= \frac{250}{3} \times 4.732$$

$$= 250 \times 1.577$$

$$= 395.25$$

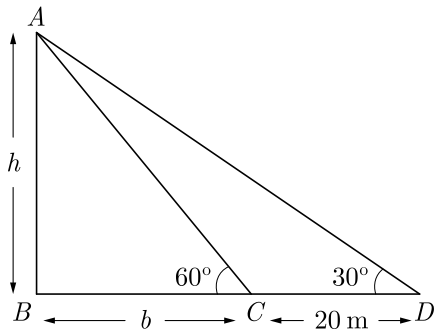
Hence, the width of river is 394 m.

Comp 2014

43. A person standing on the bank of a river, observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he retreats 20 m from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the breadth of the river.

Sol :

Let AB be the tree of height h and breadth of river be b. As per given in question we have drawn figure below. Here point C and D are the location of person .



In right ΔABC we have,

$$\frac{h}{b} = \tan 60^\circ = \sqrt{3}$$

$$h = \sqrt{3}\, b \qquad \ldots(1)$$

In right ΔABD we have

$$\frac{h}{b+20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{b+20}{\sqrt{3}} \qquad \ldots(2)$$

From (1) and (2) we have

$$b\sqrt{3} = \frac{b+20}{\sqrt{3}}$$

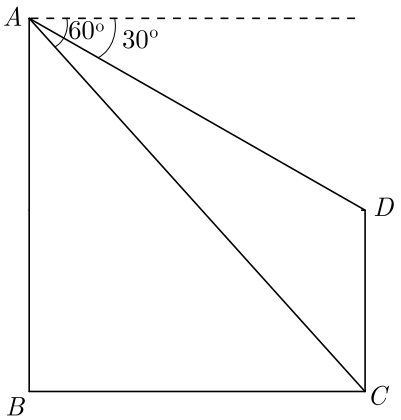
$$3b = b + 20 \Rightarrow b = 10 \text{ m}$$

$$h = b\sqrt{3} = 10 \times 1.73 = 17.3 \text{ m}$$

Thus height of tree is 17.3 m and breadth of river is 10 m.

Main 2013

44. In the given figure, from the top of building $AB = 60$ m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60°, respectively.



Find

(i) the horizontal distance observed between AB and CD.

(ii) the height of the lamp post.

Sol :

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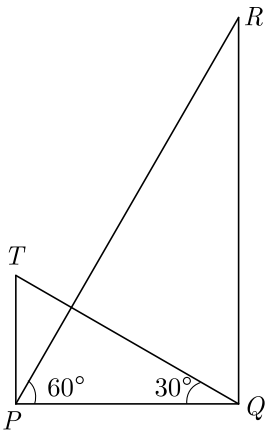
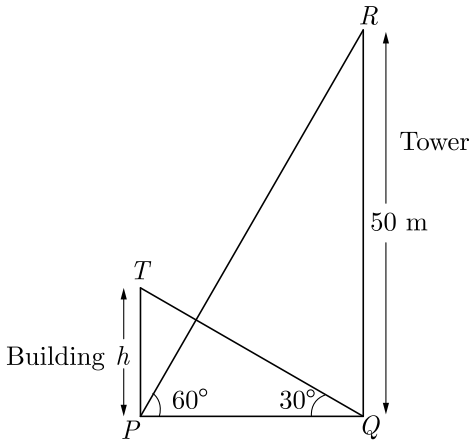


Sol :

As per given information in question we have drawn the figure below.



In ΔPQR, $\tan 60^\circ = \dfrac{RQ}{PQ}$

$$\sqrt{3} = \frac{50}{PQ}$$

$$PQ = \frac{50}{\sqrt{3}}$$

Now in ΔTPQ,

$$\tan 30^\circ = \frac{TP}{PQ}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$

$$3h = 50$$

$$h = \frac{50}{3} = 16.67$$

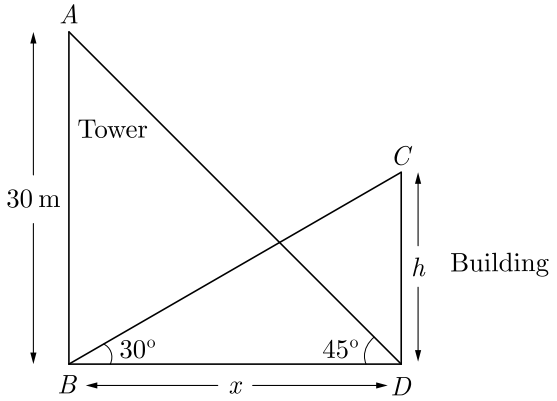
Hence, the height of the building is 16.67 m.

Comp 2018

37. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Sol :

Let the height of the building be $AB = h$. and distant between tower and building be, $BD = x$. As per given in question we have drawn figure below.



In ΔABD $\tan 45^\circ = \dfrac{AB}{BD}$

$$1 = \frac{30}{x}$$

$$x = 30 \qquad \ldots(1)$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\sqrt{3}\,h = x \Rightarrow h = \frac{x}{\sqrt{3}} \qquad \ldots(2)$$

From (1) and (2), we get

$$h = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

Therefore height of the building is $10\sqrt{3}$ m

Comp 2017

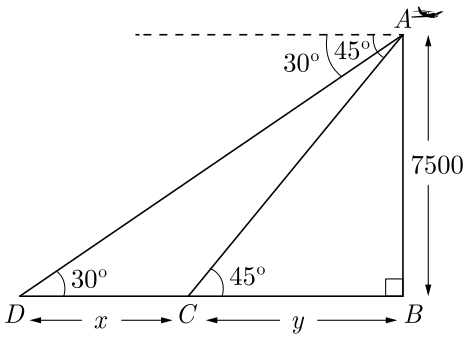
38. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

Sol :

Let AB be the building of height 7 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.

NODIA

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7500}{y} = 1$$

$$y = 7500 \qquad ...(1)$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$x + y = 7500\sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) in (2) we have

$$\begin{aligned} x + 7500 &= 7500\sqrt{3} \\ x &= 7500\sqrt{3} - 7500 \\ &= 7500(\sqrt{3} - 1) \\ &= 7500(1.73 - 1) \\ &= 7500 \times 0.73 \\ &= 5475 \text{ m} \end{aligned}$$

Hence, the distance between two ships is 5475 m.

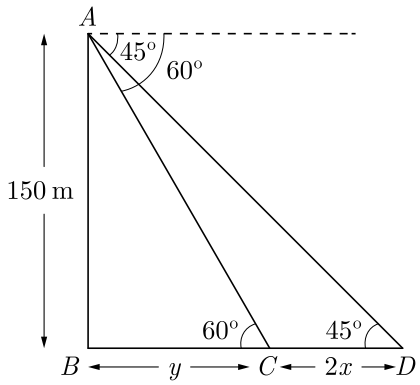
Comp 2012

47. A moving boat observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

Sol :

Let AB be the cliff of height 150 m. Let C and D be the point of boat at 60° and 45°. Let the speed of the boat be x m/min. Let BC be y

As per given in question we have drawn figure below.



Here distance covered in 2 minutes is $2x$.

Thus $CD = 2x$

In right ΔABD we have

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{150}{y} = \sqrt{3}$$

$$y = \frac{150}{\sqrt{3}} = 50\sqrt{3} \qquad ...(1)$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{150}{y+2x} = 1$$

$$y + 2x = 150 \qquad ...(2)$$

From equations (1) and (2), we get

$$\begin{aligned} 50\sqrt{3} + 2x &= 150 \\ 2x &= 150 - 50\sqrt{3} \\ 2x &= 50(3 - \sqrt{3}) \\ x &= 25(3 - \sqrt{3}) \\ \text{Speed of the boat} &= 25(3 - \sqrt{3}) \text{ m/min.} \\ &= \frac{25(3 - \sqrt{3}) \times 60}{1000} \\ &= \frac{3}{2}(3 - \sqrt{3}) \text{ km/hr.} \end{aligned}$$

Main 2011

48. A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m, the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.

Sol :

Let height of building CD be h and distance BC be x.

As per given in question we have drawn figure

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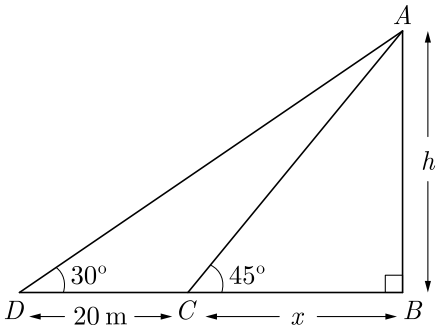


Here $CB = x$ and $DC = 20$ m

Now in right ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$
$$\frac{h}{x} = 1$$
$$h = x$$

In right ΔABD we have

$$\frac{AB}{DB} = \tan 30^\circ$$
$$\frac{h}{(20+x)} = \frac{1}{\sqrt{3}}$$
$$h\sqrt{3} = 20 + x$$

Substituting the value of x from (1) in (2)

$$h\sqrt{3} = 20 + h$$
$$h\sqrt{3} - h = 20$$
$$h(\sqrt{3}-1) = 20$$
$$h = \frac{20}{\sqrt{3}-1}$$
$$= \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{20(\sqrt{3}+1)}{3-1}$$
$$= 10(\sqrt{3}+1)$$

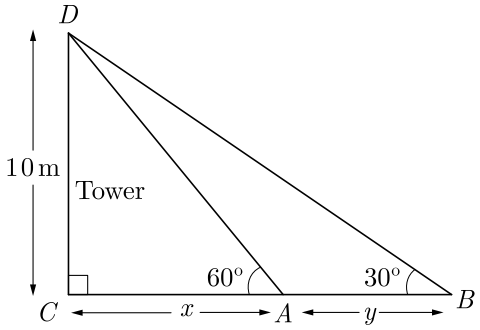
Hence, the height of tower $= 10(\sqrt{3}+1)$ m

Main 2009

53. From two points, A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60°, respectively. If the height of the building is 10 m, then find the distance between A and B correct to two decimal places.

Sol :

Let CD be the tower of height 10 m. Let A and B point on same side of tower As per given in question we have drawn figure below.



In right ΔDCA we have

$$\frac{DC}{CA} = \tan 60^\circ$$
$$\frac{10}{x} = \sqrt{3}$$
$$x = \frac{10}{\sqrt{3}}$$

In right ΔDCB we have

$$\frac{DC}{CB} = \tan 30^\circ$$
$$\frac{10}{x+y} \quad \frac{1}{\sqrt{3}}$$
$$x + y = 10\sqrt{3}$$
$$\frac{10}{\sqrt{3}} + y = 10$$
$$y = 10\sqrt{3} - \frac{10}{\sqrt{3}}$$
$$= \frac{10 \times 3 - 10}{\sqrt{3}}$$
$$= \frac{20}{\sqrt{3}} = \frac{20}{3}\sqrt{3}$$
$$= \frac{20 \times 1.732}{3}$$
$$= \frac{34.64}{3} = 11.55 \text{ m}$$

Hence, the distance between points 11.55 m

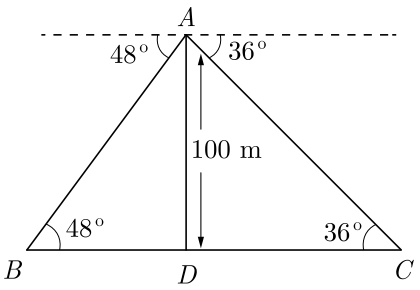
Main 2008

54. A vertical pole and a vertical are on the same level ground. From the top of the pole, the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30° . Find the height of the tower, if the height of the ple is 20 m.

Sol :

Let AB be a tower of height h and CD be pole of height CD. Let x be the distance between pole and tower. From the given information we have drawn the figure as below.

NODIA



Here B and C are the positions of two ships on opposite sides of AD.

Then, $\angle ABD = 48^\circ$ and $\angle ACD = 36^\circ$

In ΔADB, we have

$$\tan 48^\circ = \frac{AD}{BD}$$

$$1.11 = \frac{100}{BD} \qquad \tan 48^\circ = 1.11$$

[From natural tangent table, $\tan 48^\circ = 1.11$]

$$BD = \frac{100}{1.11} = 90.09 \text{ m}$$

In ΔADC, we have

$$\tan 36^\circ = \frac{AD}{DC}$$

$$0.7265 = \frac{100}{DC} \qquad \tan 36^\circ = 0.7265$$

[from natural tangent table, $\tan 36^\circ = 0.7265$]

$$DC = \frac{100}{0.7265} = 137.64 \text{ m}$$

Now, distance between two ships

$$BC = BD + DC$$

$$= 90.09 + 137.64 = 227.73 \text{ m}$$

Thus distance between two ships is 227.73 m.

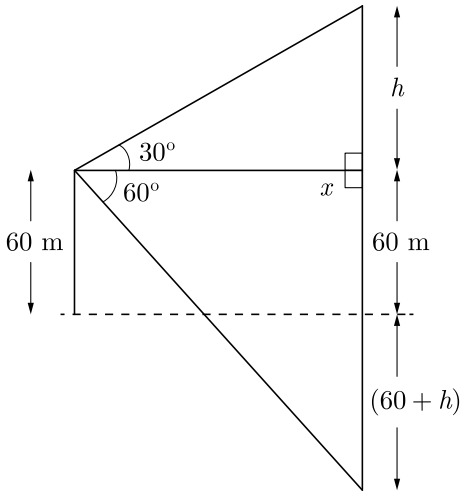
Here we need to use tangent table to get the value of $\tan 48^\circ$ and $\tan 36^\circ$.

Comp 2010

51. An angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water is 60°. Find the height of the cloud from the surface of water.

Sol :

As per given in question we have drawn figure below.



Here $\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$$x = h\sqrt{3} \qquad \ldots(1)$$

and $\frac{h + 60 + 60}{x} = \tan 60^\circ$

$$\frac{h + 120}{x} = \sqrt{3}$$

$$h + 120 = x\sqrt{3} \qquad \ldots(2)$$

From (1) and (2) we get

$$h + 120 = \sqrt{3}\, h \times \sqrt{3}$$

$$h + 120 = 3h$$

$$h = \frac{120}{2} = 60 \text{ m}$$

Hence height of cloud from surface of water

$$= 60 + 60 = 120 \text{ m}$$

Comp 2009

52. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m, towards the base of the tower, the angle of elevation of the top increase by 15°, find the height of the tower.

Sol :

Let AB be the tower of height h. Angle of elevation from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.

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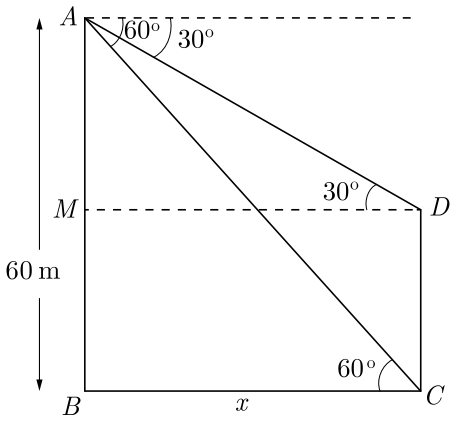
Let AB be the building of height 60 m and CD be the lamp post of height h. From the top of a building of height 60 m, the angles of depression of the top and bottom of a lamp post 30° and 60° respectively. As per given in question we have drawn figure below.



In right ΔABC we have,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}}$$

$$= 20\sqrt{3}$$

$$= 20 \times 1.732 = 34.62 \text{ m}$$

(i) Thus distance of lamp post from bottom of building is 34.6 m.

Now in ΔAMD we have

$$\tan 30^\circ = \frac{AM}{MD}$$

$$\frac{1}{\sqrt{3}} = \frac{AM}{x}$$

$$AM = \frac{x}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$$

Height lamp post $h = CD = BM$

$$= AB - AM$$

$$= 60 - 20 = 40 \text{ m}.$$

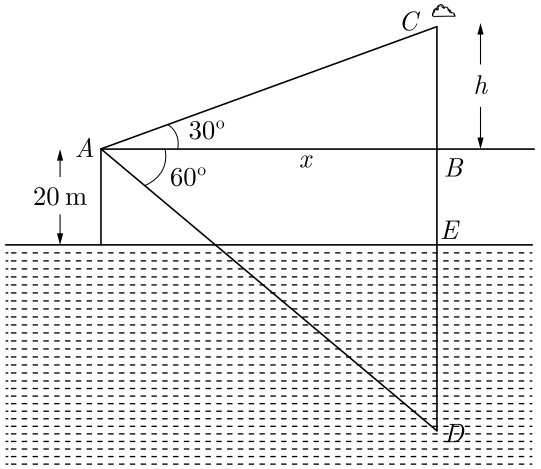
(ii) Thus height of lamp post is 40 m.

Comp 2013

45. At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30° . The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?

Sol :

As per given in question we have drawn figure below. Here cloud is at C, D is reflection of cloud in water.



In right ΔABC we have

$$\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$x = \sqrt{3}\, h \qquad \ldots(1)$$

Here $DE = EC$ because D is reflection of cloud and E is at water level.

In right ΔABD we have

$$\frac{BD}{BA} = \tan 60^\circ$$

$$\frac{DE + EB}{x} = \sqrt{3}$$

$$\frac{EC + EB}{x} = \sqrt{3}$$

$$\frac{h + 20 + 20}{x} = \sqrt{3}$$

$$h + 40 = \sqrt{3}\, x \qquad \ldots(2)$$

From (1) and (2),

$$h + 40 = \sqrt{3} \times \sqrt{3}\, h = 3h \;\Rightarrow\; h = 20 \text{ m}$$

$$x = \sqrt{3}\, h = 20\sqrt{3}$$

Now $AC = \sqrt{h^2 + x^2}$

$$= \sqrt{(20)^2 + (20\sqrt{3})^2}$$

$$= \sqrt{400 + 1200} = 40 \text{ m}.$$

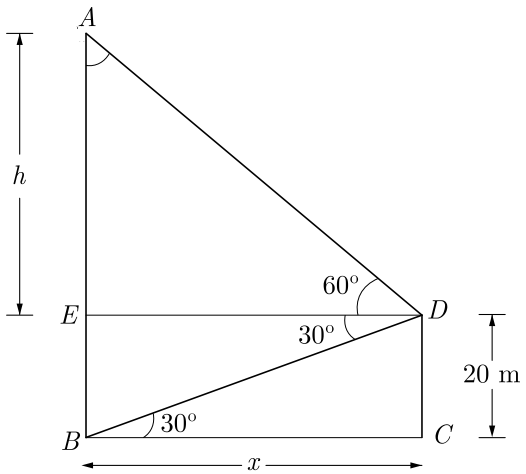
Hence distance of the cloud is 40 m.

Main 2012

46. As observed from the top of a 80 m tall light house, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40°, respectively. Find the distance between the two ships. Give your answer correct to the nearest metre.

Sol :

NODIA



Now $ED = BC = x$

In ΔBCD, $\tan 30^\circ = \dfrac{CD}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{20}{x}$$

$$x = 20\sqrt{3}$$

In ΔAED, $\tan 60^\circ = \dfrac{AE}{ED}$

$$\sqrt{3} = \frac{h}{x}$$

$$h = x\sqrt{3}$$

Substituting the value of x, we get

$$h = 20 \times \sqrt{3} \times \sqrt{3} = 60 \text{ m}$$

Height of tower $AB = AE + EB$

$$= h + 20$$

$$= 60 + 20$$

$$= 80 \text{ m}$$

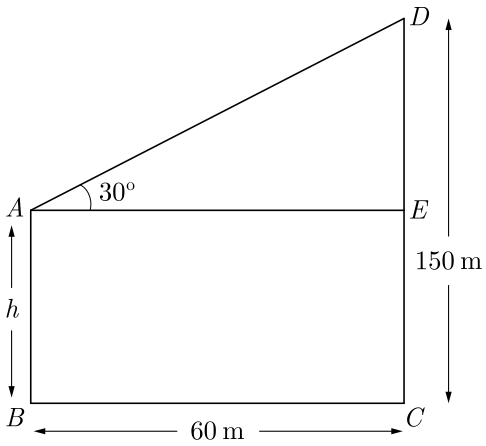
Hence, the height of the tower is 80 m.

Comp 2008

55. The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30°. If the height of the taller tower is 150 m, then find the height of the shorter tower.

Sol :

Let AB and CD be two towers. Let the height of the shorter tower $AB = h$. As per given in question we have drawn figure below.



Here $BC = AE = 60$ m

$$DE = DC - EC = (150 - h)$$

In ΔAED, $\dfrac{DE}{AE} = \tan 30^\circ$

$$\frac{150 - h}{60} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$150\sqrt{3} - h\sqrt{3} = 60$$

$$\sqrt{3}\,h = 150\sqrt{3} - 60$$

$$\sqrt{3}\,h = 150\sqrt{3} - 20\sqrt{3} \times \sqrt{3}$$

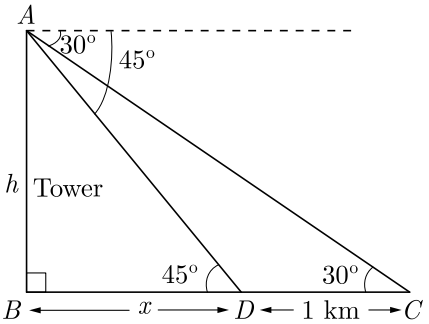
or $h = (150 - 20\sqrt{3})$ m

Main 2007

56. From the top of a hill, the angles of depression of two consecutive kilometre stones, due east are found to be 30° and 45°, respectively. Find the distance of the two stones from the foot of the hill.

Sol :

Let AB be the hill height h. Let C and D be the position of two stone where CD is 1 km.
As per given in question we have drawn figure below.



In right ΔABD we get,

$$\tan 45^\circ = \frac{AB}{BD}$$

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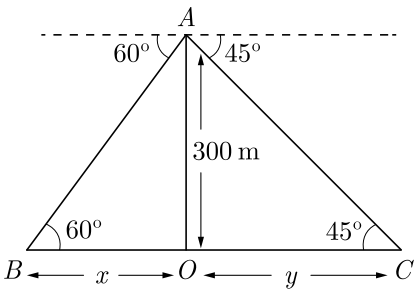


Let BO be x and OC be y.

In right ΔAOC we have

$$\frac{AO}{OC} = \tan 45^\circ$$
$$\frac{300}{y} = 1$$
$$y = 300$$

In right ΔAOB we have

$$\frac{AO}{BO} = \tan 60^\circ$$
$$\frac{300}{x} = \sqrt{3}$$
$$x\sqrt{3} = 300$$
$$x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$
$$BC = y + x$$
$$= 300 + 100\sqrt{3}$$
$$= 300 + 100 \times 1.732$$
$$= 473.2 \text{ m}$$

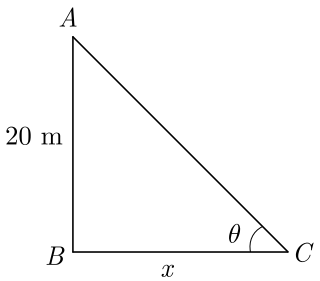
Hence, the width of river is 473.2 m.

Main 2001

66. A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0.53. How far is he standing from the foot of the tower?

Sol :

Let a man standing at the point C and an angle of elevation of the top of tower is θ. Let x be the distance between foot of the tower and man. Height of tower AB is 20 m.



Now $$\tan\theta = \frac{AB}{BC}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{20}{x} \quad (1)$$

Now $$\sin\theta = \sqrt{1 - \cos^2\theta}$$
$$= \sqrt{1 - (0.53)^2}$$
$$= \sqrt{1 - 0.281}$$
$$= \sqrt{0.719}$$
$$\sin\theta = 0.85$$

NODIA

Substituting value of $\sin\theta = 0.85$ and $\cos\theta = 0.53$ in eq (1) we have

$$\frac{0.85}{0.53} = \frac{20}{x}$$
$$x = \frac{20 \times 0.53}{0.85}$$
$$= 12.47$$

Hence, the distance between man and foot of tower is 12.5 m.

Comp 2001

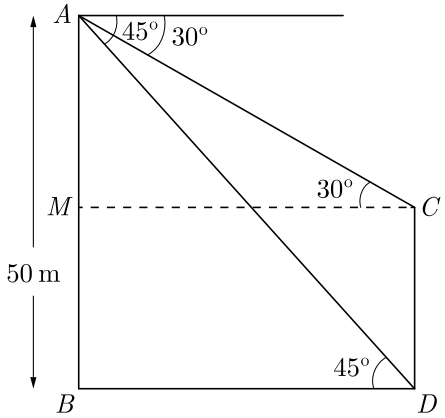
67. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. Find :

(i) How far the pole is from the bottom of the tower,

(ii) The height of the pole. (Use $\sqrt{3} = 1.732$)

Sol :

Let AB be the tower of height 50 m and CD be the pole of height h. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. As per given in question we have drawn figure below.



In right ΔABD we have,

$$\tan 45^\circ = \frac{AB}{BD} = 1$$

$$= \frac{10(\sqrt{3}+1)}{3-1}$$

$$= 5(\sqrt{3}+1)$$

$$= 5(1.732+1)$$

$$= 5 \times 2.732$$

$$= 13.66 \text{ m}$$

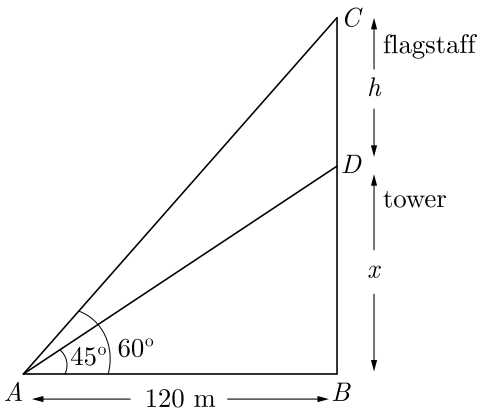
Thus height of tower is 13.66 m.

Comp 2006

59. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

Sol :

Let BD be the tower of height x and CD be flagstaff of height h. As per given in question we have drawn figure below.



Here $\angle DAB = 45^\circ, \angle CAB = 60^\circ$

and $AB = 120$ m

In right angled ΔABD we have

$$\frac{x}{AB} = \tan 45^\circ = 1$$

$$x = AB = 120 \text{ m}$$

In right angled ΔACB we have

$$\frac{h+x}{120} = \tan 60^\circ = \sqrt{3}$$

$$h + 120 = 120\sqrt{3}$$

$$h = 120\sqrt{3} - 120$$

$$= 120(\sqrt{3}-1)$$

$$= 120(1.73-1)$$

$$= 120 \times 0.73$$

$$h = 87.6 \text{ m}$$

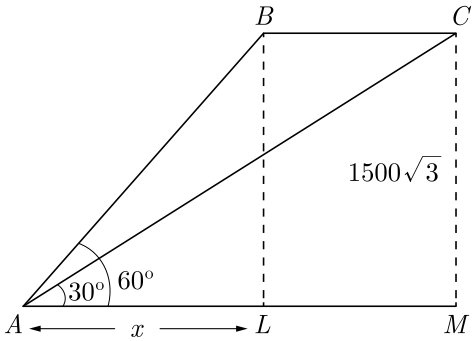
Hence, height of the flagstaff is 87.6 m.

Comp 2005

60. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changed to 30° . If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

Sol :

Let A be the point on ground, B and C be the point of location of aeroplane at height of $1500\sqrt{3}$ m. As per given in question we have drawn figure below.



In right ΔBAL

$$\frac{BL}{AL} = \tan 60^\circ$$

$$\frac{1500\sqrt{3}}{x} = \sqrt{3} \qquad BL = CM = 1500\sqrt{3}$$

$$x = 1500 \text{ m}.$$

In right ΔCAM we have

$$\frac{CM}{AL+LM} = \tan 30^\circ$$

$$\frac{1500\sqrt{3}}{x+y} = \frac{1}{\sqrt{3}}$$

$$x + y = 1500 \times 3$$

$$1500 + y = 4500$$

$$y = 3000 \text{ m}.$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{y}{t}$$

$$= \frac{3000}{15} = 200 \text{ m/s}$$

$$= \frac{200}{1000} \times 60 \times 60$$

$$= 720 \text{ km/hr}.$$

Hence, the speed of the aeroplane is 720 km/hr.

Main 2004

61. Two people standing on the same side of a tower in a straight line with it, measure the angles of

NODIA

$$82 = \frac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$$

$$82 \times 5 = 410 + 5x$$

$$410 - 410 = 5x \Rightarrow x = 0$$

Required mean is,

$$\overline{x} = \frac{130+x+126+x+68+x+50+x+1+x}{5}$$

$$= \frac{375+5x}{5}$$

$$= \frac{375+0}{5} = \frac{375}{5} = 75$$

Thus (a) is correct option.

7. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set
(a) Is increased by 2
(b) Is decreased by 2
(c) Is two times the original median
(d) Remains the same as that of the original set

Sol :

Since, $n = 9$

then, median term $= \left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ item.

Now, last four observations are increased by 2, but median is 5^{th} observation, which is remaining unchanged. Hence there will be no change in median.
Thus (d) is correct option.

8. If the coordinates of the point of intersection of less than ogive and more than ogive is (13.5,20), then the value of median is
(a) 13.5 (b) 20
(c) 33.5 (d) 7.5

Sol :

The abscissa of point of intersection gives the median of the data. So, median is 13.5.
Thus (a) is correct option.

9. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is
(a) 6 (b) 7
(c) 8 (d) 10

Sol :

Mode of the data is 8 as it is repeated maximum number of times. Thus (c) is correct option.

10. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the mean of the last three observation is
(a) $10\frac{1}{3}$ (b) $10\frac{2}{3}$
(c) $11\frac{1}{3}$ (d) $11\frac{2}{3}$

Sol :

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$

$$9 = \frac{x+x+3+x+5+x+7+x+10}{5}$$

$$9 = \frac{5x+25}{5}$$

$$x = 4$$

So, mean of last three observation,

$$= \frac{x+5+x+7+x+10}{3} = \frac{5x+22}{3}$$

$$\frac{3x+22}{3} = \frac{3\times 4+22}{3}$$

$$= \frac{12+22}{3}$$

$$= \frac{34}{3} = 11\frac{1}{3}$$

Thus (c) is correct option.

11. Observations of some data are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. If the median of the data is 4, then the value of x is
(a) 5 (b) 15
(c) 9 (d) 10

Sol :

Given observations are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. On arranging the above observations in ascending order, we get

$$\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{2x}{5}, \frac{2x}{3}, \frac{3x}{4}, x$$

Here, total number of observations are 7, which is odd.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= \left(\frac{7+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= 4^{\text{th}} \text{ observation} = \frac{2x}{5}$$

$$\text{Median} = \frac{2x}{5} = 4$$

$$x = \frac{4\times 5}{2} = 10$$

Thus (d) is correct option.

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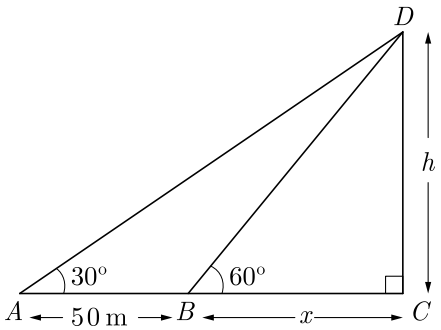
position of person after moving 30 m away from point B on bank of river. Let $BC = x$ be the width of the river.

As per given in question we have drawn figure below.



In right ΔDBC, $\dfrac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad \ldots(1)$$

In right ΔADC,

$$\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 50 \qquad \ldots(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x + 50$$

$$x = 25 \text{ m} \qquad \ldots(3)$$

Thus $h = \sqrt{3} \times 25 = 25\sqrt{3}$

$$= 25 \times 1.732$$

$$= 43.3 \text{ m}$$

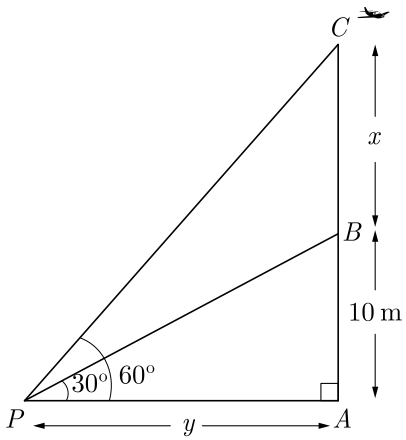
Hence, height of tree is 43.3 m.

Comp 2003

64. From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some height vertically over the top of the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

Sol :

Let AB be the building of height 10 m and the height of the helicopter from top the building be x. As per given in question we have drawn figure below.



Let the distance between point and building be y.

Height of the helicopter from ground

$$= (10 + x) \text{ m}$$

In right ΔBAP we have

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{10}{y} = \frac{1}{\sqrt{3}}$$

$$y = 10\sqrt{3} \qquad \ldots(1)$$

In right ΔCAP,

$$\frac{AC}{PA} = \tan 60^\circ$$

$$\frac{10 + x}{y} = \sqrt{3}$$

$$10 + x = y\sqrt{3} \qquad \ldots(2)$$

From (1) and (2) we have

$$10 + x = 10\sqrt{3} \times \sqrt{3} = 30$$

$$x = 20$$

Hence height of the helicopter is 20 m.

FIVE MARK QUESTIONS

Comp 2002

65. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angle of depression from the aeroplane of two points on both banks of a respectively. Find the width of the river. River in opposite direction are 45° and 60°.

Sol :

Let A be helicopter flying at a height of 300 m above the point O on ground. Let B and C be the bank of river. As per given in question we have drawn figure below.

NODIA

$$= OL - FG$$
$$= 50 - 20 = 30 \text{ m}$$

In right ΔOMG we have

$$\frac{OM}{OG} = \sin 45^\circ$$
$$\frac{OM}{OG} = \frac{1}{\sqrt{2}}$$
$$\frac{30}{OG} = \frac{1}{\sqrt{2}}$$
$$OG = 30\sqrt{2} \text{ m}$$

Thus distance of the bird from the girl is $30\sqrt{2}$ m.

END

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$$1 = \frac{50}{x} \Rightarrow x = 50 \text{ m}$$

(1) Thus distance of pole from bottom of tower is 50 m.

Now in ΔAMC we have

$$\tan 30^\circ = \frac{AM}{MC} = \frac{AM}{x}$$

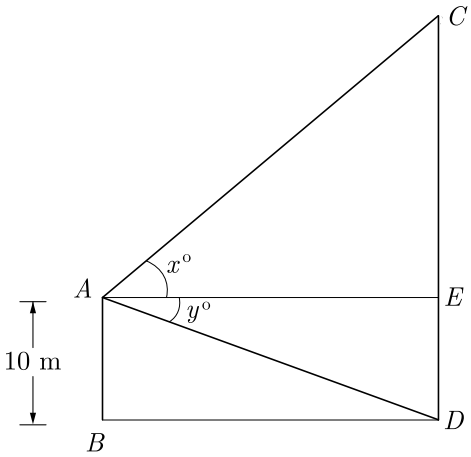
$$AM = \frac{50}{\sqrt{3}} \text{ or } 28.87 \text{ m.}$$

(2) Height pole $h = CD = BM$

$$= 50 - 28.87 = 21.13 \text{ m.}$$

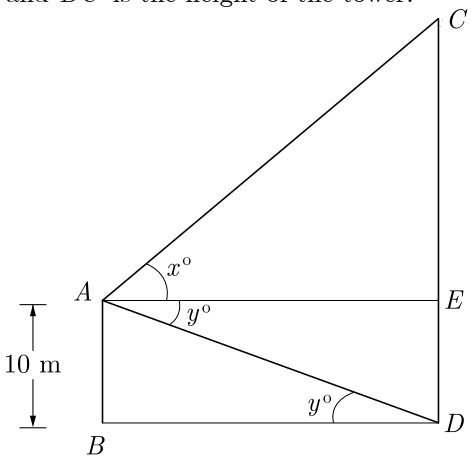
Main 2000

68. From a window A, 10 m above ground, the angle of elevation of the top C of a tower is x°, where $\tan x^\circ = \frac{5}{2}$ and the angle of depression of the foot D of the tower is y°, where $\tan y^\circ = \frac{1}{4}$, see the figure given along side. Calculate the height CD of the tower in metre.



Sol :

In the given figure, A is the position of window and DC is the height of the tower.



We have, $\tan x^\circ = \frac{5}{2}$ and $\tan y^\circ = \frac{1}{4}$

In ΔAEC, we have

$$\tan x^\circ = \frac{EC}{AE}$$

$$\frac{5}{2} = \frac{EC}{AE}$$

$$AE = \frac{2}{5} EC$$

In ΔABD, we have

$$\tan y^\circ = \frac{AB}{BD}$$

$$\frac{1}{4} = \frac{10}{BD}$$

$$\frac{1}{4} = \frac{10}{\frac{2}{5} EC}$$

$$\frac{2}{5} EC = 4 \times 10$$

$$EC = \frac{10 \times 4 \times 5}{2}$$

$$= 100 \text{ m}$$

Height of tower,

$$CD = DE + EC$$

$$= 10 + 100$$

$$= 110 \text{ m}$$

Hence, the height of the tower is 110 m.

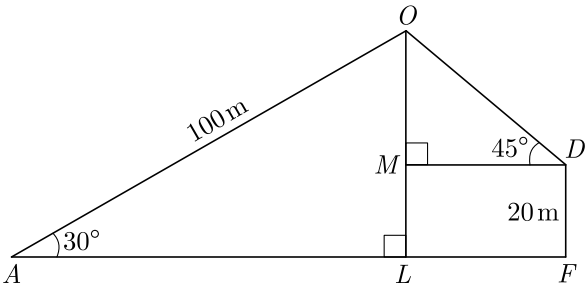
Comp 2000

69. A boy observes that the angle of elevation of a bird flying at a distance of 100 m is 30°. At the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20 m high is 45°. Find the distance of the bird from the girl.

Sol :

Let O be the position of the bird and B be the position of the boy. Let FG be the building and G be the position of the girl. As per given in question we have drawn figure below.

NODIA



In right ΔOLB we have

$$\frac{OL}{BO} = \sin 30^\circ$$

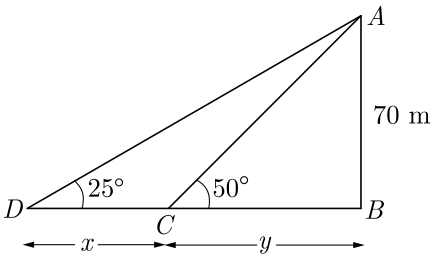
$$\frac{OL}{100} = \frac{1}{2} \Rightarrow OL = 50 \text{ m}$$

$$OM = OL - ML$$

elevation of the top of the tower at 25° and 50° , respectively. If the height of the tower is 70 m, then find the distance between the two people.

Sol :

Let AB be the tower of height 70 m. Let people at points C and D. Let x be the distance between people. As per question statement we have shown digram below.



In right ΔABC we have

$$\tan 50^\circ = \frac{AB}{BC}$$

$$\tan 50^\circ = \frac{70}{y}$$

$$\tan 50^\circ = \frac{70}{y}$$

$$y = \frac{70}{\tan 50^\circ} \qquad (1)$$

In right ΔABD we have

$$\tan 25^\circ = \frac{AB}{BD}$$

$$\tan 25^\circ = \frac{70}{x+y}$$

$$x+y = \frac{70}{\tan 25^\circ}$$

$$x = \frac{70}{\tan 25^\circ} - y \qquad (2)$$

Substituting value of y from eq (1) we have

$$x = \frac{70}{\tan 25^\circ} - \frac{70}{\tan 50^\circ}$$

$$= 70\left(\frac{1}{0.47} - \frac{1}{1.19}\right)$$

$$= 70\,(2.13 - 0.84)$$

$$= 70 \times 1.29$$

$$= 90.3$$

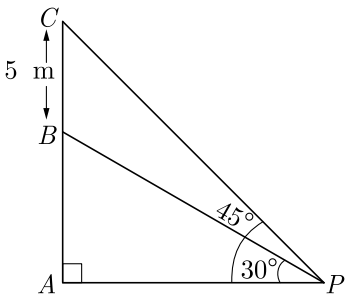
Hence, the distance between the two persons is 90.3 m.

Comp 2004

62. From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flagstaff is 45°. If height of flagstaff is 5 m, find the height of the tower. $(\sqrt{3} = 1.732)$

Sol :

Let AB denotes the height of the tower and BC denotes the height of the flag. As per given information in question we have drawn the figure as given below.



From ΔBAP, $\angle A = 90^\circ$

Now, $$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AP}$$

$$AP = \sqrt{3}\,AB \qquad \ldots(1)$$

Again from ΔCAP,

$$\angle A = 90^\circ$$

and $$\tan 45^\circ = \frac{AC}{AP}$$

$$1 = \frac{AC}{AP}$$

$$AP = AC = (AB + BC)$$

$$AP = (AB + 5) \qquad \ldots(2)$$

From equation (1) and (2), we obtain,

$$(AB+5) = \sqrt{3}\,AB$$

$$5 = \sqrt{3}\,AB - AB$$

$$AB = \frac{5}{(\sqrt{3}-1)} = \frac{5}{(1.732-1)}$$

$$= \frac{5}{0.732} = 6.8306 \text{ m.}$$

Hence, height of the tower, $AB = 6.8306$ m.

Main 2003

63. A man standing on the bank of a river observes that the angle of elevation of a tree on the opposite bank is 60°. When he moves 50 m away from the bank, then he find the angle of elevation to be 30° . Calculate

(i) the width of the river.

(ii) the height of the tree.

Sol :

Let CD be the tree of height h. Let A be the

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CHAPTER 23

MEASURE OF CENTRAL TENDENCY

MULTIPLE CHOICE QUESTIONS

Main 2024

1. The median of the following observations arranged in ascending order is 64.
$27, 31, 46, 52, x, x+4, 71, 79, 85, 90$
Find the value of x:
(a) 60 (b) 61
(c) 62 (d) 66

Sol :

Here, number of observations, $n = 10$, which is even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$64 = \frac{\left(\frac{10}{2}\right)\text{th observation} + \left(\frac{10}{2}+1\right)\text{th observation}}{2}$$

$$64 = \frac{5\text{th observation} + 6\text{th observation}}{2}$$

$$64 = \frac{x + x + 4}{2}$$

$$64 = \frac{2x+4}{2}$$

$$64 = x + 2$$

$$60 = x$$

Thus (a) is correct option.

SQP 2024

2. For the given 25 variables: $x_1, x_2, x_3, \ldots\ldots\ldots x_{25}$
Assertion (A): To find median of the given data, the variate needs to be arranged in ascending or descending order.
Reason (R): The median is the central most term of the arranged data.
(a) A is true, R is false
(b) A is false, R is true
(c) both A and R are true
(d) both A and R are false

Sol :

Median for a data set is the middle value of the set after arranging the set in ascending or descending order.
Thus both are true.
Thus (c) is correct option.

Main 2023

3. Which of the following cannot be determined graphically for a grouped frequency distribution?
(a) Median (b) Mode
(c) Quartiles (d) Mean

Sol :

Mean
Thus (d) is correct option.

4. The cumulative frequency table is useful in determining
(a) Mean (b) Median
(c) Mode (d) All of these

Sol :

Cumulative frequency is defined as a running total of frequencies. It is helpful in finding the mean, median and mode.
Thus (d) is correct option.

5. For finding the popular size of readymade garments, which central tendency is used?
(a) Mean
(b) Median
(c) Mode
(d) Both Mean and Mode

Sol :

For finding the popular size of ready made garments, mode is the best measure of central tendency.
Thus (c) is correct option.

6. If the mean of the numbers $27 + x$, $31 + x$, $89 + x$ $107 + x, 156 + x$ is 82, then the mean of $130 + x$, $126 + x$, $68 + x$, $50 + x$, and $1 + x$ is
(a) 75 (b) 157
(c) 82 (d) 80

Sol :

Given,

NODIA

$$1 = \frac{h}{x} \Rightarrow h = x$$

In right ΔABC we have,

$$\tan 30^\circ = \frac{AB}{BD+1}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

$$x+1 = \sqrt{3}\,h$$

$$x+1 = \sqrt{3}\,x$$

$$1 = \sqrt{3}\,x - 1$$

$$1 = x(\sqrt{3}-1)$$

$$x = \frac{1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}+1}{3-1}$$

$$= \frac{1.732+1}{2} = 1.366 \text{ km}$$

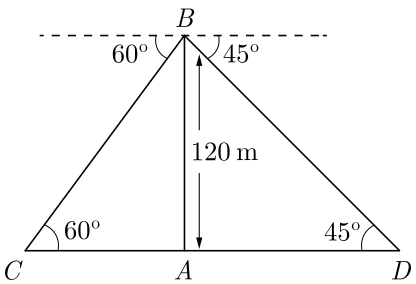
Thus distance of first stone form the foot of hill is 1.366 km and distance of 2nd stone form foot is 2.366 km

Comp 2007

57. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of repression as 60° and 45°. Find the distance between two cars.

Sol :

Let AB be the tower of height 120 m. Let C and D be location of car on opposite side of tower. As per given in question we have drawn figure below.



In right ΔBAD we have

$$\frac{AB}{AD} = \tan 45^\circ$$

$$\frac{120}{AB} = 1$$

$$AB = 120$$

In right ΔBAC we have

$$\frac{AB}{CA} = \tan 60^\circ$$

$$\frac{120}{CA} = \sqrt{3}$$

$$CA = \frac{120}{\sqrt{3}} = 40\sqrt{3}$$

$$CD = CA + AD$$

$$= 120 + 40\sqrt{3}$$

$$= 120 + 40 \times 1.732$$

$$= 189.28 \text{ m}$$

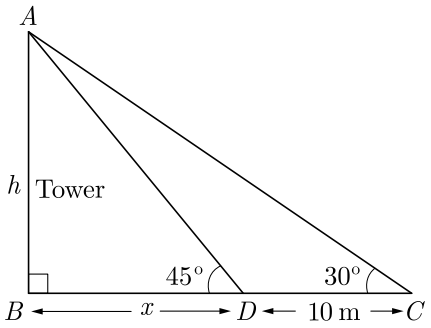
Hence the distance between two men is 189.28 m.

Main 2006, Main 2002

58. The shadow of a vertical tower on a level ground increases by 10 m, when the altitude of the sum changes from 45° to 30°. Find the height of the tower correct to two decimal places.

Sol :

Let AB be the tower of height h. Let BC be the shadow at 45° and BD be shadow at 30°.
As per given in question we have drawn figure below.



In right ΔABC we get,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{h}{x} \Rightarrow h = x$$

In right ΔABD we have,

$$\tan 30^\circ = \frac{AB}{BC+10}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+10}$$

$$x+10 = \sqrt{3}\,h$$

$$h+10 = \sqrt{3}\,h$$

$$\sqrt{3}\,h - h = 10$$

$$(\sqrt{3}-1)\,h = 10$$

$$(\sqrt{3}-1)\,h = 10$$

$$h = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

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12. If the mean of the squares of first n natural numbers is 105, then the first n natural numbers is

(a) 8 (b) 9
(c) 10 (d) 11

Sol :

Sum of square, $\Sigma x^2 = \dfrac{n(n+1)(2n+1)}{6}$

Mean of squares of first n natural numbers,

$$105 = \frac{(n+1)(2n+1)}{6}$$

$$2n^2 + 3n + 1 = 630$$

$$2n^2 + 3n - 629 = 0$$

$$2n^2 + 37n - 34n - 629 = 0$$

$$n(2n+37) - 17(2n+37) = 0$$

$$(2n+37)(n-17) = 0 \Rightarrow n = 17$$

Since, n is odd, therefore median is

$$= \left(\frac{17+1}{2}\right)^{\text{th}} = 9^{\text{th}} \text{ observation.}$$

Thus (b) is correct option.

13. While computing the mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes
(b) centred at the class marks of the classes
(c) centred at the upper limits of the classes
(d) centred at the lower limits of the classes

Sol :

While computing mean of ground data, we assume that the frequencies distribution table.
Thus (b) is correct option.

14. While computing mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes
(b) centred at the class marks of the classes
(c) centred at the upper limits of the classes
(d) centred at the lower limits of the classes

Sol :

Frequencies are centred at the class-marks of the classes.
Thus (b) is correct option.

15. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below

Class	Frequency
13.8-14	2
14-14.2	4
14.2-14.4	5
14.4-14.6	71
14.6-14.8	48
14.8-15	20

The number of athletes who completed the race in less than 14.6 second is :

(a) 11 (b) 71
(c) 82 (d) 130

Sol :

The number of athletes who completed the race in less than 14.6

$$= 2 + 4 + 5 + 71 = 82$$

Thus (c) is correct option.

16. Consider the following distribution :

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is :

(a) 3 (b) 4
(c) 48 (d) 51

Sol :

Marks obtained	Number of students
0-10	$(63 - 58) = 5$
10-20	$(58 - 55) = 3$
20-30	$(55 - 51) = 4$
30-40	$(51 - 48) = 3$
40-50	$(48 - 42) = 6$
50-60	$42 = 42$

Hence, frequency in the class interval 30-40 is 3.
Thus (a) is correct option.

17. The mean weight of 9 students is 25 kg. If one more student is joined in the group the mean is unaltered, then the weight of the 10^{th} student is

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x_i	13	15	17	18	20	λ	25
f_i	1	3	1	3	3	1	3

Frequency of 20 must be highest to be mode of the frequency distribution, $\lambda = 20$.
Now the table will be

x_i	13	15	17	18	20	25
f_i	1	3	1	3	4	3

Main 2014

38. The numbers 6, 8, 10, 12, 13 and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x.

Sol :

Given numbers in ascending order are 6, 9, 10, 12, 13 and x.
Here, number of observations, $n = 6$ which is even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{6}{2}\right)\text{th observation} + \left(\frac{6}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{3\text{rd observation} + 4\text{th observation}}{2}$$

$$= \frac{10+12}{2}$$

$$= \frac{22}{2} = 11$$

Thus median is 11. Mean is also equal to median.

$$\text{Now} \quad \overline{x} = \frac{\Sigma x_i}{N}$$

$$11 = \frac{\text{Sum of the obersevations}}{\text{Number of observations}}$$

$$11 = \frac{6+8+10+12+13+x}{6}$$

$$11 = \frac{49+x}{6}$$

$$66 = 49 + x$$

$$x = 17$$

Hence, the value of x is 17.

SQP 2014

39. Calculate the mean of the distribution given below using the shortcut method.

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Number of students	2	6	10	12	9	7	4

Sol :

We prepare following table to find mean.

Mark (CI)	Number of students f_i	Mid value x_i	$d_i = x_i - 42.5$	$f_i d_i$
11-20	2	15.5	-30	-60
21-30	6	25.5	-20	-120
31-40	10	35.5	-10	-100
41-50	12	45.5	0	0
51-60	9	55.5	10	90
61-70	7	65.5	20	140
71-80	4	75.5	30	10
Total	$\Sigma f_i = 50$			$\Sigma f_i d_i = 70$

Here, $\Sigma f_i = 50$ and $\Sigma f_i d_i = 70$
Using shortcut method,

$$\text{Mean} \quad \overline{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 45.5 + 1.4$$

$$= 46.9$$

Hence, the mean marks of the given distribution is 46.9.

Comp 2013

40. Observations of some data are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. If the median of the data is 4, then what is the value of x?

Sol :

Given observations are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. On arranging the above observations in ascending order, we get

$$\frac{x}{5},\ \frac{x}{4},\ \frac{x}{3},\ \frac{2x}{5},\ \frac{2x}{3},\ \frac{3x}{4},\ x$$

Here, total number of observations are 7, which is odd.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= \left(\frac{7+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= 4^{\text{th}} \text{ observation} = \frac{2x}{5}$$

$$\text{Median} = \frac{2x}{5} = 4$$

$$x = \frac{4 \times 5}{2} = 10$$

Main 2012

41. Find the mode and median of the following frequency distribution.

x	10	11	12	13	14	15
f	1	4	7	5	9	3

Sol :

NODIA

50-60	55	2	5	10
			$\Sigma f_i = 65$	$\Sigma f u_i = -44$

Let assumed mean, $a = 35$ and given $h = 10$.

Mean, $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 35 + \frac{-44}{65} \times 10$$

$$= 35 - 6.76 = 28.24$$

Main 2022

21. The mean of the following distribution is 50. Find the unknown frequency

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	6	f	8	12	8

Sol :

The frequency table is as given below :

Class Interval	**Frequency** (f)	**Class Mark** (x)	fx
0-20	6	10	60
20-40	f	30	$30f$
40-60	8	50	400
60-80	12	70	840
80-100	8	90	720
	$\Sigma f = 34 + f$		$\Sigma fx = 2020 + 30f$

Mean $$\overline{x} = \frac{\Sigma f x_i}{\Sigma f}$$

$$50 = \frac{2020 + 30f}{34 + f}$$

$$50(34 + f) = 2020 + 30f$$

$$1700 + 50f = 2020 + 30f$$

$$20f = 320$$

$$f = 16$$

Comp 2022

22. The mean of the following distribution is 314. Determine the missing frequency x.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	10	12	7	8

Sol :

We prepare following table to find mean.

C.I.	x	f	$u_i = \frac{x - f}{h}$	$f_i u_i$
1-10	5	5	-3	-15
10-20	15	x	-2	$-2x$
20-30	25	10	-1	-10
30-40	35	12	0	0
40-50	45	7	1	7
50-60	55	8	2	16
Total		$42 + 2$		$-2x - 2$

Let mid point of class 30-40 be assumed mean a.

$$a = 35$$

Mean $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$31.4 = 35 + \frac{-2x - 2}{42 + x} \times 10$$

$$(2x + 2)10 = (42 + x)(3.6)$$

$$20x + 20 = 151.2 + 3.6x$$

$$16.4x = 131.2 \Rightarrow x = 8$$

Main 2020

23. The mean of the following data is 16. Calculate the value of f.

Marks	Number of Students
5	3
10	7
15	f
20	9
25	6

Sol :

The frequency table is as given below :

Marks x_i	Number of students f	$f_i x_i$
5	3	15
10	7	70
15	f	$15f$
20	9	180
25	6	150
Total	$\Sigma f_i = 25 + f$	$\Sigma f_i x_i = 415 + 15f$

Given, mean of above data is 16.

$$\frac{\Sigma f_i x_i}{\Sigma f_i} = 16$$

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(a) 25 kg (b) 24 kg
(c) 26 kg (d) 23 kg

Sol :

The sum of the weights of the 9 students $= 25 \times 9 = 225$. If one more student is joined in the group, then total number of students is 10 and mean is 25.
Hence, the sum of the weights of the 10^{th} students $= 25 \times 10 = 250$.
Hence, the weight of the 10^{th} student is $250 - 225 = 25$ kg.
However we can answer this question without any calculation. If mean is not altered on adding more data, then added data must be of mean value.
Thus (a) is correct option.

18. **Assertion :** If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.
Reason : Median $= \left(\frac{n+1}{2}\right)^{th}$ value, if n is odd.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Sol :

Arranging the terms in ascending order,
0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

$$\text{Median value} = \left(\frac{11+1}{2}\right)^{th}$$

$$= 6^{th} \text{ value} = 27$$

Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

THREE MARKS QUESTIONS

Main 2023

19. The following table gives the marks scored by a set of students in an examination. Calculate the mean of the distribution by using the short cut method.

Marks	**Number of workers** (f)
0−10	3
10−20	8
20−30	14
30−40	9
40−50	4
50−60	2

Sol :

The frequency table is as given below :

Marks	**Number of Students** f_i	**Class marks** x_i	$d_i = x_i - A$	$f_i d_i$
0−10	3	5	−20	−60
10−20	8	15	−10	−80
20−30	14	25	0	0
30−40	9	35	10	90
40−50	4	45	20	80
50−60	2	55	30	60
	$\Sigma f_i = 40$			$\Sigma f_i d_i = 90$

Here $\Sigma f_i d_i = 90$, $\Sigma f_i = 40$ and $A = 25$

Mean $$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 25 + \frac{90}{40}$$

$$= 25 + 2.25$$

$$= 27.25$$

Comp 2023

20. Find the mean of the following distribution by step deviation method :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	13	20	15	7	5

Sol :

The frequency table is as given below :

Class	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
0-10	5	−3	5	−15
10-20	15	−2	13	−26
20-30	25	−1	20	−20
30-40	35	0	15	0
40-50	45	1	7	7

NODIA

Given, assumed mean, $A = 45$.
The table for the given data is as below.

Class	x_i	f_i	$d_i = x_i - A$	$f_i d_i$
10-20	15	5	-30	-150
20-30	25	2	-20	-40
30-40	35	7	-10	-70
40-50	$\boxed{45}$	9	0	0
50-60	55	2	10	20
60-70	65	5	20	100
Total		$\Sigma f_i = 30$		$\Sigma f_i d_i = -140$

Here, $\Sigma f_i d_i = -140$, $\Sigma f_i = 30$ and $A = 45$

Mean $\overline{x} = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$= 45 + \dfrac{-140}{30}$

$= 45 - \dfrac{14}{3}$

$= \dfrac{135 - 14}{3}$

$= \dfrac{121}{3} = 40.33$

SQP 2019

27. The mean of the following frequency distribution is 25. Find the value of p.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

Sol :

We prepare following table to find mean.

Class-Interval	Mid-Point x_i	f_i	$f_i x_i$
0-10	5	4	20
10-20	15	6	90
20-30	25	10	250
30-40	35	6	210
40-50	45	p	45 p
		$26 + p$	$570 + 45p$

We have $M = \dfrac{\sum f_i x_i}{\sum f_i}$

$25 = \dfrac{570 + 45p}{26 + p}$

$650 + 25p = 570 + 45p$

$650 - 570 = 45p - 25p$

Thus $p = 4$

Main 2018

28. If the mean of the following distribution is 24, find the value of a.

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

Sol :

We prepare following table to find mean.

Marks	Mid values x_i	Number of students f_i	$f_i x_i$
0-10	5	7	35
10-20	15	a	$15a$
20-30	25	8	200
30-40	35	10	350
40-50	45	5	225
		Σf_i	$\Sigma f_i x_i$

Here $\Sigma f_i = 30 + a$

and $\Sigma f_i x_i = 810 + 15a$

Mean $\overline{x} = \dfrac{\Sigma f x_i}{\Sigma f_i}$

$24 = \dfrac{810 + 15a}{30 + a}$

$24(30 + a) = 810 + 15a$

$720 + 24a = 810 + 15a$

$9a = 90$

$a = 10$

Hence, the required value of a is 10.

Comp 2018

29. The mean of the following distribution is 53. Find the missing frequency p :

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	p	13

Sol :

We prepare following table to find mean.

Class	x_i	f_i	$f_i x_i$
0-20	10	12	120

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that it is 110. Find the true mean.

Sol :

Mean, $M = \frac{\sum fx}{\sum f}$

$50 = \frac{\sum fx}{100}$

$\sum fx = 5000$

Correct, $\sum fx' = 5000 - 100 + 110$

$= 5010$

Correct Mean $= \frac{5010}{100}$

$= 50.1$

Main 2006

46. The median of the observations 11, 12, 14, 18, $x+4$, 30, 35 and 41 arrange in ascending order is 24. Find the value of x.

Sol :

Given observation are 11, 12, 14, 18, $x+4$, 30, 32, 35, 41.

Here, $n = 9$, which is odd.

Thus $\frac{n+1}{2} = \frac{9+1}{2} = 5$

Now 5th observation is $x+4$. Thus median is $x+4$.

But given that median is 24.

Thus $x+4 = 24$

$x = 20$

Main 2005

47. If the mean of the following distribution is 7.5, find the missing frequency f.

Variable	5	6	7	8	9	10	11	12
Frequency	20	17	f	10	8	6	7	6

Sol :

We prepare following table to find mean.

Variable x_i	Frequency f_i	$f_i x_i$
5	20	100
6	17	102
7	f	$7f$
8	18	80
9	8	72
10	6	60
11	7	77
12	6	72
Total	$\Sigma f_i = 74 + f$	$\Sigma f_i x_i = 563 + 7f$

Here, $\Sigma f_i = 74 + f$

and $\Sigma f_i x_i = 563 + 7f$

Mean, $\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$

$7.5 = \frac{563 + 7f}{74 + f}$

$7.5(74 + f) = 563 + 7f$

$555 + 7.5f = 563 + 7f$

$0.5f = 8$

$f = 16$

Hence, the missing frequency is 16.

Main 2004

48. For the following set of numbers, find the median.
10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9, 15

Sol :

We arrange the given data in ascending order:
3, 4, 9, 10, 12, 15, 17, 27, 47, 48, 75, 81
Here, number of observations, $n = 12$, which is even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)\text{th observation} + \left(\frac{12}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{6\text{th observation} + 7\text{th observation}}{2}$$

$$= \frac{15+17}{2}$$

$$= \frac{32}{2} = 16$$

Hence, the median of the given data is 16.

Main 2002

49. The marks of 20 students in a test were as follow
5, 6, 8, 9, 10, 11, 12, 13, 13, 14, 14, 15, 15, 15, 15, 15, 16, 16, 18, 19, 20.
Find
(i) the mean.
(ii) the median
(iii) the mode.

Sol :

Given, the marks of 20 students in a test are 5, 6, 8, 9, 10, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19 and 20.
Here, $n = 20$
(i) Mean
Sum of all observation is
$5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 +$

NODIA

160-165	10	b
165-170	c	43
170-175	5	48
175-180	2	d

Sol :

From the table,

$$12 + a = 25 \Rightarrow a = 25 - 12 = 13$$

$$25 + 10 = b \Rightarrow b = 35,$$

$$b + c = 43 \Rightarrow$$

$$c = 43 - b$$

$$= 13 - 35 = 8$$

and $48 + 2 = d \Rightarrow d = 50$

Main 2015

34. The marks obtained by 30 students in a class assessment of 5 marks is given below :

Marks	0	1	2	3	4	5
Number of students	1	3	6	10	5	5

Calculate the mean, median and mode of the above distribution.

Sol :

We prepare following table to find mean.

Marks	f_i	$x_i f_i$	cf
0	1	0	1
1	3	3	4
2	6	12	10
3	10	30	20
4	5	20	25
5	5	25	30
	$\Sigma f_i = 30$	$\Sigma f_i x_i = 90$	

Now $\text{Mean} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{90}{30} = 3$

Here $\dfrac{N}{2} = \dfrac{30}{2} = 15$

The cumulative frequency just greater than 15 is 20. The variate whose cumulative frequency is 20, is 3. Thus median is 3. Also, 3 have greatest frequency, thus mode is also 3.

Comp 2015

35. Find x and y from the following cumulative frequency distribution :

Classes	Frequency	c.f.
0-8	15	15
8-16	x	28
16-24	15	43
24-32	18	y
32-40	09	70

Sol :

From the cumulative frequency distribution,

$$15 + x = 28$$

$$x = 28 - 15 = 13$$

and $43 + 18 = y$

$$y = 61$$

Hence $x = 13$ and $y = 61$

Main 2015

36. Calculate the mean of the following distribution :

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	5	12	35	24	16

Sol :

We prepare following table to find mean.

Class interval	Class mark x_i	Frequency f_i	$x_i f_i$
0-10	5	8	40
10-20	15	5	75
20-30	25	12	300
30-40	35	35	1225
40-50	45	24	1080
50-60	55	16	880
		$\Sigma f_i = 100$	$\Sigma f_i x_i = 3600$

Mean $\overline{x} = \dfrac{\Sigma f x_i}{\Sigma f}$

$$= \frac{3600}{100} = 36$$

SQP 2015

37. Find the value of λ, if the mode of the following data is 20 :
15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20, λ, 18.

Sol :

First we prepare the following table as discrete frequency distribution.

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20-40	30	15	450
40-60	50	32	1600
60-80	70	p	$70p$
80-100	90	13	1170
Total		$\Sigma f_i = 72 + p$	$\Sigma f_i x_i = 3340 + 70p$

Mean, $\overline{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$$53 = \frac{3340 + 70p}{72 + p}$$

$$3340 + 70p = 53(72 + p)$$
$$3340 + 70p = 3816 + 53p$$
$$70p - 53p = 3816 - 3340$$
$$17p = 476$$
$$p = \frac{476}{17} = 28$$

Main 2017

30. Calculate the mean of the following distribution using step deviation method.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	10	9	25	30	16	10

Sol :

We prepare following table to find mean.

Marks	Mid-values x_i	f	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	fu_i
0-10	5	10	-20	-2	-20
10-20	15	9	-10	-1	-9
20-30	25	25	0	0	0
30-40	35	30	10	1	30
40-50	45	16	20	2	32
50-60	45	10	30	3	30
Total		$\Sigma f = 100$			$\Sigma fu_i = 63$

Here, $A = 25,\ h = 10,\ N = \Sigma f_i = 100$

and $\Sigma f_i u_i = 63$

Mean $\overline{x} = A + h\left\{\dfrac{1}{N}\Sigma f_i u_i\right\}$

$$= 25 + 10 \times \frac{1}{100} \times 63$$
$$= 25 + \frac{63}{10}$$
$$= \frac{250 + 63}{10} = 31.3$$

Comp 2017

31. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the find the mean of the last three observation

Sol :

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$
$$9 = \frac{x + x + 3 + x + 5 + x + 7 + x + 10}{5}$$
$$9 = \frac{5x + 25}{5}$$
$$x = 4$$

So, mean of last three observation,

$$= \frac{x + 5 + x + 7 + x + 10}{3}$$
$$= \frac{5x + 22}{3}$$
$$\frac{3x + 22}{3} = \frac{3 \times 4 + 22}{3}$$
$$= \frac{12 + 22}{3} = \frac{34}{3} = 11\frac{1}{3}$$

Main 2016

32. The mean of following numbers is 68. Find the value of 'x'. 45, 52, 60, x, 69, 70, 26, 81 and 94 Hence, estimate the median.

Sol :

Mean $\overline{x} = 68$

$$\overline{x} = \frac{\Sigma x_i}{N}$$
$$= \frac{\text{Sum of the obersevations}}{\text{Number of observations}}$$
$$68 = \frac{497 + x}{9}$$
$$612 = 497 + x$$
$$x = 612 - 497$$
$$= 115$$

Now, for median, we have to write the given data in ascending order as follows.

26, 45, 52, 60, 69, 70, 81, 94, 115

Here $n = 9$ is odd. Thus $\frac{9+1}{2} = 5$ th term 69 is median.

Comp 2016

33. Find the unknown entries a, b, c, d in the following distribution of heights of students in a class :

Height (in cm)	Frequency	Cumulative Frequency
150-155	12	12
155-160	a	25

NODIA

Table for cumulative frequencies is shown below :

x	f	cf
10	1	1
11	4	5
12	7	12
13	5	17
14	9	26
15	3	29
Total	$N = 29$	

Here $N = 29$ is odd.

Thus $\frac{N+1}{2} = \frac{29+1}{2} = 15$

Cumulative frequency just greater than 15 is 17.
The cumulative frequency just greater than 15 is 17. The variate whose cumulative frequency is 17, is 13. Thus median is 13.
From table, we see that highest frequency is 9 and the corresponding observation is 14. Thus mode is 14.

SQP 2012

42. The mean and median of the data a, b and c are 50 and 35 respectively, where $a < b < c$. If $c - a = 55$, then find the value of $(b - a)$.

Sol :

Since, a, b and c and are in ascending order, therefore median is b i.e $b = 35$.

Mean $\frac{a+b+c}{3} = 50$

$a + b + c = 150$

$a + c = 150 - 35$

$= 115$ ...(1)

Also, it is given that $c - a = 55$ ...(2)

Subtracting equation (2) and (1), we get

$a = 30$

Hence, $b - a = 35 - 30 = 5$

Main 2011

43. Find the mean of the following frequency distribution by assumed mean method

Class-interval	25-30	30-35	35-40	40-45	45-50	50-55	55-60
Frequency	14	12	16	6	5	3	4

Sol :

Table for product of frequency and deviation is shown below :

CI	f_i	Mid value x_i	$d_i = x_i - 42.5$	$f_i d_i$
25-30	14	27.5	-15	-210
30-35	12	32.5	-10	-120
35-40	16	37.5	-5	-80
40-45	6	42.5	0	0
45-50	5	47.5	5	25
50-55	3	52.5	10	30
55-60	4	57.5	15	60
Total	$\Sigma f_i = 60$			$\Sigma f_i d_i = -295$

Here, $\Sigma f_i = 50$ and $\Sigma f_i d_i = -295$

Using shortcut method,

Mean $\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 42.5 + \frac{-295}{60}$

$= 42.5 - 4.91$

$= 37.58$

Hence, the mean marks of the given distribution is 37.58.

Main 2009

44. Find the mean, median and mode of the following distribution. 8, 10, 7, 6, 10, 11, 6, 13, 10

Sol :

Given observation are 8, 10, 7, 6, 10, 11, 6, 13, 10.
Here, number of observations,

$n = 9$

Sum of all observation is

$8 + 10 + 7 + 6 + 10 + 11 + 6 + 13 + 10 = 81$

Now $\bar{x} = \frac{\Sigma x_i}{N}$

$= \frac{81}{9} = 9$

Now we arrange the data in ascending order.
6, 6, 7, 8, 10, 10, 10, 11, 13
Here, $n = 9$ which is odd.

Here $\frac{n+1}{2} = \frac{9+1}{2} = 5$

5th observation is 10. Thus median is 10.

In the given distribution, 10 is repeated maximum number of times. Thus mode is 10.

45. The mean of 100 observation is 50. The value of the largest observation is 100. It was later found

NODIA

$$\frac{415+15f}{25+f} = 16$$

$$415+15f = 400+16f$$

$$f = 415-400$$

$$= 15$$

SQP 2020

24. Find the mean of the following data :

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	6	8	10	12	8	6

Sol :

We prepare following table to find mean.

Classes	f_i	x_i	$f_i x_i$
0-20	6	10	60
20-40	8	30	240
40-60	10	50	500
60-80	12	70	840
80-100	8	90	720
100-120	6	110	660
	$\sum f_i = 50$		$\sum f_i x_i = 3020$

Mean, $M = \frac{\sum x_i f_i}{\sum f_i} = \frac{3020}{50}$

$= 60.4$

Main 2019

25. In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below

Marks	Number of students
1	1
2	2
3	3
4	3
5	6
6	10
7	5
8	4
9	3
10	3

Calculate the following for the given distribution.
(i) Median
(ii) Mode

Sol :

The frequency table is as given below :

Marks x	Number of students f	cf
1	1	1
2	2	3
3	3	6
4	3	9
5	6	15
6	10	25
7	5	30
8	4	34
9	3	37
10	3	40
Total	$\Sigma f_i = 40$	

(i) Median
Here, number of observations, $N = 40$, which is even.

Now $\frac{N}{2} = \frac{40}{2} = 20$

The cumulative frequency just greater than 20 is 25. The variate whose cumulative frequency is 25, is 6. Thus median is 6.
(ii) Mode
It is clear from the table that highest number of students is 10, i.e. it has highest frequency 10. Therefore corresponding value of 10 is 6. Hence, the mode is 6.

SQP 2019

26. The data on the number of patients attending a hospital in a month are given below.
Find the average (mean) number of patients attending the hospital in a month by using the shortcut method.
Take the assumed mean as 45. Give your answer correct to 2 decimal places.

Number of patients	Number of days
10-20	5
20-30	2
30-40	7
40-50	9
50-60	2
60-70	5

Sol :

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$$14+15+15+15+15+$$
$$+16+16+18+19+20=264$$

Here, number of observations,

$$n=20$$

Now $$\overline{x}=\frac{\Sigma x_i}{n}$$

$$=\frac{264}{20}$$

$$=13.2$$

(ii) Median

Here $n=20$ which is even.

$$\text{Median}=\frac{(\frac{n}{2})\text{th observation}+(\frac{n}{2}+1)\text{th observation}}{2}$$

$$=\frac{(\frac{20}{2})\text{th observation}+(\frac{20}{2}+1)\text{th observation}}{2}$$

$$=\frac{10\text{th observation}+11\text{th observation}}{2}$$

$$=\frac{14+14}{2}$$

$$=\frac{28}{2}=14$$

Hence, the median of the given data is 14.

(iii) Mode

In the given distribution, 15 is repeated maximum number of times. Thus mode is 15.

Main 2000

50. Calculate the mean, the median and the mode of the following numbers. 3, 1, 5, 6, 3, 4, 5, 3, 7, 2

Sol :

Given 3, 1, 5, 6, 3, 4, 5, 3, 7, 2.

We arrange i the given data in ascending order:

1, 2, 3, 3, 3, 4, 5, 5, 6, 7

(i) Mean

Sum of all observation is

$1+2+3+3+3+4+5+5+6+7=39$

Here, number of observations,

$$n=10$$

Now $$\overline{x}=\frac{\Sigma x_i}{n}$$

$$=\frac{39}{10}$$

$$=3.9$$

(ii) Median

Here $n=10$ which is even.

$$\text{Median}=\frac{(\frac{n}{2})\text{th observation}+(\frac{n}{2}+1)\text{th observation}}{2}$$

$$=\frac{(\frac{10}{2})\text{th observation}+(\frac{10}{2}+1)\text{th observation}}{2}$$

$$=\frac{5\text{th observation}+6\text{th observation}}{2}$$

$$=\frac{3+4}{2}$$

$$=\frac{7}{2}=3.5$$

(iii) Mode

In the given distribution, 3 is repeated maximum number of times. Thus mode is 3.

NODIA

FOUR MARK QUESTIONS

Comp 2019

51. Find the mean of the following data :

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	20	35	52	44	38	31

Sol :

Let $A=70$ be assumed mean. We prepare following table to find mean.

C.I.	f	x_i	$u_i=\frac{x_i-a}{h}$	fu_i
0-20	20	10	-3	-60
20-40	35	30	-2	-70
40-60	52	50	-1	-52
60-80	44	70	0	0
80-100	38	90	1	38
100-120	31	110	2	62
	$\sum f_i=220$			$\sum fu_i=-82$

Mean, $$\overline{x}=a+\frac{\sum f_i u_i}{\sum f_i}\times h$$

$$=70+\frac{(-82)}{220}\times 20$$

$$=70-\frac{82}{11}$$

$$=70-7.45$$

$$=62.55$$

SQP 2018

52. On annual day of a school, 400 students participated in the function. Frequency distribution showing their ages is as shown in the following table :

frequency is 20 and the corresponding class-interval is 55-60 which is model class.

Main 2008

69. The weight of 50 apples was recorded as given below. Calculate the mean weight to the nearest gram, by the step deviation method.

Weight (in grams)	Number of apples
80-85	5
85-90	8
90-95	10
95-100	12
100-105	8
105-110	4
110-115	3

Sol :

We prepare following table to find mean.

Weight	No. of apples f_i	Mid value x_i	$u_i = \frac{x_i - 97.5}{5}$	$f_i u_i$
80-85	5	82.5	-3	-15
85-90	8	87.5	-2	-16
90-95	10	92.5	-1	-10
95-100	12	$\boxed{97.5}$	0	0
100-105	8	102.5	1	8
105-110	4	107.5	2	8
110-115	3	112.5	3	9
Total	$\Sigma f_i = 50$			$\Sigma f_i u_i = -16$

Here, $\Sigma f_i = 50$, $\Sigma f_i u_i = -16$, $A = 97.5$ and $h = 5$

Using step-deviation method,

$$\text{Mean weight} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 97.5 + \frac{-16}{50} \times 5$$

$$= 97.5 - 1.6$$

$$= 95.6$$

Hence, the mean weight is 95.6 gm or 96 gm.

END

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55, 61, 71, 80
Since, 46 appear maximum number of times, mode is 46.

SQP 2017

55. The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

Expenditure (in ₹)	100-150	150-200	200-250	250-300	300-350
Number of households	4	5	12	2	2

Sol :

Let $a = 225$ be assumed mean. We prepare following table to find mean.

Daily Expenditure (in ₹)	No. of household f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-150	4	125	-2	-8
150-200	5	175	-1	-5
200-250	12	225	0	0
250-300	2	275	1	2
300-350	2	325	2	4
	$\sum f_i = 25$			$\sum f_i u_i = -7$

Mean, $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$
$$= 225 + \frac{(-7)}{25} \times 50$$
$$= 225 - 14$$
$$= 211$$

Hence, mean of daily expenditure on food is ₹211.

Compa 2016

56. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Sol :

Class	x_i	f_i	$f_i x_i$
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
Total		$\Sigma f_i = 40 + f$	$\Sigma f_i x_i = 704 + 20f$

NODIA

We have $$\sum f_i = 40 + f$$
$$\sum f_i x_i = 704 + 20f$$

Mean, $$M = \frac{\sum f_i x_i}{\sum f_i}$$
$$18 = \frac{704 \times 20f}{40 + f}$$
$$720 + 18f = 704 + 20f$$
$$f = 8$$

Main 2013, Main 2006

57. Find the mean of the following distribution.

Class-interval	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	6	8	12	5	9

Sol :

We prepare following table to find mean.

Class-interval	Frequency f_i	Mid value x_i	$f_i x_i$
20-30	10	25	250
30-40	6	35	210
40-50	8	45	360
50-60	12	55	660
60-70	5	65	325
70-80	9	75	675
Total	$\Sigma f_i = 50$		$\Sigma f_i x_i = 2480$

Here, $\Sigma f_i = 50$

and $\Sigma f_i x_i = 2480$

Mean $$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$
$$= \frac{2480}{50} = 49.6$$

Hence, the mean is 49.6.

Main 2013

58. The median of the observations 11, 12, 14, $(x-2)$, $(x+4)$, 32, 38 and 47 arranged in ascending order is 24. Find the value of x and hence find

Total	$\Sigma f_i = 36 + f$		$\Sigma f_i x_i =$ $1940 + 35f$

Here, $\Sigma f_i = 36 + f$

and $\Sigma f_i x_i = 1940 + 35f$

Given, $\overline{x} = 52$

Mean $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$52 = \frac{1940 + 35f}{36 + f}$$

$$52(36 + f) = 1940 + 35f$$

$$1872 + 52f = 1940 + 35f$$

$$17f = 68$$

$$f = 4$$

Hence, the value of f is 4.

Main 2009

64. The following table gives the wage of workers in a factory.

Wages (in) Rs.	45-50	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	5	8	30	25	14	12	6

Calculate the mean by the shortcut method.

Sol :

We prepare following table to find mean.

CI	f_i	Mid value x_i	$d_i = x_i - 62.5$	$f_i d_i$
45-50	5	47.5	-15	-75
50-55	8	52.5	-10	-80
55-60	30	57.5	-5	-150
60-65	25	62.5	0	0
65-70	14	67.5	5	70
70-75	12	72.5	10	120
75-80	6	77.5	15	90
Total	$\Sigma f_i = 100$			$\Sigma f_i d_i = -25$

Here, $\Sigma f_i = 100$ and $\Sigma f_i d_i = -25$

Using shortcut method,

Mean $\overline{x} = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$$= 62.5 + \frac{-25}{100}$$

$$= 62.5 - 0.25$$

$$= 62.25$$

Hence, the mean of wage of workers is Rs 62.25.

Main 2007

65. Find the mean of the following distribution.

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency	10	6	8	12	5

Sol :

We prepare following table to find mean.

Class-interval	Frequency f_i	Mid value x_i	$f_i x_i$
0-10	10	5	50
10-20	6	15	90
20-30	8	25	200
30-40	12	35	420
40-50	5	45	225
Total	$\Sigma f_i = 41$		$\Sigma f_i x_i = 985$

Here, $\Sigma f_i = 41$

and $\Sigma f_i x_i = 985$

Mean $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$= \frac{985}{41} = 24.02$$

Hence, the mean is 24.02.

Main 2006

66. Find the mean of the following distribution.

Class-interval	0-50	50-100	100-150	150-200	200-250	250-300
Frequency	4	8	16	13	6	3

Sol :

We prepare following table to find mean.

Class-interval	Frequency f_i	Mid value x_i	$f_i x_i$
0-50	4	25	100
50-100	8	75	600
100-150	16	125	2000
150-200	13	175	2275
200-250	6	225	1350
250-300	3	275	825

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is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0-30	30-60	60-90	90-120	120-150	150-180
Frequency	12	21	x	52	y	11

Sol :

We prepare following table to find mean.

Class	x_i	f_i	$f_i x_i$
0-30	15	12	180
30-60	45	21	945
60-90	75	x	$75x$
90-120	105	52	5460
120-150	135	y	$135y$
150-180	165	11	1815
Total		Σf_i	$\Sigma f_i x_i$

Here $\Sigma f_i = x + y + 96$

and $\Sigma f_i x_i = 8400 + 75x + 135y$

Now $x + y + 96 = 150$

$$x + y = 54 \quad \ldots(1)$$

Now $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$91 = \frac{8400 + 75x + 135y}{150}$$

$$13650 = 8,400 + 75x + 135y$$

$$75x + 135y = 5250$$

$$5x + 9y = 350 \quad \ldots(2)$$

Solving equation (1) and (2) we get $x = 34$ and $y = 20$

Main 2010

62. The distribution given below shows the marks obtained by 25 students in an aptitude test.

Marks obtained	5	6	7	8	9	10
Number of students	3	9	6	4	2	1

Find the mean, median mode of the distribution.

Sol :

We prepare following table to find mean.

NODIA

Marks x_i	Number of student f_i	$f_i x_i$	Cumulative frequency (cf)
5	3	15	3
6	9	54	12
7	6	42	18
8	4	32	22
9	2	18	24
10	1	10	25
Total	$\sum f_i = 25$	$\sum f_i x_i = 171$	

Here, $\Sigma f_i = 25$

and $\Sigma f_i x_i = 171$

Mean $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$= \frac{171}{25} = 6.84$$

Here, $N = \sum f_i = 25$, which is odd.

Here $\dfrac{N+1}{2} = \dfrac{25+1}{2} = 13$

The cumulative frequency just greater than 13 is 18. The variate whose cumulative frequency is 18, is 7. Thus median is 7.

Also, 6 have greatest frequency 9, thus mode is also 6.

Main 2010

63. The mean of the following distribution is 52 and the frequency of class-interval 30-40 is f. Find f.

Class-interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	3	f	7	2	6	13

Sol :

We prepare following table to find mean.

Class-interval	Frequency f_i	Mid value x_i	$f_i x_i$
10-20	5	15	75
20-30	3	25	75
30-40	f	35	$35f$
40-50	7	45	315
0-60	2	55	110
60-70	6	65	390
70-80	13	75	975

Ages (in years)	05-07	07-09	09-11	11-13	13-15	15-17	17-19
No. of students	70	120	32	100	45	28	5

Find mean of the above data.

Sol :

We prepare following table to find mean.

C.I.	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
05-07	70	6	-3	-210
07-09	120	8	-2	-240
09-11	32	10	-1	-32
11-13	100	12	0	0
13-15	45	14	1	45
15-17	28	16	2	56
17-19	5	18	3	15
	$\sum f_i = 400$			$\sum f_i u_i = -366$

Assumed mean, $a = 12$

$$\text{Mean, } M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 12 + \frac{-366}{400} \times 2$$

$$= 12 - \frac{183}{100} = 12 - 1.83$$

$$= 10.17$$

SQP 2017

53. The marks obtained by 110 students in an examination are given below

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Number of Students	14	16	28	23	18	8	3

Find the mean marks of the students.

Sol :

We prepare following table to find mean.

Marks	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
30-35	14	32.5	-3	-42
35-40	16	37.5	-2	-32
40-45	28	42.5	-1	-28
45-50	23	47.5	0	0
50-55	18	52.5	1	18
55-60	8	57.5	2	16
60-65	3	62.5	3	9
	$\sum f_i = 110$			$\sum f_i u_i = -59$

Let a be assumed mean, where $a = 47.5$

$$\text{Mean } M = a + \frac{\sum f_i u_i}{N} \times h$$

$$= 47.5 + \frac{(-59)}{110} \times 5$$

$$= 47.5 - 2.682$$

$$= 44.818$$

Main 2017

54. The marks of 10 students of a class in an examination arranged in ascending order is as following : 13, 35, 43, 46, x, $x+4$, 55, 61, 71, 80
If the median marks is 48, find the value of x. Hence find the mode of the given data.

Sol :

Given, marks of 10 students in ascending order as follows :
13, 35, 43, 46, x, $x+4$, 55, 61, 71, 80
Here, number of observations, $n = 10$, which is even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$48 = \frac{\left(\frac{10}{2}\right)\text{th observation} + \left(\frac{10}{2}+1\right)\text{th observation}}{2}$$

$$48 = \frac{5\text{th observation} + 6\text{th observation}}{2}$$

$$48 = \frac{x + x + 4}{2}$$

$$48 = \frac{2x+4}{2}$$

$$48 = x + 2$$

$$46 = x$$

$$x = 46$$

$$x + 4 = 46 + 4 = 50$$

Thus 5th observation is and 6th observation is 50.
Now, the given data is 13, 35, 43, 46, 46, 50, 50,

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Total	$\Sigma f_i = 50$		$\Sigma f_i x_i = 6850$

Here, $\Sigma f_i = 50$

and $\Sigma f_i x_i = 6850$

Mean $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{7150}{50} = 143$

Hence, the mean is 143.

Main 2002

67. The following table gives the weekly wages of workers in factory.

Weekly wages (in Rs.)	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
	5	20	10	10	9	6	12	8

Calculate

(i) the mean.

(ii) the model class.

(iii) the number of workers weekly, below Rs. 80.

(iv) the number of workers getting Rs. 65 or more, but less than Rs. 85 as weekly wages.

Sol :

We prepare following table to find mean.

Weekly wages (Rs.)	Mid value x_i	Number of workers f_i	$f_i x_i$	cf
50-5	52.5	5	262.5	5
55-60	57.5	20	1150.0	25
60-65	62.5	10	625.0	35
65-70	67.5	10	675.0	45
70-75	72.5	9	652.5	54
75-80	77.5	6	465.0	60
80-85	82.5	12	990.0	72
85-90	87.5	8	700.0	80
Total		$\Sigma f_i = 80$	$\Sigma f_i x_i = 5520$	

Here, $\Sigma f_i = 80$

and $\Sigma f_i x_i = 5520$

(i) Mean

Mean $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{5520}{80} = Rs.\,69$

(ii) Model Class

Class of maximum frequency is called the model class. In the above maximum frequency is 20 and the corresponding interval is 55-60. Hence, model class is 55-60.

(iii) Number of workers weekly, below Rs. 80.

Number of workers getting weekly wages below Rs. 80 are $5 + 20 + 10 + 10 + 9 + 6 = 60$ workers

(iv) Number of workers getting Rs. 65 or more, but less than Rs. 85 as weekly wages

Number of workers getting Rs. 65 or more than but less than Rs. 85 are $10 + 9 + 6 + 12 = 37$

FIVE MARK QUESTIONS

Main 2011

68. (i) Using step deviation method, calculated the mean marks of the following distribution.

Class-interval	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Frequency	5	20	10	10	9	6	12	8

(ii) State the model class.

Sol :

(i) Table for product of frequencies and step deviations is shown below :

Class-interval	Frequency f_i	Mid value x_i	$u_i = \dfrac{x_i - 76.5}{5}$	$f_i u_i$
50-55	5	52.5	-3	-15
55-60	20	57.5	-2	-40
60-65	10	62.5	-1	-10
65-70	10	$\boxed{76.5}$	0	0
70-75	9	72.5	1	9
75-80	6	77.5	2	12
80-85	12	82.5	3	36
85-90	8	87.5	4	32
Total	$\Sigma f_i = 80$			$\Sigma f_i u_i = 24$

Here $\Sigma f_i = 80$, $\Sigma f_i u_i = 24$, $A = 67.5$ and $h = 5$

Mean marks $= A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 67.5 + \dfrac{24}{80} \times 5$

$= 67.5 + \dfrac{24}{16}$

$= 67.5 + 1.5$

$= 69$

Hence, mean marks is 69.

(ii) Model class is the class, which have highest frequency. In the above table, highest

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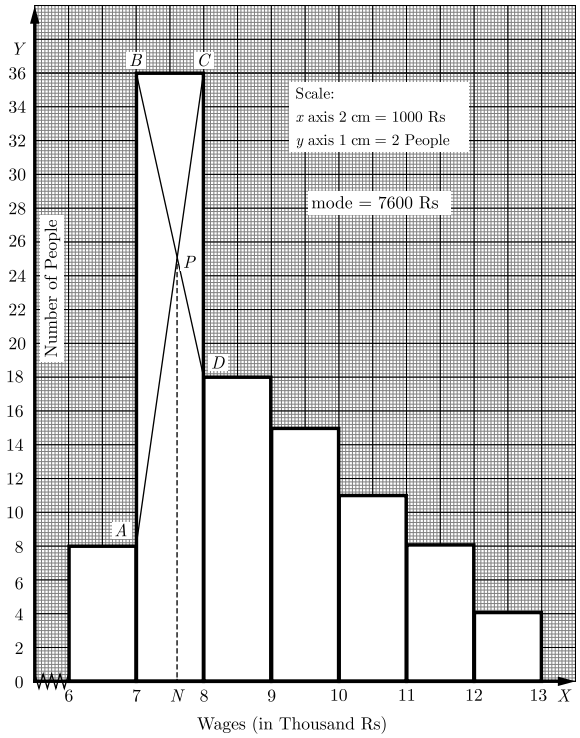

Y
B
C
Scale:
x axis 2 cm = 1000 Rs
y axis 1 cm = 2 People
mode = 7600 Rs
Number of People
P
D
A
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
6 7 N 8 9 10 11 12 13 X
Wages (in Thousand Rs)

In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 7600.
Hence, the required mode is Rs 7600.

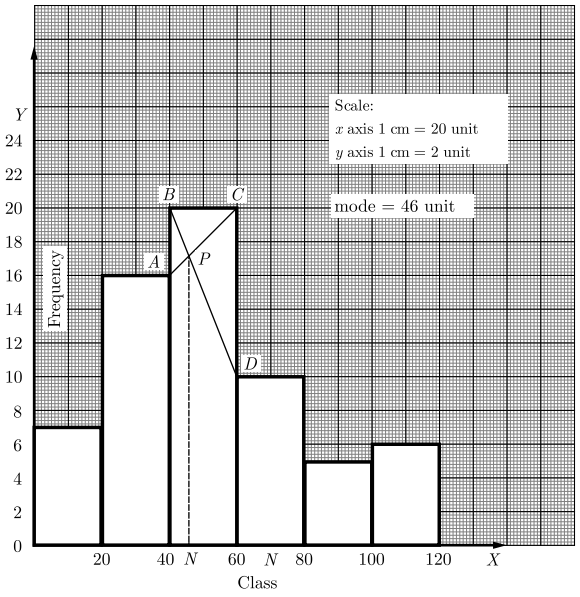
SQP 2023

3. Use graph paper to answer this question. Draw a histogram and hence estimate the mode for the following frequency distribution.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	16	20	10	5	6

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the X-axis at N. The abscissa of the point represents 46. Hence, the required mode is 46.

Main 2022

4. Marks obtained by 100 students in an examination are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students	5	15	20	28	20	12

Draw a histogram for the given data using a graph paper and find the mode.

Sol :

The histogram from the given frequency table is shown below.

NODIA

CHAPTER 24

HISTOGRAM AND OGIVE

FOUR MARK QUESTIONS

Main 2023

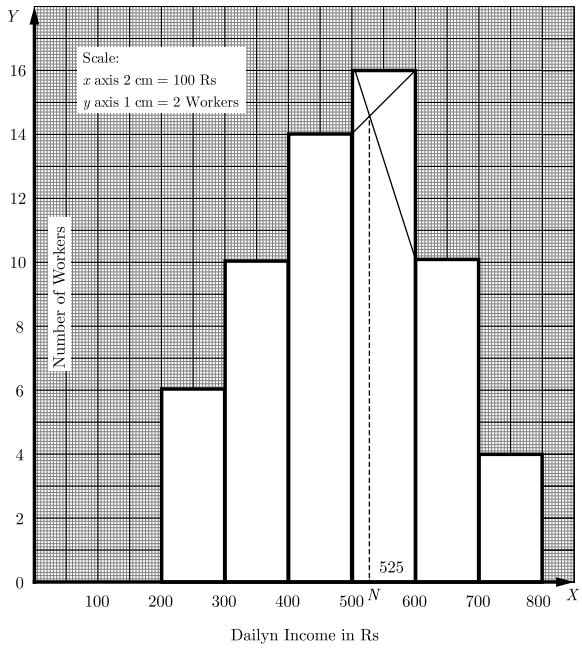
1. The following distribution gives the daily wages of 60 workers of a factory.

Daily income in ₹	**Number of workers** (f)
200−300	6
300−400	10
400−500	14
500−600	16
600−700	10
700−800	4

Use graph paper to answer this question.
Take 2 cm = ₹100 along one axis and 2 cm = 2 workers along the other axis. Draw a histogram and hence find the mode of the given distribution.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represents 525. Hence, the required mode is 525.

Comp 2023

2. Monthly Daily wages of 110 workers, obtained in a survey, are tabulated below :

Wages (in ₹)	Number of people
6000-7000	8
7000-8000	36
8000-9000	18
9000-10000	15
10000-11000	11
11000-12000	8
12000-13000	4

Use graph paper to answer this question. Draw a histogram and hence find the mode of the given distribution.

Sol :

The histogram from the given frequency table is shown below.

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the mean.

Sol :

Given observation in ascending order are 11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38 and 47.
Here, number of observation, $n=9$, which is odd.
Thus median is $\frac{n+1}{2}=\frac{9+1}{2}=5$ th observation.
The 5th observation is $(x+4)$ but median is given 24. Thus

$$x+4 = 24$$
$$x = 20$$

Thus our observation are now 11, 12, 14, 18, 24, 29, 32, 38 and 47.
Sum of observation

$$11+12+14+18+24+29+32+38+47=225$$

Now $$\overline{x} = \frac{\Sigma x_i}{n}$$
$$= \frac{\text{Sum of the obersevations}}{\text{Number of observations}}$$
$$= \frac{225}{9} = 25$$

Hence, the value of x is 20 and mean is 25.

Comp 2014

59. The mean of the following distribution is 48 and sum of all the frequency is 50. Find the missing frequencies x and y.

Class	20-30	30-40	40-50	50-60	60-70
Frequency	8	6	x	11	y

Sol :

We prepare following table to find mean.

C.I.	f_i	x_i	$u_i=\frac{x_i-a}{h}$	f_iu_i
20-30	8	25	-2	-16
30-40	6	35	-1	-6
40-50	x	45	0	0
50-60	11	55	1	11
60-70	y	65	2	$2y$
Total	$\Sigma f_i=$ $25+x+y$			$\Sigma f_iu_i=$ $2y-11$

Mean, $$M = a+\frac{\sum f_iu_i}{\sum f_i}\times h$$
$$48 = 45+\frac{2y-11}{50}\times 10$$
$$48-45 = \frac{2y-11}{5}$$
$$3\times 5 = 2y-11$$
$$15 = 2y-11 \Rightarrow y = 13$$

Also $$\sum f_i = 25+x+y = 50$$
$$x+y = 25$$
$$x = 25-13 = 12$$

Thus $x=12$ and $y=13$

Main 2012

60. Marks obtained by 40 students in a short assessment are given below, where a and b are missing data.

Marks	5	6	7	8	9
Number of students	6	a	16	13	b

If the mean of the distribution is 7.2, find a and b.

Sol :

We prepare following table to find mean.

Marks x_i	Number of students f_i	f_ix_i
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
Total	$\Sigma f_i=$ $35+a+b$	Σf_ix_i $=246+6a+9b$

Given, $\overline{x}=7.2$ and $\Sigma f_i=40$

Here, $$\Sigma f_i = 35+a+b$$
$$40 = 35+a+b$$
$$5 = a+b \quad \text{(i)}$$

and $$\Sigma f_ix_i = 246+6a+9b$$

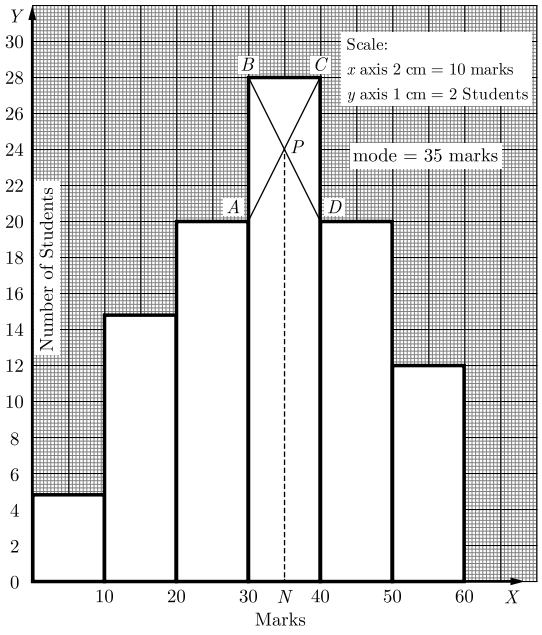
Mean, $$\overline{x} = \frac{\sum f_ix_i}{\sum f_i}$$
$$7.2 = \frac{246+6a+9b}{40}$$
$$288 = 246+6a+9b$$
$$42 = 6a+9b$$
$$14 = 2a+3b \quad \text{(ii)}$$

Solving eq (i) and (ii) we get we get $a=1$ and $b=4$.
Hence, the value of a is 1 and b is 4.

Comp 2011

61. If the mean of the following frequency distribution

NODIA



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represents 35. Hence, the required mode is 35.

SQP 2022

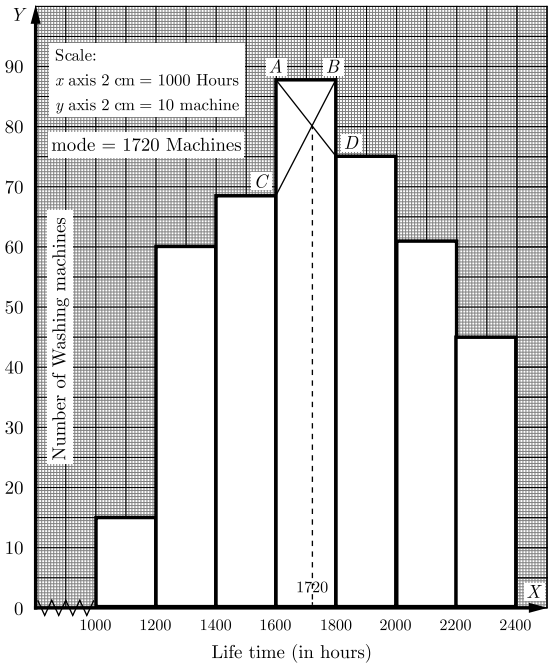
5. The following distribution gives the distribution of life times of washing machines of a certain company :

Life time (in hours)	No. of washing machines
1000-1200	15
1200-1400	60
1400-1600	68
1600-1800	86
1800-2000	75
2000-2200	61
2200-2400	45

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 17220.
Hence, the required mode is 1720 hours.

SQP 2021

6. Use a graph paper for this question, the daily pocket expenses of 100 students in a school are given below :

Pocket expenses (in Rs.)	Number of students
0-10	7
10-20	5
20-30	14
30-40	21
40-50	25
50-60	15
60-70	6
70-80	7

Draw a histogram representing the above distribution and estimate the mode from the graph.

Sol :

The histogram from the given frequency table is shown below.

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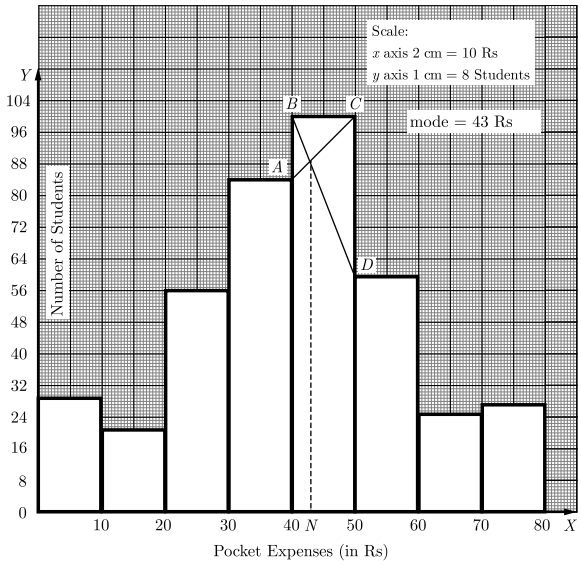


In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represents 43.
Hence, the required mode is Rs 43.

Main 2020

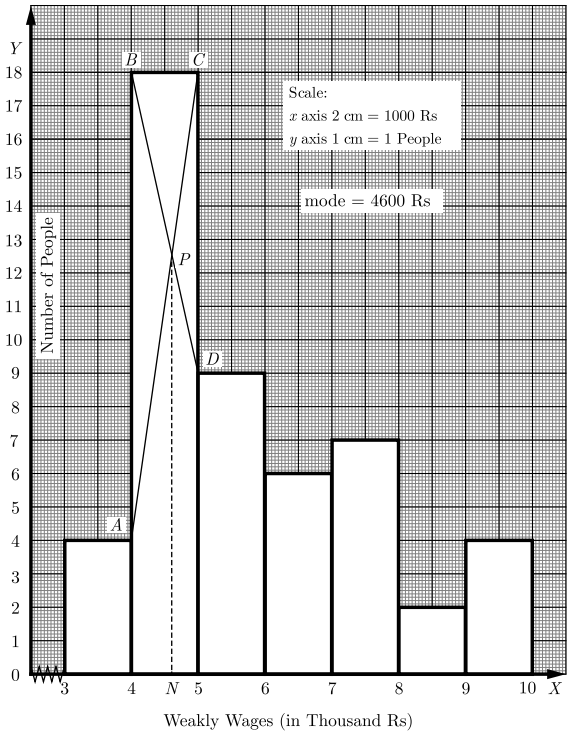
7. Draw a Histogram for the given data, using a graph paper.

Weekly wages (in ₹)	Number of people
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	6
7000-8000	7
8000-9000	2
9000-10000	4

Estimate the mode from the graph.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 4600.
Hence, the required mode is Rs 4600.

SQP 2020

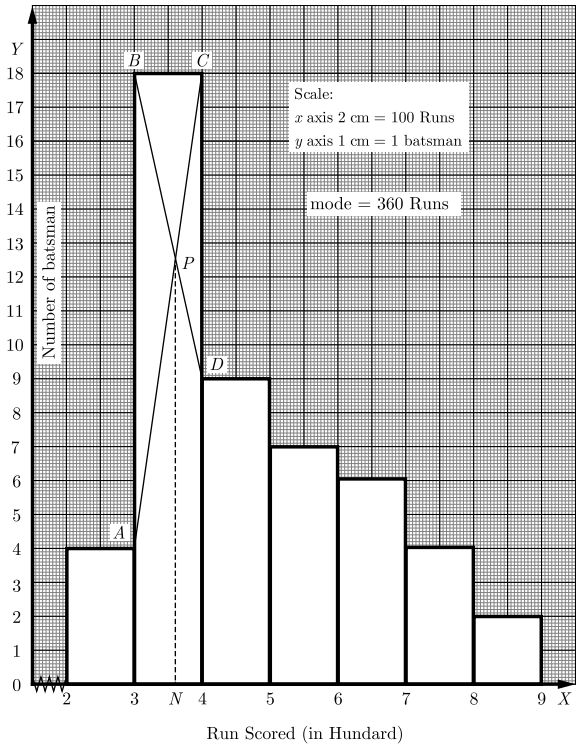
Using a graph paper to draw a histogram for the given distribution showing the number of runs scored by 50 batsman in IPL. Estimate the mode of the data.

Runs Scored	Number of batsman
200-300	4
300-400	18
400-500	9
500-600	7
600-700	6
700-800	4
800-900	2

Sol :

The histogram from the given frequency table is shown below.

NODIA



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the X-axis at N. The abscissa of the point represents 4600. Hence, the required mode is 360 runs.

Main 2019

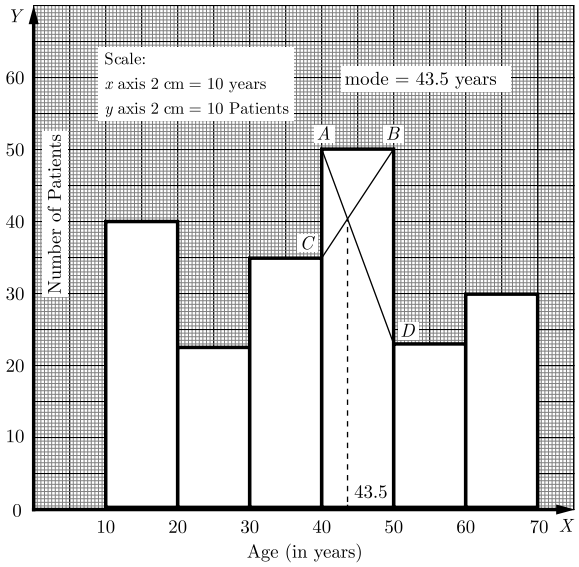
9. The following are the ages of 200 patients getting medical treatment in a hospital on a particular day :

Age (In years)	10-20	20-30	30-40	40-50	50-60	60-70
No. of Patients	40	22	35	50	23	30

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 43.5.

Hence, the required mode is 43.5 years.

Comp 2019

10. The following frequency distribution shows the distance (in meters) thrown by 70 students in a Javelin throw competition.

Distance (in m)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	6	7	13	20	14	7	3

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.

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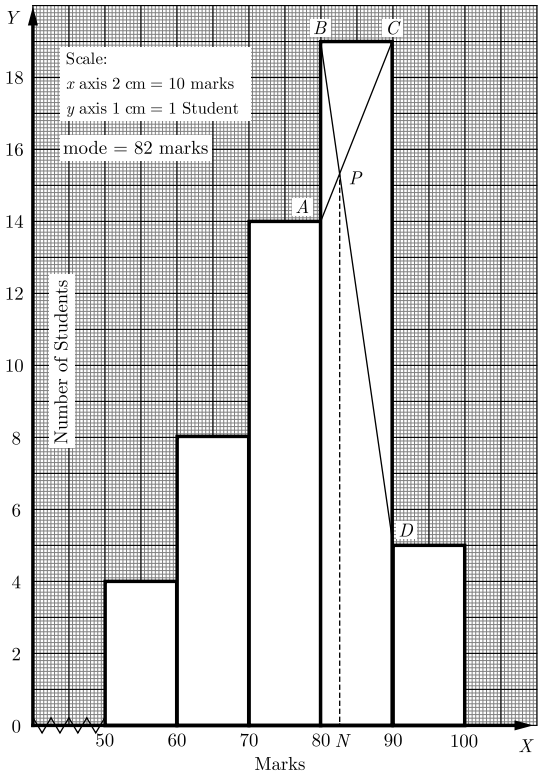


In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the X-axis at N. The abscissa of the point represents 82.6.
Hence, the required mode is 82.6.

Comp 2011

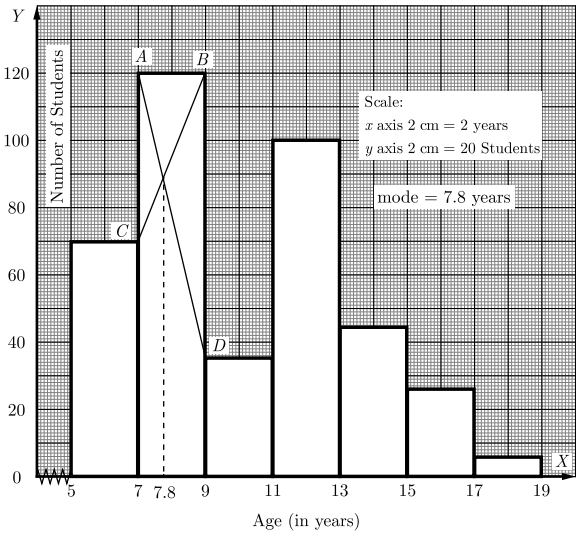
23. On annual day of a school, 400 students participated in the function. Frequency distribution showing their ages is as shown in the following table :

Ages (in years)	Number of students
5-7	70
7-9	120
9-11	32
11-13	100
13-15	45
15-17	28
17-19	5

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 7.8.
Hence, the required mode is 7.8 year.

Main 2010

24. Following frequency distribution shows the Air Quality Index of different localities of Delhi on 27th January 2024 reported by Times of India Newspaper on 28th January 2024.

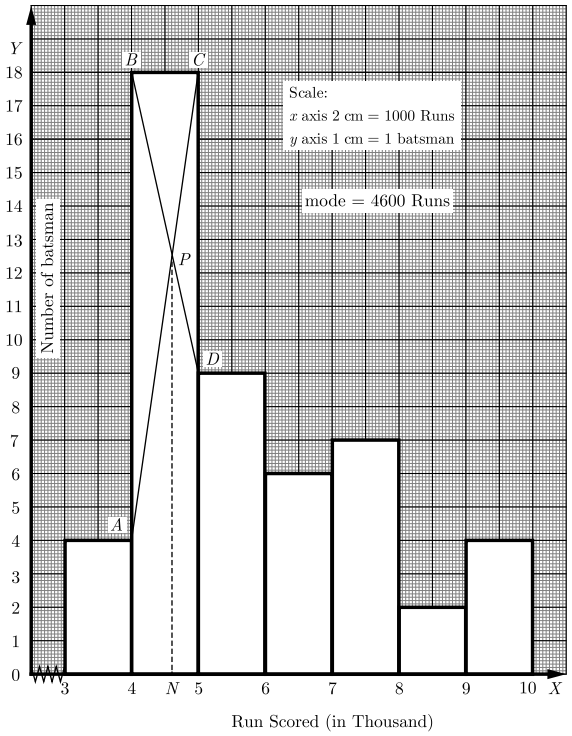
AIQ	Number of Weeks
270-280	4
280-290	10
290-300	14
300-310	20
310-320	24
320-330	8

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.

NODIA



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the X-axis at N. The abscissa of the point represents 4600. Hence, the required mode is 4600.

SQP 2018

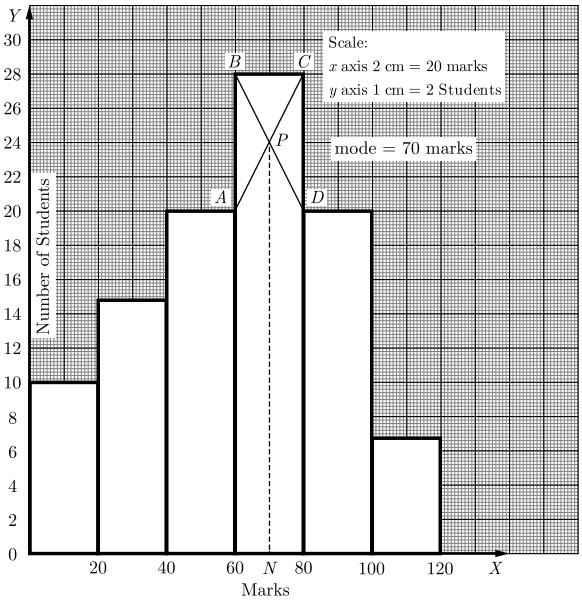
13. Marks obtained by 100 students in an examination are given below :

Marks	0-20	20-40	40-60	60-80	80-100	100-120
Number of Students	10	15	20	28	20	7

Draw a histogram for the given data using a graph paper and find the mode.

Sol :

The histogram from the given frequency table is shown below.



NODIA

In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represents 70. Hence, the required mode is 70 marks.

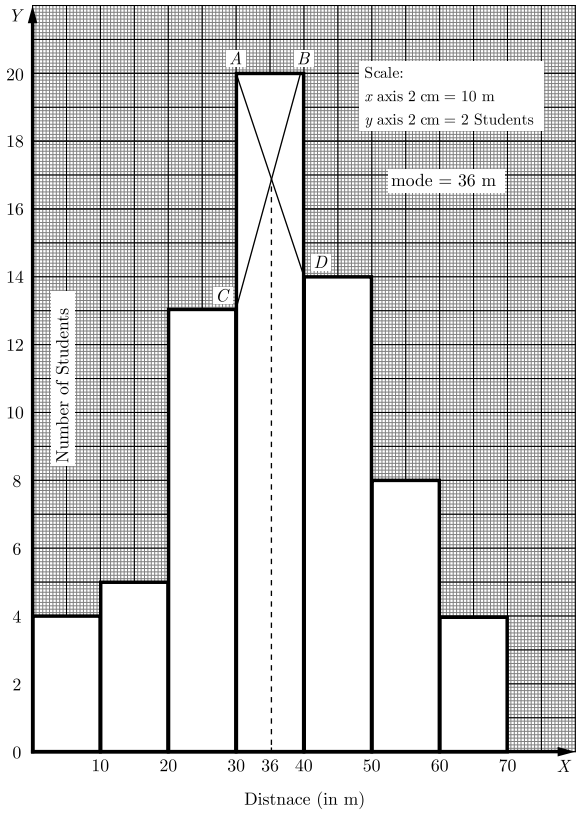
Comp 2017

14. Draw a histogram from the following frequency distribution and find the mode from the graph.

Class	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	3	10	36	28	15	12

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 36.
Hence, the required mode is 36 m.

Comp 2018

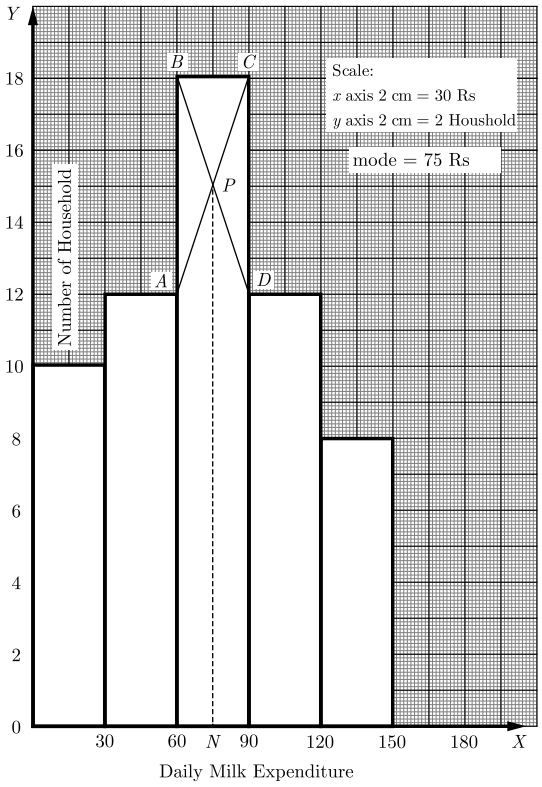
11. Following frequency distribution shows the expenditure on milk of 60 households in a locality :

Daily milk expenditure (Rs.)	Number of households
0-30	10
30-60	12
60-90	18
90-120	12
120-150	8

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 75.
Hence, the required mode is 75.

Comp 2019

12. Using a graph paper to draw a histogram for the given distribution showing the number of runs scored by 50 batsman. Estimate the mode of the data.

Runs Scored	Number of batsman
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	6
7000-8000	7
8000-9000	2
9000-10000	4

Sol :

The histogram from the given frequency table is shown below.

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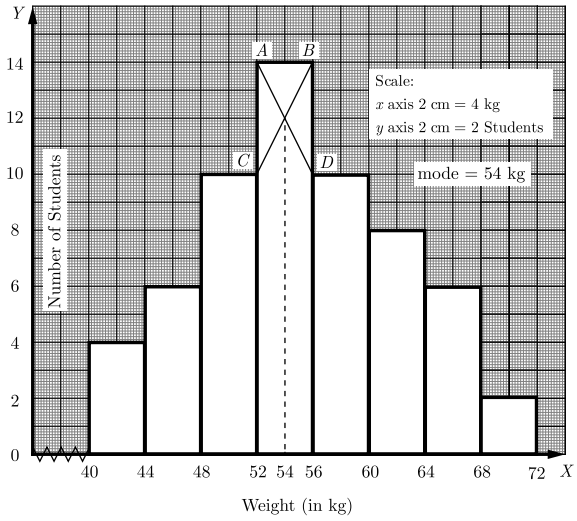


In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 54.
Hence, the required mode is 54 kg.

Main 2015

17. A Mathematics aptitude test of 100 students as follows :

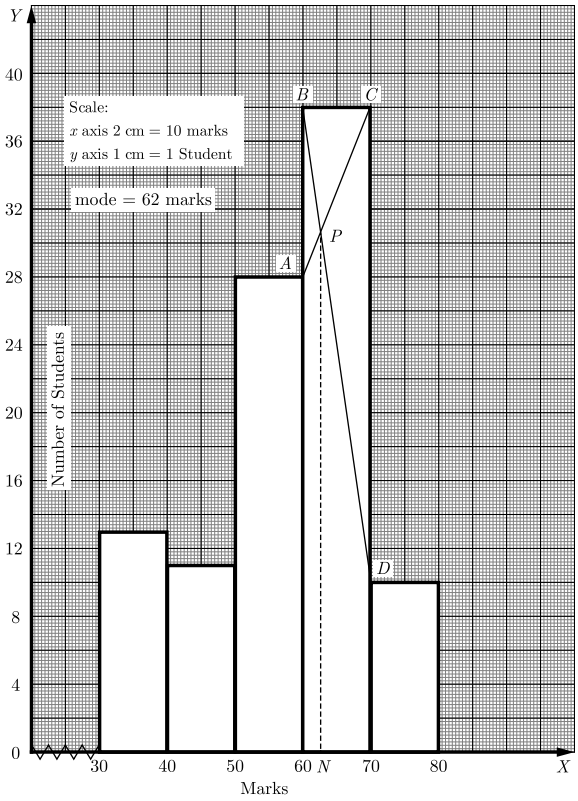
Marks	30-40	40-50	50-60	60-70	70-80
Number of students	13	11	28	38	10

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below. Here scale on x-axis starts at 30 marks, so their is some break at the origin.

NODIA



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the X-axis at N. The abscissa of the point represents 62.
Hence, the required mode is 62 marks.

Comp 2015

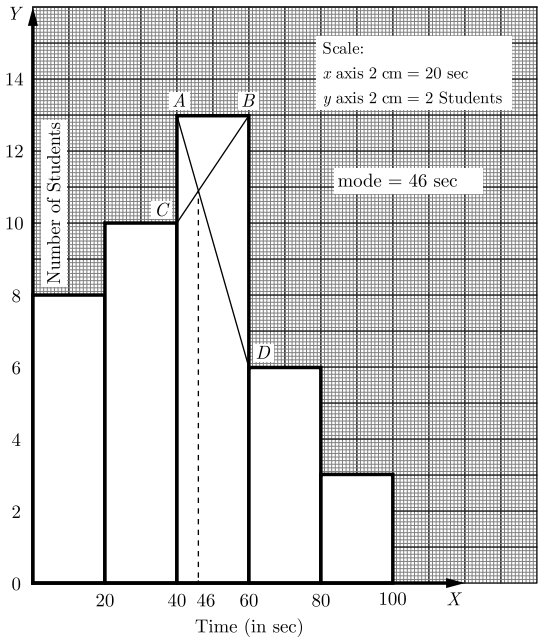
18. A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 46.
Hence, the required mode is 46 sec.

Main 2014

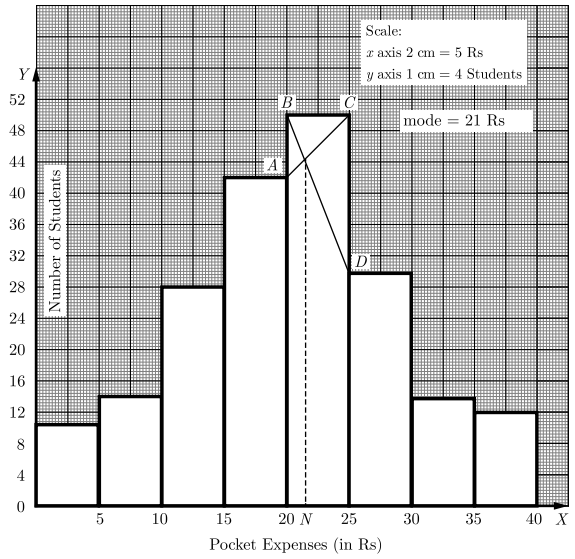
19. Use a graph paper for this question, the daily pocket expenses of 200 students in a school are given below :

Pocket expenses (in Rs.)	Number of students
0-5	10
5-10	14
10-15	28
15-20	42
20-25	50
25-30	30
30-35	14
35-40	12

Draw a histogram representing the above distribution and estimate the mode from the graph.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represents 21.
Hence, the required mode is 21.

Comp 2014

20. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 min and summarised in the table given below.

Number of cars	**Number of Periods**
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15
70-80	08

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.

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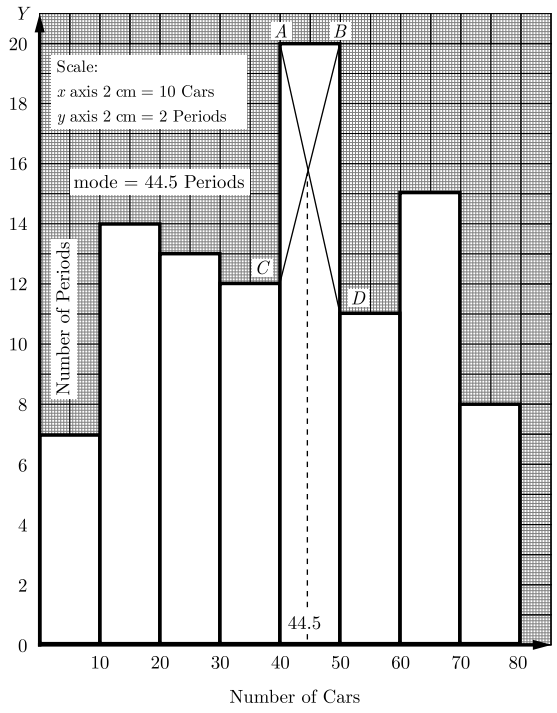


In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 44.5.
Hence, the required mode is 44.5 period.

Main 2013

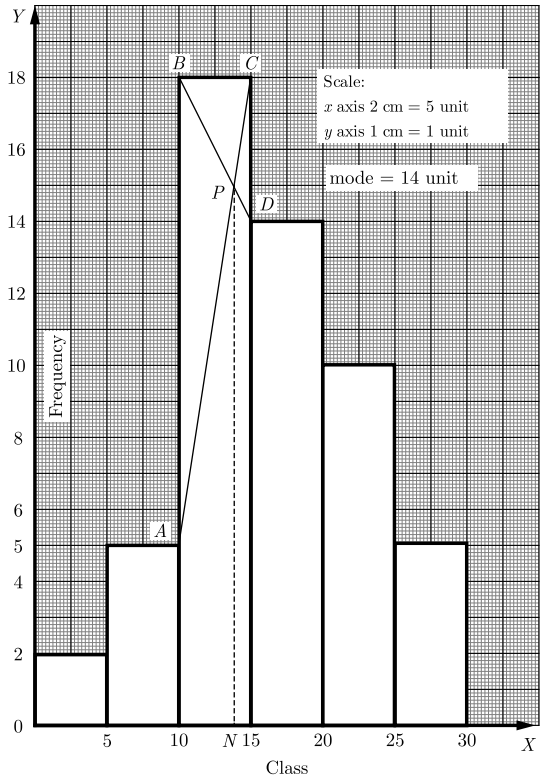
21. Draw a histogram from the following frequency distribution and find the mode from the graph.

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	5	18	14	8	5

Sol :

The histogram from the given frequency table is shown below.

NODIA



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 14.
Hence, the required mode is 14.

Main 2011

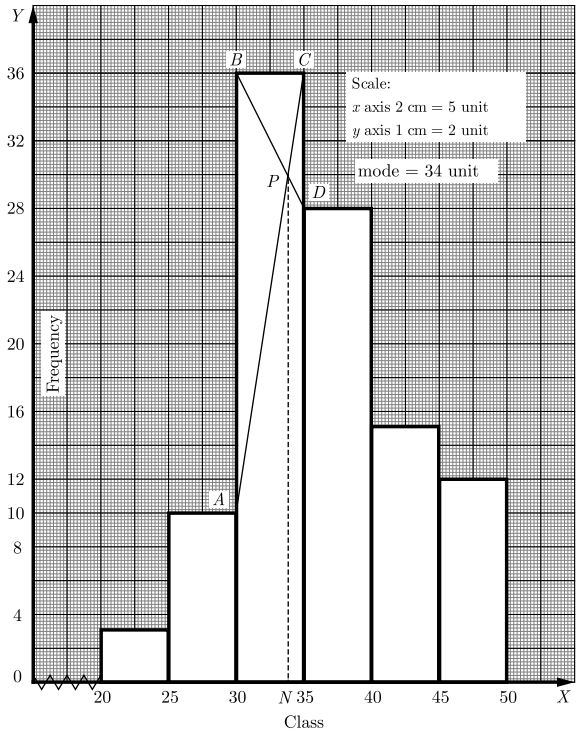
22. A Mathematics aptitude test of 50 students as follows :

Marks	50-60	60-70	70-80	80-90	90-100
Number of students	4	8	14	19	5

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below. Here scale on x-axis starts at 50 marks, so their is some break at the origin.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 34.
Hence, the required mode is 34.

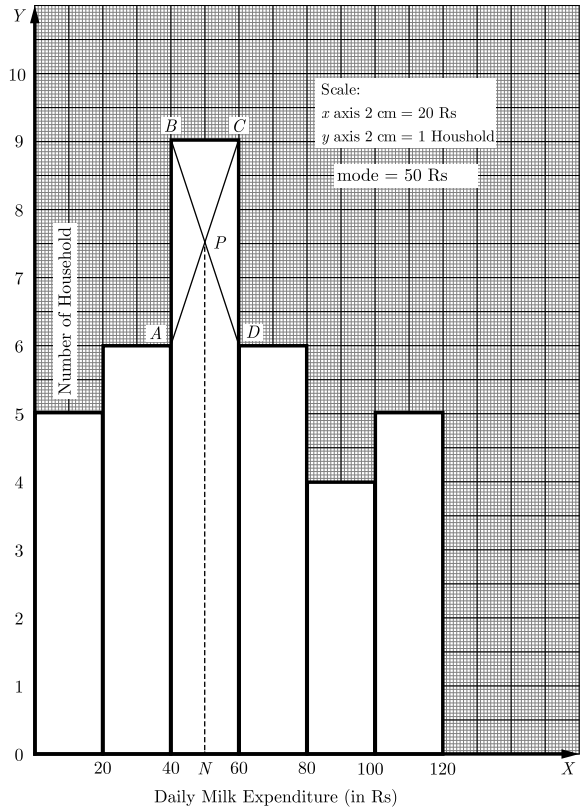
Main 2016

15. Following frequency distribution shows the expenditure on milk of 35 households in a locality :

Daily milk expenditure (Rs.)	Number of households
0-20	5
20-40	6
40-60	9
60-80	6
80-100	4
100-120	5

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :
The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 50.
Hence, the required mode is Rs 50.

Comp 2016

16. The following distribution gives the weights of 60 students of a class.

Weight (in kg)	Number of students
40-44	4
44-48	6
48-52	10
52-56	14
56-60	10
60-64	8
64-68	6
68-72	2

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :
The histogram from the given frequency table is shown below.

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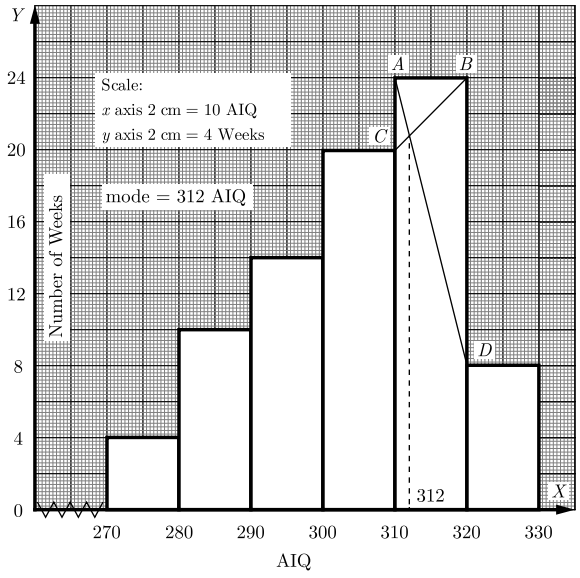


In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 312.
Hence, the required mode is 312 AIQ.

Comp 2010

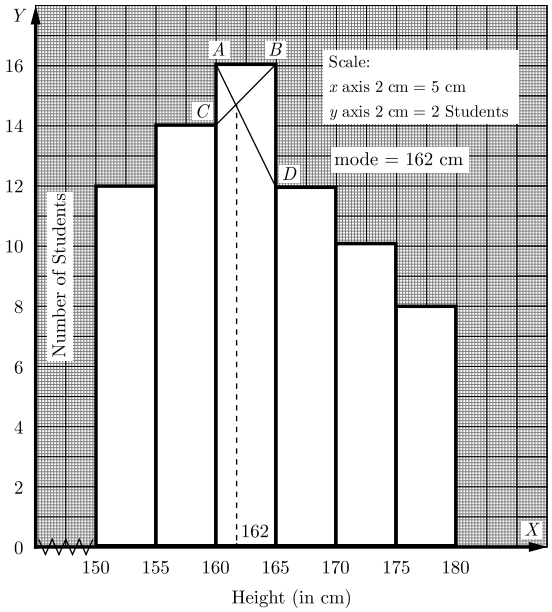
25. Consider the following frequency distribution of the heights of 70 students of a class.

Height (in cm)	Number of students
150 - 155	12
155 - 160	14
160 - 165	16
165 - 170	12
170 - 175	10
175 - 180	8

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 162.
Hence, the required mode is 162 cm.

26. For the following frequency distribution draw a histogram.

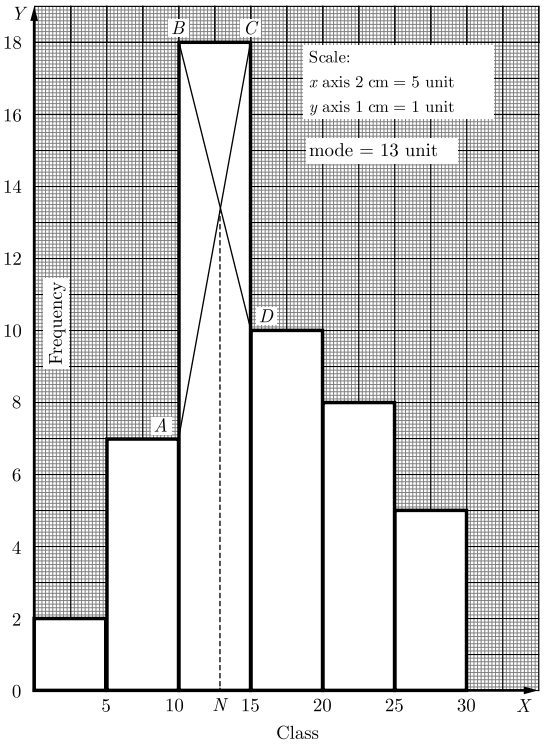
Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	7	18	10	8	5

Hence, calculate the mode.

Sol :

The histogram from the given frequency table is shown below.

NODIA



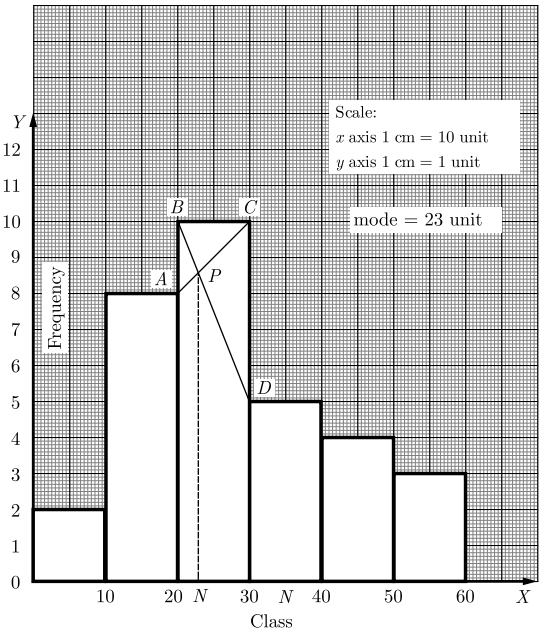
In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 13. Hence, the required mode is 13.

27. Draw a histogram and hence estimate the mode for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	2	8	10	5	4	3

Sol :

The histogram from the given frequency table is shown below.



In the highest rectangle draw two straight lines AC and BD, which intersect at point P. Through point P, draw a vertical line to meet the X-axis at N. The abscissa of the point represents 23. Hence, the required mode is 23.

Main 2001

28. The weekly observation on cost of living index in a certain city for a particular year are given below.

Cost of Living Index	Number of Weeks
140-150	5
150-160	10
160-170	20
170-180	9
180-190	6
190-200	2

Draw a histogram for the above data using a graph paper and locate the mode.

Sol :

The histogram from the given frequency table is shown below.

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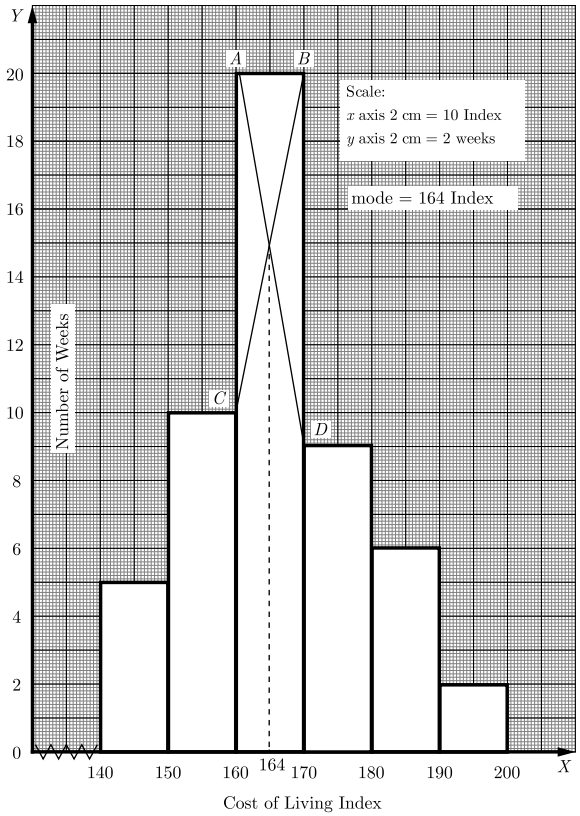


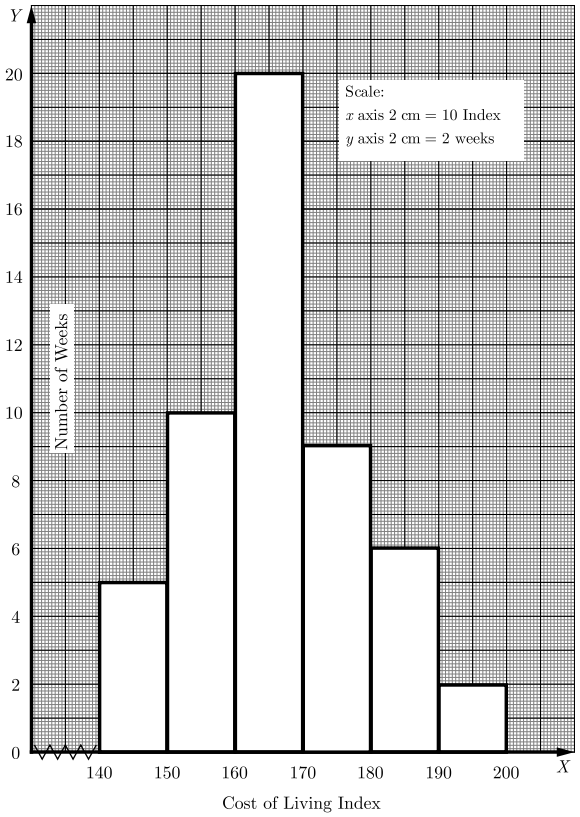
In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 164.

Hence, the required mode is 164 units.

FIVE MARK QUESTIONS

Main 2013

29. The weekly observation on cost of living index in a certain city for a particular year are given below in following histogram.



Use the above histogram to

(i) frame a frequency table

(ii) to calculate mean

(iii) to determine median class

(iv) to determine the modal class

Sol :

(i) Frequency table

From given histogram we prepare following frequency distribution table.

Cost of Living Index	Number of Weeks
140-150	5
150-160	10
160-170	20
170-180	9
180-190	6
190-200	2

(ii) Mean

We prepare the following commutative frequency distribution table.

Cost of Living Index	Number of weeks f	cf	Mid-point x_i	f_ix_i
140-150	5	5	145	725

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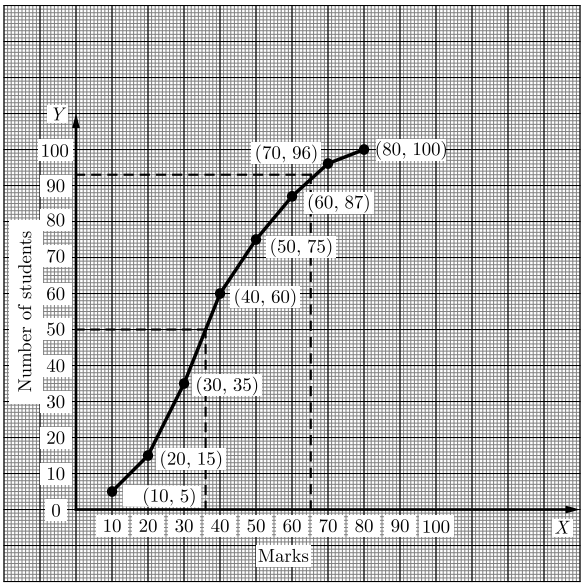


(i) Here $N = 100$ thus $\frac{N}{2} = \frac{100}{2} = 50$. Now, we locate the point on the ogive, whose y-ordinate is 50. The x co-ordinate corresponding to this ordinate is 36. Hence, median marks is 36.

(iii) Now, we locate the point on the ogive, whose x co-ordinate is 65. The y co-ordinate corresponding to this ordinate is 92. It means 92 students are having score below 65. The number of students who score above 65 is $100 - 92 = 8$

Main 2001

39. Using the data given below construct the cumulative frequency table and draw the ogive.

Marks	Number of students
0-10	3
10-20	8
20-30	12
30-40	14
40-50	10
50-60	6
60-70	5
70-80	2

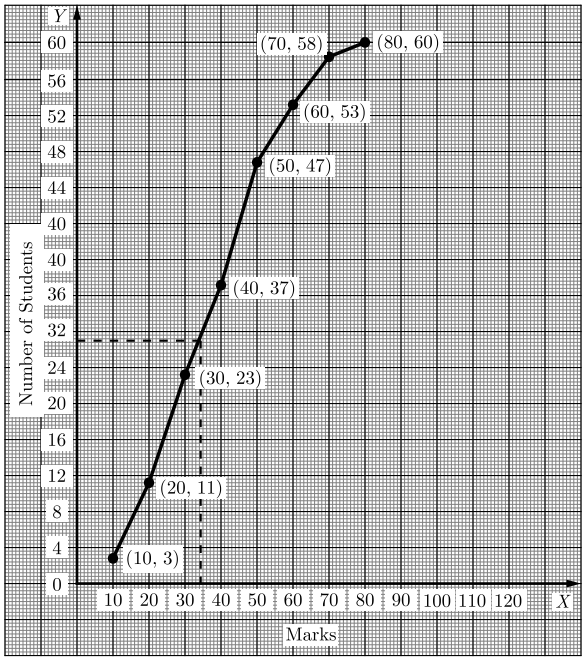
From the ogive, determine the median.

Sol :

Cumulative frequency distribution for the given data is shown below :

Marks	Number of students	Cumulative frequency cf
0-10	3	3
10-20	8	11
20-30	12	23
30-40	14	37
40-50	10	47
50-60	6	53
60-70	5	58
70-80	2	60

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



Here $N = 60$ thus $\frac{N}{2} = \frac{60}{2} = 30$. Now, we locate the point on the ogive, whose y-ordinate is 30. The x co-ordinate corresponding to this ordinate is 35. Hence, median marks is 35.

Main 2000

40. The following tables shows the distribution of the heights of a group of factory workers.

Height (in cm)	Number of workers
150-155	6
155-160	12
160-165	18
165-170	20
170-175	13
175-180	8

NODIA

Sol :

(i) Frequency table

From given histogram we prepare following frequency distribution table.

AIQ	Number of weeks f
270-280	4
280-290	10
290-300	14
300-310	20
310-320	24
320-330	8
Total	80

(ii) Mean

We prepare the following commutative frequency distribution table.

AIQ	Number of weeks f	cf	Mid-point x_i	f_ix_i
270-280	4	4	275	1100
280-290	10	14	285	2850
290-300	14	28	295	4130
300-310	20	48	305	6100
310-320	24	72	315	7560
320-330	8	80	325	2600
Total	80			24340

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{24340}{80} = 304.25$$

(iii) Median class

Cumulative frequency just greater than $\frac{N}{2} = \frac{80}{2} = 40$ is 48 and the corresponding class is 300-310. Thus median class is 300-310.

(iv) Modal class

Since 310-320 has highest frequency i.e. 24, thus 310-320 is model class.

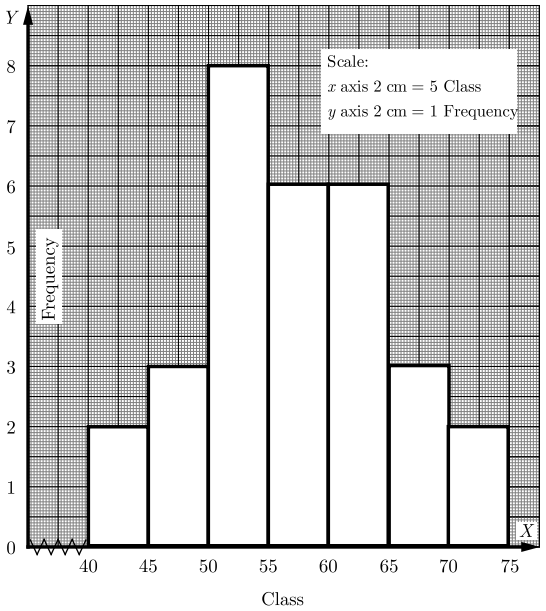
Comp 2008

32. Use the following histogram to

(i) frame a frequency table

(ii) to calculate median class

(iii) to determine the modal class



Sol :

(i) Frequency table

From given histogram we prepare following frequency distribution table.

Class	Frequency	$c.f.$
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
	$N = 30$	

We have $N = 30$; $\frac{N}{2} = 15$

(ii) Median

Cumulative frequency just greater than $\frac{N}{2}$ is 19 and the corresponding class is 55-60. Thus median class is 55-60.

(iii) Modal class

We observe that the class 50-55 has maximum frequency 8. Therefore, this is the modal class.

Comp 2002

33. The following histogram represents the number of plants per house in a colony.

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90-100	3

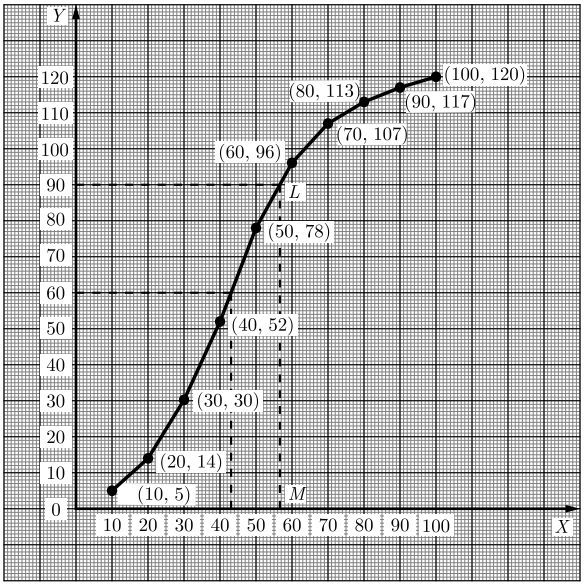
Draw the ogive and hence, estimate
(i) the median marks.
(ii) the number of students who did not pass the test if the pass percentage was 50.
(iii) the upper quartile marks.

Sol :

Cumulative frequency distribution for the given data is shown below :

Marks	Number	Cumulative frequency (cf)
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
Total	120	

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 120$ thus $\frac{N}{2} = \frac{120}{2} = 60$. Now, we locate the point on the ogive, whose y-ordinate is 100. The x co-ordinate corresponding to this ordinate is 57. Hence, median marks is 57.

(ii) We locate the point on the ogive, whose x co-ordinate is 50. The y co-ordinate corresponding to this ordinate is 78. It means that 78 students got marks less than 50. Thus 78 students did not pass exam if pass mark was 50.

(iii) Now $\frac{3N}{4} = \frac{3 \times 120}{4} = 90$. Now, we locate the point on the ogive, whose y-ordinate is 90. The x co-ordinate corresponding to this ordinate is 57. Hence upper quartile, Q_3 is 57 m.

Main 2011

37. Marks obtained by 200 students in an examination are given below :

Marks	Number of students
0-10	
10-20	
20-30	10
30-40	
40-50	
50-60	
60-70	
70-80	
80-90	
90-100	

Draw an ogive for the given distribution taking 2 cm =10 marks on one axis and 2 cm =20 students on the other axis.
Using the graph, determine.
(i) the median marks.
(ii) the number of students who failed, if minimum marks required to pass is 40.
(iii) if scoring 85 and more marks is considered as grade on, find the number of students who secured grade one in the examination.

Sol :

Cumulative frequency distribution for the given data is shown below :

Marks	Frequency f	Cumulative frequency cf
0-10	5	5
10-20	11	16
20-30	10	26
30-40	20	46

NODIA

(iii) Modal class

We observe that the class 18-24 has maximum frequency 12. Therefore, 18-24 is the modal class.

Main 2020

35. Students enter for a game of shot-put competition. The distance thrown (in metres) is recorded below

Distance in m	Number of Students
12-13	3
13-14	9
14-15	12
15-16	9
16-17	4
17-18	2
18-19	1

Use a graph paper to draw an ogive for the above distribution. Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis.

Hence using your graph find

(i) the median

(ii) Upper Quartile

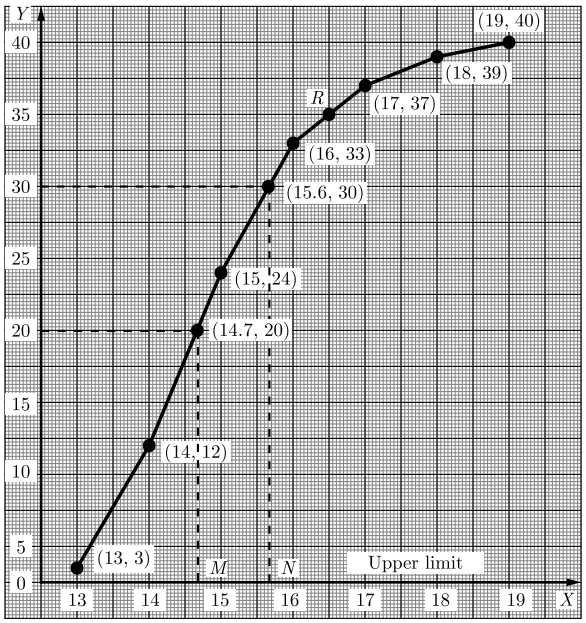
(iii) Number of students who cover a distance which is above $16\frac{1}{2}$ m.

Sol :

Cumulative frequency distribution for the given data is shown below :

Distance	Frequency f	Cumulative frequency cf
12-13	3	3
13-14	9	12
14-15	12	24
15-16	9	33
16-17	4	37
17-18	2	39
18-19	1	40

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 40$ thus $\frac{N}{2} = \frac{40}{2} = 20$. Now, we locate the point on the ogive, whose y-ordinate is 20. The x co-ordinate corresponding to this ordinate is 14.7. Hence, median distance is 14.7.

(ii) Now $\frac{3N}{4} = \frac{3 \times 40}{4} = 30$. Now, we locate the point on the ogive, whose y-ordinate is 30. The x co-ordinate corresponding to this ordinate is 15.7. Hence upper quartile, Q_3 is 15.7 m.

(iii) We locate the point on the ogive, whose x-ordinate is $16\frac{1}{2}$. The y co-ordinate corresponding to this ordinate is 35. It means that distance of 35 students is less than $16\frac{1}{2}$ m. Number of students who cover distance above $16\frac{1}{2}$ is $40 - 35 = 5$.

Main 2019

36. Use graph paper for this question. The marks obtained by 120 students in an English test are given below

Marks	Number of students
0-10	5
10-20	9
20-30	16
30-40	22
40-50	26
50-60	18
60-70	11
70-80	6
80-90	4

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Cost of Living Index	Number of weeks f	cf	Mid-point x_i	fx_i
150-160	10	15	155	1550
160-170	20	35	165	3300
170-180	9	44	175	1575
180-190	6	50	185	1110
190-200	2	52	195	390
Total	52			860

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{8650}{52} = 166.4$$

(iii) Median class

Cumulative frequency just greater than $\frac{N}{2} = \frac{52}{2} = 26$ is 35 and the corresponding class is 160-170. Thus median class is 160-170.

(iv) Modal class

Since maximum frequency is 20 and corresponding class is 160-170 which is modal class.

Main 2011

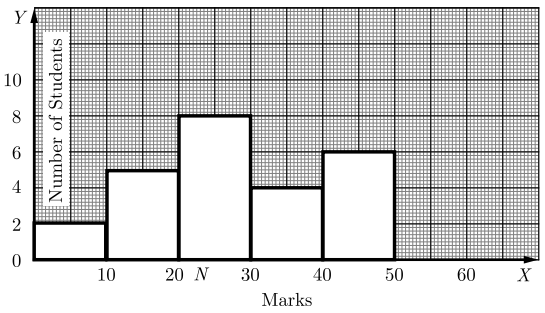
30. The histogram adjacent represents the scores obtained by 25 students in a Mathematics mental test.

Use the data to

(i) frame a frequency table

(ii) to calculate mean

(iii) to determine the modal class



Sol :

(i) Frequency Table :

From given histogram we can get following frequency table

Scores	Number of students
0-10	2
10-20	5
20-30	8
30-40	4
40-50	6
Total	25

(ii) Mean :

To calculate mean, we make the frequency distribution following table :

Class	f_i	Class mark x_i	$f_i x_i$
0-10	2	5	10
10-20	5	15	75
20-30	8	25	200
30-40	4	35	140
40-50	6	45	270
	$\sum f_i = 25$		$\sum f_i x_i = 695$

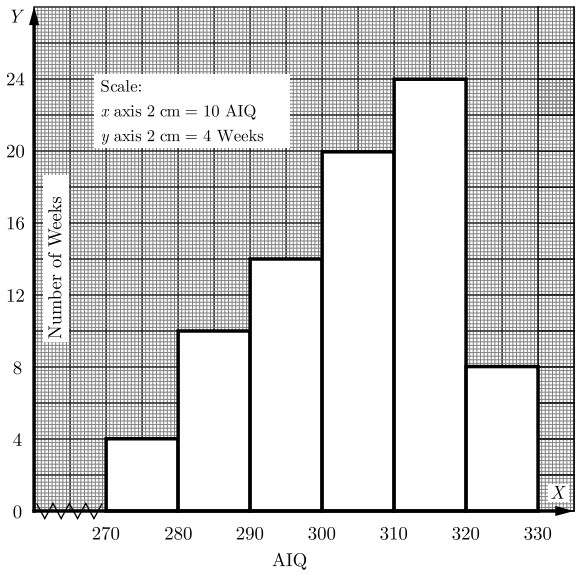
Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{695}{25} = 27.8$

(ii) Modal Class

In frequency distribution, the class which have maximum frequency is called model class. Here class 20-30 has frequency 8, which is maximum amongst all given frequencies. Thus class 20-30 is modal class.

Comp 2011

31. Following frequency distribution shows the Air Quality Index of different localities of Delhi on 27th January 2024 reported by Times of India Newspaper on 28th January 2024.



Use the above histogram to

(i) frame a frequency table

(ii) to calculate mean

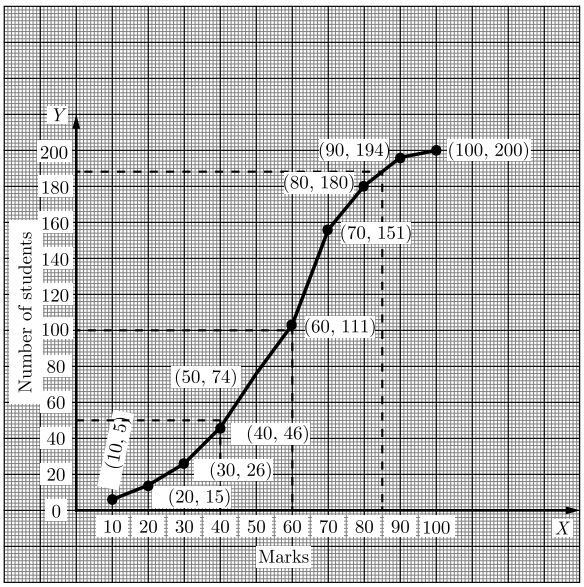
(iii) to determine median class

(iv) to determine the modal class

NODIA

40-50	28	74
50-60	37	111
60-70	40	151
70-80	29	180
80-90	14	194
90-100	6	200

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 200$ thus $\frac{N}{2} = \frac{200}{2} = 100$. Now, we locate the point on the ogive, whose y-ordinate is 100. The x co-ordinate corresponding to this ordinate is 57. Hence, median marks is 57.

(ii) Suppose, the minimum marks required to pass is 40. Now, we locate the point on the ogive, whose x co-ordinate is 40. The y co-ordinate corresponding to this ordinate is 46. Hence, the number of students who failed is 46, if minimum marks required to pass is 40.

(iii) Suppose scoring 85 and more marks is considered as grade one. Now, we locate the point on the ogive, whose x co-ordinate is 85. The y co-ordinate corresponding to this ordinate is 190. It means 190 students are below grade one and number of students who secured grade in the examination $200 - 190 = 10$

Main 2008

38. Using a graph paper, draw an ogive for the following distribution which shows the marks obtained in the General Knowledge paper by 100 students.

Marks	Number of students
0-10	5
10-20	10
20-30	20
30-40	25
40-50	15
50-60	12
60-70	9
70-80	4
Total	100

Use the ogive to estimate

(i) the median.

(ii) the number of students, who score marks above 65.

Sol :

Cumulative frequency distribution for the given data is shown below :

Marks	Number of students	Cumulative frequency *cf*
0-10	5	5
10-20	10	15
20-30	20	35
30-40	25	60
40-50	15	75
50-60	12	87
60-70	9	96
70-80	4	100
Total	100	

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.

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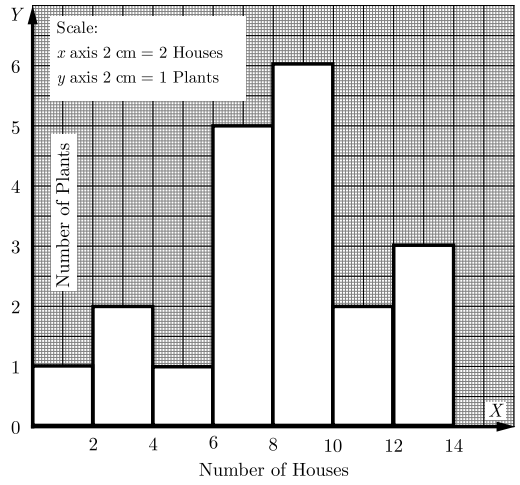


Use the data to
(i) frame a frequency table
(ii) to calculate mean
(iii) to determine the modal class

Sol :

(i) Frequency table

From given histogram we prepare following frequency distribution table.

Number of plants	Number of houses
0-2	1
2-4	2
4-6	1
6-8	5
8-10	6
10-12	2
12-14	3

(ii) Mean

From frequency distribution table we prepare following cumulative distribution table.

Class	x_i	f_i	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
Total		$\Sigma f_i = 20$	$\Sigma f_i x_i = 162$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

Mean number of plants per house is 8.1.

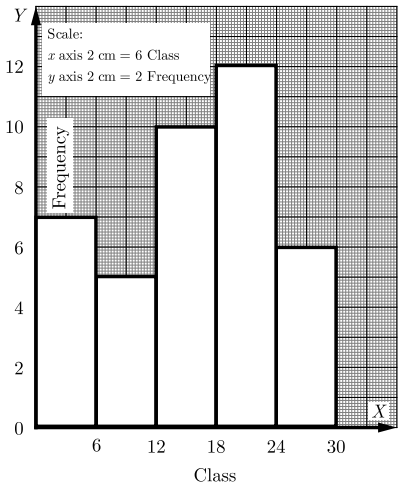
(iii) Modal class

We observe that the class 8-10 has maximum frequency 6. Therefore, 8-10 is the modal class.

Main 2011

34. Use the following histogram to
(i) frame a frequency table
(ii) to calculate mean
(iii) to determine median class
(iv) to determine the modal class

NODIA



Sol :

(i) Frequency table

From given histogram we prepare following frequency distribution table.

Class	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

(ii) Mean

From frequency distribution table we prepare following cumulative distribution table.

Classes	x_i	f_i	$f_i x_i$
0-6	3	7	21
6-12	9	5	45
12-18	15	10	150
18-24	21	12	252
24-30	27	6	162
		$\sum f_i = 40$	$\sum f_i x_i = 630$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{630}{40} = 15.75$$

180-185	6

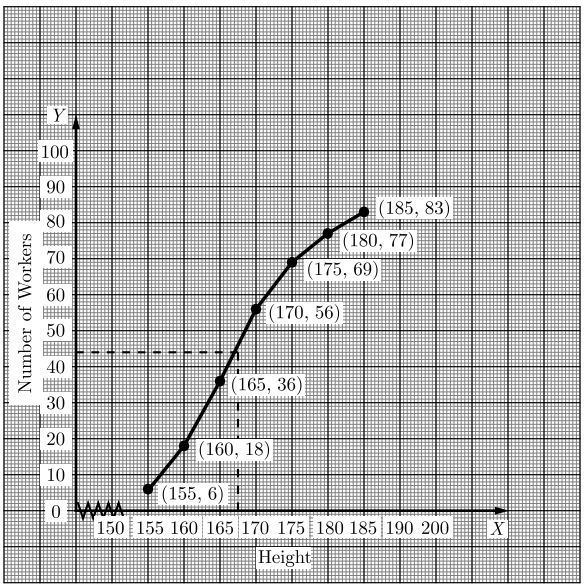
(i) Determine the cumulative frequencies.
(ii) Draw the cumulative frequency curve on a graph paper.
Use 2 cm = 5 cm height on one axis and 2 cm = 10 workers on the other.
(iii) From your graph, write down the median height in cm.

Sol :

(i) Cumulative Frequency Distribution
Cumulative frequency distribution for the given data is shown below :

Height	Frequency f	Cumulative frequency cf
150-155	6	6
155-160	12	18
160-165	18	36
165-170	20	56
170-175	13	69
175-180	8	77
180-185	6	83

The cumulative frequency are 6, 18, 36, 56, 69, 77, 83.
(ii) On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(iii) Here $N = 83$ thus $\frac{N}{2} = \frac{83}{2}$. Now, we locate the point on the ogive, whose y-ordinate is $\frac{83}{2}$. The x co-ordinate corresponding to this ordinate is 166.3 cm. Hence, median height is 166.3 cm.

SIX MARK QUESTIONS

Main 2023, Comp 2009

41. Use graph paper to answer this question. During a medical checkup of 60 students in a school, weights were recorded as [6]

Weight (in kg)	**Number of students**
28–30	2
30–32	4
32–34	10
34–36	13
36–38	15
38–40	9
40–42	5
42–44	2

Taking 2 cm – 2 kg along axis and 2 cm = 10 students along the other axis draw an ogive. your graph to find the :
(a) median
(b) upper Quartile.
(c) number of students whose weight is above 37 kg.

Sol :

Cumulative frequency distribution for the given data is shown below :

Weight (in kg)	Number of Students	Cumulative Frequency
28–30	2	2
30–32	4	6
32–34	10	16
34–36	13	29
36–38	15	44
38–40	9	53
40–42	5	58
42–44	2	60

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.

NODIA

70-80	10
80-90	8
90-100	7

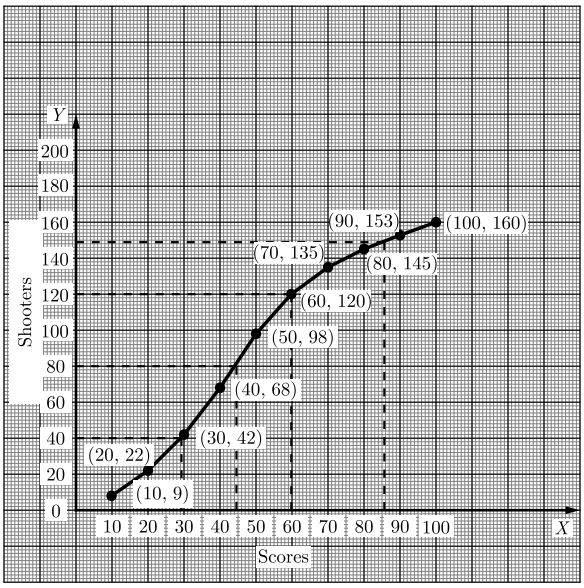
Use your graph to estimate the following
(i) The median
(ii) The interquartile range
(iii) The number of shooters who obtained a score of more than 85 %.

Sol :

We first prepare the cumulative frequency table as given below

Scores	Number of shooters	Cumulative frequency
0-10	9	9
10-20	13	22
20-30	20	42
30-40	26	68
40-50	30	98
50-60	22	120
60-70	15	135
70-80	10	145
80-90	8	153
90-100	7	160
Total	$N = 160$	

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 160$ thus $\frac{N}{2} = \frac{160}{2} = 80$. Now, we locate the point on the ogive, whose y-ordinate is 80. The x co-ordinate corresponding to this ordinate is 44. Hence, median score is 44.

(ii) Here $N = 160$ thus $\frac{N}{4} = \frac{160}{4} = 40$. Now, we locate the point on the ogive, whose y-ordinate is 40. The x co-ordinate corresponding to this ordinate is 29. Hence lower quartile, Q_1 is 29. To find third quartile, take $\frac{3N}{4} = \frac{3 \times 160}{4} = 120$. Now, we locate the point on the ogive, whose y-ordinate is 120. The x co-ordinate corresponding to this ordinate is 60. Hence third quartile, Q_3 is 60.
Interquartile range, $Q_3 - Q_1 = 60 - 29 = 31$

(iii) We locate the point on the ogive, whose x-ordinate is 85. The y co-ordinate corresponding to this ordinate is 148. It means that 148 shooter score less than 85. Number of shooters above 85 % are $160 - 148 = 12$

Main 2015, Comp 2006

45. The weight of 50 workers is given below :

Weight (kg)	Number of workers
50-60	4
60-70	7
70-80	11
80-90	14
90-100	6
100-110	5
110-120	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use a graph to estimate the following :
(i) the upper and lower quartiles.
(ii) if weighing 95 kg and above is considered overweight, find the number of workers who are overweight.

Sol :

We first prepare the following cumulative frequency table :

Weight (kg)	Frequency f	Cumulative frequency cf
50-60	4	4
60-70	7	11
70-80	11	22
80-90	14	36

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ordinate is 146.8. Hence lower quartile, Q_1 is 146.8.

(iii) We locate the point on the ogive, whose x-ordinate is 158. The y co-ordinate corresponding to this ordinate is 50. It means that 50 students are having height less than 158 cm. Thus number of students having height greater than 158 cm is $60 - 50 = 10$. Hence tall boys in class are 10 students.

Main 2017, Comp 2011

43. The daily wages of 80 workers in project are given below :

Wages (in Rs)	Number of workers
400-450	2
450-500	6
500-550	12
550-600	18
600-650	24
650-700	13
700-750	5

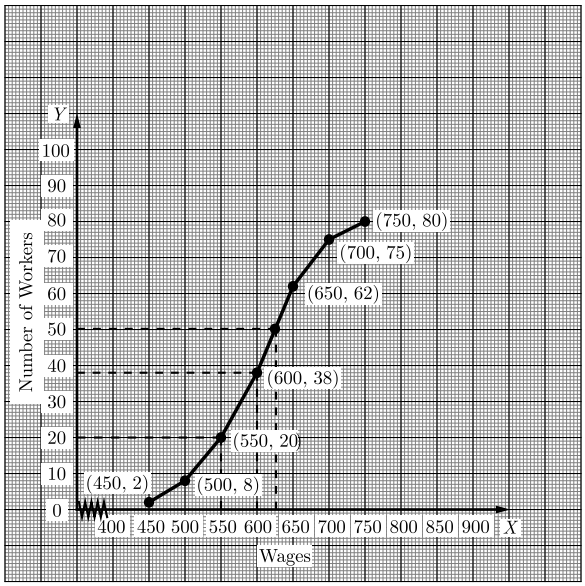
Use a graph paper to draw an ogive for the above distribution.
(Use a scale of 2 cm = Rs. 50 on x-axis and 2 cm = 10 wokers on y-axis).
Use your ogive to estimate :
(i) the median wage of the workers.
(ii) the lower quartile wage of workers.
(iii) the number of workers who earn more than Rs. 625 daily.

Sol :

We first prepare the following cumulative frequency table :

Wages (in Rs)	Number of workers	Cumulative frequency cf
400-450	2	2
450-500	6	8
500-550	12	20
550-600	18	38
600-650	24	62
650-700	13	75
700-750	5	80

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



NODIA

(i) Here $N = 80$ thus $\frac{N}{2} = \frac{80}{2} = 40$. Now, we locate the point on the ogive, whose y-ordinate is 40. The x co-ordinate corresponding to this ordinate is 605. Hence, median wage is Rs 605.

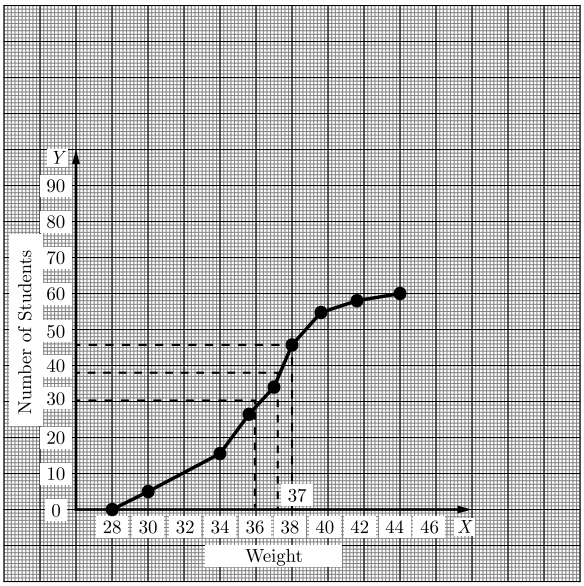
(ii) Here $N = 80$ thus $\frac{N}{4} = \frac{80}{4} = 20$. Now, we locate the point on the ogive, whose y-ordinate is 20. The x co-ordinate corresponding to this ordinate is 550. Hence lower quartile, Q_1 is Rs 550.

(iii) We locate the point on the ogive, whose x-ordinate is 625. The y co-ordinate corresponding to this ordinate is 30. It means that 30 worker earn less that Rs 625 as daily wages. The number of workers who earn more than Rs. 625 are $80 - 30 = 50$

Main 2016

44. The table shows the distribution of the scores obtained by 160 shooters in a shooting competition. Use a graph and draw an ogive for the distribution.
(Take 2 cm = 10 scores on the x-axis and 2 cm = 20 shooters on the y-axis).

Scores	Number of Shooters
0-10	9
10-20	13
20-30	20
30-40	26
40-50	30
50-60	22
60-70	15



(i) Here $N = 60$ thus $\frac{N}{2} = \frac{60}{2} = 30$. Now, we locate the point on the ogive, whose y-ordinate is 30. The x co-ordinate corresponding to this ordinate is 36. Hence, median weight is 36.

(ii) Now $\frac{3N}{4} = \frac{3 \times 60}{4} = 45$. Now, we locate the point on the ogive, whose y-ordinate is 45. The x co-ordinate corresponding to this ordinate is 38.2. Hence upper quartile, Q_3 is 38.2 kg.

(iii) We locate the point on the ogive, whose x-ordinate is 37. The y co-ordinate corresponding to this ordinate is 37. It means that weight of 37 students is less than 37 kg. Number of students having weight 37 kg $60 - 37 = 23$.

Main 2018

42. Use graph paper for this question. A survey regarding height (in cm) of 60 boys belonging to class 10 of a school was conducted. The following data was recorded.

Heights (in cm)	Number of boys
135-140	4
140-145	8
145-150	20
150-155	14
155-160	7
160-165	6
165-170	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph of estimate the following

(i) the median

(ii) lower quartile

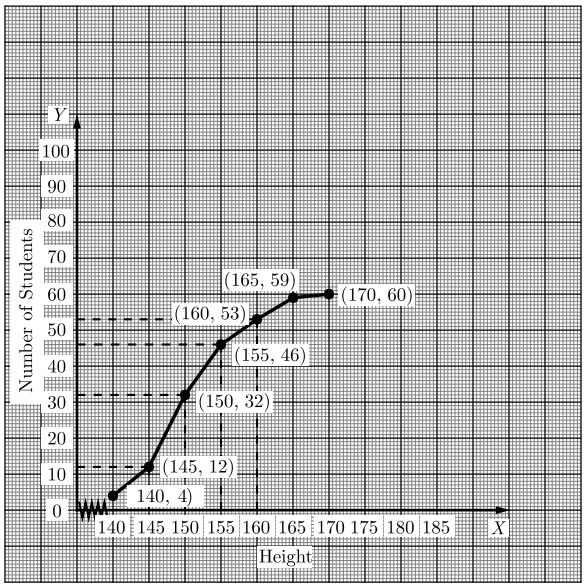
(iii) If above 158 cm is considered as the tall boys of the class, find the number of boys in the class who are tall.

Sol :

We first prepare the cumulative frequency table by less than method as given below

Heights (in cm)	Number of boys	Cumulative frequency cf
135-140	4	4
140-145	8	12
145-150	20	32
150-155	14	46
155-160	7	53
160-165	6	59
165-170	1	60
	$N = 60$	

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 60$ thus $\frac{N}{2} = \frac{60}{2} = 30$. Now, we locate the point on the ogive, whose y-ordinate is 30. The x co-ordinate corresponding to this ordinate is 149.5. Hence, median height is 149.5 cm.

(ii) Here $N = 60$ thus $\frac{N}{4} = \frac{60}{4} = 15$. Now, we locate the point on the ogive, whose y-ordinate is 15. The x co-ordinate corresponding to this

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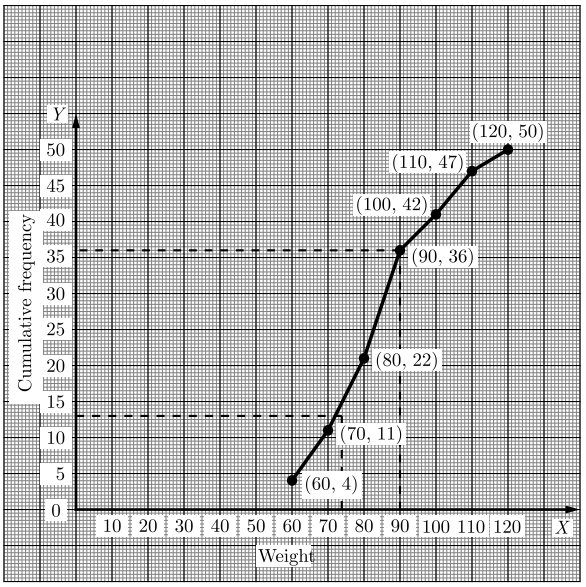
90-100	6	42
100-110	5	47
110-120	3	50

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Upper and lower quartiles.
Here $N = 50$ thus $\frac{N}{4} = \frac{50}{4} = 12.5$. Now, we locate the point on the ogive, whose y-ordinate is 12.5. The x co-ordinate corresponding to this ordinate is 71.3. Hence lower quartiles Q_1 is 71.3.
To find third quartile, take $\frac{3N}{4} = \frac{3\times 50}{4} = 37.5$
Now, we locate the point on the ogive, whose y-ordinate is 37.5. The x co-ordinate corresponding to this ordinate is 92.5. Hence third quartile, Q_3 is 92.5.
Interquartile range, $Q_3 - Q_1 = 60 - 29 = 31$

(ii) We locate the point on the ogive, whose x-ordinate is 95. The y co-ordinate corresponding to this ordinate is 40. It means that 40 worker are underweight. Therefore number of worker overweight are $50 - 40 = 10$

Main 2014

46. The marks obtained by 100 students in a Mathematics test are given below :

Marks	Number of Students
0-10	3
10-20	7
20-30	12
30-40	17
40-50	23
50-60	14
60-70	9
70-80	6
80-90	5
90-100	4

Draw an ogive for the given distribution on a graph sheet.
(use a scale of 2 cm = 10 units on both axes).Use the ogive to estimate the
(i) Median.
(ii) lower quartile.
(iii) number of students who obtained more than 85% marks in the test.
(iv) number of students who did not pass in the test, if the pass percentage was 35.

Sol :

The cumulative frequency table for the given continuous distribution is given below

Marks	Number of students	Cumulative frequency cf
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.

NODIA

locate the point on the ogive, whose y-ordinate is 40. The x co-ordinate corresponding to this ordinate is 151.3. Hence lower quartile, Q_1 is 151.3.

To find third quartile, take $\frac{3N}{4} = \frac{3 \times 160}{4} = 120$. Now, we locate the point on the ogive, whose y-ordinate is 120. The x co-ordinate corresponding to this ordinate is 164.1. Hence third quartile, Q_3 is 164.1 cm.

Interquartile range,

$$Q_3 - Q_1 = 164.1 - 151.3 = 12.8$$

(iii) We locate the point on the ogive, whose x-ordinate is 172. The y co-ordinate corresponding to this ordinate is 147. It means that length of 147 students is less than 172 cm. Number of students above 172 cm $160 - 147 = 13$

Main 2010

49. The monthly income of a group of 320 employees in a company is given below :

Monthly income	Number of employees
6000-7000	20
7000-8000	45
8000-9000	65
9000-10000	95
10000-11000	60
11000-12000	30
12000-13000	5

Draw an ogive of the given distribution of a graph sheet taking 2 cm = Rs. 1000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine

(i) the median wage.

(ii) the number of employees whose income is below Rs. 8500.

(iii) if the salary of a senior employee is above is Rs. 11500, find the number of senior employees in the company.

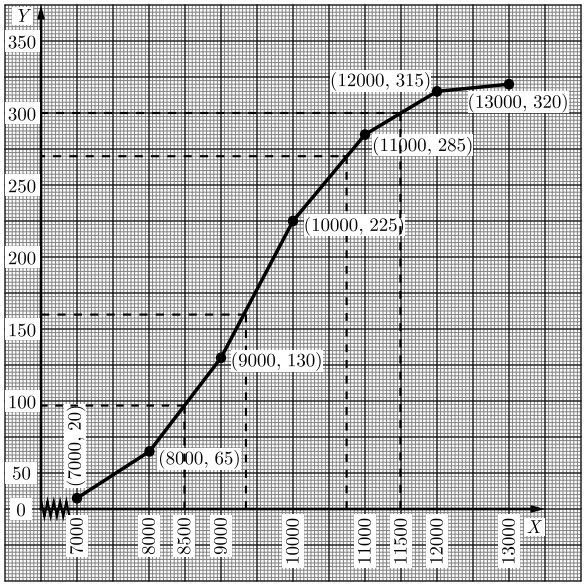
(iv) the upper quartile.

Sol :

The cumulative frequency table for the continuous distribution is given below :

Monthly income	Number of employees f	Cumulative frequency cf
6000-7000	20	20
7000-8000	45	65
8000-9000	65	130
9000-10000	95	225
10000-11000	60	285
11000-12000	30	315
12000-13000	5	320

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 320$ thus $\frac{N}{2} = \frac{320}{2} = 160$. Now, we locate the point on the ogive, whose y-ordinate is 160. The x co-ordinate corresponding to this ordinate is Rs 9316. Hence, median monthly income is Rs 9315.

(ii) We locate the point on the ogive, whose x-ordinate is 8500. The y co-ordinate corresponding to this ordinate is 95. The number of employed, whose income is below Rs. 8500 is 95.

(iii) We locate the point on the ogive, whose x-ordinate is 11500. The y co-ordinate corresponding to this ordinate is 300. It means that 300 employees income is less than Rs 11500. Thus number of senior employees having salary above Rs. 11500 in the company is $320 - 300 = 20$.

(iv) Here $N = 320$ thus $\frac{N}{4} = \frac{3 \times 360}{4} = 270$. Now, we locate the point on the ogive, whose y-ordinate is 270. The x co-ordinate corresponding to this ordinate is 10250 Hence upper quartile, Q_3 is Rs 10250.

Main 2009

50. Attempt this question on graph paper. Marks

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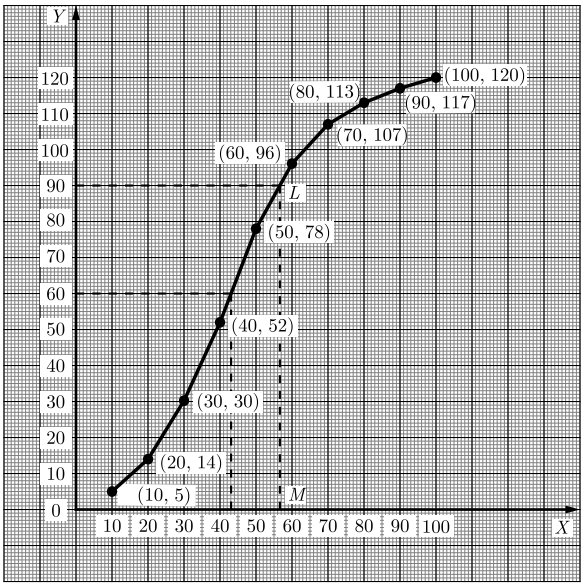


(i) Here $N = 120$ thus $\frac{N}{2} = \frac{120}{2} = 60$. Now, we locate the point on the ogive, whose y-ordinate is 60. The x co-ordinate corresponding to this ordinate is 43. Hence, median marks is 43.

(ii) 75% marks = 75% of 100 = 75 marks
We locate the point on the ogive, whose x-ordinate is 75. The y co-ordinate corresponding to this ordinate is 110 students . It means that 110 students score less than 75 %. Number of students who obtained more than 75% in the test are $120 - 110 = 10$.

(iii) 45% marks = 40% of 100 = 40
We locate the point on the ogive, whose x-ordinate is 40. The y co-ordinate corresponding to this ordinate is 52 students . It means that 52 students score less than 30%. Hence, the number of students, who did not pass the test is 52.

Main 2012

48. The following distribution represents the height of 160 students of a school.

Height	Number of students
140-145	12
145-150	20
150-155	30
155-160	38
160-165	24
165-170	16
170-175	12
175-180	8

Draw an ogive for the given distribution taking 2 cm = cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

(i) the median height.

(ii) the inter quartile range.

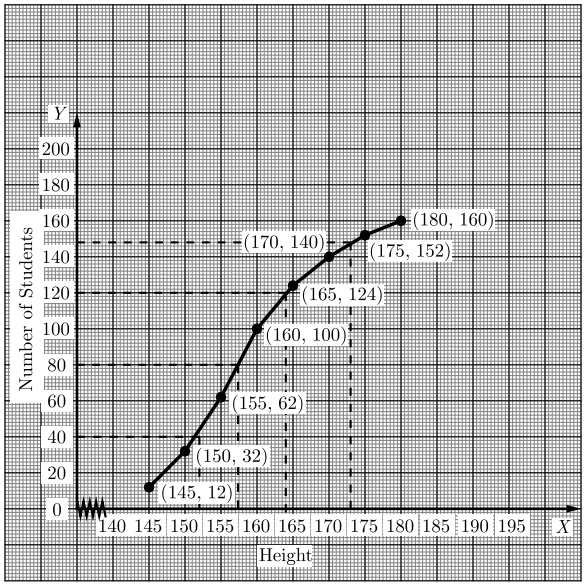
(iii) the number of students, whose height is above 172 cm.

Sol :

The cumulative frequency table for the continuous distribution is given below :

Height	Number of students	Cumulative frequency cf
140-145	12	12
145-150	20	32
150-155	30	62
155-160	38	100
160-165	24	124
165-170	16	140
170-175	12	152
175-180	8	160

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 160$ thus $\frac{N}{2} = \frac{160}{2} = 80$. Now, we locate the point on the ogive, whose y-ordinate is 80. The x co-ordinate corresponding to this ordinate is 157.3. Hence, median height is 157.3 cm.

(ii) Here $N = 160$ thus $\frac{N}{4} = \frac{160}{4} = 40$. Now, we

NODIA

50-60	18
60-70	11
70-80	6
80-90	4
90-100	3

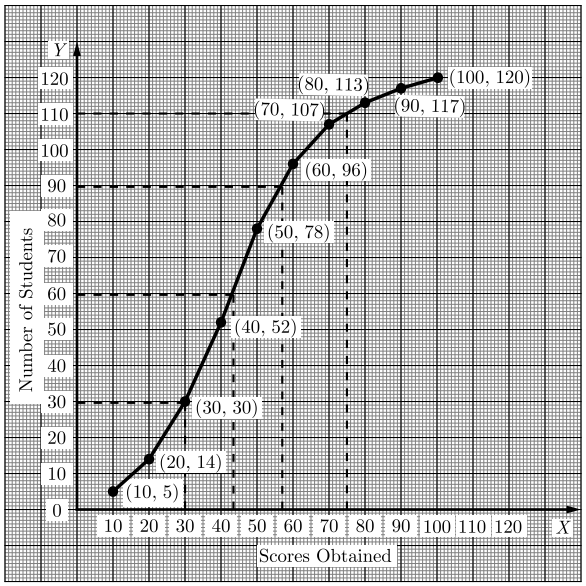
Use your ogive to estimate
(i) the median
(ii) the inter quartile range.
(iii) the number of shooters who obtained more than 75% scores.

Sol :

The cumulative frequency table for the continuous distribution is given below :

Scores	Number of shooters	Cumulative frequency (Cf)
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 120$ thus $\frac{N}{2} = \frac{120}{2} = 60$. Now, we locate the point on the ogive, whose y-ordinate is 60. The x co-ordinate corresponding to this ordinate is 43. Hence, median score is 43.

(ii) Here $N = 120$ thus $\frac{N}{4} = \frac{120}{4} = 30$. Now, we locate the point on the ogive, whose y-ordinate is 30. The x co-ordinate corresponding to this ordinate is 30. Hence lower quartile, Q_1 is 30. To find third quartile, take $\frac{3N}{4} = \frac{3 \times 120}{4} = 90$. Now, we locate the point on the ogive, whose y-ordinate is 90. The x co-ordinate corresponding to this ordinate is 56.7. Hence third quartile, Q_3 is 56.7.
Interquartile range,
$$Q_3 - Q_1 = 56.7 - 30 = 26.7$$

(iii) 75% marks = 75 of 100 = 75 scores. We locate the point on the ogive, whose x-ordinate is 75. The y co-ordinate corresponding to this ordinate is 110. It means that 110 shooter score less than 75. Number of shooters above 75 score are $120 - 110 = 10$

Main 2006

52. The daily wages of 160 workers in a building project are given below :

Wages (in Rs.)	Number of workers
0-10	12
10-20	20
20-30	30
30-40	38
40-50	24
50-60	16

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obtained by 200 students in examination are given below :

Marks	Number of students
0-10	5
10-20	10
20-30	14
30-40	21
40-50	25
50-60	34
60-70	36
70-80	27
80-90	16
90-100	12

Draw an ogive for the given distribution taking 2 cm = 10 marks on one exis and 2 cm = 20 students on the other axis. Find the graph find

(i) the median

(ii) the upper quartile.

(iii) number of students scoring above 65 marks.

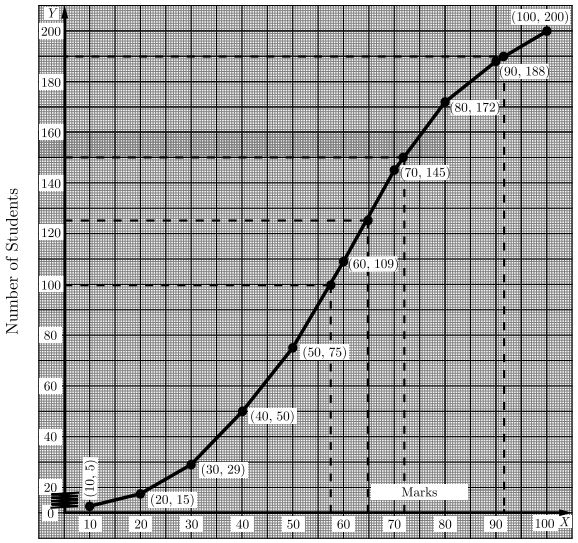
(iv) if 10 students qualify for merit scholarship, find the minimum marks required to qualify.

Sol :

The cumulative frequency table for the continuous distribution is given below :

Marks	Frequency f	Cumulative frequency cf
0-10	5	5
10-20	10	15
20-30	14	29
30-40	21	50
40-50	25	75
50-60	34	109
60-70	36	145
70-80	27	172
80-90	16	188
90-100	12	200

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 200$ thus $\frac{N}{2} = \frac{200}{2} = 100$. Now, we locate the point on the ogive, whose y-ordinate is 100. The x co-ordinate corresponding to this ordinate is 57.3. Hence, median marks is 57.3.

(ii) Here $N = 200$ thus $\frac{N}{4} = \frac{3 \times 200}{4} = 150$. Now, we locate the point on the ogive, whose y-ordinate is 150. The x co-ordinate corresponding to this ordinate is 72. Hence upper quartile, Q_3 is 72 marks

(iii) 65% marks = 65 of 100 = 65 marks
We locate the point on the ogive, whose x-ordinate is 65. The y co-ordinate corresponding to this ordinate is 128 students It means that 128 students score less than 65 % marks. Number of students who obtained more than 65% marks are $200 - 128 = 72$.

(iii) Only 10 students qualify i.e 190 students does not qualify. We locate the point on the ogive, whose y-ordinate is 190. The x co-ordinate corresponding to this ordinate is 92 marks. Thus minimum marks to qualify is 92.

Main 2007

51. The table shows the distribution of the scores obtained by 120 shooters in a shooting competition. Using a graph sheet, draw an ogive for the distribution.

Scores obtained	Number of Shooters
0-10	5
10-20	9
20-30	16
30-40	22
40-50	26

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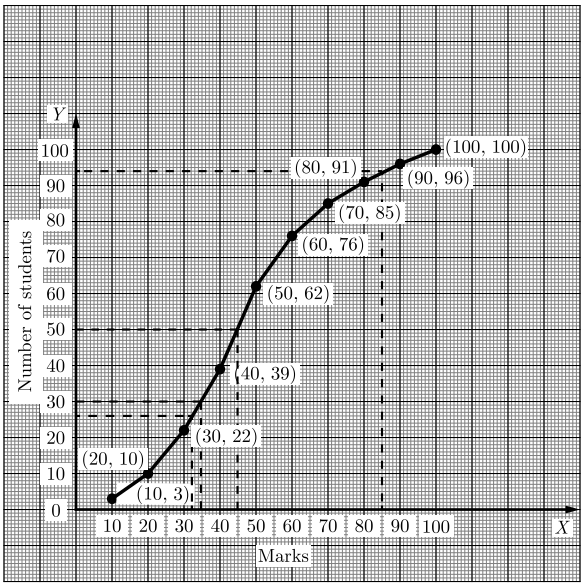

(i) Here $N = 100$ thus $\frac{N}{2} = \frac{100}{2} = 50$. Now, we locate the point on the ogive, whose y-ordinate is 50. The x co-ordinate corresponding to this ordinate is 45. Hence, median marks is 45.

(ii) Here $N = 100$ thus $\frac{N}{4} = \frac{100}{4} = 25$. Now, we locate the point on the ogive, whose y-ordinate is 25. The x co-ordinate corresponding to this ordinate is 32. Hence lower quartile, Q_1 is 32 marks.

(iii) 85% marks = 85% of 100 = 85 marks
We locate the point on the ogive, whose x-ordinate is 85. The y co-ordinate corresponding to this ordinate is 95 students . It means that 95 students score less than 85 %. Number of students who obtained more than 85% in the test are $100 - 95 = 5$.

(iv) 35% marks = 35% of 100 = 35
We locate the point on the ogive, whose x-ordinate is 35. The y co-ordinate corresponding to this ordinate is 30 students . It means that 30 students score less than 35 %. Hence, the number of students, who did not the pass test is 30.

Main 2012, 2003

47. The marks obtained by 120 students in a test are given below

Marks	Number of students
0-10	5
10-20	9
20-30	16
30-40	22
40-50	26
50-60	18
60-70	11
70-80	6
80-90	4
90-100	3

Draw an ogive for the given distribution on a graph sheet.
Use suitable scale for ogive to estimate the following
(i) the median.
(ii) the number of students who obtained more than 75% marks in the test.
(iii) the number of students who did not pass the test, if minimum marks required to pass is 40.

Sol :

The cumulative frequency table for the given continuous distribution is given below

Marks	Frequency f	Cumulative frequency cf
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.

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60-70	12
70-80	8

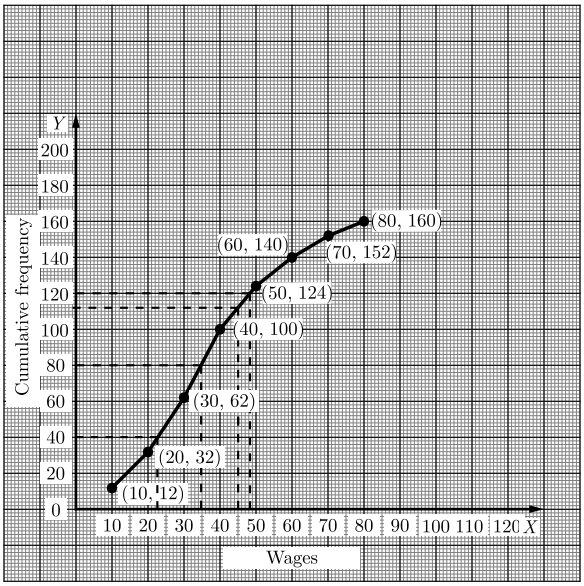
Using a graph paper, draw an ogive for the above distribution.
Use your ogive to estimate
(i) the median wage of the workers.
(ii) the upper quartile wage of the workers.
(iii) the lower quartile wage of the workers.
(iv) the percentage of workers, who earn more than Rs.45 a day.

Sol :

The cumulative frequency table for the continuous distribution is given below :

Wages (in Rs.)	Frequency f	Commutative Frequency f
0-10	12	12
10-20	20	32
20-30	30	62
30-40	38	100
40-50	24	124
50-60	16	140
60-70	12	152
70-80	8	160

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) Here $N = 160$ thus $\frac{N}{2} = \frac{160}{2} = 80$. Now, we locate the point on the ogive, whose y-ordinate is 80. The x co-ordinate corresponding to this ordinate is 35. Hence, median wage is Rs 35.

(ii) Here $N = 160$ thus $\frac{N}{4} = \frac{3 \times 160}{4} = 120$. Now, we locate the point on the ogive, whose y-ordinate is 120. The x co-ordinate corresponding to this ordinate is 49. Hence upper quartile, Q_3 is Rs 49.

(iii) Here $N = 160$ thus $\frac{N}{4} = \frac{160}{4} = 40$. Now, we locate the point on the ogive, whose y-ordinate is 40. The x co-ordinate corresponding to this ordinate is 23. Hence lower quartile, Q_1 is Rs 33.

(iii) We locate the point on the ogive, whose x-ordinate is Rs 45. The y co-ordinate corresponding to this ordinate is 112. It means that 112 worker earn less than Rs 45 as daily wages. The number of workers who earn more than Rs. 45 are $160 - 112 = 48$
The percentage of workers, who earn more than Rs. 45 a day is $\frac{48}{160} \times 100 = 30$ %.

Main 2005

53. Using a graph paper, draw an ogive for the following distribution that show a record of the weight in kilograms of 200 students.

Weight	Frequency
40-45	5
45-50	17
50-55	22
55-60	45
60-65	51
65-70	31
70-75	20
75-80	9

Use your ogive to estimate the following.
(i) the percentage of students weighing 55 kg or more.
(ii) the weight above which the heaviest 30% of the students fall.
(iii) the number of students who are (1) underweight and (2) overweight, if 55.70 kg is considered as standard weight.

Sol :

The cumulative frequency table for the continuous distribution is given below :

Weight	Frequency f	Cumulative frequency cf
40-45	5	5

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Number of multiples of 4 between 1 to 15 are 4, 8, 12 i.e. 3 favorable outcome.

$$n(E) = 3$$

Required Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{15} = \frac{1}{5}$

Thus (d) is correct option.

13. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Sol :

All possible outcomes are {HH, HT, TH, TT}.

Thus $n(S) = 4$

Favorable outcomes are {HT, TH, TT}.

$$n(E) = 3$$

Probability of getting at most one head,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

Thus (d) is correct option.

14. If an event cannot occur, then its probability is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) 0

Sol :

The event which cannot occur is said to be impossible event and probability of impossible event is zero.

Thus (d) is correct option.

15. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 240 (b) 480

(c) 3000 (d) 3480

Sol :

Total number of sold tickets are 6000. Let she bought x tickets.

Now $n(S) = 6000$

$$n(E) = x$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$0.08 = \frac{x}{6000}$$

$$x = 0.08 \times 6000 = 480$$

Hence, she bought 480 tickets.

Thus (b) is correct option.

16. 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. What is the probability that the number on the drawn ticket is a multiple of 3 or 7 ?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{5}$

Sol :

Total number of cases,

$$n(S) = 20$$

Favorable outcome,

$$E = \{3,6,7,9,12,14,15,18\}$$

Number of favorable cases,

$$n(E) = 8$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

Thus (d) is correct option.

17. Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective?

(a) $\frac{47}{50}$ (b) $\frac{3}{50}$

(c) $\frac{47}{100}$ (d) $\frac{53}{100}$

Sol :

Total number of bulbs,

$$n(S) = 200$$

Number of favorable cases,

$$n(E) = 200 - 12 = 188$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{188}{200} = \frac{47}{50}$$

Thus (a) is correct option.

18. Which of the following cannot be the probability of an event?

(a) $\frac{1}{3}$ (b) 0.1

(c) 3% (d) $\frac{17}{16}$

Sol :

Probability of an event always lies between 0 and 1.

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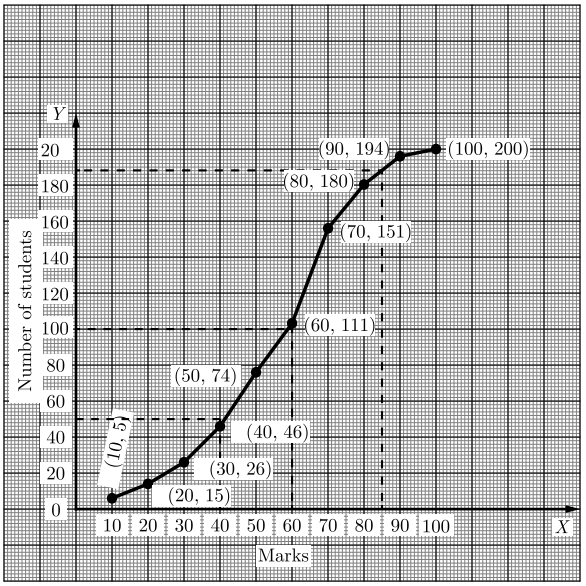
is shown on the graph paper.



(i) Here $N = 200$ thus $\frac{N}{2} = \frac{200}{2} = 100$. Now, we locate the point on the ogive, whose y-ordinate is 100. The x co-ordinate corresponding to this ordinate is 57. Hence, median marks is 57.

(iii) Here $N = 200$ thus $\frac{N}{4} = \frac{200}{4} = 50$. Now, we locate the point on the ogive, whose y-ordinate is 50. The x co-ordinate corresponding to this ordinate is 41.5. Hence lower quartile, Q_1 is 41.5.

(iii) 80% marks = 80 % of = 80 marks
We locate the point on the ogive, whose x-ordinate is 80. The y co-ordinate corresponding to this ordinate is 180 students . It means that 180 students score less than 80 %. Number of students who obtained more than 80 % marks are $200 - 180 = 20$.

(iv) 35% marks = 35% of 100 = 35 mark
We locate the point on the ogive, whose x-ordinate is 35. The y co-ordinate corresponding to this ordinate is 35 students . It means that 35 students score less than 35 %. Hence, the number of students, who did not the pass test is 35.

END

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(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Sol :

$$n(S) = 6$$
$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Thus (d) is correct option.

26. Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Sol :

One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.

$$n(S) = 4$$
$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

Thus (b) is correct option.

27. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, then the probability of drawing a blue marble is:

(a) $\frac{2}{5}$ (b) $\frac{1}{4}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Sol :

There are 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways. Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

$$n(S) = 5$$
$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$$

Thus (b) is correct option.

28. A single letter is selected at random from the word PROBABILITY. The probability that the selected letter is a vowel is

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) 0

Sol :

There are 11 letter in word PROBABILITY. Out of these 11 letter, 4 letter are vowels.

$$n(S) = 11$$
$$n(E) = 4$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{11}$$

Thus (c) is correct option.

29. A fair die is thrown once. The probability of getting a composite number less than 5 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) 0

Sol :

The outcomes are 1, 2, 3, 4, 5, 6. Out of these, 4 is the only composite number which is less than 5.

$$n(S) = 6$$
$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Thus (b) is correct option.

30. If a letter is chosen at random from the letter of English alphabet, then the probability that it is a letter of the word DELHI is

(a) $\frac{1}{5}$ (b) $\frac{1}{26}$

(c) $\frac{5}{26}$ (d) $\frac{21}{26}$

Sol :

The English alphabet has 26 letters in all. The word DELHI has 5 letter, so the number of favorable outcomes is 5.

$$n(S) = 26$$
$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{26}$$

Thus (c) is correct option.

31. The probability that a two digit number selected at random will be a multiple of 3 and not a multiple of 5 is

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options 0.0001 is closest to zero.
Thus (a) is correct option.

6. The $P(A)$ denotes the probability of an event A , then

(a) $P(A) < 0$ (b) $P(A) > 1$
(c) $0 \le P(A) \le 1$ (d) $-1 \le P(A) \le 1$

Sol :

Probability of an event always lies between 0 and 1.
Thus (c) is correct option.

7. If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$
(c) $\frac{2}{13}$ (d) $\frac{1}{2}$

Sol :

In a deck of 52 cards, there are 12 face cards i.e., 6 red and 6 black cards.

$$n(S) = 52$$

$$n(E) = 6$$

So, probability of getting a red face card,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

Thus (a) is correct option.

8. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favorable to E is

(a) 4 (b) 13
(c) 48 (d) 51

Sol :

In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.
Hence, the number of outcomes favorable

$$n(E) = 52 - 1 = 51$$

Thus (d) is correct option.

9. When a die is thrown, the probability of getting an odd number less than 3 is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) 0

Sol :

Odd number less than 3 is 1 only.

$$n(S) = 6$$

$$n(E) = 1$$

So, probability of getting an odd number less than 3,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Thus (a) is correct option.

10. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(a) 7 (b) 14
(c) 21 (d) 28

Sol :

We have $n(S) = 400$

$$n(E) = x$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$0.035 = \frac{x}{400}$$

$$x = 0.035 \times 400 = 14$$

Thus (b) is correct option.

11. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 40 (b) 240
(c) 480 (d) 750

Sol :

Total number of sold tickets are 6000. Let she bought x tickets.

Now $n(S) = 6000$

$$n(E) = x$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$0.08 = \frac{x}{6000}$$

$$x = 0.08 \times 6000 = 480$$

Hence, she bought 480 tickets.
Thus (c) is correct option.

12. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is

(a) $\frac{4}{15}$ (b) $\frac{2}{15}$
(c) $\frac{1}{15}$ (d) $\frac{1}{5}$

Sol :

Total possible outcome, $n(S) = 15$

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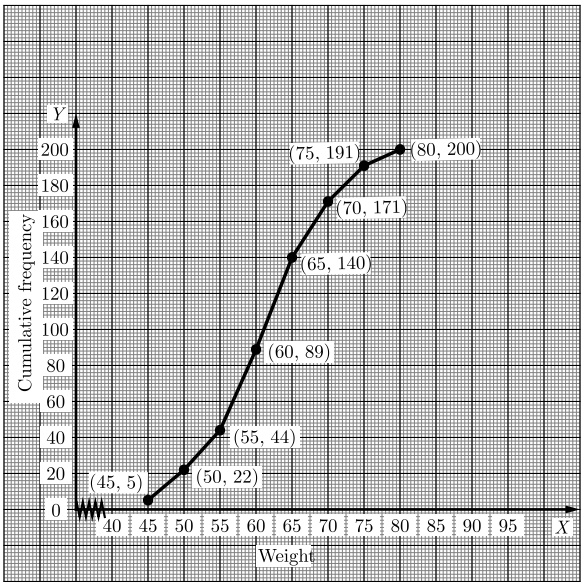
45-50	17	22
50-55	22	44
55-60	45	89
60-65	51	140
65-70	31	171
70-75	20	191
75-80	9	200

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.



(i) We locate the point on the ogive, whose x-ordinate is 55. The y co-ordinate corresponding to this ordinate is 44 students . It means that 44 students are having weight less than 55 kg. Number of students whose weight are $200 - 44 = 156$.
The percentage of students weight 55 kg or more

$$= \frac{156}{200} \times 100 = 78\% \text{ (approx).}$$

(ii) 30 % of 200 students i.e 60 heaviest students has to be selected. It means we have to select the $200 - 60 = 140$ lower weight students. We locate the point on the ogive, whose x-ordinate is 140. The y co-ordinate corresponding to this ordinate is 65. It means that 140 students have weight less than 65 kg. Thus heaviest 30 % of students fall above 65 kg.

(ii) We locate the point on the ogive, whose x-ordinate is 55.7. The y co-ordinate corresponding to this ordinate is 45. It means that 45 students are underweight. Number of students overweight are $200 - 45 = 155$.

Main 2004

54. The marks obtained by 200 students in an examination are given below :

Marks	Number of students
0-10	5
10-20	10
20-30	11
30-40	20
40-50	27
50-60	38
60-70	40
70-80	29
80-90	14
90-100	6

Using a graph paper, draw an ogive for the above distribution. Use your ogive to estimate
(i) the median.
(ii) the lower quartile.
(iii) the number of students, who obtained more than 80% marks in the examination.
(iv) the number of students, who did not pass, if the pass percentage was 35.
Use the scale as 2 cm = 10 marks on one axis and 2 cm = 20 cm students on the other axis.

Sol :

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The cumulative frequency table for the continuous distribution is given below :

Marks	Frequency f	Cumulative frequency cf
0-10	5	5
10-20	10	15
20-30	11	26
30-40	20	46
40-50	27	73
50-60	38	111
60-70	40	151
70-80	29	180
80-90	14	194
90-100	6	200

On the graph paper, we plot all points and join all these points by free drawing. The required ogive

Thus (d) is correct option.

19. If the probability of an event is p, then the probability of its complementary event will be

(a) $p-1$ (b) p

(c) $1-p$ (d) $1-\frac{1}{p}$

Sol :

Since,

$$P(E)+P(\overline{E})=1$$
$$P(E)=1-P(\overline{E})$$
$$=1-p$$

Thus (c) is correct option.

20. The probability expressed as a percentage of a particular occurrence can never be

(a) less than 100

(b) less than 0

(c) greater than 1

(d) anything but a whole number

Sol :

Sol :

We know that the probability expressed as a percentage always lie between 0 and 100. So, it cannot be less than 0.

Thus (b) is correct option.

21. A die is thrown once. What is the probability of getting a number less than 3?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) 0

Sol :

There are 6 possible outcome for a die.

$$n(S)=6$$

Favorable outcome are 1 and 2 i.e. two outcomes.

$$n(E)=2$$

P(number less than 3)

$$P(E)=\frac{n(E)}{n(S)}=\frac{2}{6}=\frac{1}{3}$$

Thus (a) is correct option.

22. A number is chosen at random from the numbers $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$. Then the probability that square of this number is less than or equal to 1 is

(a) $\frac{8}{11}$ (b) $\frac{3}{11}$

(c) $\frac{6}{11}$ (d) $\frac{7}{11}$

Sol :

Given numbers are $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ and their squares are 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25.

Total number of outcomes $n(S)=11$.

Favorable outcome are -1, 0, 1, thus number of favorable outcomes is $n(E)=3$.

Required probability, $P(E)=\frac{n(E)}{n(S)}=\frac{3}{11}$

Thus (b) is correct option.

23. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{1}{3}$

Sol :

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35 and 40 thus 8 outcome.

$$n(S)=40$$
$$n(E)=8$$
$$P(E)=\frac{n(E)}{n(S)}=\frac{8}{40}=\frac{1}{5}$$

Thus (a) is correct option.

24. Someone is asked to take a number from 1 to 100. The probability that it is a prime, is

(a) $\frac{8}{25}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{13}{50}$

Sol :

Prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97, i.e 25 outcome.

$$n(S)=100$$
$$n(E)=25$$
$$P(E)=\frac{n(E)}{n(S)}=\frac{25}{100}=\frac{1}{4}$$

Thus (c) is correct option.

25. The probability of getting a number greater then 3 in throwing a die is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

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37. If a card is selected from a deck of 52 cards, then what is the probability of its being a red face card?

(a) $\frac{23}{26}$ (b) $\frac{3}{26}$

(c) $\frac{6}{13}$ (d) $\frac{7}{13}$

Sol :

In a deck of 52 cards, there are 12 face cards i.e., 6 red and 6 black cards.

$$n(S) = 52$$

$$n(E) = 6$$

So, probability of getting a red face card,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

Thus (b) is correct option.

38. If a number x is chosen at random from the numbers $-2, -1, 0, 1, 2$. Then, the probability that $x^2 < 2$ is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) $\frac{3}{5}$

Sol :

Total number of possible outcomes are 5.
We observe that $x^2 < 2$ when x takes anyone of the following three values -1, 0 and 1.

We have $n(S) = 5$

$$n(E) = 3$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{5}$$

Thus (d) is correct option.

39. Which of the following relationship is the correct?

(a) $P(E) + P(\overline{E}) = 1$
(b) $P(\overline{E}) - P(E) = 1$
(c) $P(E) = 1 + P(\overline{E})$
(d) None of these

Sol :

Sol :

$$P(E) + P(\overline{E}) = 1$$

Thus (a) is correct option.

40. Two dice are thrown together. The probability that sum of the two numbers will be a multiple of 4, is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Sol :

Total number of outcomes is 36.
Here, all possible outcome is (1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2) and (6, 6),

$$n(S) = 36$$

$$n(E) = 9$$

P(sum of two numbers will be multiple of 4)

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

Thus (d) is correct option.

41. A letter is chosen at random from the letters of the word ASSASSINATION, then the probability that the letter chosen is a vowel is in the form of $\frac{6}{2x+1}$, then x is equal to

(a) 5 (b) 6

(c) 7 (d) 8

Sol :

There are 13 letters in the word ASSASSINATION out of which one letter can be chosen in 13 ways. Hence, total number of outcomes are 13. There are 6 vowels in the word ASSISSINATION. So, there are 6 ways of selecting a vowel.

$$n(S) = 13$$

$$n(E) = 6$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{13}$$

But given that, $\frac{6}{2x+1} = \frac{6}{13}$

$$2x + 1 = 13 \Rightarrow x = 6$$

Thus (b) is correct option.

42. Ramesh buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random a tank containing 5 male fish and 9 female fish. Then, the probability that the fish taken out is a male fish, is

(a) $\frac{5}{13}$ (b) $\frac{5}{14}$

(c) $\frac{6}{13}$ (d) $\frac{7}{13}$

Sol :

There are $14 = (5 + 9)$ fish out of which one can be chosen in 14 ways.
There are 5 male fish out of which one male fish

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(a) $\frac{2}{15}$ (b) $\frac{4}{15}$

(c) $\frac{1}{15}$ (d) $\frac{4}{90}$

Sol :

24 out of the 90 two digit numbers are divisible by 3 and not by 5.

$$n(S) = 90$$

$$n(E) = 24$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{24}{90} = \frac{4}{15}$$

Thus (b) is correct option.

32. If in a lottery, there are 5 prizes and 20 blanks, then the probability of getting a prize is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) 1

Sol :

We have $n(S) = 20 + 5 = 25$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

Thus (c) is correct option.

33. Someone is asked to take a number from 1 to 100. What is the probability that it is a prime ?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{5}$

Sol :

Prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97, i.e 25 outcome.

$$n(S) = 100$$

$$n(E) = 25$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{25}{100} = \frac{1}{4}$$

Thus (b) is correct option.

34. Out of one digit prime numbers, one number is selected at random. What is the probability of selecting an even number?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{5}$

Sol :

One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.

$$n(S) = 4$$

$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

Thus (b) is correct option.

35. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, what is the probability of drawing a blue marble ?

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) $\frac{3}{5}$

Sol :

There are 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways. Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

$$n(S) = 5$$

$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$$

Thus (a) is correct option.

36. A die is thrown once. What is probability of getting "at most 2."?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

Sol :

All possible outcome i.e. sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Number of all possible outcome,

$$n(S) = 6$$

Favorable outcomes,

$$E = \{1,\ 2\}$$

Number of favorable outcome,

$$n(E) = 2$$

Thus $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Thus (b) is correct option.

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CHAPTER 25

PROBABILITY

MULTIPLE CHOICE QUESTIONS

Main 2024

1. Assertion (A): A die is thrown once and the probability of getting an even number is $\frac{2}{3}$.
Reason (R): The sample space for even numbers on a die is $\{2,4,6\}$
(a) A is true, R is false.
(b) A is false, R is true.
(c) Both A and R are true.
(d) Both A and R and false.

Sol :

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

Favorable cases is $\{2, 4, 6\}$.
No. of favorable outcomes,

$$n(E) = 3$$

Probability of getting an even number,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Thus A is false but R is true.
Thus (b) is correct option.

SQP 2024

2. Event A: The sun will rise from east tomorrow.
Event B: It will rain on Monday.
Event C: February month has 29 days in a leap year.
Which of the above event(s) has probability equal to 1?
(a) all events A, B and C
(b) both events A and B
(c) both events B and C
(d) both events A and C

Sol :

Event A and C are certain event. The probability of a certain event is 1.
Thus (d) is correct option.

Main 2023

3. A letter is chosen at random from all the letters of the English alphabets. The probability that the letter chosen is a vowel, is :

(a) $\frac{4}{26}$ (b) $\frac{5}{26}$

(c) $\frac{21}{26}$ (d) $\frac{5}{24}$

Sol :

Total number of alphabets in English are 26.

$$n(S) = 26$$

Out of 26 alphabet vowels are a, e, i, o, u.
No. of favorable outcomes,

$$n(E) = 5$$

Probability that the letter chosen is a vowel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{26}$$

Thus (b) is correct option.

Main 2022 Sem II

4. The probability of getting a number divisible by 3 in throwing a dice is :

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Sol :

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

Favorable cases is $\{3, 6\}$.
No. of favorable outcomes,

$$n(E) = 2$$

Probability of getting a number divisible by 3,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Thus (b) is correct option.

5. An event is very unlikely to happen. Its probability is closest to
(a) 0.0001 (b) 0.001
(c) 0.01 (d) 0.1

Sol :

The probability of an event which is very unlikely to happen is closest to zero and from the given

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can be chosen in 5 ways.

$$n(S) = 14$$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{14}$$

Thus (b) is correct option.

43. A number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9 then the probability that the product xy of the two numbers will be less than 9 is

(a) $\frac{3}{7}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{7}{9}$

Sol :

Number x can be selected in three ways and corresponding to each such way there are three ways of selecting number y.

Therefore two numbers can be selected in 9 ways as (1, 1), (1, 4), (1, 9), (2, 1), (2, 4), (2, 9), (3, 1), (3, 4), (3, 9) So, total numbers of possible outcomes are 9.

The product xy will be less than 9, if x and y are chosen in one of the following ways: (1, 1), (1, 4), (2, 1), (2, 4), (3, 1)

$$n(S) = 9$$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{9}$$

Thus (c) is correct option.

44. There are 1000 sealed envelopes in a box. 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well-shuffled and an envelope is picked up out, then the probability that is contains no cash prize is

(a) 0.65 (b) 0.69

(c) 0.54 (d) 0.57

Sol :

Total number of envelopes in the box = 1000

Number of envelopes containing cash prize

$$= 10 + 100 + 200 = 310$$

Number of envelopes containing no cash

$$= 1000 - 310 = 690$$

Now $n(S) = 1000$

$$n(E) = 690$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{690}{1000} = 0.69$$

Thus (b) is correct option.

45. **Assertion :**A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of getting a face card is 1.1.

Reason : Probability of an event always lies between 0 and 1.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

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THREE MARKS QUESTIONS

Main 2023

46. A bag contains 25 cards, numbered through 1 to 25. A card is drawn at random. What is the probability that the number on the card drawn is:

(a) multiple of 5

(b) a perfect square

(c) a prime number?

Sol :

Given, cards marked with number 1 to 25. We have 25 cards and thus there are 25 possible outcomes.

$$n(S) = 25$$

(a) Multiple of 5

Multiple of 5 from 1 to 25 are {5, 10, 15, 20, 25}

No. of favorable outcomes,

$$n(E_1) = 5$$

P(Multiple of 5),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

(b) a perfect square

Perfect square number are {4, 9, 16, 25}

No. of favorable outcomes,

, p. They are put in a box and shuffled. A boy is assumed to draw a card the box. What is the probability that the card drawn is
(i) a vowel?
(ii) a consonant?
(iii) none of the letters of the word median?

Sol :

Total number of cards $= 16$
No. of all possible outcomes

$$n(S) = 16$$

Clearly, the cards labelled a, e, i, o are vowels rest 12 are consonant.
(i) A vowel
No. of favorable outcomes,

$$n(E_1) = 4$$

Probability of drawing a vowel,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{16} = \frac{1}{4}$$

(ii) A consonant
No. of favorable outcomes,

$$n(E_2) = 12$$

Probability of drawing a constant,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{12}{16} = \frac{3}{4}$$

(iii) None of the letters of the word median?
Word median includes 6 letter and all letter are in our sample space.
Thus number of cards not labelled with letters of the word median is $16 - 6 = 10$
No. of favorable outcomes,

$$n(E_3) = 10$$

Probability of drawing one of the letters of the word median

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{10}{16} = \frac{5}{8}$$

Comp 2016

79. Cards marked with numbers 3, 4, 5, ...50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is :
(i) Divisible by 7.
(ii) A perfect square.
(iii) A multiple of 6.

Sol :

We have 48 cards and thus there are 48 possible outcomes.

$$n(S) = 48$$

(i) Divisible by 7.
Number of cards divisible by 7 are 7, 14, 21, 28, 35, 35, 42 and 49.
No. of favorable outcomes,

$$n(E_1) = 7$$

P(cards divisible by 7),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{7}{48}$$

(ii) A perfect square.
Number of cards having a perfect square are 4, 9, 16, 25, 36 and 49.
No. of favorable outcomes,

$$n(E_2) = 6$$

P(cards having a perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{48} = \frac{1}{8}$$

(iii) A multiple of 6.
Number of multiples of 6 from 3 to 50 are 6, 12, 24, 30, 36, 42, and 48.
No. of favorable outcomes,

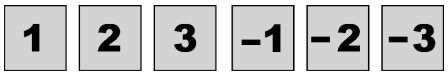
$$n(E_3) = 6$$

P(multiple of 6 from 3 to 50),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{8}{48} = \frac{1}{6}$$

Main 2014

80. A die has 6 faces marked by the given numbers as shown below:



The die is thrown once, what is the probability of getting
(i) a positive integer?
(ii) an integer greater than -3?
(iii) the smallest integer?

Sol :

Total possible outcomes, $n(S) = 6$
(i) A positive integer
Positive integer are $\{1, 2, 3\}$
No. of favorable outcomes,

$$n(E_1) = 3$$

Probability of drawing a positive integer,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) An integer greater than -3?
Integers greater than -3 are $\{1,2,3,-1,-2\}$.
No. of favorable outcomes,

$$n(E_2) = 5$$

Probability of drawing a integer greater than -3

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{6}$$

(iii) Smallest integer

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alphabet which appears in the given word?
(iii) one of the last 9 letters of the English alphabet which appears in the given word?

Sol :

Total letters in AUTHORIZES word are 10. So we have 10 identical circular discs.

$$n(S) = 10$$

(i) A vowel
Here vowels are A, U, O, I, E i.e 5 vowels.
No. of favorable outcomes,

$$n(E_1) = 5$$

P(A vowel),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

(ii) One of the first 9 letters
From first 9 letters of the English alphabet in the given word are A, H, I, E.
No. of favorable outcomes,

$$n(E_1) = 4$$

P(One of the first 9 letters),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{10} = \frac{2}{5}$$

(iii) One of the last 9 letters
From last 9 letters of the English alphabet in the given word are U, T, R, Z, S.
No. of favorable outcomes,

$$n(E_3) = 5$$

P(One of the last 9 letters),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

Comp 2020

52. In a family of three children, find the probability of having at least two boys.

Sol :

If there are three children in family all possible outcome are {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}.
So, the total number of outcomes,

$$n(S) = 2^3 = 8$$

At-least two of them are boys means all those cases in which we have either 2 or 3 boys. Thus favorable outcome are {BBB, BBG, BGB, GBB}
Number of favorable outcome,

$$n(E) = 4$$

The probability of having at least two boys

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Main 2019

53. There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box. Find the probability that the number on the disc is
(i) an odd number.
(ii) divisible by 2 and 3 both.
(iii) a number less than 16.

Sol :

Given, discs marked with number 1 to 25. We have 25 discs and thus there are 25 possible outcomes.

$$n(S) = 25$$

(i) an odd number.
Number of discs having odd numbers 1, 3, 5 ...., 25 i.e. 13 discs
No. of favorable outcomes,

$$n(E_1) = 13$$

P(an odd number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{25}$$

(ii) divisible by 2 and 3 both.
Number of discs divisible by both 2 and 3 are 6, 12, 18 and 24
No. of favorable outcomes,

$$n(E_2) = 4$$

P(divisible by 2 and 3 both),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{25}$$

(iii) Number less than 16.
Number less than 16 are 1 to 15.
No. of favorable outcomes,

$$n(E_3) = 15$$

P(Number less than 16),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{15}{25} = \frac{3}{5}$$

Comp 2019

54. Two dice are tossed simultaneously. Find the probability of getting
(i) an even number on both dice.
(ii) the sum of two numbers more than 9.

Sol :

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) an even number on both dice.
Favorable outcome are (2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4) and (6, 6).
Number of favorable outcomes

$$n(E_1) = 9$$

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Main 2009

69. A die is thrown once. What is the probability that the
(i) number is even?
(ii) number is greater than 2?

Sol :

When a die is thrown, then outcomes are 1, 2, 3, 4, 5, 6.

Total outcomes $n(S) = 6$

(i) Number is even

Even numbers are 2, 4 and 6.

$$n(E_1) = 3$$

P(Number is even),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Number is greater than 2

Number greater than 2 are 3, 4, 5 and 6

$$n(E_2) = 4$$

P(Number is greater than 2),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Comp 2009

70. Two different dice are tossed together. Find the probability:
(i) of getting a doublet
(ii) of getting a sum 10, of the numbers on the two dice.

Sol :

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) of getting a doublet

Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6) which are 6 doublets.

Number of favorable outcomes,

$$n(E_1) = 6$$

P(doublet), $\quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) of getting a sum 10, of the numbers on the two dice.

Favorable outcomes are (4, 6), (5, 5), (6, 4) i.e., 3.

Number of favorable outcomes,

$$n(E_2) = 3$$

P(sum 10), $\quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

Main 2007

71. A child has a die whose six faces show the letters as shown below:

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{D}$ $\boxed{E}$ $\boxed{A}$

The die is thrown once. What is the probability of getting (i) A, (ii) D?

Sol :

Total possible outcomes, $n(S) = 6$

(i) Probability of getting letter A,

$$n(E_1) = 2.$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Probability of getting letter D,

$$n(E_2) = 1$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

Comp 2005

72. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.

Sol :

Let x be blue balls.

Total balls $\quad n(S) = 6 + x$

$$n(R) = 5 \text{ and } n(B) = x$$

P(red ball) $\quad P(R) = \frac{n(R)}{n(S)} = \frac{6}{6+x}$

P(blue ball) $\quad P(R) = \frac{n(B)}{n(S)} = \frac{x}{6+x}$

As per question we have

$$\frac{x}{6+x} = \frac{2 \times 6}{6+x} \Rightarrow x = 12$$

Thus there are 12 blue balls.

Main 2003

73. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5.

Sol :

Number of possible outcomes,

$$n(S) = 6^2 = 36$$

The favorable outcomes are (sum less than 5) {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2) and (3, 1)} i.e. 6 outcomes.

Number of favorable outcome,

$$n(E) = 6$$

P (have sum less than 5)

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Comp 2003

74. Two dice are rolled simultaneously. Find the probability that the sum of numbers appearing is 10.

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(ii) number divisible by 5.

Sol :

Total number of discs in the box are 90.
Thus we have 90 possible outcomes.

$$n(S) = 90$$

(i) a two digit number,
Discs with two digit number are 10, 11, ...89 and 90 which are 81 numbers.
No. of favorable outcomes,

$$n(E_1) = 81$$

P(a disc with two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) number divisible by 5
The numbers divisible by 5 between 1 to 90 are 5, 10, 15 ..... 85 and 90 which are 18 numbers.
No. of favorable outcomes,

$$n(E_2) = 18$$

P (a disc with a number divisible by 5)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{18}{90} = \frac{1}{5}$$

Main 2015

59. A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is :
(i) a green ball.
(ii) a white or a red ball.
(iii) neither a green ball nor a white ball.

Sol :

Number of white ball $n(W) = 5$

Number of red ball $n(R) = 6$

Number of green ball $n(G) = 9$

Total ball, $n(S) = 5 + 6 + 9 = 20$

(i) Green ball
No. of favorable outcomes,

$$n(E_1) = n(G) = 9$$

P(Green ball),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{20}$$

(ii) White or a red ball
Number of white or red is $5 + 6 = 11$.
No. of favorable outcomes,

$$n(E_2) = 11$$

P(White or a red ball),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{11}{20}$$

(iii) Neither a green ball nor a white ball
Neither a green ball nor a white ball i.e. only red ball.
No. of favorable outcomes,

$$n(E_3) = n(R) = 6$$

P(Neither a green ball nor a white ball),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{20} = \frac{3}{10}$$

Comp 2015

60. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the numbers 25.

Sol :

Peter throws two dice together. There are $6^2 = 36$ total number of possible outcomes.

$$n(S) = 36$$

He get 25 only when he gets (5, 5),
No. of favorable outcomes,

$$n(E_1) = 1$$

P(getting the numbers of product 25),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Rina throws one dice. There are 6 total number of all possible outcomes.

$$n(S) = 6$$

The number where square is 25 is 5.
No. of favorable outcomes,

$$n(E_2) = 1$$

P (getting a number whose square is 25)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

$$P(E_2) > P(E_1)$$

Hence, Rina has better chances to the number square 25.

Comp 2014

61. Two unbiased coins are tossed simultaneously. Find the probability of getting :
(i) at least one head,
(ii) almost one head,
(iii) no head.

Sol :

There are 4 possible outcome when two unbiased coins are tossed simultaneously.

Sample space $S = \{HH, HT, TH, TT\}$

$$n(S) = 4$$

(i) at least one head,
Favorable outcomes are {HT, TH, HH}.

$$n(E_1) = 3$$

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$$n(E_2) = 4$$

P(a perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{25}$$

(c) a prime number

Prime numbers from 1 to 25 are {2, 3, 5, 7, 11, 13, 17, 19, 23,} i.e 9 numbers

No. of favorable outcomes,

$$n(E_3) = 9$$

P(a prime number),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{9}{25}$$

Comp 2023

47. A bag contains cards numbered 1 to 49. Find the probability that the number on the drawn card is :

(i) an odd number

(ii) a multiple of 5

(iii) Even prime

Sol :

Total cards, $n(S) = 49$

(i) an odd number

Odd number are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47 and 49.

Total odd number, $n(E_1) = 25$

P(odd number), $P(E_1) = \frac{n(E_1)}{n(S)}$

$$= \frac{n(O)}{n(S)} = \frac{25}{49}$$

(ii) a multiple of 5

Multiple of 5 are 5, 10, 15, 20, 25, 30, 35, 40 and 45.

Total multiple of 5 number,

$$n(E_2) = 5$$

P(multiple of 5), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{9}{49}$

(iii) Even prime

Only 2 is even prime number. Therefore

$$n(E_3) = 1$$

P(even prime), $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{49}$

Main 2022 Sem II

48. A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball?

Sol :

There are total 5 + 2 + 3 = 10 ball in bag.

Thus $n(S) = 10$

No. of red ball i.e. favorable outcomes,

$$n(E) = 2$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{10} = 0.2$$

Main 2022 Sem II

49. A letter of the word SECONDARY is selected at random. What is the probability that the letter selected is a vowel?

Sol :

Total letters in SECONDARY word are 9.

$$n(S) = 9$$

Vowels = 3 (E, O, A)

No. of favorable outcomes,

$$n(E) = 3$$

Required probability of getting a vowel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

Comp 2022

50. An integer is chosen at random between 1 and 100. Find the probability that it is:

(i) divisible by 8.

(ii) not divisible by 8.

Sol :

Total number of outcomes,

$$n(S) = 100 - 2 = 98$$

(i) divisible by 8.

Favorable outcomes are 8, 16, 24, ..., 96, i.e., 12.

Number of favorable outcomes,

$$n(E_1) = 12$$

P(Divisible by 8),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{98} = \frac{6}{49}$$

(ii) not divisible by 8.

P(not divisible by 8),

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{6}{49} = \frac{43}{49}$$

Main 2020

51. Each of the letters of the word 'AUTHORIZES' is written on identical circular disc and put in a bag. They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is

(i) a vowel?

(ii) one of the first 9 letters of the English

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$$n(E_1) = 3 \times 4 = 12$$

Probability of drawing a face card

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{40} = \frac{3}{10}$$

(ii) An even numbered red card

Even number card are 2, 4, 6, 8 and 10. Since, there are two suits of red cards,

No. of favorable outcomes,

$$n(E_2) = 2 \times 5 = 10$$

Probability of drawing an even numbered red card

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{40} = \frac{1}{4}$$

Comp 2011

66. A box contains cards, number 1 to 90. A card is drawn at random from the box. Find the probability that the selected card bears a :

(i) Two digit number.

(ii) Perfect square number

Sol :

We have 90 cards and thus there are 90 possible outcomes.

$$n(S) = 90$$

(i) No. of cards having 2 digit number $90 - 9 = 81$.

Number of favorable outcomes,

$$n(E_1) = 81$$

P(selected card bears two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect square number between 1 to 90 are 1, 4, 9, 16, 25, 36, 49, 64 and 81 i.e. 9 numbers.

No. of favorable outcomes,

$$n(E_2) = 9$$

P(perfect square numbers)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{9}{90} = \frac{1}{10}$$

Main 2010

67. Cards marked with numbers 1, 2, 3, 4, ..., 20 are well shuffled and a card is drawn at random. What is probability that the number of the cards is

(i) a prime number?

(ii) divisible by 3?

(iii) a perfect square?

Sol :

Given, cards marked with number 1, 2, 3, 4, ..., 20. We have 20 cards and thus there are 20 possible outcomes.

$$n(S) = 20$$

(i) A prime number

Number of cards having a prime number is {2, 3, 5, 7, 11, 13, 17, 19}

No. of favorable outcomes,

$$n(E_1) = 8$$

P(A prime number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

(ii) Divisible by 3.

Number of cards divisible by 3 are 3, 6, 9, 12, 15 and 18.

No. of favorable outcomes,

$$n(E_2) = 6$$

P(Divisible by 3),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{20} = \frac{3}{10}$$

(iii) a perfect square?

Number of cards having a perfect square are 1, 4, 9 and 16.

No. of favorable outcomes,

$$n(E_3) = 4$$

P(a perfect square),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

Comp 2010

68. What is the probability that a randomly taken leap year has 52 Sundays?

Sol :

Number of days in a leap year= 366

$$\text{Number of weeks} = \frac{366}{7} = 52.28$$

So, there will be 52 weeks and 2 days

So, every leap year has 52 Sundays

Now, the probability depends on remaining 2 days

The Possible pairing of days are

Sunday – Monday
Monday – Tuesday
Tuesday – Wednesday
Wednesday – Thursday
Thursday – Friday
Friday – Saturday
Saturday – Sunday

There are total 7 pairs and out of 7 pairs, only 2 pairs have Sunday. The remaining 5 pairs does not include Sunday.

$$n(S) = 7$$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{7}$$

Therefore, the probability of only 52 Sunday in a Leap year is $\frac{5}{7}$.

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P(an even number on both dice),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) sum of two numbers more than 9

Favorable outcome are (4, 6), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).

Number of favorable outcomes

$$n(E_2) = 6$$

P (sum of two numbers more than 9),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Comp 2018

55. A box contains 20 cards from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Sol :

We have 20 cards and thus there are 20 possible outcomes.

$$n(S) = 20$$

(i) divisible by 2 or 3

Number divisible by 2 or 3 are 6, 12, 18.

Number of favorable outcomes,

$$n(E_1) = 3$$

P(divisible by 2 or 3),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{20}$$

(ii) a prime number

Prime numbers are 2, 3, 5, 7, 11, 13, 17 and 19 i.e 8 numbers.

Number of favorable outcomes,

$$n(E_2) = 8$$

P(a prime no.),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

Comp 2017

56. Three digit number are made using the digits 4, 5, 9 (without repetition). If a number among them is selected at random, what is the probability that the number will :

(i) be a multiple of 5 ?

(ii) be a multiple of 9 ?

(iii) will end with 9 ?

Sol :

Total number of three digit numbers are 459, 495, 549, 594, 945 and 954. Thus we have 6 possible outcomes.

$$n(S) = 6$$

(i) be a multiple of 5

Multiple of 5 are 495 and 945.

$$n(E_1) = 2$$

P(multiple of 5),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) be a multiple of 9

All are multiple of 9.

$$n(E_2) = 6$$

P(multiple of 9),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{6} = 1$$

(iii) will end with 9

Numbers 459 and 549 ends with 9.

$$n(E_3) = 2$$

P(ending with 9),

$$P(E_3) = \frac{n(E_2)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Main 2016

57. A game of numbers has cards marked with 11, 12, 13, .., 40. A card is drawn at random. Find the probability that the number on the card drawn is

(i) a perfect square. (ii) divisible by 7.

Sol :

Given, cards marked with number 11, 12, 13, 14, ..., 40. We have 30 cards and thus there are 30 possible outcomes.

$$n(S) = 30$$

(i) A perfect square.

Number of cards having a perfect square are 16, 25 and 36.

No. of favorable outcomes,

$$n(E_1) = 3$$

P(cards having a perfect square),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{30} = \frac{1}{10}$$

(ii) Divisible by 7.

Number of cards divisible by 7 are 14, 21, 28, and 35.

No. of favorable outcomes,

$$n(E_2) = 4$$

P(cards divisible by 7),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{30} = \frac{2}{15}$$

Comp 2016

58. A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two digit number,

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Sol :

When two dice are thrown, we have $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

Favorable outcomes are $(4, 6)$, $(6, 4)$ and $(5, 5)$. In these outcomes, sum of numbers appearing is 10.
No. of favorable outcomes

$$n(E) = 3$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

Main 2001

75. A bag contains, white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

Sol :

We have $P(W) = \frac{3}{10}$

$$P(B) = \frac{2}{5}$$

$$P(R) = 1 - \left(\frac{3}{10} + \frac{2}{5}\right) = \frac{3}{10}$$

Now $P(B) = \frac{n(B)}{n(S)}$

Substituting $P(B) = \frac{2}{5}$ and $n(B) = 20$ in above equation we have

$$\frac{2}{5} = \frac{20}{n(S)} \Rightarrow n(S) = \frac{20 \times 5}{2} = 50$$

Thus there are 50 total balls.

Comp 2001

76. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of blackballs in the bag.

Sol :

Let x be black balls and 15 white balls.

Total balls, $n(S) = 15 + x$

Let $P(B)$ be the probability of drawing black ball and
$P(W)$ be the probability of drawing white ball.

Now $P(B) = 3 \times P(W)$

$$\frac{x}{(15+x)} = 3 \times \frac{15}{(15+x)}$$

$$x = 45$$

Thus there are 45 black balls in the bag.

FOUR MARK QUESTIONS

Main 2018

77. Cards bearing numbers 2, 4, 6, 8, 10, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting card which is
(i) a prime number.
(ii) a number divisible by 4.
(iii) a number that is a multiple of 6.
(iv) an odd number.

Sol :

Given, cards marked with number 2, 4, 6, 8, 10, 14, 16, 18 and 20. We have 10 cards and thus there are 10 possible outcomes.

$$n(S) = 10$$

(i) A prime number
Only 2 is prime number in given card numbers.
No. of favorable outcomes,

$$n(E_1) = 1$$

P(A prime number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{10}$$

(ii) Divisible by 4.
Number of cards divisible by 4 are 4, 8, 12, 16 and 20.
No. of favorable outcomes,

$$n(E_2) = 5$$

P(Divisible by 3),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

(iii) Number that is a multiple of 6
Number of cards having a multiple of 6 are perfect square are 6,12 and 18.
No. of favorable outcomes,

$$n(E_3) = 3$$

P(a perfect square),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{10}$$

(iv) Odd number
All number are even numbers. There is no any odd number.
No. of favorable outcomes,

$$n(E_3) = 0$$

P(a perfect square),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{0}{10} = 0$$

Main 2017

78. Sixteen cards are labelled as a, b, c, ..., m, n, o

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$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{4}$$

(ii) almost one head,
Favorable outcomes are {HT, TH, HH}.

$$n(E_2) = 3$$
$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{4}$$

(iii) no head.
Favorable outcomes is {TT} only.

$$n(E_3) = 1$$
$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{4}$$

Main 2013

62. A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, then find the number of black balls in the box.

Sol :

Number of white ball $n(W) = 30$

Number of black ball $n(B) = x$

Total ball, $n(S) = x + 30$

Probability of drawing a black ball,

$$P(B) = \frac{n(B)}{n(S)} = \frac{x}{x+30}$$

Probability of drawing a white ball,

$$P(W) = \frac{n(W)}{n(S)} = \frac{30}{x+30}$$

Since the probability of drawing a black ball is two-fifths of a white ball,

$$P(B) = \tfrac{2}{5} P(W)$$

$$\frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30}$$

$$x = \frac{2}{5} \times 30 = 12$$

Hence, the number of black ball in the box is 12.

Comp 2013

63. An integer is chosen between 70 and 100. Find the probability that it is
(i) a prime number (ii) divisible by 7

Sol :

There are 29 integer from 70 to 100. Total number of outcomes are 29 in both case.

$$n(S) = 29$$

(i) There are 6 prime numbers between 70 and 100 as 71, 73, 79, 83, 89 and 97 i.e. 6 favorable outcome.

$$n(E_1) = 6$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{29}$$

(ii) There are 4 numbers between 70 and 100 which are divisible by 7 as 77, 84, 91 and 98 i.e. 4 favorable outcome.

$$n(B) = 4$$
$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{29}$$

Main 2012

64. Two coins are tossed once. Find the probability of getting
(i) 2 heads.
(ii) atleast 1 tail.

Sol :

Sample space for two coins tossed is {HH, HT, TH, TT}.

$$n(S) = 4$$

(i) Sample space for 2 heads is {HH}
Number of favorable outcomes,

$$n(E_1) = 1$$

Probability of 2 heads,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{4}$$

(ii) Sample space for atleast 1 tail is {HT, TH, TT}
Number of favorable outcomes,

$$n(E_2) = 3$$

Probability of atleast 1 tail,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{4}$$

Main 2011

65. From a pack of 52 playing cards, all cards, whose numbers are multiples of 3, are removed. A card is now drawn at random.
What is the probability that the card drawn is
(i) a face card (king, jack or queen)?
(ii) an even numbered red card?

Sol :

There are total 52 cards out of which 6 cards are removed.
Multiples of 3 in each suit are 3, 6 and 9. In a pack of cards, there are four suits.
Number of remove cards $= 3 \times 4 = 12$
Total number of cards left $= 52 - 12 = 40$
Total number of all possible outcomes,

$$n(S) = 40$$

(i) A face card king,
Face cards are 3 in each suit and there are 4 suit.
No. of favorable outcomes,

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Smallest integer is -3 only
No. of favorable outcomes,

$$n(E_3) = 1$$
$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{6}$$

Comp 2013

81. Five cards, ten, Jack, Queen, King and Ace of diamonds are well shuffled. One card is picked up from them.
(i) Find the probability that the drawn card is Queen.
(ii) If Queen is put aside, then find the probability that the second card drawn is an ace.

Sol :

We have 5 cards and thus there are 5 possible outcomes.

$$n(S) = 5$$

(i) Drawn card is queen
No. of favorable outcomes,

$$n(E_1) = 1$$
$$P(\text{queen}), \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{5}$$

(ii) second card drawn is an ace
Since, queen was kept, number of all possible outcomes

$$n(S) = 5 - 1 = 4$$

Number of favorable outcomes

$$n(E_2) = 1$$

$P(\text{second card drawn is an ace})$,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{4}$$

Comp 2012

82. The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

Sol :

Let x and y be the number of blue and black marbles. No of green marbles $= 11$
Total number of marbles $= x + y + 11$
According to the problem,

$$P(\text{black marbles}) = \frac{1}{4}$$
$$\frac{y}{x+y+11} = \frac{1}{4}$$
$$x = 3y - 11 \qquad \ldots(1)$$

Again, $P(\text{blue marble}) = \frac{1}{5}$

$$\frac{x}{x+y+11} = \frac{1}{5}$$
$$5x = x + y + 11$$
$$x = \frac{y+11}{4} \qquad \ldots(2)$$

From equation (1) and (2), we have

$$3y - 11 = \frac{y+11}{4}$$
$$12y - 44 = y + 11$$
$$12y - y = 11 + 44$$
$$11y = 55 \Rightarrow y = 5$$

From equation (1) we have

$$x = 3 \times 5 - 11 = 4$$

Hence, total number of marbles in the jar,

$$x + y + 11 = 4 + 5 + 11 = 20$$

Comp 2011

83. Cards numbered 2 to 101 are placed in a box. A card is selected at random from the box, find the probability that the card selected :
(i) has a number which is a perfect square.
(ii) has an odd number which is not less than 70.

Sol :

Since box consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) has a number which is a perfect square.
Perfect squares are 4, 9, 16, 25, 36, 49, 64, 81 and 100.
Number of favorable outcomes,

$$n(E_1) = 9$$

$P(\text{Perfect square})$,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{100}$$

(ii) has an odd number which is not less than 70.
Favorable outcomes are 71, 73, 75, ...101.
Number of favorable outcomes,

$$n(E_2) = 16$$

$P(\text{odd number not less than 70})$,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{16}{100} = \frac{4}{25}$$

Comp 2009

84. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears,
(i) a one digit number.
(ii) a number divisible by 5.
(iii) an odd number less than 30.
(iv) a composite number between 50 and 70.

Sol :

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We have $70-5=65$ cards and thus there are 65 possible outcomes.

$$n(S)=65$$

(i) a one digit number.

One digit numbers are 6, 7, 8 and 9.

Number of favorable outcomes

$$n(E_1)=4$$

P(one digit number),

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{4}{65}$$

(ii) a number divisible by 5.

Number divisible by 5 are 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65 and 70 i.e. 13 numbers.

Number of favorable outcomes,

$$n(E_2)=13$$

P(a number divisible by 5),

$$P(E_2)=\frac{n(E_2)}{n(S)}=\frac{13}{65}=\frac{1}{5}$$

(iii) an odd number less than 30.

Odd number less than 30 are 7, 9, 11, 13, 15, 17, 19 23, 25, 27 and 29.

Number of favorable outcomes,

$$n(E_3)=12$$

P(a odd number less than 30),

$$P(E_3)=\frac{n(E_3)}{n(S)}=\frac{12}{65}$$

(iv) a composite number between 50 and 70

Composite number between 50 and 70 are 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68 and 69.

Number of favorable outcomes,

$$n(E_4)=15$$

P(a composite number between 50 and 70)

$$P(E_4)=\frac{n(E_4)}{n(S)}=\frac{15}{65}=\frac{3}{13}$$

Main 2008

85. A box contains 125 shirts of which 110 are good 12 have minor defects and 3 have major defects. Ram Lal will buy only those shirts which are good while Naveen will reject only those which have major defects. A shirt is taken out at random from the box. Find the probability that :

(i) Ram Lal will buy it

(ii) Naveen will buy it

Sol :

For both case total shirt,

$$n(S)=125$$

(i) Ram Lal will buy it

Ramlal will buy only a good shirt.

No. of all possible outcomes,

$$n(E_1)=110$$

P(Ramlal will buy a shirt),

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{110}{125}=\frac{22}{25}$$

(ii) Naveen will buy it

Naveen will reject the shirt which have major defects and will buy all other shirts.

No. of favorable outcomes,

$$n(E_2)=125-3=122$$

P(Naveen will buy the shirt)

$$P(E_2)=\frac{n(E_2)}{n(S)}=\frac{122}{125}$$

Comp 2008

86. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh will win the game if all the tosses show the same result, (i.e either all three heads or all three tails) and loses the game otherwise. Find the probability that Ramesh will lose the game.

Sol :

There are 8 possible outcome when one coin is tossed three times : {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

$$n(S)=8$$

In the case of same result on all the tosses,

$$E=\{\text{HHH, TTT}\}$$

$$n(E)=2$$

P(Ramesh will win the game)

$$P(E)=\frac{n(E)}{n(S)}=\frac{2}{8}=\frac{1}{4}$$

P(Ramesh will loose the game)

$$P(\overline{E})=1-P(E)$$

$$=1-\frac{1}{4}=\frac{3}{4}$$

Main 2007

87. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

Sol :

Let x be blue balls.

Total balls, $n(S)=5+x$

$$n(R)=5 \text{ and } n(B)=x$$

P (red ball), $P(R)=\frac{n(R)}{n(S)}=\frac{5}{5+x}$

P (blue ball), $P(R)=\frac{n(B)}{n(S)}=\frac{x}{5+x}$

As per question we have

$$\frac{x}{5+x}=\frac{3\times 5}{5+x}$$

Thus $x = 15$

Hence, bag contains 15 blue balls.

Main 2006

88. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting :

(i) not a white ball,

(ii) neither a green nor a red ball.

Sol :

Bag contains 5 red, 8 green and 7 white balls i.e. total 20 ball.

Total number of possible outcomes,

$$n(S) = 20$$

(i) not a white ball,

There are 5 red and 8 green balls which are not white.

Thus number of favorable outcome,

$$n(E_1) = 13$$

P (not a white ball),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{20}$$

(ii) neither a green nor a red ball.

There are 7 white balls which are neither a green nor a red ball.

$$n(E_2) = 7$$

P (neither a green nor a red ball)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{20}$$

Comp 2006

89. A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number of the drawn card is

(i) A perfect square number

(ii) A multiple of 7.

Sol :

Total number of all possible outcomes,

$$n(S) = 113$$

(i) Perfect square numbers between 11 to 123 are 16, 25, 36, 49, 64, 81, 100 and 121.

No. of all favorable outcomes

$$n(E_1) = 8$$

P(Number drawn is perfect square),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{8}{113}$$

(ii) No. of multiples of 7 from 11 to 123 is 16 i.e 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112 and 119.

No. of all favorable outcomes.

$$n(E_2) = 16$$

P(number drawn card is multiple of 7)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{16}{113}$$

Comp 2005

90. A bag contains 15 balls of which x are blue and the remaining are red. If the number of red balls are increased by 5, the probability of drawing the red balls doubles. Find :

(i) P(red ball)

(ii) P(blue ball)

(iii) P(blue ball if of 5 extra red balls are actually added)

Sol :

Total ball, $n(S) = 15$

Blue ball $n(B) = x$

Red ball $n(R) = 15 - x$

Now probability of drawing red ball,

$$P(R) = \frac{n(R)}{n(S)} = \frac{15 - x}{15} \quad ...(1)$$

If the number of red balls are increased by 5, i.e. total the probability of drawing the red balls doubles.

In this case, number of total ball,

$$n(S') = 15 + 5 = 20$$

and number of red ball,

$$n(R') = 15 - x + 5 = 20 - x\,.$$

Now in this case probability of drawing red ball,

$$P(R') = \frac{n'(R)}{n'(S)} = \frac{20 - x}{20}$$

According to the question, we have

$$P(R') = 2P(R)$$

$$\frac{20 - x}{20} = 2\left(\frac{15 - x}{15}\right)$$

$$1 - \frac{x}{20} = 2 - \frac{2x}{15}$$

$$\frac{2x}{15} - \frac{x}{20} = 2 - 1$$

$$\frac{8x - 3x}{60} = 1$$

$$5x = 60 \Rightarrow x = 12$$

(i) P(red ball)

$$P(R) = \frac{n(R)}{n(S)} = \frac{15 - 12}{15} = \frac{3}{15} = \frac{1}{5}$$

(ii) P(blue ball)

$$P(R) = \frac{n(B)}{n(S)} = \frac{12}{15} = \frac{4}{5}$$

(iii) P(blue ball if of 5 extra red balls are actually added)

$$P'(R) = \frac{n'(R)}{n'(S)} = \frac{3 + 5}{15 + 5} = \frac{8}{20} = \frac{2}{5}$$

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Number of good shirts

$$n(E_1) = 88$$

P(Ramesh buys the shirt).

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{88}{100} = \frac{22}{25}$$

(ii) Kewal will buy the selected shirt ?

Number of shirts without major defect,

$$n(E_2) = 88 + 8 = 96$$

P(Kewal buys a shirt)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{96}{100} = \frac{24}{25}$$

Comp 2002

95. A box contains 100 cards marked from 1 to 100. If one card is drawn at random from the box, find the probability that it bears :

(i) a single digit number

(ii) a number which is a perfect square

(iii) a number which is divisible by 7

Sol :

Since box consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) a single digit number

Number of favorable outcomes,

$$n(E_1) = 9$$

P(single digit number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{100}$$

(ii) a number which is a perfect square

Perfect square number are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

No. of favorable outcomes,

$$n(E_2) = 10$$

P(perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

(iii) a number which is divisible by 7

Number divisible by 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98 i.e. 14 numbers.

No. of favorable outcomes,

$$n(E_3) = 14$$

P(a number divisible by 7),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{14}{100}$$

Main 2000

96. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card.

(i) is divisible by 9 and is a perfect square.

(ii) is a prime number greater than 80.

Sol :

Since bag consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) is divisible by 9 and is a perfect square.

Number divisible by 9 and perfect square are 9, 36 and 81 i.e. 3 numbers.

$$n(E_1) = 3$$

Required probability,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{100}$$

(ii) is a prime number greater than 80.

Prime numbers greater than 80 and less than 100 are 83, 89 and 97 i.e 3 numbers.

$$n(E_2) = 3$$

Required probability,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{100}$$

Comp 2000

97. All red face cards are removed from a pack of playing cards. The remaining cards are well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is :

(i) a red card

(ii) a face card

(iii) a card of clubs

Sol :

Since red face cards are removed, number of all possible outcomes are $52 - 6 = 46$

$$n(S) = 46$$

(i) a red card

No. of remaining red cards,

$$n(E_1) = 26 - 6 = 20$$

P(red card), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{20}{46} = \frac{10}{26}$

(ii) a face card

Number of remaining face cards,

$$n(E_2) = 12 - 6 = 6$$

P(a face card),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{46} = \frac{3}{23}$$

(iii) a card of clubs

Number of cards clubs

$$n(E_3) = 13$$

P(a card of clubs),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{13}{46}$$

END

NODIA

Main 2004

91. Three different coins are tossed together. Find the probability of getting
(i) exactly two heads.
(ii) at least two heads
(iii) at least two tails.

Sol :

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$$n(S) = 8$$

(i) Exactly two heads

Sample space $E_1 = \{HHT, HTH, THH\}$

$$n(E_1) = 3$$
$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{8}$$

(ii) At least two heads.

Sample space $E_2 = \{HHT, HTH, THH, HHH\}$

$$n(E_2) = 4$$
$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iii) At least two tails,

Sample space $E_3 = \{TTH, THT, HTT, TTT\}$

$$n(E_3) = 4$$
$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Comp 2004

92. In a single throw of a pair of different dice, what is the probability of getting
(i) a prime number on each dice ?
(ii) a total of 9 or 11 ?

Sol :

When two dice are thrown there are $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

(i) a prime number on each dice ?
Favorable outcomes are (2, 2) (2, 3) (2, 5) (3, 2) (3, 3) (3, 5) (5, 2) (5, 3) and (5, 5) i.e. 9 outcomes.

$$n(E_1) = 9$$

P(a prime number on each die)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) a total of 9 or 11 ?
Favorable outcomes are (3, 6) (4, 5) (5, 4) (6, 3) (5, 6) and (6, 5) i.e. 6 outcomes.

$$n(E_1) = 6$$

P(a total of 9 or 11)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Comp 2003

93. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is
(i) a black king,
(ii) a card of red colour,
(iii) a card of black colour.

Sol :

There are total 52 cards out of which 6 cards are removed.
Total number of all possible outcomes,

$$n(S) = 52 - 6 = 46$$

Number of black king,

$$n(E_1) = 2$$

(i) a black king,
Probability of drawing black king

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{46} = \frac{1}{23}$$

(ii) a card of red colour,
Total red card,

$$n(E_2) = 26 - 6 = 20$$

Probability of drawing red colour card

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{20}{46} = \frac{10}{23}$$

(iii) a card of black colour.
Total card of black colour,

$$n(E_3) = 26$$

Probability of drawing black colour card

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{26}{46} = \frac{13}{23}$$

Main 2002

94. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but Kewal another shopkeeper will not buy shirts with major defects. A shirt is taken out of the box at random. What is the probability that :
(i) Ramesh will buy the selected shirt ?
(ii) Kewal will buy the selected shirt ?

Sol :

Since box consists of 100 shirts, there are 100 possible outcomes.

$$n(S) = 100$$

(i) Ramesh will buy the selected shirt ?

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